A New Two-Stage Stochastic Model for Reverse Logistics Network Design under Government Subsidy and Low-Carbon Emission Requirement

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Abstract – **Nowadays, increasing number of companies incorporates the reverse logistics decisions into their supply chain design in order to cope with the enforced international and national legislation and improve the resource efficiency and public image. This paper investigates a new stochastic optimization model for designing a single-period multi-product multi-level reverse logistics system under government subsidy and low-carbon emission requirement. In order to resolve the stochastic optimization problem, a modified multi-criteria scenario-based approach is proposed to maximize the profit generation while simultaneously improve the stability of the decision-making under uncertainty. The model and solution method are tested with several numerical experiments, and managerial insights are obtained with respect to the carbon emission requirement, governmental subsidy, economy of scale, and system flexibility.**

Keywords – **Reverse logistics, supply chain management, carbon emission requirement, operational research, stochastic optimization, scenario-based solution**

I. INTRODUCTION

Today, with the enforced international and national legislation and the public concern on circular economy and sustainable development, reverse logistics has become one of the most important means for the value re-creation and recovery from end-of-use products. As defined by the Reverse Logistics Executive Council, reverse logistics refers to the process of managing the material, cash and information flows starting from the end customers towards the raw material suppliers for the value re-creation and recovery from the end-of-use products through reuse, re-fabrication, remanufacturing, recycling, recovery, and proper disposal [1, 2].

Network design is one of the most important strategic decisions in supply chain management, which has significant influence on the profitability of a supply chain [3]. In order to improve the decision-making on reverse logistics network design, this paper proposes a new stochastic optimization model for reverse logistics network design under government subsidy and low-carbon emission requirement. The rest of the paper is organized as follows. Section II describes the problem and establishes the mathematical model. Section III introduces the solution method. Section IV provides numerical experiments for testing the proposed model and solution method. Section V summarizes the paper.

II. PROBLEM DEFINATION AND MODELING

Fig. 1 shows the schematic of the reverse logistics network. As shown in the figure, the reverse flow of end-of-use products starts from the customer zones, and via the intermediate collection/inspection centers at which the end-of-use products are checked and separated for further distribution, towards the respective facilities for recovery and proper disposal.
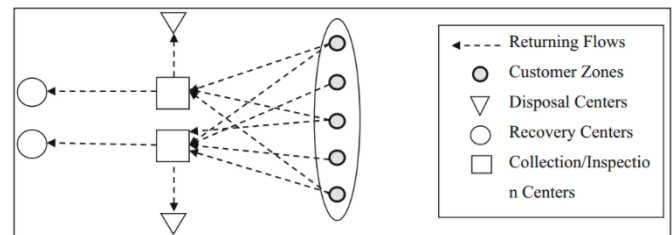

Fig. 1. Reverse logistics network [4].

The objective of the model is to maximize the profit of the reverse logistics system through optimally determining the locations of collection/inspection centers and recovery centers as well as the transportation plan. In order to account the uncertainty issues, the generation of end-of-use products at the customer zones and the selling price of the recovered produces are considered stochastic parameters. The sets, parameters and variables used in the model are first presented, and then the model is formulated.

A. Sets

$cz=1,\dots,CZ$	Set of customer zones
$dc=1,\dots,DC$	Set of disposal centers
$rc=1,\dots,RC$	Set of recovery centers
$cc=1,\dots,CC$	Set of collection/inspection centers
$sc=1,\dots,SC$	Set of scenarios
$ty=1,\dots,TY$	Set of types of end-of-use products

B. Parameters

$P_{rc,ty,sc}$	Profit for recovering one unit of product ty in scenario sc
$Q_{rc,ty}$	Subsidy for recovering one unit of product ty
F_{cc}	Fixed cost for collection/inspection centers

F_{rc}	Fixed cost for recovery centers	$W_{cc,dc,ty,sc}$	Amount transported between collection/inspection centers and disposal centers

F_{rc} — Fixed cost for recovery centers

$V_{cc,ty}$ — Variable cost for collecting one unit of product ty

$V_{rc,ty}$ — Variable cost for recovering one unit of product ty

$Gf_{dc,ty}$ — Entrance fee for disposing one unit of product ty

$T_{cz,cc,ty}$ — Transportation cost between customer zones and collection/inspection centers

$T_{cc,rc,ty}$ — Transportation cost between collection/inspection centers and recovery centers

