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Magnetic Actuator Design Using Level Set Based Topology Optimization

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This paper presents a novel design methodology for optimum structural design of magnetic actuators using a level set based topology optimization method where the level set method can represent the precise boundary shape of a structure and also deal with complex topological changes during the optimization process. The distribution of ferromagnetic material is represented by introducing a level set function into the definition of the magnetic reluctivity. The optimization problem is defined to obtain optimal configurations that maximize the magnetic energy of actuators under a minimum bound of total volume. The movement of the implicit moving boundaries of the structure is driven by a transformation of design sensitivities of the objective and the constraints into speed functions that govern the level set propagation. The proposed method is applied to the structural design of magnetic actuators, and is confirmed to be useful for achieving optimal configurations that deliver higher performance and lighter weight designs.

Index Terms—**Level set method, magnetic actuator, structural boundary design, topology optimization.**

I. Introduction

MAGNETIC actuators are widely used in electro-mechanical industries since they offer relatively large actuation motion in comparison with other actuators such as piezoelectric actuators [1]. However, achieving desired improvements in actuation performance has been elusive. To mitigate this problem, topology optimization which determines optimal distribution of material has been applied to the design of magnetic devices since the work of Dyck and Lowther [2]. Element based topology optimization methods often result in mesh-dependent boundaries and grayscale elements due to numerical instabilities [3], and such features greatly influence the performance of magnetic devices due to discontinuities in the magnetic flux. As a result, topology optimization methods are currently restricted to the design of basic magnetic devices having simple shapes, due to the low resolution of the boundary expressions [4].

Level set based topology optimization has recently emerged as an attractive alternative to overcome the shortcomings of conventional methods [5], [6]. The level set method is a numerical technique for tracking interfaces and shapes using an implicit function, the so-called level set function [7]. Sethian and Wiegmann [8] firstly introduced the level set method to topology optimization to represent material boundaries, and were able to design a cantilever beam with distinct boundaries free from mesh-dependencies and grayscale elements. Thus, the goal of this paper is to develop a systematic approach for the optimum structural design of magnetic actuators by adopting the concept of the level set method. The versatile handling of drastic changes in topology leads to novel configurations of magnetic actuators and the exact shape extraction of the optimized design can be easily integrated with CAD/CAE systems.

In this work, we focus on designing a yoke of magnetic actuators using level set based topology optimization to improve the magnetic force on the armature while reducing its weight. To formulate the optimization problem, the objective function is set to maximize the magnetic energy in the domain between the yoke and armature, since the magnetic force on the armature becomes larger as this increases and the amount of material used in the domain is constrained to limit the weight of the structure. The movement of the structure's implicit moving boundaries is driven by a transformation of the objective and the constraint into speed functions that govern the level set propagation. The normal velocity is derived from the optimality and convergence conditions of the level set equation and calculated using the sensitivities of the objective function and the constraint, where the adjoint variable method is employed since the objective function is implicitly dependent with respect to the level set function.

II. Level Set Based Topology Optimization in Magnetic Fields

The ferromagnetic structure to be optimized is implicitly represented through an embedded level set function and the structural boundary is propagated along its normal direction with speed expressed in terms of the magnetic energy and the amount of material used in the design.

A. Material Representation

The level set function is introduced to distinguish the material boundaries between the ferromagnetic material domain and another domain filled with air, as shown in Fig. 1. The computational design domain can be represented by the level set function $\phi(\mathbf{x})$ which is defined as

$$\begin{aligned} &\phi(\mathbf{x}) > 0 \quad \text{for} \quad {}^{\forall}\mathbf{x} \in \Omega^{+} \quad (\text{material}) \\ &\phi(\mathbf{x}) = 0 \quad \text{for} \quad {}^{\forall}\mathbf{x} \in \partial\Omega \quad (\text{boundary}) \\ &\phi(\mathbf{x}) < 0 \quad \text{for} \quad {}^{\forall}\mathbf{x} \in \Omega^{-} \quad (\text{air}) \end{aligned} \tag{1}$$

where $\mathbf{x}$ stands for an arbitrary position in the design domain.

