

# Design and Fabrication of Permanent Magnet Quadrupole Lens 

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#### Abstract

Three kinds of permanent magnet quadrupole lens were fabricated and investigated．Two of which are the trapezoidally segmented geometries and another is the circuler pole geometry． Aperture fields were measured with rotating coils for these models．Multipole field harmonics were obtained from the measurment and compared with theoretical calculations．Calculations by com－ puter code＂PANDIRA＂developed at LANL are going on．


## KEY WORDS：Permanent magnet／Quadrupole magnetic lens／Proton linear accelerator／

## 1．INTRODUCTION

Recently permanent magnet quadrupole（PMQ）lens had been proposed for proton linear accelerator at several laboratories．，${ }^{1,2)} \mathrm{K}$ ．Halbach showed the optimum condition for high coercive material ${ }^{3}$ ，such as REC（Rare Earth Cobalt）or Strontium ferrite． The condition can be realized with trapezoidal segmentation．

Figure 1 shows the geometrical cross sections of the three designs investigated in the present work．The first one has eight trapezoidal segments of permanent magnet material，which were magnetized along the arrows．The second has sixteen trapezoidal segments．The third has eight circular poles of permanent magnet material，magnetized along a diameter of the pole．Aperture field harmonics were measured with rotating

（a）

（b）

（c）

Fig．1．Geometrical cross sections of three designs．（a）eight trapezoidal segments，（b）sixteen segments，（c）eight circular pole tips．Arrows represent directions of magnetization．

[^0]coils for these models. Multipole field harmonics were obtained from the measurement and compared with theoretical calculations.

The calculations of magnetic fields are going on by the computer code PANDIRA ${ }^{5}$ which was developed at LANL. One analysis by PANDIRA will be stated in this paper.

## 2. FABRICATION OF PMQ

Permanent magnet material of segments of the trapezoidally segmented geometry was anisotropic strontium ferrite and that of the circular pole tips was isotropic barium ferrite. These materials were made by sintering at Sumitomo Special Metals Co., Ltd.. Shaping and magnetzation of the materials were also done at the company.
A) Because of the magnetic force, it is difficult to locate segments directly. The assembling process of trapezoidally segmented geometries was as follows: The dummy segments made from stainless steel were positioned in the segment holder and they were replaced with real segments one by one. A brass rod, which has the same diameter as the lense bore, was inserted into the aperture. Position of each segment was adjusted with screws so that all segments could just touch the rod.
B) Assembling of the circular pole geometry was done as follows: A pole tip which was supported freely for the rotation around the main axis was put into uniform magnetic field. So, the pole tip turned parallel to the field. Like fig. 2(a) a long acrilic adjusting bar was fixed to a side of the pole tip and a line was marked on the bar. Each pole tip was positioned in a holder. To adjust the tip angle, the line on the bar was made parallel to the alignment line drawn on the holder as fig. 2(b). And each tip was fixed with a screw.

(a)

(b)

Fig. 2. Assembling of the circular pole PMQ. (a) determination of the direction of magnetization. (b) assembled eight pole tips.

## 3. MULTIPOLE COMPONENT OF PMQ FIELD

For the theoretical convenience, we assumed that the permeability of the material
equals that of vacuum and that the direction of magnetization is restricted in a plane. The multipole expansion of the field in the aperture can be written by ${ }^{3 /}$

$$
\begin{equation*}
B^{*}\left(Z_{0}\right)=B_{r} \sum_{\nu=0}^{\infty}\left(\frac{Z_{0}}{r_{1}}\right)^{n-1} F_{n} \quad(n=N+\nu M, \nu=0,1,2, \cdots \cdots) \tag{I}
\end{equation*}
$$

where $B$ is complex magnetic field and superscript asterisk means complex conjugate, $Z_{0}$ is a point in the complex plane, $B_{r}$ is the magnitude of the remanent field of the material, $r_{1}$ is the bore radius, $F_{n}$ is the coefficient of the multipole expansion, $M$ is the number of the segments or poletips and $N$ is the order of the desired multipole such as $N=1$ for dipole, $N=2$ for quadrupole and so on.

