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The Statistics of Velocity Difference Vs. the Statistics of Dissipation in Isotropic Turbulence

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From the data of a 3D numerically simulated isotropic turbulence, it is found that the statistics of longitudinal velocity difference in distance r is different from that of the velocity scale derived from dissipation averaged over a domain of scale r, and then intermittency exponents cannot exactly be related with exponents of structure functions. The probability distributions in both statistics are comparatively studied from the viewpoint of scale-similarity and modelling to involve multifractality.

The multifractality of isotropic turbulence was first argued by Frisch and Parisi[1] in relevance with longitudinal velocity difference Δu_r in distance r. They assume the scale-similarity described as

$$\Delta u_r \sim r^h, \tag{1}$$

where h is singularity strength at each location of the flow field. The iso-h set in space is considered to be multifractal with dimension D(h). The spatial distribution of h is now claimed to have been observed by a wavelet analysis[2], but the D-h spectrum to be derived from that is not yet publicized to the authors' knowledge.

On the other hand, the $f-\alpha$ formalism originating in the theory of strange attractors was applied by Meneveau and Sreenivasan[3)]to the dissipation measure of isotropic turbulence. They assume in place of (1)

$$\varepsilon_r \sim r^{\alpha-1}, \tag{2}$$

where ε_r is energy dissipation rate averaged over a domain of scale r. Similarly, the iso-α set in space is multifractal with dimension $f(\alpha)$. From the experimental data in a 1D cut, they investigated the f-α spectrum through generalized dimensions[4)] and proposed a theoretical model, called the p model, to explain the intermittent and multifractal structure of isotropic turbulence.[5)] Later, the present authors did the same investigation based on the data of a 3D direct numerical simulation to get similar but clearly distinct pictures of the f-α spectrum as well as generalized dimensions.[6-8)] The senior author (IH) contrived a generalized class of Cantor set models (including the p model) to find a model most suitable for these results at hand.[9)]

From the viewpoint of scale-similarity[1, 3)], (1) and (2) can be interrelated with each other. Since the velocity scale to be related with ε_r is

$$v_r = (r\varepsilon_r)^{1/3}, \tag{3}$$

the equality:

$$h = \alpha/3 \tag{4}$$

may naturally be expected by a presumption that $\Delta u_r \sim v_r$.[8)] If it is true, the equivalence of the D-h spectrum and the f-α spectrum is obvious. However, it may not really be so. A symptom is the dilemma that we have no negative value of α in the f-α spectrum so far, while negative values of h are frequently seen in the result of the wavelet analysis[2)]. Therefore, it is essential at this stage to investigate whether the correspondence:

$$\Delta u_r \sim v_r \tag{5}$$

is valid or not.

Before doing so, we should like to point out that there are many ways to represent the velocity difference associated with a disjoint subbox of scale r.

In a most plain way, Δu_r can be defined as the absolute difference of the x components of velocity at the centers of the opposite yz surfaces of the subbox. But another quantity, $\Delta \mathbb{v}_r$, which is the magnitude of the vectorial velocity difference just there, may be better. Or, $\langle \Delta \mathbb{v}_r \rangle$, which is the three-directional average of the magnitude of the vectorial velocity difference averaged over the whole yz surfaces, may be much better.

For the purpose of comparing these velocity differences with v_r, we utilize the same data of a 3D numerical direct simulation of fully-developed isotropic turbulence as used for calculating the f-α spectrum[6-8], the details of which were described in Refs. 10 and 11. We plot v_r and Δu_r at all locations of the subboxes in Fig. 1 for r/L = 8/128, which is in the inertial range; L is the scale of the main box. Here, v_r is normalized by $(L \varepsilon_L)^{1/3}$, and Δu_r by such a particular constant that the averages of the two normalized quantities exactly coincide. In Figs. 2 and 3 are seen $\Delta \mathbb{v}_r$ and $\langle \Delta \mathbb{v}_r \rangle$ against v_r, normalized just in the same way as above. From Fig. 1 we can judge that Δu_r is independent of v_r and it is impossible to presume any individual correspondence between them. Any statistical resemblance is not found, either. Then, (5) and then (4) are not exactly valid, and there is no equivalence of the D-h spectrum (if it exists) and the f-α spectrum. We should notice that Fig. 1 hardly supports the 1962 Kolmogorov theory on refined similarity hypothesis[12] (specifically Eq. (7) in Ref. 12), because we cannot see even a statistical proportional dependence of Δu_r on v_r. As a corollary, the often used relation between intermittency exponents and the scaling exponents of velocity structure functions:

$$\zeta_p = p/3 - \mu_{p/3} \tag{6}$$

does not exactly hold; since it is based on (1)-(4) through the definitions, $\langle |\Delta u_r|^p \rangle \sim r^{\zeta p}$ and $\langle \varepsilon_r{}^q \rangle \sim r^{-\mu q}$, the angular bracket denoting the ensemble average. In Figs. 2 and 3 we can see that $\Delta \mathbb{v}_r$ is closer to v_r in the variation range than Δu_r, and $\langle \Delta \mathbb{v}_r \rangle$ is closest to v_r. In fact, a considerable statistical resemblance between $\langle \Delta v_r \rangle$ and v_r is expected, and it may well judged in Fig. 4, where the probability densities of all the observed quantities are compared. Then, the 1962 Kolmogorov theory can be only statistically true, if Δu_r is replaced by $\langle \Delta v_r \rangle$. These results are invariant to a different r which belongs to the inertial range.

What can be noticed in Fig. 1 is that there are many points beyond $\Delta u_r = 1$ and a dense population exists merging in $\Delta u_r = 0$. The former fact suggests the existence of negative values of h, since Δu_r corresponds to r^h. The probability density of Δu_r plotted in Fig. 4 might well be exponential $\sim e^{-kx}$. This is transformed in terms of h to

$$P(h;r) = kc\exp(-kcr^h)r^h|\ln r|. \qquad (7)$$

Here and hereafter, r should read as r/L, and c (const) = $\Delta u_r/r^h$. The peak of P appears at $r^h = (kc)^{-1}$. If we assume $h = 1/3$ (the 41 Kolmogorov value) at the peak and replace kc by $r^{-1/3}$, we may have a universal expression of P, which supports the result of the wavelet analysis[2] considerably well. However, this P yields straightforwardly $\zeta_p = p/3$, which is the same trend as the 1941 Kolmogorov theory. Therefore, it is unlikely that a purely exponential form fully involves intermittency and possible multifractality. A valid probability density should contain a D(h) in the form of (10) in place of the $f(\alpha)$. But the function D(h) is still unknown.

In contrast, we know that the lognormal distribution[13] of dissipation $\varepsilon_r/\varepsilon_L$ $(= r^{\alpha-1})$, written in terms of α, as

$$P(\alpha;r) = r^{(\alpha-1-\mu/2)/2\mu}[|\ln r|/(2\pi\mu)]^{1/2} \qquad (8)$$

gives exactly the fractal dimension of the iso-α set as

$$f(\alpha) = -(\alpha - 1 - \mu/2)^2/2\mu + 3, \qquad (9)$$

where $\mu = \mu_2$; $\mu = 0.2$ is a current setting. Indeed, this probability well suits to the general expression required from multifractality of the iso-α set:

$$P(\alpha;r) = r^{3-f(\alpha)}[|f''(\alpha)\ln r|/(2\pi)]^{1/2}, \text{ as } r \to 0. \qquad (10)^{14)}$$