The magnetic quantities are calculated by linear magnetostatic finite element analysis under the assumption that the material property has a linear relationship between magnetic field

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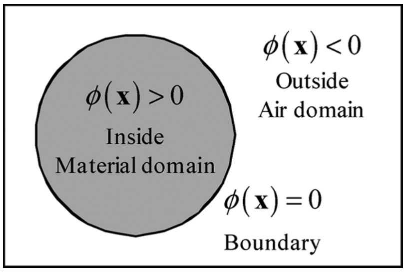

Fig. 1. Material boundary representation using level set function.

intensity and flux density. To represent the distribution of ferromagnetic material, the magnetic reluctivity of each element can be defined using a level set function as

$$\nu(\phi(\mathbf{x})) = \nu_0 [\nu_{air} + (\nu_{mat} - \nu_{air}) H(\phi(\mathbf{x}))] \tag{2}$$

where ν_0, ν_{air}, and ν_{mat} are respectively the magnetic reluctivities in free space, air, and material, and $H(\mathbf{x})$ is the smooth Heaviside function [9] where it is continuous near the boundary, to avoid numerical instability.

B. Problem Formulation

For the design of a magnetic actuator it is common to consider the magnetic force or the magnetic flux as a performance index. Since the magnetic force is proportional to the magnetic flux and the magnetic energy can be defined as the square of the magnetic flux, we can maximize the magnetic force by maximizing the magnetic energy. Therefore, the objective function is set to maximize the magnetic energy in the domain Ω_{obj} between a yoke and an armature, and this is defined in a discretized form by

$$\begin{aligned}(\text{Magnetic Energy}) &= \frac{1}{2}\sum_{\Omega_{obj}} \mathbf{B}_e^{\mathrm{T}}\mathbf{H}_e \\ &= \frac{1}{2}\sum_{\Omega_{obj}} \nu(\phi(\mathbf{x}))\,\mathbf{B}_e^{\mathrm{T}}\mathbf{B}_e \\ &= \frac{1}{2}\sum_{\Omega_{obj}} \mathbf{A}_e^{\mathrm{T}}\mathbf{K}_e(\phi(\mathbf{x}))\,\mathbf{A}_e \end{aligned} \tag{3}$$

where $\mathbf{B}_e$, $\mathbf{H}_e$, $\mathbf{A}_e$, and $\mathbf{K}_e$ are the magnetic flux density, field intensity, vector potential and element stiffness, respectively.

The amount of ferromagnetic material in the design domain Ω_{design} is constrained to a specific value $\overline{\Omega}$ to limit the weight of the magnetic actuator by

$$(\text{Volume}) = \sum_{\Omega_{design}} H(\phi(\mathbf{x})) \le \overline{\Omega}. \tag{4}$$

Therefore, the level set based topology optimization in a magnetic field can be formulated as

$$\begin{aligned} &\text{Find} && \{\mathbf{x}(t) \mid \phi(\mathbf{x}(t), t) = 0\} \\ &\text{maximize} && f = \frac{1}{2}\sum_{\Omega_{obj}} \mathbf{A}_e^{\mathrm{T}}\mathbf{K}_e(\phi(\mathbf{x}(t), t))\,\mathbf{A}_e \\ &\text{subject to} && g = \sum_{\Omega_{design}} H(\phi(\mathbf{x}(t), t)) - \overline{\Omega} \le 0 \end{aligned} \tag{5}$$

C. Boundary Propagation

Since it is assumed in the level set method that the motion of the interface is important only in the normal direction, and the level set function is defined as a signed distance function, the level set equation that represents the material boundary changes can be obtained by differentiating the level set function with respect to time, as

$$\frac{d\phi(\mathbf{x}(t), t)}{dt} = \frac{\partial\phi(\mathbf{x}(t), t)}{\partial t} + \mathbf{v}_n = 0 \tag{6}$$

where $\mathbf{v}_n$ is the normal velocity of the boundary.

The propagation of the boundary is controlled by solving a time-dependent initial value problem, and the normal velocity is required to satisfy the optimality conditions when the boundary becomes stationary. Therefore, the normal velocity can be derived from both the KKT conditions of the optimization problem (5) and the convergence criteria of the level set (6) as

$$\mathbf{v}_n = -\left(\frac{df}{d\phi} + \lambda\frac{dg}{d\phi}\right) \tag{7}$$

where λ is the Lagrange multiplier.

