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# RF Characteristics of Coupled Split Coaxial Lines for RFQ Structure

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The radio frequency quadrupole (RFQ) structure for injection of low velocity ions into the linear accelerator was studied. The structure was consisted from the coupled split coaxial lines.

A full scale model of the rf cavity was made and the resonant frequencies and the rf field distributions were measured. At the lowest resonance, the transverse electric quadrupole was observed along the central axis of the cavity. The resonant frequency, the $Q$ value and the shunt impedance of the cavity was investigated.

In conclusion, the coupled split coaxial RFQ structure was as usable as the four-vane RFQ structure, and the fabrication of the former is easier than that of the latter.

KEY WORDS : RFQ / Injector / Linear accelerator / Transmission line /

## INTRODUCTION

An idea of ions acceleration by the RFQ structure had been suggested by I. M. Kapchinskij and V. A. Teplyakov,[1] and the four-vane RFQ[2] for the injection of low velocity ions into the linear accelerator was developed successfuly at Los Alamos.

The RFQ structure is very powerful, because it has the abilities of focusing as well as accelerating and bunching. As for the four-vane RFQ,[6] however, it is not easy to machine vanes and to assemble them into the cavity. While the coupled split coaxial RFQ structure which was proposed by a group of Frankfurt Univ. and GSI[3,4,5] has a simple configuration. It consists of four modulated circular rods in a cylindrical cavity, and the fabrication of the cavity seems to be much easier. We studied on the rf characteristics of the coupled split coaxial cavity using a full scale model of the buncher for the proton linear accelerator. The investigated cavity is made of four non-modulated circular rods inside a cylindrical tube. Two opposite rods were terminated to one of the end plates of the tube and the other two opposite rods were terminated to the other end plate. (see fig. 1) The cavity had a resonant mode, RFQ mode in which the transverse electric field was approximately quadrupole whose strength was almost uniform along the central axis of the tube. In this mode the wave propagation on a pair of the opposite rods coupled strongly with the wave propagation on the other pair. The resonant frequency became lower when the coupling was strengthened. To account for the fact, we applied the coupled mode theory of the transmission lines to the analysis of the resonant mode of the coupled split coaxial cavity. The results almost agreeded with the experimental values. And also we obtained the Q-value and the shunt impedance

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of the cavity from the measurement of the reflection coefficients and the field distributions in the model cavity, respectively. As the case of the real buncher, four rods in the cavity will be modulated[7] to produce the electric fields which give the accelerating force on particles. Such modulation of the inner rods will not change greatly the rf characteristics of the cavity.

## DESIGN OF COUPLED SPLIT COAXIAL CAVITY MODEL

The full scale model of the coupled split coaxial cavity which is used for the low power level test is shown in fig. 1. This model cavity has a structure of two coupled $\lambda/4$ resonators, each has two split inner rods, arranged at an angle 90°. The cavity is made of copper and the length $l$ is 30 cm. The radius and the position of inner rods is varied. Dimensions of the configuration is shown in table I. The cavity which has the dimensions in table I yields a desirable RFQ mode with resonant frequency of about 100 MHz. To feed rf power into the cavity, a capacitive coupler which is able to change the coupling strength is connected to a position near the open end of one of the inner rods.

The cavity is connected with the signal generator and with the network analyzer through the directional bridge as shown in fig. 2.

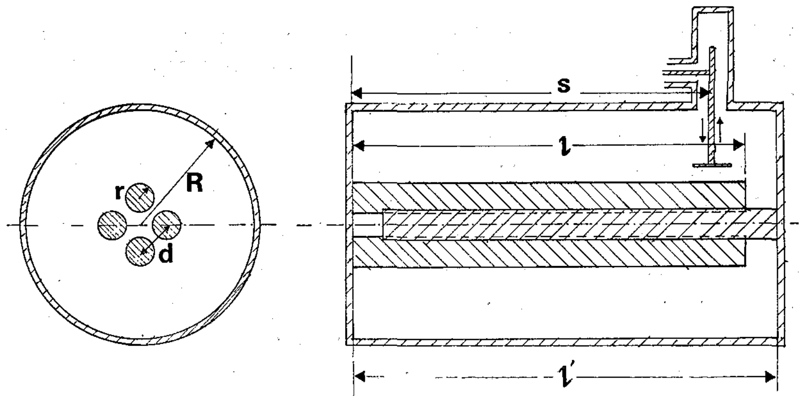


Fig. 1. Full scale model of "the coupled split coaxial cavity".