As a meaningful alternative model, we here introduce a generalized Cantor set model[9] which has a simple expression of μ_q as

$$\mu_q = \log_A[(B^q + C^q)/2] \qquad (11)$$

with $A = 2^{1/3}$, $B = 1.2175$ and $C = 0.7825$. These settings of the parameters better fit to our 3D numerical results than the p model in which $A = 2$, $B = 1.4$ and $C = 0.6$ [9] (This fact may be seen in Fig. 4 in comparison with v_r. Note in particular that the range of v_r is narrower in the p model, because of the narrower range of α just as described below.) This Cantor set model gives exactly

$$f(\alpha) = 3 - \{[\ln(A^{\alpha-1}B)/\ln(C/B)]\ln[\ln(B/C)/\ln(A^{\alpha-1}B) - 1]$$
$$- \ln[2\ln(A^{\alpha-1}C)/\ln(C/B)]\}/\ln A \qquad (12)$$

for $\alpha_{min} < \alpha < \alpha_{max}$, where $\alpha_{min} = 1 - \ln B/\ln A$ and $\alpha_{max} = 1 - \ln C/\ln A$. The lognormal model is very approximate to this model [(10)-(12)], as is seen in Fig. 4, but violates the Novikov condition[15] while the Cantor set model does not, as μ_q in (11) is asymptotically linear in q. It is interesting to note that the probability density of v_r in this model has a longer exponential tail for smaller r, as is assured by a computer. (This is more exponential than $\varepsilon_r^{1/2}$. See Ref. 9.)

In conclusion, the statistics of Δu_r and that of v_r are separate matters in an exact sense. The error of (6) will be large for p large; it may be most readily estimated by comparing the ζ_p by Vincent and Meneguccі[16] and the μ_q by Hosokawa and Yamamoto[7]. Here seems to be one of the reasons why the latter deviats from the prediction using (6) from Anselmet et al's experiment[17]. It is to be noted that μ_q exists for $q < 0$, as was already shown[6-8], while ζ_p for $p < 0$ cannot exist because we have a non-vanishing probability of $\Delta u_r = 0$. The probability density of Δu_r collaborated with its multifractality is still unclear, while a consistent model with multifractality exists for ε_r and then

v_r. Finally, it should be pointed out that the same problem can exist for the statistics of temperature difference in distance r and that of the temperature scale derived from temperature dissipation averaged over in a domain of scale r and ε_r in isotropic turbulence.