$T_{cc,dc,ty}$ — Transportation cost between collection/inspection centers and disposal centers

$DE_{cz,ty,sc}$ — End-of-use product generated at customer zones in scenario sc

$C_{cc,ty}$ — Capacity of collection/inspection centers

$C_{rc,ty}$ — Capacity of recovery centers

$C_{dc,ty}$ — Capacity of disposal centers

$E_{cc,ty}$ — CO_2 emission for collecting one unit of product ty

$E_{rc,ty}$ — CO_2 emission for recovering one unit of product ty

$E_{dc,ty}$ — CO_2 emission for disposing one unit of product ty

$E_{cz,cc,ty}$ — CO_2 emission of the transportation between collection/inspection centers and recovery centers

$E_{cc,rc,ty}$ — CO_2 emission of the transportation between collection/inspection centers and recovery centers

$E_{cc,dc,ty}$ — CO_2 emission of the transportation between collection/inspection centers and disposal centers

$EM_{rq,sc}$ — CO_2 emission requirement

C. Variables

$D_{cc,sc}$ — $\begin{cases} 1 \text{ if } cc \text{ is open in scenario } s \\ 0 \text{ otherwise} \end{cases}$

$D_{rc,sc}$ — $\begin{cases} 1 \text{ if } rc \text{ is open in scenario } s \\ 0 \text{ otherwise} \end{cases}$

$W_{cc,ty,sc}$ — Amount collected at collection/inspection centers in scenario sc

$W_{rc,ty,sc}$ — Amount recovered at recovery centers in scenario sc

$W_{dc,ty,sc}$ — Amount disposed at disposal centers in scenario sc

$W_{cz,cc,ty,sc}$ — Amount transported between customer zones and collection/inspection centers in scenario sc

$W_{cc,rc,ty,sc}$ — Amount transported between collection/inspection centers and recovery centers in scenario sc

$W_{cc,dc,ty,sc}$ — Amount transported between collection/inspection centers and disposal centers

D. Model

The objective function and constraints of the proposed model are presented in Eqs. (1)-(10).

$$\max f = \sum_{rc=1}^{RC} \sum_{ty=1}^{TY} (P_{rc,ty,sc} + Q_{rc,ty}) W_{rc,ty,sc}$$
$$- \sum_{cc=1}^{CC} \sum_{ty=1}^{TY} (F_{cc} D_{cc,sc} + V_{cc,ty} W_{cc,ty,sc})$$
$$- \sum_{rc=1}^{RC} \sum_{ty=1}^{TY} (F_{rc} D_{rc,sc} + V_{rc,ty} W_{rc,ty,sc})$$
$$- \sum_{dc=1}^{DC} \sum_{ty=1}^{TY} Gf_{dc,ty} W_{dc,ty,sc}$$
$$- \sum_{cz=1}^{CZ} \sum_{cc=1}^{CC} \sum_{ty=1}^{TY} T_{cz,cc,ty} W_{cz,cc,ty,sc}$$
$$- \sum_{cc=1}^{CC} \sum_{rc=1}^{RC} \sum_{ty=1}^{TY} T_{cc,rc,ty} W_{cc,rc,ty,sc}$$
$$- \sum_{cc=1}^{CC} \sum_{dc=1}^{DC} \sum_{ty=1}^{TY} T_{cc,dc,ty} W_{cc,dc,ty,sc},$$
$$For\ sc = 1, \dots, SC \tag{1}$$

Subject to:

$$DE_{cz,ty,sc} = \sum_{cc=1}^{CC} W_{cz,cc,ty,sc}, For\ cz = 1, \dots, CZ, ty = 1, \dots, TY, sc = 1, \dots, SC \tag{2}$$

$$W_{cc,ty,sc} \leq C_{cc,ty} D_{cc,sc}, For\ cc = 1, \dots, CC, ty = 1, \dots, TY, sc = 1, \dots, SC \tag{3}$$

$$W_{rc,ty,sc} \leq C_{rc,ty} D_{rc,sc}, For\ rc = 1, \dots, RC, ty = 1, \dots, TY, sc = 1, \dots, SC \tag{4}$$

$$W_{dc,ty,sc} \leq C_{dc,ty}, For\ dc = 1, \dots, DC, ty = 1, \dots, TY, sc = 1, \dots, SC \tag{5}$$

