

# **A multi-objective mathematical programming for sustainable reverse logistics network design. Part I: Model formulation**

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## **Abstract**

Reverse logistics has received more and more attention during the past decade due to the increasing public awareness of sustainable development. Because of the fluctuation in both quantity and quality of the reverse material flow, design and planning of reverse logistics network is much more complicated comparing with the forward ones. Therefore, it is important to develop decision support tools for designing reverse logistics network in an economically efficient and environment-friendly manner. This research proposes a novel multi-objective mixed integer programming model in order to justify the relationship between the cost and sustainability of reverse logistics system, and the weighted sum utility method is employed for combining the two objective functions. This research is presented in a series of two papers. Part I formulates the conceptual framework of reverse logistics network and the mathematical programming for the minimization of the overall system cost and environmental influence. Part II introduces the weighted sum utility method for combining the two objective functions, and the application and analysis are also given in this part.

*Keywords: multi-objective programming, mixed integer programming, reverse logistics, network design.*

## **1 Introduction**

Reverse logistics has received more and more attention during the past decade due to the increasing concern of environmental pollution, resource reduction, and limited capacity of landfill [1]. Reverse logistics refers to the process and activities starting from the end customers for capturing the remaining values of



used products. Through effectively and efficiently managing the reverse logistics network, enterprises can not only improve the utilization of resources and customer service level [2], but also provide a better public image to take into account sustainable development.

Logistics network design has significant and long-term influence on the profitability and sustainability of all players within a supply chain, so it is one of the most important strategic decisions in supply chain management [3]. A great number of previous studies have contributed in both theoretical development and computational optimization models in this field [4]. However, compared to the configuration of forward logistics network, the design and planning of reverse logistics network is much more complicated due to the fluctuation in both quantity and quality of the reverse material flow. Thereby, development of comprehensive decision-making tools for reverse logistics network design in a cost-efficient and environment-friendly manner is of importance.

In this research, a novel mathematical programming is proposed for reverse logistics network design considering both economic and environmental factors. This research is structured as a series of two papers. Part I focuses on model formulation, while Part II concentrates on the application and analysis of the proposed model. The rest of Part I is organized as follows. Section 2 gives an extensive literature study on the computational models of reverse logistics network design. Section 3 presents the theoretical framework of the reverse logistics system, and a novel mathematical programming based upon the theoretical framework is also formulated in this section. Section 4 summarizes the highlights of this paper.

## 2 Literature survey

The concept of reverse logistics was first introduced in the early 1990s, and during the past two decades, a great number of literatures have been published to account for the network design, planning, and optimization of both independent and integrated reverse logistics system for reuse, refabricating, remanufacturing, and recycling of used products. Demirel and Gokcen [1] proposed a mixed integer programming model for the network design of a remanufacturing system. The objective of the model is to minimize the overall cost for establishing and operating the remanufacturing system. Pishvae and Kianfar [3] formulated a single objective mixed integer linear programming for minimizing the total cost of a general reverse supply chain network with multiple levels of players, and a simulated annealing approach for model computation was also developed in this paper. Zhou and Wang [5] reported a computational optimization model for an integrated supply chain network design with the consideration of both forward and reverse logistics.

Taking into account a specific industry, Sasikumar et al. [6] proposed a multi-period mixed integer programming for maximizing the total profit generated by the remanufacturing system of truck tyre. A single objective mathematical model for the reverse logistics network planning of end-of-life vehicle recovery was formulated by Zarei et al. [7]. The objective of this model is to minimize the total



cost for the network configuration of the reverse logistics system, and a genetic algorithm was also proposed for resolving the model in an efficient and reliable manner. Liu [8] reported a network optimization model for managing the reverse material flow of E-commerce, and a genetic algorithm was developed accordingly for model computation in this paper. Yu et al. [9] established a decision support system based upon linear programming for sustainable management of the reverse logistics network of municipal solid waste.