Table I. Dimensions of "the coupled split coaxial cavity" shown in fig. 1. (unit is cm.)

| | | |
|---|---|---|
| $l'$ | : cavity length | 30 |
| $R$ | : cavity radius | 7.5 |
| $l$ | : electrode length | 29.5 ± 0.05 |
| $r$ | : electrode radius | 1.0 |
| $d$ | : distance between electrodes | 2.55 ± 0.01 (for type A) |
| | | 3.39 ± 0.01 (for type B) |
| | | 3.82 ± 0.01 (for type C) |
| $s$ | : position of coupler | 25.0 ± 0.1 |

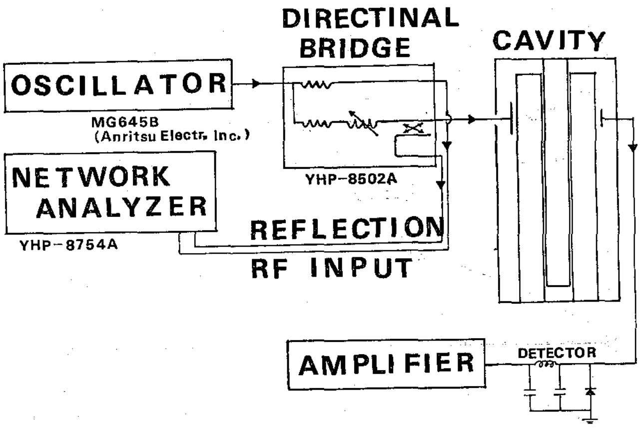


Fig. 2. Block diagram of the measurement system.

## ANALYSIS OF COUPLED SPLIT COAXIAL LINE BY COUPLED MODE THEORY OF TRANSMISSION LINE

In a well known quarter wave coaxial resonator, the wave length of the resonant wave in TEM mode is given by

$$\lambda_n = \frac{4l}{2n-1}, \quad (n=1,\ 2,\ 3,\ \ldots), \tag{1}$$

where $l$ is the length of the cavity and $n$ is the number of the nodes of the wave along the axis of the cavity. In the free space the wave length is related to the resonant frequency:

$$f_n = \frac{c}{\lambda_n}, \tag{2}$$

where $c$ is the velocity of light ($c=3.0\times10^{10}$ cm).

Now let's consider the case where two split inner electrodes are put into the cavity. Under the situation, there are two independent TEM resonant modes according to the order of $n$. At two dominant modes ($n=1$), the potential along the electrodes distribute as shown in fig. 3. (a), (b). A mode may be called even mode from the symmetric nature if the potential of electrodes is in the same phase (fig. 3. (a)), and odd mode if the potential of the electorodes is at a phase $\pi$ (fig. 3. (b)). The coupling is strong in the odd mode, but is weak in the even mode. If a coupling is inductive, the resonant mode has a higher frequency but if a coupling is capacitive, the resonant mode has a lower frequency.

The resonant frequency of the quarter wave coaxial cavity $f_1$ and that of the split coaxial cavity $f_{1a}$, $f_{1b}$ were measured using the model cavity. In the case that the cavity is the length of 30 cm, and the radius of 7.5 cm and electrodes is the length of 29.5 cm, and the radius of 1 cm, and the distance of the split electrodes is 2.6 cm, $f_1$, $f_{1a}$ and $f_{1b}$ is listed in table II. One must note that the effect that the open end of the electrodes is not completely open tends to lower the resonant frequency.