$$D_{cc,sc}, D_{rc,sc} = \{0,1\}, For\ cc = 1, \dots, CC, sc = 1, \dots, SC, rc = 1, \dots, RC \tag{6}$$

$$W_{cc,ty,sc} = \sum_{cz=1}^{CZ} W_{cz,cc,ty,sc}, For\ cc = 1, \dots, CC, ty = 1, \dots, TY, sc = 1, \dots, SC \tag{7}$$

$$W_{rc,ty,sc} = \sum_{cc=1}^{CC} W_{cc,rc,ty,sc}, For\ rc = 1, \dots, CC, ty = 1, \dots, TY, sc = 1, \dots, SC \tag{8}$$

$$W_{dc,ty,sc} = \sum_{cc=1}^{CC} W_{cc,dc,ty,sc}, For\ dc = 1, \ldots, CC, ty \tag{9}$$
$$= 1, \ldots, TY, sc = 1, \ldots, SC$$

$$\sum_{cc=1}^{CC}\sum_{ty=1}^{TY} E_{cc,ty}W_{cc,ty,sc} + \sum_{rc=1}^{RC}\sum_{ty=1}^{TY} E_{rc,ty}W_{rc,ty,sc}$$
$$+ \sum_{dc=1}^{DC}\sum_{ty=1}^{TY} E_{dc,ty}W_{dc,ty,sc}$$
$$+ \sum_{cz=1}^{CZ}\sum_{cc=1}^{CC}\sum_{ty=1}^{TY} E_{cz,cc,ty}W_{cz,cc,ty,sc} \tag{10}$$
$$+ \sum_{cc=1}^{CC}\sum_{rc=1}^{RC}\sum_{ty=1}^{TY} E_{cc,rc,ty}W_{cc,rc,ty,sc}$$
$$+ \sum_{cc=1}^{CC}\sum_{dc=1}^{DC}\sum_{ty=1}^{TY} E_{cc,dc,ty}W_{cc,dc,ty,sc} \le EM_{rq,sc}, For\ sc$$
$$= 1, \ldots, SC$$

Eq. (1) is the objective function of the model, which maximizes the total profit generated by the reverse logistics system. The first part is the total income from the recovery of end-of-use products, and it includes the sales revenue and subsidy which is provided by the government in order to divert the end-of-use products from landfill and promote the recovery activities. The second and third parts are the fixed and variable cost for opening and operating the collection/inspection centers and recovery center. The fourth part is the entrance fee for using the disposal centers, and the rest parts are the transportation cost of the end-of-use products and disassembled parts.

Eq. (2) is the demand constraint which guarantees the demand for treating the end-of-use products at each customer zone is fulfilled. Eqs. (3), (4) and (5) are capacity constraints of collection/inspection centers, recovery centers and disposal centers, respectively. Eq. (6) requires the variables $D_{cc,sc}$ and $D_{rc,sc}$ are binary variables. Eqs. (7), (8) and (9) are mass balance constraints at collection/inspection centers, recovery centers and disposal centers, respectively. Eq. (10) ensures the maximum limit of CO_2 emissions from the reverse logistics system, which is used as the indicator to improve the environmental sustainability.

In addition, the generation of end-of-use products at customer zones and the selling price are considered with great uncertainty and formulated as stochastic variables, and all the decision variables should be non-negative in this model.

III. SOLUTION METHOD

Stochastic optimization is a well-developed approach for complex decision-making problems with uncertainties, and it has been widely applied in many industries, i.e., finance [5], medical supply chain planning [6], energy

planning [7], etc. Due to the simplicity and applicability, scenario-based approach is extensively used to resolve the stochastic optimization problems. The optimal solutions to a scenario-based stochastic program should be robust to withstand all the random events while simultaneously be flexible to adapt the changes in order to improve the performance of decision-making [8].

In this paper, a multi-criteria approach is developed to find the optimal solution of the scenario-based stochastic program, which aims at improving both profit expectation of the reverse logistics system and the stability of decision-making. Even if stability may not be suitable for an evaluation indicator in many stochastic optimization problems due to the different technical and managerial function required [8], the supply chain network is a strategic decision which determines the long-term economic performance and environmental impact, so it requires higher level of confidence and stability in order to minimize the risk of decision-making.
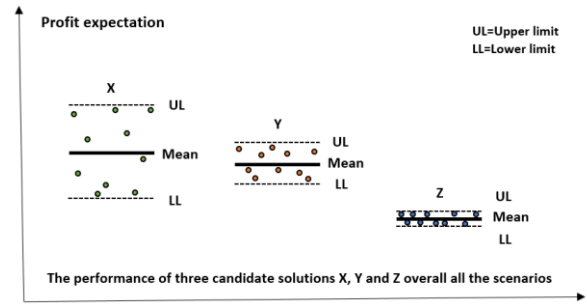

Fig. 2. Schematic of the problem caused by performance evaluation with the reciprocal of coefficient of variation.

