

Title	Computer Program for Three Body Break Up Reaction (Commemoration Issue Dedicated to Professor Sakae Shimizu on the Occasion of his Retirement)
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Citation	Bulletin of the Institute for Chemical Research, Kyoto University (1979), 57(1): 92-101
Issue Date	1979-03-31
URL	<a href="http://hdl.handle.net/2433/76814">http://hdl.handle.net/2433/76814</a>
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

## Computer Program for Three Body Break Up Reaction

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Received October 11, 1978

A FORTRAN program is developed to calculate the energy spectrum for three body break up reaction at intermediate energy. Energy spectra are calculated on the basis of AGS (Alt, Grassberger and Sandhas) formalism assuming quasi free scattering with final state interaction. The calculated curves reproduced well energy spectra of  ${}^3\text{He}$  in coincidence with protons for  ${}^3\text{He}(\tau, \tau p)$  reaction and deuterons for  ${}^3\text{He}(\tau, \tau d)$  reaction at 120 MeV.

KEY WORDS Nuclear Reaction  ${}^3\text{He}(\tau, \tau p)$  at 120 MeV /  
Program for QF Scattering with FSI/

### I INTRODUCTION

Three body problems could be treated exactly on the basis of Faddeev's equations with given two body interaction potentials and the energy spectrum of nucleons from break up reaction of deuterons by nucleons could be successfully explained using a separable potential for nucleon-nucleon interaction<sup>1)</sup>. For the case where components of high angular momentum contribute to the two body interaction in the break up reaction, the problem is not so easy because that realistic separable potential has not yet been obtained and a long computer time needs to solve the integral equations. Furthermore if composite particles are concerned in the final state complicated reaction mechanism is presumed. At intermediate energies, however, continuous energy spectra of emitted particles show rather simple shape with large peaks due to quasi free scattering and some times with peaks for final state interaction appearing at low relative energies of the interacting two particles<sup>2)</sup>. Then it is reasonable to assume some simplifications in calculation for the break up reaction. Alt *et al.*<sup>3)</sup> have derived one dimensional integral equations for the three body scattering problem and derived a scattering amplitude at the special case where, in the first place, a quasi free process produces three particles and in the final state any two particles interact. We have obtained a computer program to calculate this amplitude and using it evaluated energy spectra for the break up reaction of  ${}^3\text{He}$  nucleus by  ${}^3\text{He}$  beam at 120 MeV.

### II EQUATIONS

Break up amplitude for three body break up reaction can be calculated using Eq. (5. 2) in Ref. 3 as follows,

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$$X_{p_1', 1m}(z) = - \sum_{r=1} \langle \mathbf{p}_1' | T'_r(z) G_0(z) | 1, m; z \rangle + \sum_{r=1} \langle \mathbf{p}_1' | T_1(z) G_0(z) T'_r(z) G_0(z) | 1, m; z \rangle, \quad (1)$$

where  $T'_r$  are non separable "amplitude" and small compared with the two body amplitude  $T_1$ . In the calculation the following conditions are assumed. 1) two body amplitude  $T_1$  and  $T'_r$  have not any singularity on the energy shell, 2) the Green's function  $G_0$  at the last position in the second term does not give any singularity because the cancellation of amplitudes at two integration angles  $\mathbf{p}_1''$  and  $-\mathbf{p}_1''$ , and 3) a pole approximation is applied to integrate the other Green's function in the second term. Then the cross section is given as follows,

$$\frac{d^3\sigma}{dQ_1 dQ_2 dE_1} = \frac{2\pi}{\hbar^2} \cdot \frac{\mu_{in}}{k_{in}} \rho |M_+ + M_-|^2, \quad (2)$$