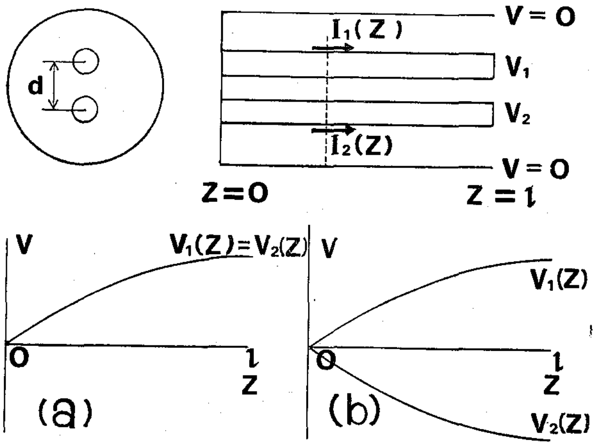


Fig. 3. Potential on the electrodes of "the split caxial cavity". (a) is even mode in which $V_1$and $V_2$ are in the same phase. (b) is odd mode in which $V_1$ and $V_2$ are at a phase $\pi$.

Table II. Resonant freqency of "the quarter wave coaxial cavity", $f_1$ and those of "the split coaxal cavity", $f_{1a}$ and $f_{1b}$.

| | |
|---|---|
| $f_1$ | 217 (MHz) |
| $f_{1a}$ | 231 |
| $f_{1b}$ | 208 |

Then we consider that two quarter wave split coaxial lines are coupled at an angle 90°, as shown in fig. 1. There are four independent modes as shown in fig. 4, and each mode is given by different coupling of the $n=1$ TEM waves.

In the case of fig. 4 (a), the transverse electric field is approximately quadrupole around the center of four electrodes, and the quadrupole strength is almost uniform along the central axis of the cavity. That is RFQ mode which we coucern in.

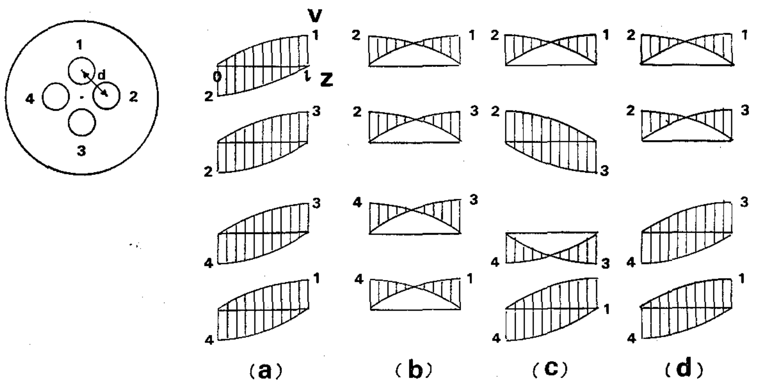


Fig. 4. Potentil on the electrodes of "the coupled split coaxial cavity". The electrodes 1 and 3 are shorted at z=0, the electrodes 2 and 4 are shorted at $z=l$. (a) is even-even $3\pi/2$ mode. (b) is even-even $\pi/2$ mode. (c) is odd-odd mode (d) is even-odd mode.

