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Analysis of the Dynamics of Coral Diseases 'Band Syndromes' : Approach to Mathematical Model

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1 Introduction

Between 18 and 30 diverse coral diseases have been recognized since they were first discovered in the early 1970s ([2], [4], [5], [6]). There is still little known about the causes and effects of coral disease although diseases of coral can be caused by bacteria, fungi, algae and worms. The causes of coral diseases are most likely a combined result of global warming, ozone depletion, overfishing, eutrophication, poor land-use practices and other manifestations of human activities.

The first coral disease, black band syndrome, reported in 1973 consisted of a dark band that is present between apparently healthy coral tissue and freshly exposed coral skeleton. The second coral diseases, white band syndrome, reported in 1977 appears as a sharp boundary between coral tissue and exposed skeleton, with no apparent microbial biomass ([5], [6]). Brown band syndrome is a new coral disease characterized by a local accumulation of yet-unidentified ciliates migrating as a band along the branches of coral colonies ([7], [9]). The characterization of the three coral diseases is to progress with a sort of band shape. Moreover, all three diseases progressively destroy coral tissue at rates of several millimeters per day ([6], [7]).

Coral reef communities have a wide variety of mutualistic associations none more important than the relationship between corals and their symbiotic dinoflagellates, commonly referred to as zooxanthellae. Scleractinian corals' ecological success is directly related to the acquisition of dinoflagellate endosymbionts that enable the symbiosis to survive in oligotrophic and high solar irradiance habitats ([3]). Under normal conditions, most corals are various colors of green or brown, which primarily due to the intracellular symbiotic zooxanthellae. Bleaching of corals implies the loss of zooxanthellae in corals, and thus white coral skeleton is visible through the coral tissue ([8]).

Healthy corals live in close association with zooxanthellae and almost all coral species host a range of endolithic organisms such as algae, fungi and bacteria, especially the siphonaceous green algae, Ostreobiem ([4]). All members of the consortium live in equilibrium but environmental stress can disrupt this balance. The endolithic community lives in a sheltered environment within the coral skeleton where less than 5% of the photosynthetically active radiation penetrates ([1]).

WBS has no any bacteria on the progressing white band and the biomass of endolitics increase dramatically with increasing distance from the white band after the syndrome progresses ([1]), which are not appeared in BBS and BrBs. Why the endolithic algae blooms are not as common in the other syndromes, BBS and BrBs, with a distinct and fast progressing lesion ? In this study, we notice this point with the dynamics of the three band syndromes by mathematical models. The mathematical modeling of the syndromes could suggest some clues to treat the three band syndromes properly.

2 Models

In coral ecosystem, various unknown microorganisms as well as zooxanthellae live together with some associations, even if zooxanthellae is the most important endosymbiont. Moreover, such an unknown microorganisms may do an important function to corals through indirect interactions. Let us assume microorganism which give positive effect to corals but gain a negative effect from corals. We assume here the microorganisms obtains some benefits from endolithic community but give some disadvantage to zooxanthellae, and they give some disadvantage to the microorganisms.

Under the assumptions, we first consider a four-species model of healthy coral tissues, zooxanthellae, microendoliths and some microorganisms which denote $S(x,t)$, $Z(x,t)$, $P(x,t)$ and $B(x,t)$, respectively. The model is as follows:

$$\begin{aligned} S_t &= (-\gamma_1 + \alpha_1 Z + \alpha_2 B)S, \\ Z_t &= (-\gamma_2 + \beta_1 S - \beta_2 B)Z, \\ P_t &= (\gamma_3 - k_p P - \tau_1 S - \tau_2 Z - \tau_3 B)P + D_p \Delta P, \\ B_t &= (-\gamma_4 - \mu_1 S - \mu_2 Z + \mu_3 P)B + D_b \Delta B \end{aligned} \tag{1}$$

where all of parameters are positive constants. Here, we suppose that corals and zooxanthellae can increase infinitely if their environment maintains good condition continuously because the sea is large enough spacially. However, we suppose that microendoliths have inter-specific competition because they are habitable on corals' skeleton and are on poor radiation environment. The diffusion of corals and zooxanthellae can be considered to be slower enough than that of other species, thus we assume the diffusion coefficients of them are zero in the model.