$$\begin{aligned} M_\pm &= \langle q_1 | X_\pm | q_0 \rangle \\ &= - \sum_{r=1} \left( \frac{1}{S_{r1}} \right)^3 \langle \mathbf{p}_1' | T'_r(z - q_1^2) | \mathbf{p}_r \rangle \frac{1}{\{ \mathbf{p}_1' - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 \mp \mathbf{q}_0) \}^2 - Q} \\ &\quad \times \langle \mathbf{p}_1' - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 - \mathbf{q}_0) | 1, m; z \rangle \\ &+ \sum_{r=1} \left( \frac{1}{S_{r1}} \right)^3 \int dQ'' \langle \mathbf{p}_1' | T_1(z - q_1^2) | \mathbf{p}_1'' \rangle \frac{\pi i p_1''}{2} \langle \mathbf{p}_1'' | T'_r(z - q_1^2) | \mathbf{p}_r \rangle \\ &\quad \times \frac{1}{\{ \mathbf{p}_1' - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 - \mathbf{q}_0) \}^2 - Q} \langle \mathbf{p}_1' - \frac{C_{r1}}{S_{r1}} (\mathbf{q}_1 \mp \mathbf{q}_0) | 1, m; z \rangle, \end{aligned} \quad (3)$$

$$p_1''^2 = z - q_1^2,$$

where  $\rho$  is the phase space for factor three body system in the final state.  $M_+$  and  $M_-$  are matrix elements for the target break up process and projectile break up process, respectively. The variable  $p_1^2$  and  $q_1^2$  have a dimension of energy and relate to the usual wave vector  $\mathbf{k}$  and mass  $m$ ,

$$\begin{aligned} p_1 &= [2m_2 m_3 (m_2 + m_3)]^{-1/2} (m_3 \mathbf{k}_2 - m_2 \mathbf{k}_3), \\ q_1 &= [2m_1 (m_2 + m_3) (m_1 + m_2 + m_3)]^{-1/2} [(m_1 (\mathbf{k}_2 + \mathbf{k}_3) - (m_2 + m_3) \mathbf{k}_1)], \end{aligned} \quad (4)$$

and the variables in the  $i-jk$  system are related to the variables in the  $j-ki$  system through the following relations,

$$\begin{aligned} \mathbf{p}_2 &= C_{21} \mathbf{p}_1 + S_{21} \mathbf{q}_1, \\ \mathbf{q}_2 &= -S_{21} \mathbf{p}_1 + C_{21} \mathbf{q}_1. \end{aligned} \quad (5)$$

where

$$\begin{aligned} C_{21} &= - \left[ \frac{m_1 m_2}{(m_1 + m_3) (m_2 + m_3)} \right]^{1/2} = C_{12}, \\ S_{21} &= \sqrt{1 - C_{21}^2}, \end{aligned} \quad (6)$$

Four types of potential are assumed in the calculation of the initial interaction, that is Gaussian type, Yukawa type, Woods-Saxon type and Woods-Saxon for real potential with the first derivative of Woods-Saxon for imaginary potential,

$$\langle \mathbf{p}_r | T_r(z - q_r^2) | \mathbf{p}'_r \rangle = T_r(\mathbf{p}'_r - \mathbf{p}_r), \quad (7)$$

$$= -\pi^{3/2} \frac{V}{\alpha^3} \exp \left[ -\frac{(\mathbf{p}'_r - \mathbf{p}_r)^2}{4\alpha^2} \right], \quad \text{for Gaussian} \quad (8)$$

$$= -4\pi V \left[ \frac{1}{\alpha_1^2 + (\mathbf{p}'_r - \mathbf{p}_r)^2} + \frac{\eta}{\alpha_2^2 + (\mathbf{p}'_r - \mathbf{p}_r)^2} \right], \quad \text{for Yukawa} \quad (9)$$

$$= \int d\mathbf{r}^3 V(r) e^{i(\mathbf{p}_r - \mathbf{p}'_r)r}, \quad \text{for Woods Saxon } V(r). \quad (10)$$

The form factor of the target nucleus is assumed as,

$$\begin{aligned} \langle \mathbf{p}_1 | l, m; z \rangle &= (z - q_1^2) \phi_{lm}(\mathbf{p}_1), \\ \phi_{lm}(\mathbf{p}_1) &= \frac{\sqrt{\alpha}}{\pi} \frac{1}{\alpha^2 + \mathbf{p}_1^2}. \end{aligned} \quad (11)$$