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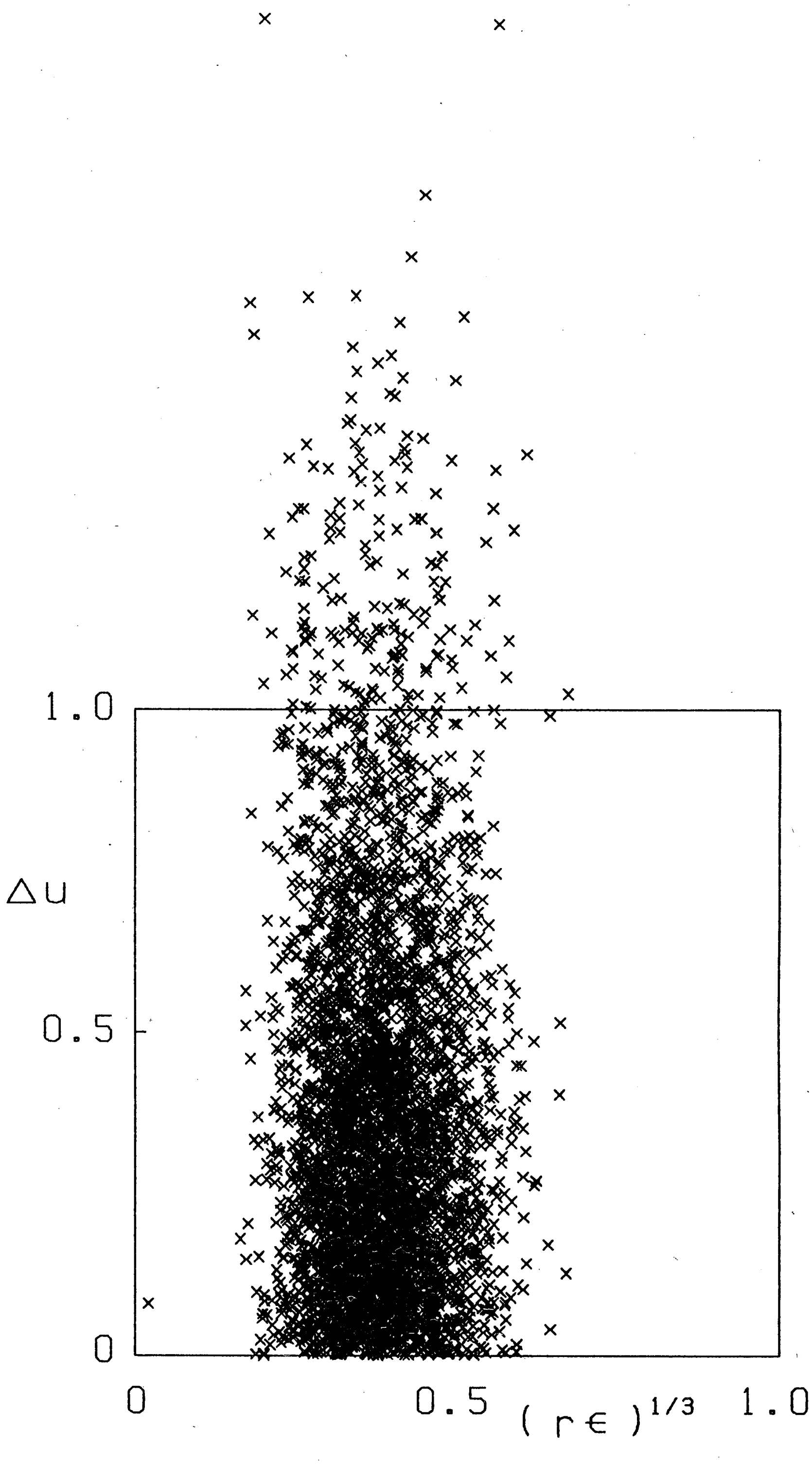
Figure Captions

Fig. 1 Correlation of Δu_r and v_r for r/L = 8/128.

Fig. 2 Correlation of $\Delta \mathbb{v}_r$ and v_r for r/L = 8/128.

Fig. 3 Correlation of $\langle \Delta \mathbb{v}_r \rangle$ and v_r for r/L = 8/128.

Fig. 4 Normalized histograms of v_r (= $(r\varepsilon_r)^{1/3}$), Δu_r, $\Delta \mathbb{v}_r$, and $\langle \Delta \mathbb{v}_r \rangle$ for r/L = 8/128. Theoretical probability densities: ——— the Cantor set model quoted from Ref. 9, based on (10)-(12), ········ the p model, based on (10)-(12), and —— · —— the lognormal model with μ = 0.2 based on (8).



