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## ***P*-WAVE $\Lambda N$ - $\Sigma N$ COUPLING AND THE SPIN-ORBIT SPLITTING OF ${}^9_{\Lambda}\text{Be}$**

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We reexamine the spin-orbit splitting of  ${}^9_{\Lambda}\text{Be}$  excited states in terms of the  $SU_6$  quark-model baryon-baryon interaction. The previous folding procedure to generate the  $\Lambda\alpha$  spin-orbit potential from the quark-model  $\Lambda N$   $LS$  resonating-group kernel predicted three to five times larger values for  $\Delta E_{\ell s} = E_x(3/2^+) - E_x(5/2^+)$  in the model FSS and fss2. This time, we calculate  $\Lambda\alpha$   $LS$  Born kernel, starting from the  $LS$  components of the nuclear-matter  $G$ -matrix for the  $\Lambda$  hyperon. This framework makes it possible to take full account of an important  $P$ -wave  $\Lambda N$ - $\Sigma N$  coupling through the antisymmetric  $LS^{(-)}$  force involved in the Fermi-Breit interaction. We find that the experimental value,  $\Delta E_{\ell s}^{\text{exp}} = 43 \pm 5$  keV, is reproduced by the quark-model  $G$ -matrix  $LS$  interaction with a Fermi-momentum around  $k_F = 1.0$  fm<sup>-1</sup>, when the model FSS is used in the energy-independent renormalized RGM formalism. On the other hand, the model fss2 gives too large splitting of almost 200 keV, owing to the uncanceled contribution of the scalar-meson exchange  $LS$  components.

*Keywords:* Quark-model baryon-baryon interaction; spin-orbit splitting of  $\Lambda$  hypernuclei

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### **1. Introduction**

In view of rich experimental data accumulated for the light  $\Lambda$ -hypernuclei,<sup>1,2</sup> it is important to examine if various models of the fundamental hyperon-nucleon ( $YN$ ) interactions can reproduce these experimental data or not. For few-body systems, this program is most reliably carried out by detailed Faddeev calculations for the hypertriton ( ${}^3_{\Lambda}\text{H}$ ),  ${}^3_{\Lambda}\text{}^4\text{H}$  and  ${}^4_{\Lambda}\text{He}$ ,<sup>4</sup> using some versions of the Nijmegen models<sup>5</sup> and Jülich potentials.<sup>6</sup> The knowledge of the  $\Lambda N$  interaction learned from these calculations, however, is mainly about the central part of the interaction and features of the  $\Lambda N$ - $\Sigma N$  coupling of the  ${}^3S_1 + {}^3D_1$  state due to the one-pion exchange tensor force. For the  $p$ -shell  $\Lambda$ -hypernuclei, some kinds of models inevitably need to be assumed so far, to connect properties of the  $\Lambda$ -hypernuclei and the under-

lying  $YN$  interactions. For example, the small spin-orbit ( $\ell s$ ) splitting commonly observed in many of the light  $\Lambda$ -hypernuclei <sup>2</sup> is typically manifested in the excited states of  ${}^9_{\Lambda}\text{Be}$ , for which a simple  $\Lambda + \alpha + \alpha$  three-cluster model is usually employed with appropriate  $\Lambda\alpha$  and  $\alpha\alpha$  potentials. <sup>7</sup> In the framework of this model, the origin of the  $\ell s$  splitting for the  $5/2^+$  and  $3/2^+$  excited states is the spin-orbit potential between  $\Lambda$  and one of the  $\alpha$  clusters, which is known to be very small due to the strong cancellation between the symmetric ( $LS$ ) and antisymmetric ( $LS^{(-)}$ )  $LS$  forces of the  $\Lambda N$  interaction.