III. Optimization Method

The optimization process here can be implemented as a mathematical programming problem. Since the normal velocity drives the design boundary into the optimal configuration, boundary variation sensitivity is derived from the objective function and the constraint.

A. Design Sensitivity

Material boundaries are driven by speed functions that govern the level set propagation. The normal velocity is derived from the optimality and convergence conditions of the level set equation and calculated using design sensitivities.

For the sensitivity of the constraint, this can be obtained directly by differentiating the constraint with respect to ϕ as follows:

$$\frac{dg}{d\phi} = \sum_{\Omega_{design}} \frac{dH(\phi(\mathbf{x}))}{d\phi} = \sum_{\Omega_{design}} \delta(\phi(\mathbf{x})) \tag{8}$$

where $\delta(\mathbf{x})$ is the Dirac-delta function.

As for the objective function, $\mathbf{K}_e$ and $\mathbf{A}_e$ have explicit and implicit dependencies with the level set function, respectively, which makes it infeasible to derive the sensitivities analytically. Therefore, the adjoint variable method is employed to calculate the sensitivity of the objective function, where the adjoint variable is introduced. Using the objective function and the governing equation for magnetic fields, the functional $\hat{f}$ can be defined as

$$\hat{f} = f + \mathbf{z}^{\mathrm{T}}(\mathbf{KA} - \mathbf{J}) \tag{9}$$

where the adjoint variable $\mathbf{z}^{\mathrm{T}}$ can be arbitrary since $\mathbf{KA} = \mathbf{J}$ is always valid. Considering the fact that the current density $\mathbf{J}$

is not related to the level set function, the sensitivity of $\widehat{f}$ with respect to ϕ is obtained as follows:

$$\frac{d\widehat{f}}{d\phi} = \frac{\partial f}{\partial \phi} + \mathbf{z}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial \phi}\mathbf{A} + \left(\mathbf{z}^{\mathrm{T}}\mathbf{K} + \frac{\partial f}{\partial A}\right)\frac{d\mathbf{A}}{d\phi}. \qquad (10)$$

To cancel out the last term, the adjoint variable $\mathbf{z}^{\mathrm{T}}$ is determined to satisfy the following adjoint equation:

$$\mathbf{z}^{\mathrm{T}}\mathbf{K} + \frac{\partial f}{\partial A} = \mathbf{0}. \qquad (11)$$

Finally, the sensitivity of the objective function f with respect to ϕ is given as a function of the magnetic potential vector, the sensitivity of the magnetic stiffness and the adjoint variable as

$$\begin{aligned}\frac{df}{d\phi} = \frac{d\widehat{f}}{d\phi} &= \frac{\partial f}{\partial \phi} + \mathbf{z}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial \phi}\mathbf{A} \\ &= \frac{1}{2}\sum_{\Omega_{obj}} \mathbf{A}_e^{\mathrm{T}}\frac{\partial \mathbf{K}_e}{\partial \phi}\mathbf{A}_e + \mathbf{z}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial \phi}\mathbf{A}. \qquad (12)\end{aligned}$$

B. Lagrange Multiplier

A Lagrange multiplier is required to calculate the normal velocity for the movement of the boundary. The gradient projection method is employed to estimate the Lagrange multiplier, assuming that the constraint for the use of the material is active. Thus, differentiation of the constraint with respect to time must always be zero, and hence

$$\frac{\partial g}{\partial t} = \frac{\partial g}{\partial \phi}\frac{\partial \phi}{\partial t} = \frac{\partial g}{\partial \phi}\left(\frac{df}{d\phi} + \lambda\frac{dg}{d\phi}\right) = 0. \qquad (13)$$

Thus, the evaluation of the Lagrange multiplier yields the following:

$$\lambda = -\frac{df}{d\phi}\cdot\frac{\partial g}{\partial \phi}\Big/\frac{dg}{d\phi}\cdot\frac{\partial g}{\partial \phi}. \qquad (14)$$

C. Algorithm

The proposed method can be summarized as follows:

Step 1: Define the design domain in which the material is distributed and the objective function domain where the magnetic energy is calculated.
Step 2: Define the material boundary by initializing the level set function.
Step 3: Perform the magneto-static analysis and evaluate the magnetic energy and the amount of ferromagnetic material.
Step 4: Perform the design sensitivity analysis and calculate the normal velocity
Step 5: Solve the level set equation and propagate the boundary.
Step 6: Repeat the Steps 3–5 until the evolution of the material boundary is converged.