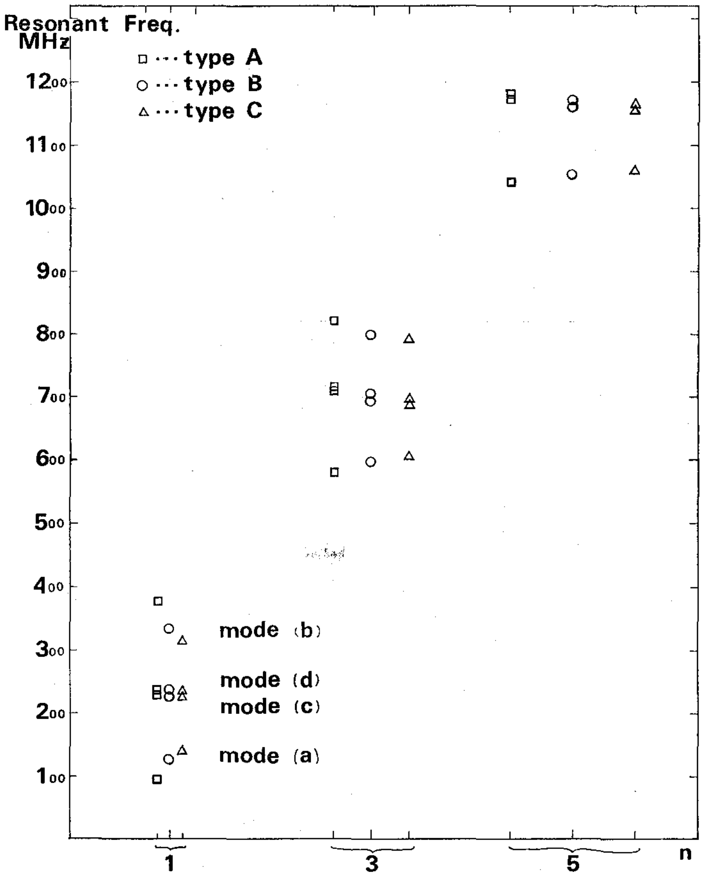


Fig. 5. Resonant frequencies of "the coupled split coaxial cavity." The cavity has the dimensions as shown in table 1. In the figure, points belong to the same type are horizontally shifted at same distance The modes which are shown for $n=1$ are also same for the higher modes. n is the number of the nodes of original TEM wave.

Figure. 5 shows the frequencies of the resonant mode as the function of the number of nodes, $n$ of the original TEM waves which can concern the coupling. The field distribution between electrodes was measured by the bead perturbation method[9]. The resonant frequencies of the modes as shown in figs. 4 (a) and (b) greatly depends on the relative position of the electrodes. The resonant freqency of the mode (a) becomes lower and that of the mode (b) becomes higher when the distance of the electlodes is extended. To analyze the mode (a), RFQ mode, we applied the coupled mode theory of the transmission lines. We assume that the coupling is mainly capacitive in RFQ mode, that the open end of the electrodes is perfectly open and that the cavity is lossless. Thus the equivalent circuit of RFQ mode is given in fig. 6. Since the potential of the opposite electrodes are in the same phase, they can be treated as a single LC line.

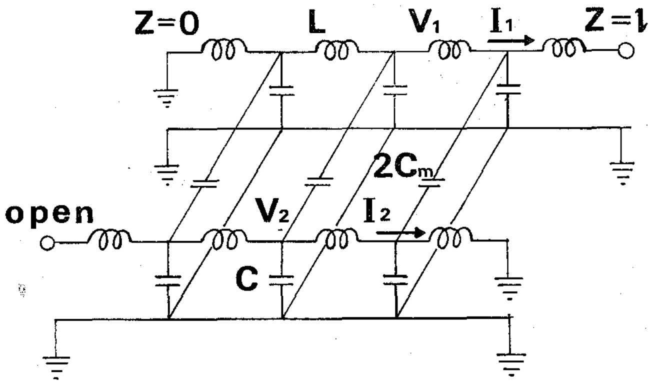


Fig. 6. Equivalent circuit of RFQ mode. $L$ is the inductance and $C$ is the capacitance of the line per unit length. $2C_m$ is the coupling capacitance between the lines. $V_1$, $I_1$ are the potential and the current on the lines 1 and 3, and $V_2$, $I_2$ are those on the lines 2 and 4.

In fig. 6, $L$ and $C$ is the inductance and the capacitance of the line per unit length, respectively. Both of the $LC$ lines are coupled by the capacitance $2C_m$, where $C_m$ is the mutual capacitance between the neighboring electrodes, $V_1$, $I_1$ and $V_2$, $I_2$ are the potential and the current on the lines 1, 3 and on the lines 2, 4, respectively. $V_1$, $I_1$, $V_2$ and $I_2$ are related to

$$\left.\begin{aligned} &\frac{\partial V}{\partial Z}=-L\frac{\partial I}{\partial t}, \\ &\frac{\partial I}{\partial Z}=-\begin{pmatrix} C+2C_m & -2C_m \\ -2C_m & C+2C_m \end{pmatrix}\frac{\partial V}{\partial t}, \\ &V=\begin{pmatrix} V_1 \\ V_2 \end{pmatrix},\ I=\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}. \end{aligned}\right\} \qquad (3)$$