Black band syndrome (BBS) usually observed in polluted environments is considered one of the major diseases impacting coral reefs worldwide. It is composed of a mixed microbial mat that is dominated by cyanobacteria and comprises sulfur-reducing and sulfur-oxidizing bacteria as well as a number of other microorganisms. Usually, it is known that cyanobacteria eats microendoliths. We suppose that microorganisms $B(x,t)$ has predatory association with the cyanobacteria because it is one of natural supposition that cyanobacteria cannot invade on normal environmental condition by their natural enemy. That is, we assume that microorganisms $B(x,t)$ is natural enemy of cyanobacteria. Then, from the model (1), the model of BBS is given by

$$\begin{aligned} S_t &= (-\gamma_1 + \alpha_1 Z + \alpha_2 B - \alpha_3 C)S, \\ Z_t &= (-\gamma_2 + \beta_1 S - \beta_2 B)Z, \\ P_t &= (\gamma_3 - k_p P - \tau_1 S - \tau_2 Z - \tau_3 B - \tau_4 C)P + D_p \Delta P, \\ B_t &= (-\gamma_4 - \mu_1 S - \mu_2 Z + \mu_3 P + \mu_4 C)B + D_b \Delta B, \\ C_t &= (\gamma_5 - k_c C + \delta_1 S + \delta_2 P - \delta_3 B)C + D_c \Delta C \end{aligned} \tag{2}$$

where $C(x,t)$ denotes the density of cyanobacteria and all of parameters are positive constants.

Brown band syndrome (BrBS) is caused by the invasion of some ciliates. The ciliates do not become compromised during the progression of the brown band zone where a high photosynthetically active radiation absorptivity is observed ([7]). The ciliates graze healthy coral tissue, and they engulf zooxanthellae at the same time. Let us denotes the density of the ciliates by $D(x,t)$. The model of BrBS is as follows:

$$\begin{aligned} S_t &= (-\gamma_1 + \alpha_1 Z + \alpha_2 B - \alpha_4 D)S, \\ Z_t &= (-\gamma_2 + \beta_1 S - \beta_2 B - \beta_3 SD)Z, \\ P_t &= (\gamma_3 - k_p P - \tau_1 S - \tau_2 Z - \tau_3 B - \tau_5 D)P + D_p \Delta P, \\ B_t &= (-\gamma_4 - \mu_1 S - \mu_2 Z + \mu_3 P + \mu_5 D)B + D_b \Delta B, \\ D_t &= (\gamma_6 - k_d D + \omega_1 S + \omega_2 P - \omega_3 B + \omega_4 ZS)D + D_d \Delta D. \end{aligned} \tag{3}$$

where all of parameters are positive constants.

Finally, we formulate the model of white band syndrome (WBS). WBS has no apparent microbal biomess on the progression of the white band zone. There are no report about the direct cause of WBS yet. We can conjecture that it is not caused by the invasion of alien species but an internal cause of corals. We here just suppose that WBS is caused by a virus infection of microorganisms $B(x,t)$ by a combined result of global warming, ozone depletion, manifestations of human activities and so on. Let us denote the density of a virus by $I(x,t)$. Then, the model of WBS is given by

$$\begin{aligned} S_t &= (-\gamma_1 + \alpha_1 Z + \alpha_2 B)S, \\ Z_t &= (-\gamma_2 + \beta_1 S - \beta_2 B)Z, \\ P_t &= (\gamma_3 - k_p P - \tau_1 S - \tau_2 Z - \tau_3 B)P + D_p \Delta P, \\ B_t &= (-\gamma_4 - \mu_1 S - \mu_2 Z + \mu_3 P)B - \gamma BI + D_b \Delta B, \\ I_t &= (-\gamma_7 + \gamma_8 B)I + D_i \Delta I. \end{aligned} \tag{4}$$

where all of parameters are positive constants.