S wave final state interaction between particle 2 and particle 3 relates to the effective range parameters at zero energy limit in the two body scattering,

$$|\langle \mathbf{p}_1 | T_1(z - q_1^2) | \mathbf{p}'_1 \rangle|^2 = \frac{C^2(\eta)}{4\pi^2} \cdot \frac{1}{C^2(\eta) q^2 + \left( -\frac{1}{a} + \frac{1}{2} r q^2 \right)}, \quad (12)$$

$$q^2 = (\mathbf{p}'_1 - \mathbf{p}_1)^2,$$

$$C^2(\eta) = 2\pi\eta / [\exp(2\pi\eta) - 1],$$

$$\eta = z_2 z_3 e^2 / \hbar v,$$

where  $C(\eta)$  is the Coulomb penetration factor. For the  $l$ -th partial wave interaction in the final state Brait-Wigner type resonance form is assumed as follows,

$$|\langle \mathbf{p}_1 | T_1(z - q_1^2) | \mathbf{p}'_1 \rangle|_l^2 = A \frac{\Gamma_l^2}{(z - q_1^2 - E_l)^2 + \frac{1}{4} \Gamma_l^2} P_l(\cos\omega). \quad (13)$$

The angular integral of Eq. (3) is done analytically where the angular momentum of 1 is less than or equal to 2. The values of parameters in Eq. (8), (9), and (11) are transformed from the radial parameters in fm using following relation,

$$\alpha^2 \text{ MeV} = \frac{r}{\mu a_{fm}}, \quad r = \frac{41.6}{2},$$

$\mu$ : reduced mass

### III COMPUTER PROGRAM

The computer program "AGSIA" calculates the three body break up cross section on the basis of AGS formalism. Seven types of card have to read in the format of free field as the input data. Three body kinematics is calculated using a subprogram "KINEMA" which is the same as the program developed by Ohlsen<sup>4</sup>, and the numerical integration of Eq. (10) is made using a sub-program "SMPSON" which is a usual program of Simpson Integral. The matrix element in Eq. (3) is calculated using a sub-program "XMAT". Four types of initial interaction can be selected by **KCODE4** and five print out format can be selected by **KCODE3**. All calculated energy spectra are print out in units of  $10^{-26} \text{ cm}^2/\text{sr}^2 \cdot \text{MeV}$  as a function of the laboratory energy of particle 1. The FORTRAN programs are given in an Appendix.

The energy spectrum of  ${}^3\text{He}$  in coincidence with protons for  ${}^3\text{He}(\tau, \tau p)d$  reaction

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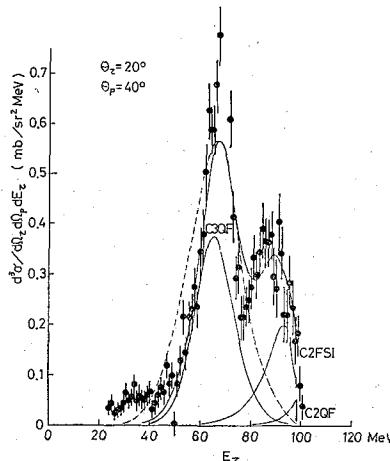


Fig. 1. Energy spectrum projected for the  ${}^3\text{He}(\tau, \tau p)d$  reactions at  $\theta_\tau = 20^\circ$  and  $\theta_p = 40^\circ$ . All solid curves are calculated on the basis of the AGS formalism. The dashed curve shows the calculated one of the simple spectator model.