In our previous study of the  ${}^9_{\Lambda}\text{Be}$  spectrum, <sup>8</sup> we have carried out the  $\Lambda\alpha\alpha$  three-cluster Faddeev calculation, trying to reproduce the very small  $\ell s$  splitting of the  $5/2^+$  and  $3/2^+$  excited states,  $\Delta E_{\ell s}^{\text{exp}} = 43 \pm 5$  keV, <sup>2</sup> experimentally observed. As a first step, Ref. 8 directly used the quark-model (QM)  $\Lambda N$   $LS$  resonating-group kernel (RGM kernel) to generate the  $\Lambda\alpha$   $LS$  potential by a simple procedure of the  $\alpha$ -cluster folding. In this approach, the QM  $\Lambda N$   $LS$  interaction of FSS or fss2 predicts 3 to 5 times larger values for  $\Delta E_{\ell s}$ , which is not much improved in comparison with the results of Nijmegen simulated potentials. <sup>7</sup> It was pointed out in Ref. 8 that a further reduction is possible in the model FSS, if one can properly take into account the short-range correlation of the  $P$ -wave  $\Lambda N$ - $\Sigma N$  coupling by the  $LS^{(-)}$  force. This was conjectured through the analysis of the Scheerbaum factors for the single-particle (s.p.) spin-orbit potentials, calculated in the  $G$ -matrix formalism.

## 2. Computational Procedure

Following the above suggestion, we here generate  $\Lambda\alpha$   $LS$  Born kernel from the  $LS$  component of the nuclear-matter  $G$ -matrix for the  $\Lambda$  hyperon. Our calculation consists of the following three steps.

1. Solve the  $G$ -matrix equation for the  $\Lambda$ -hyperon in symmetric nuclear matter with an appropriate Fermi momentum  $k_F$  and determine the s.p. potentials for  $N$ ,  $\Lambda$  and  $\Sigma$ . <sup>9,10</sup>
2. The  $LS$  components of the  $\Lambda N$   $G$ -matrices with definite momenta  $K$  and starting energies  $\omega$  are converted to the  $\Lambda\alpha$  Born kernel by the folding procedure recently developed for the  $\Lambda\alpha$  system. <sup>11</sup>
3. Solve  $\Lambda\alpha\alpha$  three-cluster system in the Faddeev formalism for composite particles. <sup>8</sup>

We generate  $\Lambda\alpha$   $LS$  Born kernel from our QM baryon-baryon interactions, FSS and fss2. <sup>12</sup> For the  $(0s)^4$   $\alpha$ -cluster folding, a new method developed in Ref. 11 is used to derive the direct and knock-on terms of the interaction Born kernel from the  $\Lambda N$   $G$ -matrix, with explicit treatments of the nonlocality and the center-of-mass motion between  $\Lambda$  and  $\alpha$ . The  $G$ -matrix calculations are carried out by assuming a constant value of the Fermi momentum,  $k_F = 1.07, 1.20,$  and  $1.35$  fm<sup>-1</sup> (the normal saturation density  $\rho_0$ ), since the local density approximation does not seem to work in light nuclear systems. The  $G$ -matrix equation is solved for the energy-

Table 1. The Scheerbaum factor  $S_{\Lambda}$  for symmetric nuclear matter and the  $\ell s$  splitting of the  ${}^9_{\Lambda}\text{Be}$  excited states predicted by the quark-model  $G$ -matrix  $\Lambda\alpha$   $LS$  Born kernel. In the last column, “ $\Lambda N$  Born” implies the previous results,<sup>8</sup> in which the  $\Lambda N$  single-channel RGM kernel is used for the  $S_{\Lambda}$  calculation and the  $\alpha$ -cluster folding.

	$\rho/\rho_0$ $k_F$ (fm $^{-1}$ )	0.5	0.7	1	$\Lambda N$ Born
$G$ -matrix	fss2 (cont)	-11.8	-12.1	-12.3	-10.9
$S_{\Lambda}$ (MeV fm $^5$ )	FSS (cont)	-4.1	-5.2	-6.3	-7.8
Faddeev	fss2 (cont)	206	216	223	198
$\Delta E_{\ell s}$ (keV)	FSS (cont)	55	85	114	137
$\Delta E_{\ell s}^{\text{exp}}$ (keV)		$43 \pm 5$			

independent QM baryon-baryon interaction, by using the renormalized RGM kernel,<sup>13</sup> and the continuous prescription for intermediate spectra. A similar procedure of the renormalized RGM is also used for the microscopic  $\alpha\alpha$  interaction,<sup>14</sup> for which the Pauli forbidden states between the two  $\alpha$ -clusters are completely eliminated in the three-cluster RGM formalism using the two-cluster RGM kernels.