The multi-criteria solution approach is developed based upon a recently published work by Soleimani, et al. [9], which includes four main steps. First, the test scenarios are generated in an effective and mathematically efficient way so that a large diversion of possible situations can be represented and the problem can be resolved within a reasonable computational time, and the methods for scenario generation is provided in King and Wallace [8] and Kaut and Wallace [10]. Second, the test scenarios are resolved independently as a deterministic mixed integer program and the optimal results of them are the candidates of the optimal solution of the stochastic program. Third, the candidates are tested through all the scenarios and the performance is evaluated through the mean and standard deviation (SD). In this phase, the first-stage decisions of facility locations of each candidate will not change, but the second-stage decisions will be flexible in accordance with the random events. Finally, the performance of the candidates is measured with the reciprocal of coefficient of variation (1/CV).

The method aims at maximizing the profit expectation of the reverse logistics system (mean) while minimizing the risk of decision-making (SD). As shown in Fig.2, although solution X has a higher profit expectation than that of solution Y, solution Y has a much better performance in the stability of decision-making, which

minimizes the risk and consequence at the low-profitability scenarios, say, the expected profit will not be so bad even if the market is fluctuate and with a lot of uncertainties. Due to this reason, solution Y is the optimal solution of the stochastic optimization with the evaluation by 1/CV.

Even through the multi-criteria method developed by Soleimani, et al. [9] has the benefit to account both profit and stability, the performance measurement with 1/CV may yield weak-stable solutions. As illustrated in Fig. 2, solution Z has a much lower profit expectation than that of solutions X and Y, and the expected profit in the best-case scenarios in solution Z is lower than that in the worst-case scenarios in solutions X and Y. Hence, it is obviously that solution Z should not be considered as the optimal solution of the stochastic optimization problem. However, with the performance measurement by 1/CV, solution Z may be selected as the optimal solutions due to is much smaller SD, and this is a weak-stable solution which has lower profit expectation but is extremely stable. In addition, there are some other mathematical and managerial problems related to the method, as discussed by Yu and Solvang [11].

$$w\frac{(mean_{max} - mean)}{(mean_{max} - mean_{min})} + (1-w)\frac{(CV - CV_{min})}{(CV_{max} - CV_{min})}$$
(11)

Obviously, it is of great importance to avoid the weak-stable solutions as well as other problems in stochastic optimization, so the performance measurement of the original multi-criteria method is improved by using a weighed sum shown in Eq. (11). Besides, the evaluation of the risk by SD is also replaced by the indicator CV in order to eliminate the influence of the mean on data dispersion, and detail introduction is given in Green, et al. [12]. The maximum and minimum values are used as the benchmark for the normalization, and the weights represent the importance of the relevant parts in decision-making. If the profit expectation varies significantly, a higher weight will be given to the first part in order to eliminate weak-stable solutions; otherwise, the stability may be emphasized.

IV. NUMERICAL EXPERIMENTATION

Numerical experimentations are conducted in this section for illustrating the application of the stochastic program and solution method in reverse logistics network design. The reverse logistics system includes 15 customer zones, 8 potential locations for collection/inspection centers, 10 recovery centers and 1 disposal center. Two types of end-of-use products are considered in the numerical experimentation, and the parameters are generated using uniform distribution.

We first generate the benchmark scenarios for the basic, best and worst situations with different combination of the mean, upper limit and lower limit of the stochastic parameters. Then, two scenarios of the generation of end-of-use products and four scenarios of the selling price are evenly generated on each side of their means, respectively. Hence, in total, 11 test scenarios are generated.

Fig. 3. Comparison of the candidate solutions 1 and 5.