at 120 MeV was calculated using the program of "AGSIA". Figure 1 shows the calculated curves at the angular set  $\theta_\tau = 20^\circ$ ,  $\theta_p = 40^\circ$  and  $\varphi_p = 0^\circ$  (opposite side of, and in coplanar with, the beam) and shows the experimental energy spectrum which was obtained from the data in Ref. 2. The curves denoted as C2QF and C2FSI are calculated using only the first term at  $\gamma = 2$  and the second term at  $\gamma = 2$  in Eq. (1). The former corresponds to the cross section assuming the quasi free scattering of particle 1 ( ${}^3\text{He}$ ) and particle 3(unobserved particle) and the latter corresponds to that for quasi scattering with the final state interaction of particle 2 and particle 3. The dashed curve is calculated using a simple impulsive spectator model. Nevertheless a normalization constant was needed to reproduce the experimental energy spectrum, the calculated energy spectrum is reproduced well using a reasonable parameter set.

### ACKNOWLEDGMENT

Authors would like to thanks to Mr. T. Ohsawa, Dr. S. Tanaka, and Mr. A. Okihana for their useful discussions and cooperations to obtain the data of  ${}^3\text{He}(\tau, \tau p)d$  reaction. Numerical calculation was made by FACOM 230 computer at the Institute for Chemical Research of Kyoto University.

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### Appendix

#### Instructions for use of "AGSIA"

Card 1 (general conditions in the calculation)

:KCODE1, KCODE2, KCODE3, KCODE4, IK, NO, FACT

- KCODE1 = 1 print subtitle "XM3 IS NOT A COMPOSITE PARTICLE"
- 2 print subtitle "XM3=XM4+XM5 AND BREAKS TO SINGLET DEUTERON"
- 3 print subtitle "XM3=XM4+XM5 AND BREAKS TO TRIPLET p+n SYSTEM"

if any number is chosen, the program does not change to calculate the reaction

KCODE2 = 0 calculate the target breakup reaction ; M=M<sub>+</sub>

1 calculate the projectile breakup reaction ; M=M<sub>-</sub>

2 calculate the target breakup plus the projectile breakup reaction ; M=M<sub>+</sub>+M<sub>-</sub>

KCODE3 = 0 print out the kinematics of three body breakup reaction

1 print out all reaction amplitudes and phase space factor

2 print out the final cross sections

3 print out the cross sections for (1-2) quasi free, the cross sections for (1-3) quasi free and the final cross sections

4 print out all cross sections of components in Eq. (3) and the final cross sections

KCODE4 = 0 initial interaction of Gaussian type

1 initial interaction of double Yukawa type

2 initial interaction of Woods-Saxon type

3 initial interaction of Woods-Saxon (real) with the first derivative of the Woods-Saxon (imaginary)

LK number of resonance in the final state interaction

NO number of angular set

FACT factor of the second term by the first term in Eq. (3), and set 1 in the usual calculation.

Card 2 (kinematical parameters)

:XMP, XMT, XM1, XM2, XM3, Q, EPL, DE1L, ZZ

XMP mass of the projectile in atomic mass unit

XMT Mass of the target in atomic mass unit

XM1 Mass of the particle 1 in atomic mass unit

XM2 mass of the particle 2 in atomic mass unit

XM3 mass of the particle 3 in atomic mass unit

Q reaction Q value in MeV

EPL incident energy in MeV

DE1L energy increment in MeV

ZZ product of electric charges Z<sub>2</sub> and Z<sub>3</sub>

Card 3 (form factor and effective range parameters)

:ALPHTO, ALPHAO, ERANGT

ALPHTO width of the form factor in fm

ALPHAO scattering length in fm

ERAMGT effective range in fm

Card 4 (potential parameters for the first step interaction between particle 1 and particle 2.)