Considering the uncertainties of the reverse material flow, Roghanian and Pazhoheshfar [10] developed a single objective model with stochastic input parameters for optimizing the cost of a multi-product and multi-level reverse logistics system, and the model was solved by a priority-based genetic algorithm. Kim and Lee [11] proposed a dynamic integer programming based upon capacitated facility location problem for planning the refuse collection network, and the model was resolved by two independent methods: multi-stage branch and bound heuristic and modified drop heuristic. Salema et al. [12] reported a single objective mixed integer programming model for the network design of a capacitated multi-product reverse logistics system under uncertain environment, and the uncertainties related to the input parameters were formulated through a multi-scenario method. Alumur et al. [13] established a multi-period dynamic model for determining the optimal configuration of a general reverse logistics network.

Taking into consideration multiple influencing factors simultaneously, Ramezani et al. [14] developed a multi-objective model for balancing the profitability, responsiveness as well as the rate of defective products of an integrated forward/reverse logistics system. Li et al. [15] proposed a multi-objective reverse logistics network design model for determining the optimal trade-off among three conflicting objectives including the minimization of cost, minimization of tardiness of cycle time, and maximization of the coverage of customer demand points. Lee et al. [16] reported a multi-objective hybrid genetic method for optimizing both overall system cost and delivery responsiveness of a general reverse logistics network.

The most frequently used mathematical tools in previous studies for reverse logistics network design and planning include linear programming, integer programming, mixed integer programming, stochastic programming, and multi-period programming. Many attempts are made for taking into account both deterministic and stochastic input parameters. Most previous models for reverse logistics network design are single objective models with consideration of the minimization of cost or maximization of profit, but only a few previous models consider the balance of several conflicting objectives. This has been proved by a recently published review of reverse logistics and closed-loop supply chain (Ref. [17]). Further, previous multi-objective models for reverse logistics network design mainly focus on the balance between profitability and customer service level, but it is hardly to find a previous network design model accounting for the environmental influence of the reverse logistics activities themselves. Improper treatment of used products and waste items in a reverse logistics system may lead to secondary pollution to the environment, so it is important to consider the environmental issue when a reverse



logistics network is planned. In order to solve the literature gap, a novel reverse logistics network design model is formulated in this paper for taking into account both system operating cost and environmental influence, and the environmental influence is evaluated by carbon emissions.

### 3 Model formulation

#### 3.1 Conceptual framework

Figure 1 illustrates the conceptual framework of a general reverse logistics system. In a forward supply chain, the direction of material flow usually from raw material supplier via producer, distributor, wholesaler, and retailer towards the end customer. However, the material flow starts from the customers in a reverse logistics system. The used products are first collected at the collection centre and then disassembled into several parts and components. The quality of the parts and components is also examined at the collection centres and then transported to relevant facilities for reuse, repair, remanufacturing, and recycling. The reused and repaired components are targeted on secondary market, while the remanufactured and recycled products will be sold in primary market. The waste items will be processed and properly disposed of at incineration plants or landfills.

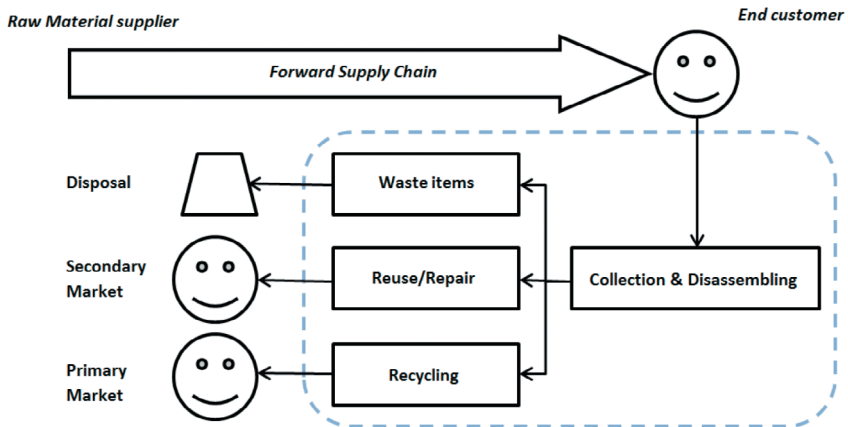


Figure 1: Conceptual framework of reverse logistics network.

In this paper, the focus is on the design of reverse logistics network for reuse, recycling, and proper disposal of used products. Both the locations of different facilities and the amount of used products or disassembled components treated at different facilities are determined. It is required that the locations of end customers, amounts of used products collected at customer locations as well as the other necessary information are known in advance.