It is assumed that $V_1$, $V_2$, $I_1$ and $I_2$ vary sinusoidally as $e^{\pm i\omega t}$ with time, where $\omega$ is the anguler frequency and vary as $e^{i\beta z}$ with $z$, where $\beta=\pm 2\pi/\lambda$ is the phase constant. The necessary condition that there exists a significant solution for eq. (3) is,

$$det\left\{\beta^2-\omega^2 L\begin{pmatrix} C+2C_m & -2C_m \\ -2C_m & C+2C_m \end{pmatrix}\right\}=0. \qquad (4)$$

The eigen-values are given as follows:

$$\left.\begin{aligned} v_1&=\frac{\omega_1}{\beta}=\pm\{L(C+4C_m)\}^{-1/2}, \\ v_2&=\frac{\omega_2}{\beta}=\pm(LC)^{-1/2}. \end{aligned}\right\} \qquad (5)$$

We are interested in the solution which belongs to $v_1$. Taking account of the boundary conditions, we decide the phase factor $e^{i\phi}$ between $V_1$ and $V_2$. So we get the solutions for the standing wave,

$$\left.\begin{aligned} V_1 &= V_0 \quad \sin\beta z \ \cos\omega t, \\ V_2 &= \pm V_0 \ \cos\beta z \ \cos\omega t, \\ I_1 &= -\frac{1}{Z_0} V_0 \ \cos\beta z \ \sin\omega t, \\ I_2 &= \pm\frac{1}{Z_0} V_0 \ \sin\beta z \ \sin\omega t, \end{aligned}\right\} \tag{6}$$

where $Z_0=\{L/(C+4C_m)\}^{1/2}$ is the characteristic impedance. RFQ mode is represented by the equations with under signs in eq. (6).

When the capacitive coupling and also the inductive coupling is taken into considerations and four coupled lines are dealt with, the theory mentioned above will be generalized to other modes including RFQ mode or the higher modes.

In eq. (5), $v_1$ is rewritten to $v_1=f_{1a}\lambda_1$, where $\lambda_1$ is the wave length of the quarter wave which is given by eq. (1). To consider the dependency of $C_m$ on $f_{1a}$, we express the relation of $f_{1a}$ to the distance between the neighboring electrodes in table III.

Table III. Resonant freqency, $f_{1a}$ in RFQ mode and distance of neighboring electrodes, $d$.

| | $d$ (cm) | $f_{1a}$ (MHz) |
|---|---|---|
| type A | 2.55±0.01 | 95.54 |
| type B | 3.39 | 125.8 |
| type C | 3.82 | 139.4 |

For the first equation in eq. (3), if we put $1/\sqrt{LC}\sim c$ (the velocity of light), the equation is reduced to

$$f_{1a}=(1+4C_m/C)^{-1/2}f_1, \tag{7}$$

where $f_1$ is given by eq. (2). In eq. (7), $C$ is the electrostatic capacitance between two parallel cylinders, one of those is put inside the other, and $C_m$ is the capacitance as that between two same parallel lines at the distance $d$. Therefore we may write them as follows:

$$\left.\begin{aligned} & C=2\pi\varepsilon_0/\{\ln(r^2+R^2-x^2+\sqrt{(r^2+R^2-x^2)^2-4r^2R^2}\}/2rR, \\ & \text{and} \\ & C_m=\pi\varepsilon_0/\ln(d/r), \\ & \text{where } x=d/\sqrt{2}. \end{aligned}\right\} \tag{8}$$

In the case of the cavity in fig. 1, $C_m$ is equal to $C$ in the order. Making a rough estimate, we get that $f_{1a}\sim\frac{1}{\sqrt{5}}f_1$. If $R$ is large enough, $C$ is not depend on $d$ and we find the relation of $(f_{1a}^{-2})\propto(\ln d)^{-1}$. In fig. 7, the square inverse of the resonant frequency: $(f_{1a}^{-2})$ is plotted against the inverse of $\ln d$ for the model cavity (type A, B and C).