The flowchart in Fig. 2 illustrates the overall procedure of the proposed method and figures in each step show the design process for a simple C-core actuator.

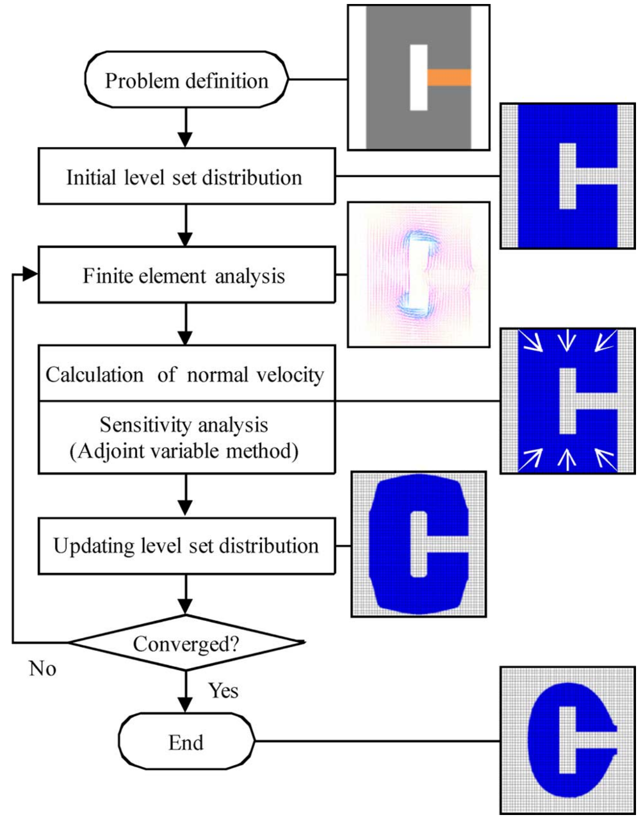

Fig. 2. Flowchart of level set based topology optimization in magnetic field.

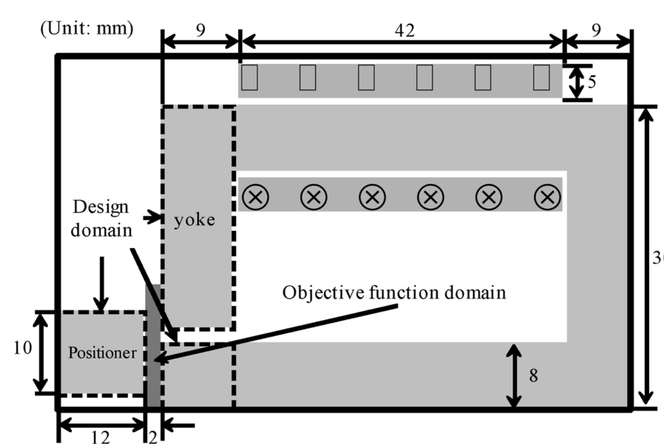

Fig. 3. Initial design of a magnetic coupler.

IV. Numerical Example

The proposed method is applied to design a yoke of magnetic coupler which enables the transmission of mechanical energy through a magnetic field interaction. The design objective is to maximize the magnetic force on the positioner so that it can be used as a locking device.

The initial design of the magnetic coupler is shown in Fig. 3. The areas of the positioner and neighboring yokes are considered as the design domain and the amount of ferromagnetic material distributed is limited to 70% of the total design domain. The air gap between the positioner and yokes is specified as an

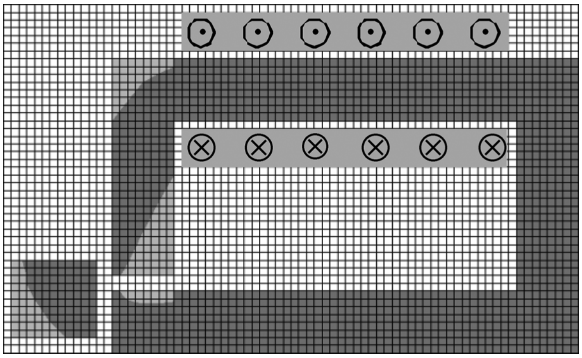

Fig. 4. Optimal design of a magnetic coupler.