### 3.2 Parameters and variables

The definitions of the parameters and variables for model formulation are first given as follows:

$F, f$	Set and index of customers
$C, c$	Set and index of collection centres
$W, w$	Set and index of waste treatment facilities
$RP, rp$	Set and index of repair facilities
$RC, rc$	Set and index of recycling facilities
$V_c, V_w, V_{rp}, V_{rc}$	Fixed cost for opening collection centre, waste treatment facility, repair facility and recycling facility
$u_c, u_w, u_{rp}, u_{rc}$	Binary decision variables for determining if a facility is opened at a potential location
$G_c, G_w, G_{rp}, G_{rc}$	Variable cost at collection centre, waste treatment facility, repair facility and recycling facility
$Q_{fc}, Q_{cw}, Q_{crp}, Q_{crc}$	Amount of used products or disassembled components transported between different facilities
$X_{fc}, X_{cw}, X_{crp}, X_{crc}$	Unit transportation cost of in each itinerary
$C_{fc}, C_{cw}, C_{crp}, C_{crc}$	Carbon emission indicator in each itinerary
$A_{fc}, A_{cw}, A_{crp}, A_{crc}$	Proximity between different facilities
$OI_c, OI_w, OI_{rp}, OI_{rc}$	Capacity of each facility
$\emptyset$	An infinite positive number
$P_f$	Generation of used products at each customer location
$T_{cw}, T_{crp}, T_{crc}$	Conversion rate of used products to disassembled components for different treatment

### 3.3 Objective function

$$\begin{aligned}
 \text{Min Cost} = & \sum_{c \in C} u_c \left( V_c + G_c \sum_{f \in F} Q_{fc} \right) + \sum_{w \in W} u_w \left( V_w + G_w \sum_{c \in C} Q_{cw} \right) \\
 & + \sum_{rp} u_{rp} \left( V_{rp} + G_{rp} \sum_{c \in C} Q_{crp} \right) \\
 & + \sum_{rc} u_{rc} \left( V_{rc} + G_{rc} \sum_{c \in C} Q_{crc} \right) \\
 & + \sum_{f \in F} \sum_{c \in C} X_{fc} Q_{fc} + \sum_{c \in C} \sum_{w \in W} X_{cw} Q_{cw} \\
 & + \sum_{c \in C} \sum_{rp \in RP} X_{crp} Q_{crp} + \sum_{c \in C} \sum_{rc \in RC} X_{crc} Q_{crc}. \tag{1}
 \end{aligned}$$

The objective function is formulated based upon mixed integer programming, and the reverse logistics network is designed for recovering a single type of used products. Equation (1) minimizes the overall system cost of reverse logistics system. The first four parts of this equation represent the fixed and variable costs for opening collection centre, waste treatment facility, repair facility, and recycling facility. The other four parts of the equation formulate the transportation cost of used products or disassembled components between different facilities. It is noted that the used products from the end customers are initially collected and disassembled at the collection centres, so the direct transportation of used products between end customers and treatment facilities is prohibited.

$$\begin{aligned} \text{Min Carbon} = & \sum_{c \in C} C_{fc} A_{fc} \sum_{f \in F} Q_{fc} + \sum_{w \in W} C_{cw} A_{cw} \sum_{c \in C} Q_{cw} \\ & + \sum_{rp \in RP} C_{crp} A_{crp} \sum_{rp \in RP} Q_{crp} + \sum_{rc \in RC} C_{crc} A_{crc} \sum_{rc \in RC} Q_{crc}. \end{aligned} \quad (2)$$

Equation (2) minimizes the overall carbon emissions of all itineraries within the reverse logistics system. The carbon emissions of the transportation of used products and disassembled components in each itinerary are proportional to the proximity between two facilities and the amount of used products transported. Carbon emission indicator reflects the average carbon-equivalent emissions of each itinerary, and this is mainly determined by the type and fuel consumption of transport vehicles.