8 Fig. 1

Fig. 2

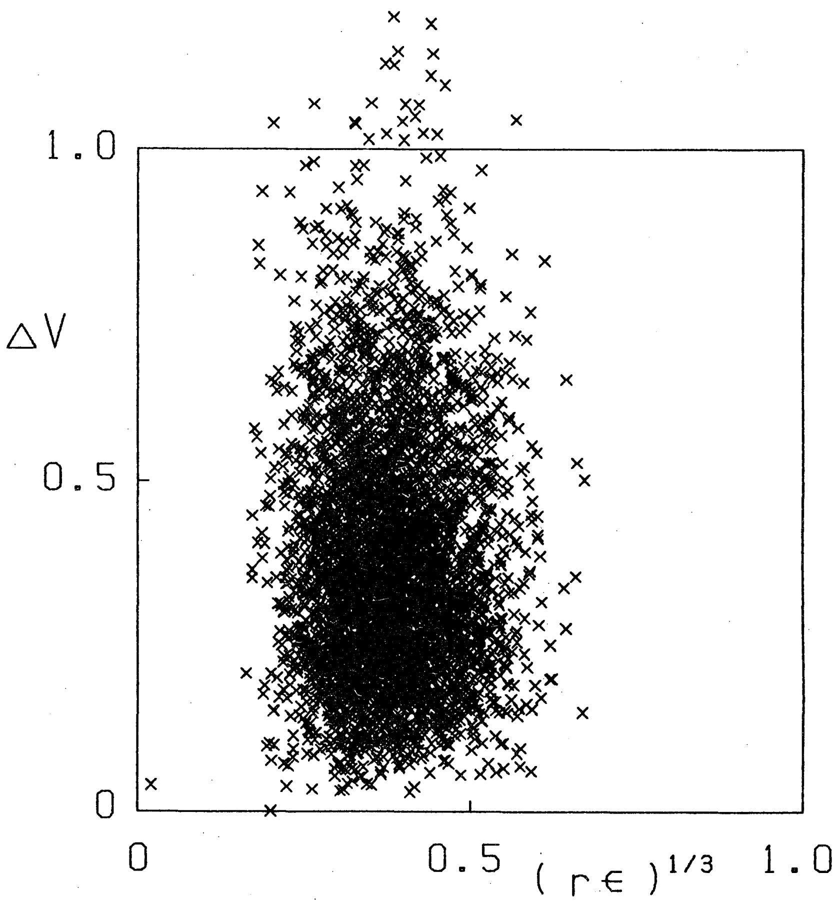
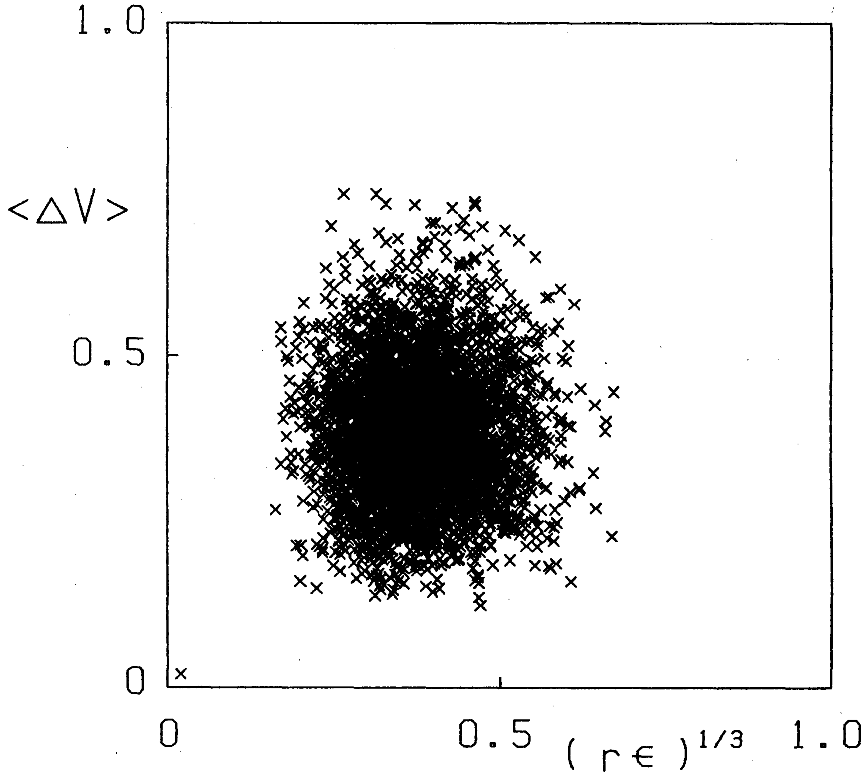