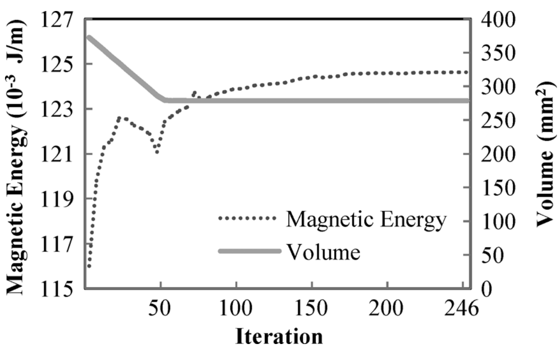

Fig. 5. Optimization history in magnetic coupler design.

TABLE I
COMPARISON BETWEEN INITIAL AND OPTIMAL DESIGN

	Initial Design	Optimal Design
Volume in design domain (mm^2)	344	240 (30%↓)
Magnetic force on armature (N/m)	44.2	48.4 (9.5%↑)

objective function domain where the magnetic energy is calculated. The whole domain is discretized into 3300 QUAD4 finite elements and the Dirichlet boundary condition is applied along the outer boundary. The electric current density in the coil domain is set to 2 A/mm^2 and the relative magnetic reluctivities of the ferromagnetic material and air are set to 0.001 and 1, respectively.

Fig. 4 shows the optimal topology layout of a magnetic coupler that has clear-cut boundaries and the exact shape extraction of the optimal design can be easily integrated with CAD/CAE systems. The optimization history for both magnetic energy and material volume is shown in Fig. 5, as is the fluctuation in magnetic energy near iteration 40. It is noted that the rapid change in yoke shape to satisfy the constraint results in a temporary decrease in the objective function. Table I summarizes that the optimal design provides a 10% higher magnetic force and 30% less material usage than the initial design.

V. CONCLUSION

We developed a novel methodology for boundary design of ferromagnetic structures, implemented the algorithm with the use of the level set method, and applied this to the design of a magnetic actuator.

It is expected that level set-based topology optimizations capable of handling magnetic fields will provide useful tools considering flexible representation of structural boundaries for the design of magnetic actuators. This will lead to better designs for lighter magnetic devices that can deliver superior mechanical performance.

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REFERENCES

[1] D. Howe, "Magnetic actuators," *Sensors Actuators A: Phys.*, vol. 81, pp. 268–274, 2000.
[2] D. N. Dyck and D. A. Lowther, "Automated design of magnetic devices by optimizing material distribution," *IEEE Trans. Magn.*, vol. 32, pp. 1188–1193, 1996.
[3] O. Sigmund and J. Petersson, "Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima," *Structural Optimization*, vol. 16, pp. 68–75, Aug. 1998.
[4] J. Yoo and H. J. Soh, "An optimal design of magnetic actuators using topology optimization and the response surface method," *Microsyst. Technol.*, vol. 11, pp. 1252–1261, 2005.
[5] X. Wang, M. Y. Wang, and D. Guo, "Structural shape and topology optimization in a level-set-based framework of region representation," *Structural and Multidisciplinary Optimization*, vol. 27, pp. 1–19, 2004.
[6] G. Allaire and F. Jouve, "A level-set method for vibration and multiple loads structural optimization," *Computer Methods in Applied Mechanics and Engineering*, vol. 194, pp. 3269–3290, 2005.
[7] S. Osher and J. A. Sethian, "Fronts propagating with curvature-dependent speed—Algorithms based on hamilton-jacobi formulations," *J. Computational Phys.*, vol. 79, pp. 12–49, Nov. 1988.
[8] J. A. Sethian and A. Wiegmann, "Structural boundary design via level set and immersed interface methods," *J. Computational Phys.*, vol. 163, pp. 489–528, 2000.
[9] S. Yamasaki, S. Nishiwaki, K. Izui, and M. Yoshimura, "Mechanical systems structural optimization of mechanical structures based on the level set method: Construction of a new re-initialization method and its application to the stiffness maximization problem," *Trans. Jpn. Soc. Mechan. Eng. Series C*, vol. 73, pp. 72–79, 2007.

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