### 3. Results and Discussion

Table 1 shows the  $\ell s$  splitting of the  ${}^9_{\Lambda}\text{Be}$  excited states, predicted by the  $\Lambda\alpha\alpha$  Faddeev calculations, using the QM  $G$ -matrix  $\Lambda\alpha$   $LS$  Born kernel. The Scheerbaum factor  $S_{\Lambda}$  is also listed to indicate the strength of the spin-orbit potentials of the  $\Lambda$  hyperon in symmetric nuclear matter. The Fermi momenta  $k_F = 1.07, 1.20,$  and  $1.35$  fm $^{-1}$  correspond to the densities  $\rho = 0.5\rho_0, 0.7\rho_0,$  and  $\rho_0$ , respectively, with  $\rho_0 = 0.17$  fm $^{-3}$  being the normal saturation density. The final values for the  $\ell s$  splitting of the  $5/2^+$  and  $3/2^+$  excited states are  $\Delta E_{\ell s} = 55 - 114$  keV for FSS and 206 - 223 keV for fss2, depending on the  $k_F$  values in the range of 1.07 - 1.35 fm $^{-1}$ . A smaller  $k_F$  value gives a smaller  $\ell s$  splitting. If we compare these results with the experimental value  $\Delta E_{\ell s}^{\text{exp}} = 43 \pm 5$  keV, we find that the model FSS can reproduce the experimental value if the  $k_F$  value around 1.02 fm $^{-1}$  is used. We find the strong cancellation between the  $LS$  and  $LS^{(-)}$  forces taking place in the QM Fermi-Breit interaction by the  $P$ -wave  $\Lambda N$ - $\Sigma N$  coupling in the  ${}^1P_1$ - ${}^3P_1$  state, when the  $G$ -matrix equation is solved especially in low-density nuclear matter. This is most prominently exhibited in the model FSS. The spin-orbit contribution from the effective-meson exchange potentials in fss2 does not lead to the sufficiently small  $\ell s$  splitting of the  $\Lambda$  hyperon, since the scalar-meson exchange  $LS$  force contains only the ordinary  $LS$  and does not produce the  $LS^{(-)}$  force.

### 4. Summary

We have carried out  $\Lambda\alpha\alpha$  Faddeev calculations by employing the  $\Lambda\alpha$   $LS$  Born kernel generated from the  $LS$  components of the nuclear-matter  $G$ -matrix for the  $\Lambda$

hyperon. One of our  $SU_6$  QM baryon-baryon interactions, FSS, can reproduce the very small  $\ell s$  splitting of  ${}^9_\Lambda\text{Be}$  excited states,  $\Delta E_{\ell s}^{\text{exp}} = 43 \pm 5$ , when an appropriate  $k_F$  value corresponding to almost half of the normal saturation density is employed in the  $G$ -matrix calculation. The explicit value of  $k_F$  depends on the model construction even within the framework of the  $\Lambda\alpha$  cluster model;  $k_F = 1.02 \text{ fm}^{-1}$  for the model FSS, when the energy-independent renormalized RGM kernels are used for the  $\alpha\alpha$  RGM kernel and for the QM baryon-baryon interaction. On the other hand, the model fss2 gives too large splitting of almost 200 keV, which is traced back to the uncanceled contribution of the scalar-meson exchange  $LS$  components. An essential ingredient of the present formalism is to take into account an important  $P$ -wave  $\Lambda N$ - $\Sigma N$  coupling through the antisymmetric  $LS^{(-)}$  force involved in the Fermi-Breit interaction. The present results indicate that the spin-orbit contribution from the effective meson-exchange potentials in fss2 needs to be improved to reproduce the small spin-orbit interaction of the  $\Lambda$  hyperon in the nuclear medium. A new model for the  $\Lambda N$  interaction with consistent central and  $LS$  components is strongly desired.

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