The model is programmed and solved with Lingo 11.0 optimization solver. First, the model is tested without the CO_2 emission constraint and evaluated by both 1/CV (candidate 1 selected) and weighted sum (candidate 5 selected). Fig. 3 shows the comparison of the results over all the test scenarios, which shows the result from weighted sum is much better in high demand scenarios and slightly weaker in low demand scenarios. This reveals that the solution calculated by weighted sum has better performance than that determined by 1/CV.

The model is then tested under the CO_2 emission requirements changing incrementally from 10% to 50% reduction with the step by 10% each, and infeasible solutions are observed due to the capacity limitation in some candidate solutions over high demand scenarios, so adjustments are made to generate meaningful comparison. In addition, two tests are also performed using uncapacitated model with 100% and 200% increase on the fixed cost, and in total, 928 rounds of calculations are performed in this research. The results are presented in Figs. 4, 5 and 6, respectively.

It is observed that the CO_2 emission requirement will negatively affect the profitability of the reverse logistics system, and the influence becomes significant after the requirement on CO_2 emission reduction increased to more than 30%. Fig. 6 shows the CO_2 emission/profit ratio which indicates how much CO_2 emission for generating one unit of profit, and the test problem 3 has the best performance as shown in the figure. This information provides decision-makers with best selection of policy instruments.
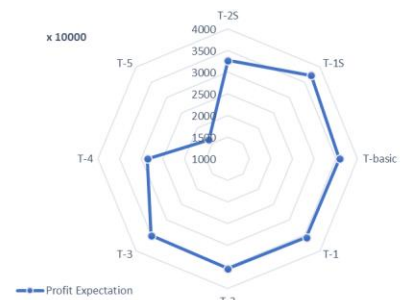

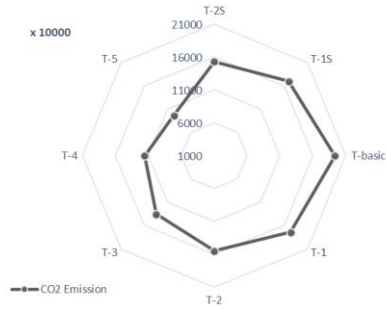Fig. 4. Comparison of the profit expectation of the test problems.

Fig. 5. Comparison of the average CO_2 emission of the test problems.

In the numerical experimentation, the violation of capacity limitation is observed only for one type of product, but there will usually be idle capacity for the other type of product, so the flexibility of reverse logistics should be improved in order to utilize the facilities and resources in a more effective and efficient manner. In addition, the relaxation of capacity constraint also shows that using larger facilities (economy of scale) may improve both economic and environmental performance if the cost increase is maintained at a certain level.
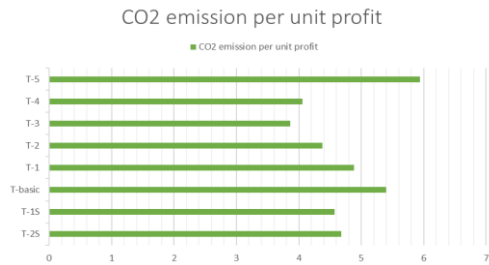

Fig. 6. Comparison of the CO_2 emission/profit ratio of the test problems.
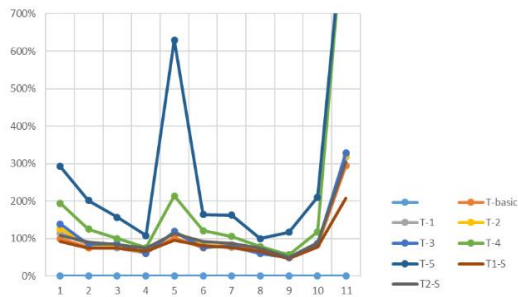

Fig. 7. Comparison of the subsidy/profit ratio of the test problems over all the scenarios.

Fig. 7 shows the subsidy/profit ratio of the test problems over all the scenarios, and it is observed that the government subsidy becomes more important to maintain the profitability of the reverse logistics system with the more stringent requirement on CO_2 emission reduction.

V. CONCLUSION

This paper has presented a two-stage stochastic optimization model for network design of a single-period multi-product multi-level reverse logistic system under government subsidy and low-carbon emission requirement. A multi-criteria scenario-based solution method is improved in order to account both profit expectation and stability of decision-making. Finally, the model and solution method are tested with several numerical experimentations, and managerial insights are obtained with respect to the carbon emission requirement, governmental subsidy, economy of scale, and system flexibility. For future research, the improvement on computational efficiency is suggested.

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