Fig. 3



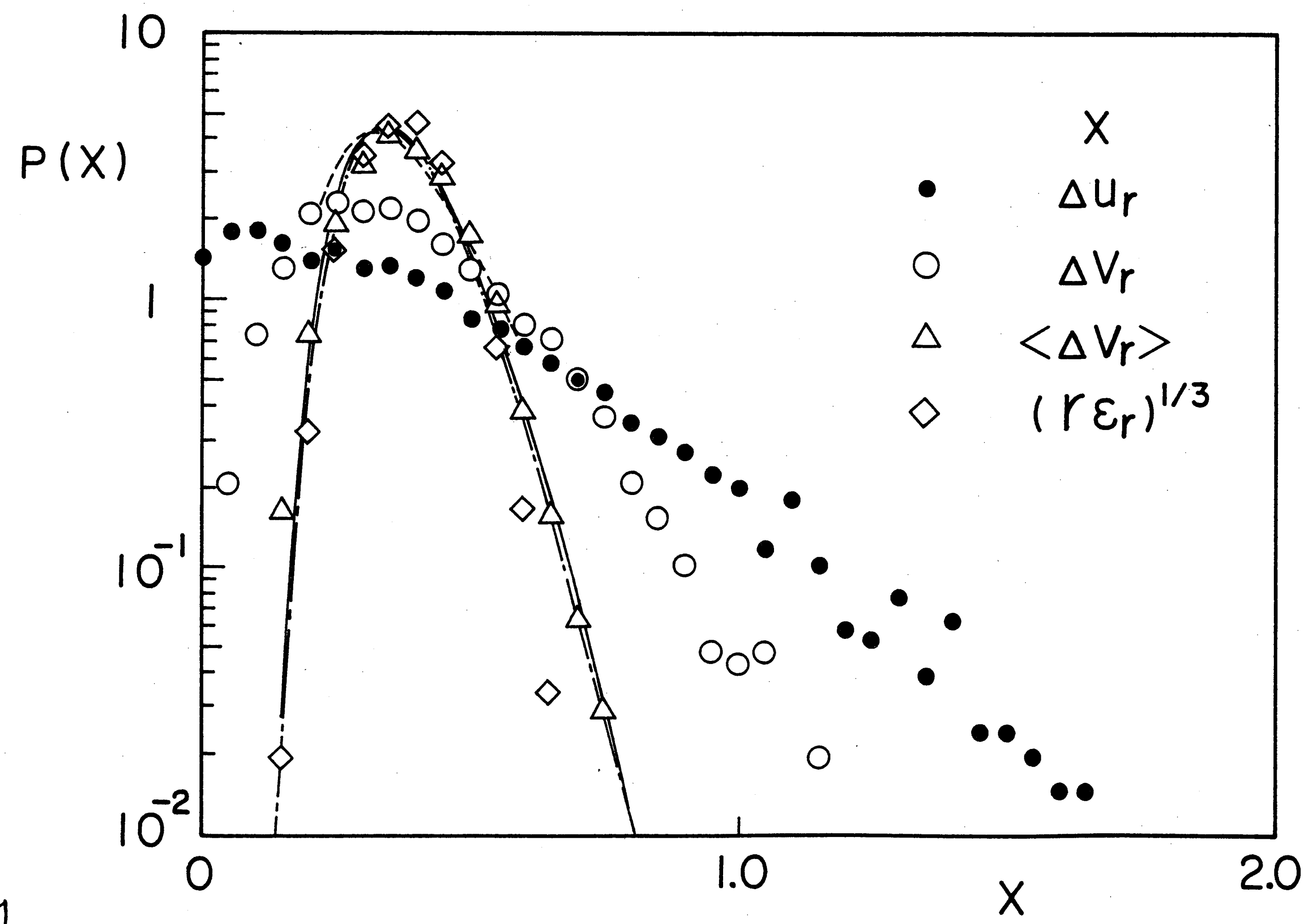

10
1
10^{-1}
10^{-2}
P(X)
0
1.0
2.0
X
X
Δu_r
ΔV_r
$<\Delta V_r>$
$(r\varepsilon_r)^{1/3}$

Fig 4