3 Results

In what follows, we show numerical simulation results of (2), (3) and (4) with the initial condition given by the positive equilibrium (S_0, Z_0, P_0, B_0) of the model (1), and $C(x,0) = C_0\delta(x - x_0)$, $D(x,0) = D_0\delta(x)$ and $I(x,0) = I_0\delta(x)$, respectively.

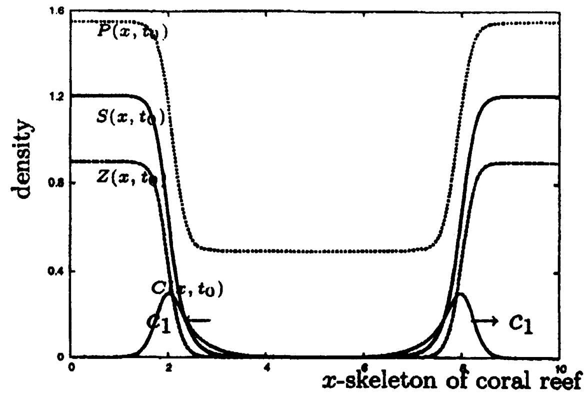

Figure 1: Black band syndrome. The case that cyanobacteria invades in the middle of coral reef. Cyanobacteria progresses with a constant speed c_1.

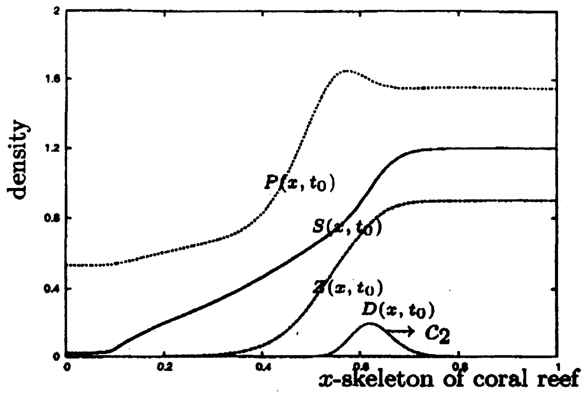

Figure 2: Brown band syndrome. The case that ciliates invade the edge of coral reef. Ciliates progress with a constant speed c_2.

Figure 1 and Figure 2 show that cyanobacteria and ciliate exist only in a small range and progress with a constant speed. Moreover, microendolithes $P(x, t)$ maintain a low density level after healthy coral tissues disappear off.

Now, we show the results of the white band syndrome. A characteristic of the white band syndrome is the bloom of microendolithes where healthy coral tissues disappear off. Figure 3 shows that healthy coral tissues disappear with a constant speed and endolithes increase dramatically.

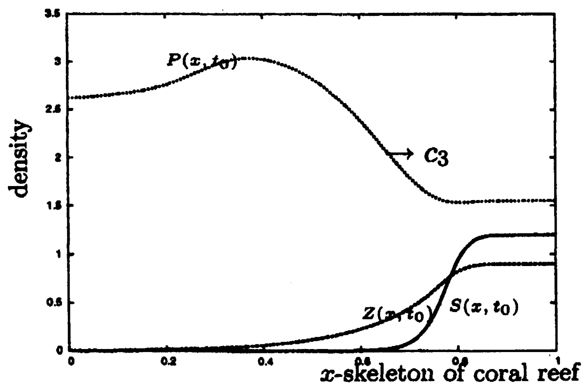

Figure 3: White band syndrome. Healthy coral tissues decrease and the white band syndrome progress with a constant speed c_3.

From the simulation results of the three models, we know that a microorganism $B(x, t)$ is a very important element to maintain a normal coral ecosystem even if the microorganism gives a negative effect to healthy coral and zooxanthellae. In the white band syndrome, we can see that healthy coral tissues decrease on ahead, then zooxanthellae decrease. This implies that the loss of zooxanthellae in the white band syndrome is caused by the loss of healthy coral tissue.

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