(a)

(b)

Fig. 3. Cross section of pieces. (a) trapezoidal segment, (b) circular pole tip. Arrows represent direction of line integration.

For trapezoidally segmented PMQ, the multiple component $F_{n}$ in eq. (1) is written by ${ }^{3}$

$$
\begin{align*}
& F_{1}=\log \left(\frac{r_{2}}{r_{1}}\right) \cos \left(\frac{\pi}{M}\right) \sin \left(\frac{\pi}{M}\right) / \pi, \\
& F_{n}=\frac{n}{n-1}\left[1-\left(\frac{r_{1}}{r_{2}}\right)^{n-1}\right] \cos ^{n}\left(\frac{\pi}{M}\right) \sin \left(\frac{n \pi}{M}\right) /\left(\frac{n \pi}{M}\right), \\
& \tag{2}
\end{align*}
$$

where $r_{2}$ is outer radius of each segment as shown in fig. 3(a). In the case of the circuler pole geometry. $F_{n}$ is written by

$$
\begin{equation*}
F_{n}=M \frac{1}{4 \pi_{i}} r_{1}^{n-1} \oint \frac{d z^{*}}{z^{n}}, \tag{3}
\end{equation*}
$$

where $i$ is $\sqrt{-1}, z$ is a complex variable and the path of the integration is shown in fig. 3(b). The integral was obtained numerically.

## 4. MEASUREMENT OF MULTIPOLE COMPONENT

The multipole components of the magnetic field in the aperture of PMQ were measured by the rotating coil method.

Using eq. (1), the output voltage from the rotating coil as fig. 4 is written by

$$
\begin{equation*}
V=T L r_{1} \omega B_{r} \sum_{n=1}^{\infty}\left(\frac{r_{c}}{r_{1}}\right)^{n} F_{n} \cos (n \theta) \tag{4}
\end{equation*}
$$

where $T$ is the number of turns of the coil, $L$ is the length of the coil, $\omega$ is the angular velocity of the coil, $r_{c}$ is the radius of the coil. From eq(4), the multipole component of the aperture field is written by

$$
\begin{equation*}
F_{n}=\frac{1}{T L r_{1} \omega B_{r}}\left(\frac{r_{1}}{r_{c}}\right)^{n} V_{n} \tag{5}
\end{equation*}
$$

where $V_{n}$ is the n-th harmonic component of output voltage.


Fig. 4. Scheme of sence coil.

## 5. MEASUREMENT SYSTEM

Schematic diagram of measurment system is shown in fig. 5. The $P M Q$ was fixed on the table that can be moved in the plane parpendicular to $z$-axis by five micrometers. The sense coil in the aperture of the PMQ were consisted of two rectangular loops as fig. 6. The returen paths of coils are on the axis of rotation. The coil system was rotated constantly at 180 rpm by a hysteresis synchronous motor. The coil signal was amplified by a preamplifier on the coil assembly. The amplified signal went through a mercury contact slip ring and a low pass filter to a 12 -bit analog-digital converter.


Fig. 5. Schematic diagram of measurement system.


Fig. 6. Structure of sense coil. Two loops are wound in one plane.

The digitized data were recorded on a floppy disk via the microcomputer ( $\mathrm{PC}-8800$ ). In this measurement, the angular position of the coil was read from a photo rotaryencoder connected to the coil system. The number of data was 300 per turn.

Measured data were transferred to the FACOM M-382 and processed by Fourier analysis.

## 6. MEASUREMENTS

The sequence of a measurement was as follows:

1) The PMQ was fixed on the measuring table. Offset and gain of main amplifier were adjusted.
2) The sense coil was rotated. The output voltage signal was measured at 300 points per turn. The data were recorded on floppy disk and plotted by PC-8800. A typical plotted data of the voltage is shown in fig. 7.
3) These data were transferred to the host computer (FACOM M-382) and analyzed by FFT to obtain the multipole components.