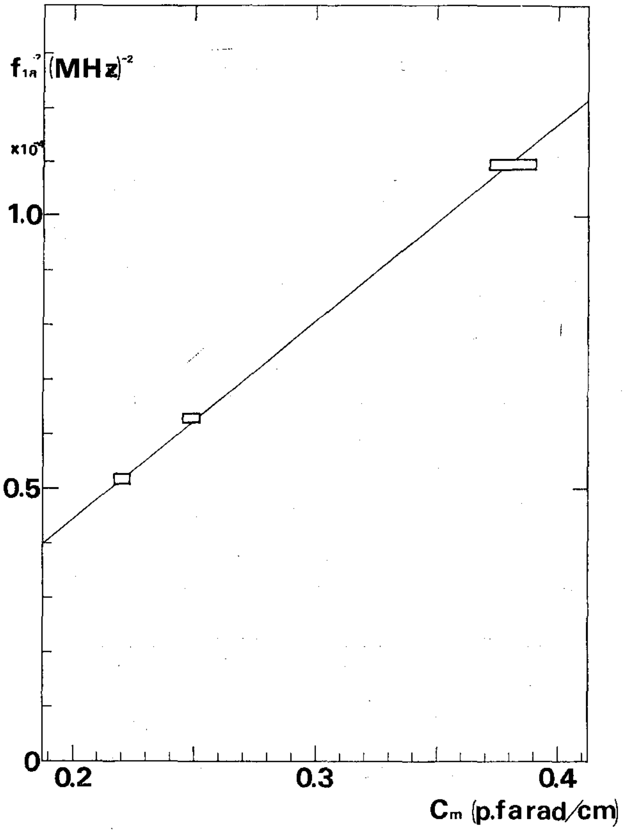


Fig. 7. Dependency of the resonant freqency of RFQ mode to the distance of the electrodes

## ESTIMATION OF Q VALUE AND SHUNT IMPEDANCE

1. $Q$-Value of Model Cavity

The measurement system for the rf characteristics of the coupled split coaxial model cavity is given in fig. 2. The $Q$ value of the model cavities, $Q_0$ for the type A, B and C (used in the previous section) is measured as following.

Near by the resonance the input impedance $Z_{in}$ of the cavity is related to the characteristic impedance $Z$ of the external system by[8]

$$\frac{Z_{in}}{Z}=\frac{1/Q_{ex}}{i(\omega/\omega_0-\omega_0/\omega)+1/Q_0}, \tag{9}$$

where $Q_{ex}$ is based on the loss of the external system, and $\omega_0$ is the resonant anguler frequency. And the quantity $\delta$ is definded as follows,

$$\delta=Q_{ex}\left(\frac{\omega}{\omega_0}-\frac{\omega}{\omega_0}\right)\sim 2Q_{ex}\left(\frac{\omega-\omega_0}{\omega}\right). \tag{10}$$

We put the anguler frequencies to $\omega_1$ and $\omega_2$ for $\delta=\pm 1$. Therefore we get

$$Q_{ex}=\frac{\omega_0}{\Delta\omega}, \quad \Delta\omega=|\omega_1-\omega_2|. \tag{11}$$

If $\omega_0$ and $[Z_{in}/Z]_{\delta=0}$ are measured and $[Z_{in}/Z]_{\delta=\pm1}$ is calculated from eq. (11), we can obtain $\omega_1$ and $\omega_2$ from the measurments. Using eqs. (11) and (10), we can get $Q_{ex}$ and $Q_0$.

We measured the reflection coefficient instead of $Z_{in}/Z$. For the model cavities of type $A$, $B$ and $C$, we obtained $Q_0$ in RFQ mode, and the dependency of $Q_0$ to the configuration of the electrodes. The results is shown in table IV.