### 3.4 Restrictions

In order to fulfil the material flow balance as well as other requirements of the reverse logistics system, restrictions of the model are established in the following formulas:

$$OI_c \geq \sum_{f \in F} Q_{fc}, \quad \forall c \in C, \quad (3)$$

$$OI_w \geq \sum_{c \in C} Q_{cw}, \quad \forall w \in W, \quad (4)$$

$$OI_{rp} \geq \sum_{c \in C} Q_{crp}, \quad \forall rp \in RP, \quad (5)$$

$$OI_{rc} \geq \sum_{c \in C} Q_{crc}, \quad \forall rc \in RC, \quad (6)$$

$$u_{c\emptyset} \geq \sum_{f \in F} Q_{fc}, \quad \forall c \in C, \quad (7)$$

$$u_{w\emptyset} \geq \sum_{c \in C} Q_{cw}, \quad \forall w \in W, \quad (8)$$

$$u_{rp} \varnothing \geq \sum_{c \in C} Q_{crp}, \quad \forall rp \in Rp, \quad (9)$$

$$u_{rc} \varnothing \geq \sum_{c \in C} Q_{crc}, \quad \forall rc \in Rc, \quad (10)$$

$$P_f = \sum_{c \in C} Q_{fc}, \quad \forall f \in F, c \in C, \quad (11)$$

$$T_{cw} \sum_{f \in F} Q_{fc} = \sum_{w \in W} Q_{cw}, \quad \forall c \in C, w \in W, \quad (12)$$

$$T_{crp} \sum_{f \in F} Q_{fc} = \sum_{rp \in RP} Q_{crp}, \quad \forall c \in C, rp \in RP, \quad (13)$$

$$T_{crc} \sum_{f \in F} Q_{fc} = \sum_{rc \in RC} Q_{crc}, \quad \forall c \in C, rc \in RC, \quad (14)$$

$$T_{cw} + T_{crp} + T_{crc} = 1, \quad \forall c \in C, \quad (15)$$

$$u_c, u_w, u_{rp}, u_{rc} \in \{0,1\}, \quad \forall c \in C, w \in W, rp \in RP, rc \in RC, \quad (16)$$

$$Q_{fc}, Q_{cw}, Q_{crp}, Q_{crc} \geq 0, \quad \forall f \in F, \forall c \in C, w \in W, rp \in RP, rc \in RC. \quad (17)$$

Equations (3)–(6) ensure the capacities of different facilities in the reverse logistics network will not be exceeded. Equations (7)–(10) restrict the used products or disassembled components such that they will not be sent to an unselected candidate location for treatment. Equations (11)–(14) are the material flow balance requirement of the reverse logistics system. It is noted that eqn (17) emphasizes all the used products generated in every customer location are collected and treated. Equation (15) ensures all the used products handled by the reverse logistics system will be first disassembled at the collection centres for further treatment. Equation (16) is the binary constraint for decision variable  $u_c$ ,  $u_w$ ,  $u_{rp}$ , and  $u_{rc}$ . Equation (17) guarantees that the amount of used products or disassembled parts in each itinerary is a positive number.

## 4 Summary

Reverse logistics activities are becoming more and more important in recent years due to the increasing public awareness for resource recovery and environmental pollution. A great number of previous studies are contributed to the development of decision-aided systems for designing reverse logistics network in an economically efficient manner. This paper, on the other hand, presents a novel multi-objective mixed integer programming for reverse logistics network planning, which takes into consideration of both economic efficiency and environmental influence of the reverse logistics activities. Comparing with previous location models for reverse logistics network design, the most significant contribution of this research is that the environmental impact of reverse logistics activities is accounted for as one of the most important indicators for the assessment of the overall performance



of reverse logistics system. Reverse logistics, as it is defined, aims at recovering remaining values of the used products so as to maximize the utilization of resources and minimize the negative environmental influence. However, improper treatment of used products may lead to secondary pollution to the environment, so taking into account environmental impact of reverse logistics activities themselves is of significance.

Being the first stage of the research, this paper presents the formulation of conceptual framework and mathematical optimization model for reverse logistics network design. The mathematical model formulated in this paper aims at seeking the optimal trade-off between the overall system cost and the negative impact on the environment. The negative environmental impact is measured by the carbon emissions in this research. At the second stage of this research, the weighted sum utility method for combining multiple objective functions, model application, and result analysis, and suggestions for future development, will be presented.

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