Fig. 7. Output voltages from coil. (a) 8 segments, (b) 16 segments, (c) circular pole tip. The upper show signal from a coil and the lower from two coils.

## Circular pole PMQ

(c)

Fig. 8. Normalized dipole components $F_{1} / F_{2}$ for various position.


The steps (2) and (3) were repeated to minimize the dipole component by adjusting the position of the PMQ. The multipole components of the field were measured at the minimized position. Dipole components $F_{1} / F_{2}$ for various position are plotted in fig. 8. The results of the measurements are shown in fig. 9.

## 7. DISCUSSION

Fig. 9 shows the measured multipole components for three types of PMO . Figures (a-1), (b-l), (c-l) are the results measured by one coil. Figures (a-2), (b-2), (c-2) are the results measured by two coils which were connected to cancell the quadrupole component.

In fig. 9, two obvious hyperharmonics $\mathrm{n}=10$ and 14 (a-1), and many harmonics for $\mathrm{n}=6,10,14,22$ and 30 , (a-2), can be seen. The series of components for $\mathrm{n}=10$, 18 and 26 are caused by the fact that the PMQ consists of finite number of segments. The $n=6$ component is produced by errors of fabrication and/or magnetization. The components for $\mathrm{n}=14,22$ and 30 are higher order harmonics of the $\mathrm{n}=6$ component.

Two harmonics $\mathrm{n}=6$ and 10 , (b-1), and many harmonics for $\mathrm{n}=6,10,14,18,22$, 26 and 30 , (b-2), are seen. The $\mathrm{n}=6$ component of 16 -segmented geometry bigger than that of the 8 -segmented geometry. The reason is that the errors of fabrication and magnetization for 16 -segmented geometry were much bigger than that for 8 -segmented geometry.

In fig. 9 ( $c-1$ ), the components for $n=8,1018,2226$ and 30 can be seen. The $\mathrm{n}=8$ component is caused by fabrication errors. The $\mathrm{n}=10$ component is caused by segmentation and is bigger than that of the trapezoidal segmented geometries. But

(a-1)

(a-2)


Fig. 9. Multipole components (a) 8 segments (b) 16 segments (c) circular pole tips. The upper figuves show harmonics from one coil and the lower from two coils.
other hyperharmonics are almost same.
In these figures, the hyperharmonics are smaller than theoretical calculation for all types. The reason may be that the actual size and position of the coils are different from the disigned values.

For the 8 -and 16 -segmented geometries, the field gradients were 3.8 and $5.5 \mathrm{kG} /$ cm at $\mathrm{r}_{1}=10 \mathrm{~mm}$ respectively. For the 8 circular pole geometry, it was $1.5 \mathrm{kG} / \mathrm{cm}$.

The manufacturing cost of PMQ is decided by the material and the easiness of shaping. For trapezoidally segmented geometries, the tolerance of easy axis is $\pm 0.5^{\circ}$. For the circular pole geometry, the easy axis of pole tips can be determined after shaping. From these, the circular pole geometry is the cheapest and the 8 -segmented geometry is next.

## 8. CALCULATION BY PANDIRA

The computer code PANDIRA ${ }^{5}$ ) developed at LANL can solve anisotropic magnetic circuits by a "direct method" algorithm. The magnetic field gradients and multipole field harmonics can be calculated by PANDIRA.


Fig. 10. Geometry of 8 segmented PMQ which requires only one kind of easy axis.


Fig. 11. PANDIRA plot of magnet field.

Another variation of 8 -segmented PMQ shown in fig. 10 was calculated by PANDIRA. In this geometry, only one kind of easy axis is required. The flux plot is shown in fig. 11. The analysis is going on.

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