2. Shunt Impedunce of Model Cavity

The shunt impedance of the accelerator is defined as follows:

$$R_s=\frac{E_0^2}{W/l}, \tag{12}$$

where $E_0$ is the maximum accelerating field and $W$ is the input power and $l$ is the length of the accelerating path. If the split coaxial electrodes are modulated by the way that is proposed by $K$-$T$,[1] the obtained accelerating field will be

$$E_z=\frac{kAV_0}{2}\sin(kz)\ I_0(kr), \quad k=\omega/\beta_s c, \tag{13}$$

$$A=\frac{m^2-1}{m^2 I_0(ka)+I_0(kma)}, \tag{14}$$

where $\beta_s$ is the velocity of the synchronous particles, $V_0$ is the potential of the electrodes, $a$ is the distance from the central axis to the hill of the modulation, and $ma$ is the distance to the valley of the modulation. From these equations, the shunt impedance:

$$R_s=\frac{kA^2}{\pi}\frac{V_0^2}{W} \tag{15}$$

is obtained.

In the experiment, $V_0^2/W$ was measured instead of $R_s$. At the case of the coupled split coaxial cavity,

Table IV. $Q$ value and estimated shunt impedance for "the coupled split coaxial cavity". Resonant freqency: $f_{1a}$, unloaded $Q$ value of model cavity: $Q_0$, voltage; V (absolute value) of electrode at z=23 cm. These values are evaluated by the experiment, in which the input rf power (W=$5.0\times10^{-5}$ watt) is fed into the cavity. Shunt impedance: $R_s/A^2$, which is estimated roughly by Eqs. (15) and (16) for $\beta_s=6.7\times10^{-3}$.

| | $f_{1a}$ (MHz) | $Q_0$ | $V$ (volt) | $R_s/A^2$ (ohm/cm) (estimated) |
|---|---|---|---|---|
| type A | 95.5 | 3,400 | $1.6\pm0.2$ | $4.8\times10^4$ |
| type B | 126. | 4,100 | $1.4\pm0.2$ | $5.1\times10^4$ |
| type C | 139. | 4,000 | $1.5\pm0.2$ | $6.2\times10^4$ |

$$V_0=\frac{V_1-V_2}{2}, \tag{16}$$

where $V_1-V_2$ is the voltage between the neighboring electrodes measured along the path in fig. 8 by the bead perturbation method.[9] The results are shown in table IV.

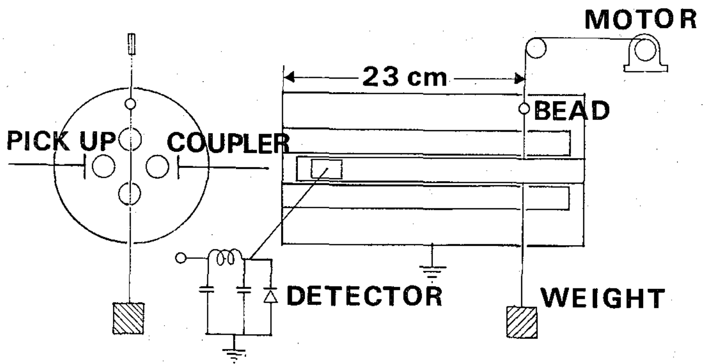


Fig. 8. The field is measured by the bead perturbation method. The electrode voltage at $z=23$ cm is obtained ly the measurmert

## DISCUSSION

The structure of the coupled split coaxial RFQ is simple compared with that of the four-vane RFQ, and assembling the electrodes into the cavity is rather easy.

Since the RFQ mode of the coupled split coaxial cavity is given by the coupling of TEM waves, it could be analyzed by the simple coupled mode theory which had been described in the previous section. The dependency of the resonant frequency of RFQ mode to the dimension of the cavity was estimated. The resonant frequency was expressed roughly as$\sim\frac{c}{4\sqrt{5}}\,\frac{1}{l}$ for the length of cavity $l$, and also it depended on the configuration of the electrodes inside the cavity. As for the radius of the cavity, it was not necessary to use the large one because the field mainly, distributes around the central part of the cavity.

One of the probrems of the coupled split coaxial RFQ is that the distortion of the quadrupole field near the open end of the electrodes occurs. However it may be avoided by proper reshaping the open end of electrodes.

$Q$ value of the cavity obtained was about 3400$\sim$4000. And also the shunt impedance was estimated. The voltage between the neighboring electrodes is about 10 $kV$ when the rf power of 5 $kW$ is fed into the cavity.

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