:VTPO1, RHTPO1, DHTPO1, YETAP1, VTPO2, RHTPO2, DHTPO2, YETAP2

VTPO1 real well depth in MeV

RHTPO1 real well width in fm

DHTPO1 real well diffuseness (for Gaussian or Woods-Saxon) or second well width in fm  
(for double Yukawa)

YETAP1 ratio of second Yukawa potential to the first Yukawa potential

VTPO2 imaginary well depth in MeV

RHTPO2 imaginary well width in fm

DHTPO2 imaginary well diffuseness (for Gaussian or Woods-Saxon) or the second well

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width in fm (for double Yukawa)

YETAP2 ratio of the imaginary depth of second Yukawa to that of first Yukawa  
Card 5 (potential parameters for the first step interaction between particle 1 and particle 3)  
:VTDO1, RHTDO1, DHTDO1, YETAD1, VTDO2, RHXDO2, DHTDO2, YETAD2  
similar to the Card 4

Card 6 (resonance parameters for the S wave final state interaction)

:COEF(1), ER(1) WIDTH(1)

COEF(1) coefficient of the Brait-Wigner resonance

ER(1) resonance energy in MeV

WIDTH(1) resonance width in MeV

If the resonances for the P wave and the D wave final state interactions are calculated, the Card 7 and Card 8 are used for the resonance parameters. If any resonance is not calculate, the COEF(1) in the Card 6 must be equal to zero. And the Card of angular set follows to the resonance Card.

Card 7 (angular set)

:TH1R, TH2R, PHI2

TH1R polar angle of particle 1 in degrees in the laboratory

TH2R polar angle of particle 2 in degrees

PHI2 azimuthal angle of particle 2 in degrees. Set zero when the detectors for particle 1 and particle 2 are set opposite side of, in coplaner, with, the incident beam.



Program for Three Body Break Up Reaction

( 99 )

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0051 ,GL2=SU+(L1*(L1-1)*T1)+(GL1*(L2+SU*GL1*(A65(G)),L1+T1*GAMA12,L1-1))*
10,L2=GAM11*ALPH1*(L1*T1+GAMA11*L1))/((ALPH1*T1+GAMA11*L1)*(ALPH1*T1+GAMA11*L1)*
0052 ,10*ALPH1*T1+GAMA11*L1))+(ALPH1*T1+GAMA11*L1)*((ALPH1*T1+GAMA11*L1)*
AL1*(X12)*(C11*5#PA1+FACT12)*FACT12*(FLM12)+T12*(C12)*(LW2)+T12*(C13))+GL2
0053 ,AL1*(X12)*(C11*5#PA1+FACT12)*FACT12*(FLM12)+T12*(C12)*(LW2)+T12*(C13))+GL2
0054 ,E=AL1*(X12)*(C11*5#PA1+FACT12)*FACT12*(FLM12)+T12*(C12)*(LW2)+T12*(C13))+GL2
0055 ,AL1*(X12)*(C11*5#PA1+FACT12)*FACT12*(FLM12)+T12*(C12)*(LW2)+T12*(C13))+GL2
0056 ,AL1*(X12)*(C11*5#PA1+FACT12)*FACT12*(FLM12)+T12*(C12)*(LW2)+T12*(C13))+GL2
0057 ,AL1*(X12)*(C11*5#PA1+FACT12)*FACT12*(FLM12)+T12*(C12)*(LW2)+T12*(C13))+GL2
0058 ,FND ;-----  
AGS02200
```

```

0001 , FUNCTION GLCA+ALPHA+P+T1*GAMA11*L1
0002 ,+ANGULAR INTEGRAL
0003 , P=1/(3*144*PI*277)
0004 , PI=(3.141592653589793)
0005 , PI=PI*(L1+L2+L3)
0006 , R=PI*(L1+L2+L3)
0007 , E=BETA2+MET2A-(T1+AV)+A1+A0*GAMA11*GAMA11+B1*B1
0008 , F=-4.0*BETA2*GAMA11*GAMA11
0009 , G=5*BETA2*BETA2
0010 , S1=AVS(E+F+G)
0011 , S2=AVS(E-F-G)
0012 , SN=Sqrt(529)
0013 , SG=Sqrt(15)
0014 , G=3.141592653589793
0015 , IF(G<18.0) GOTO 10
0016 , GO=GA+ALOG1(10+2.*SG/(BETA2-SG))
0017 , GO TO 10
0018 , 10 GO=1.0*PA1/BETA2
0019 , 10 CONTINUE
0020 , L=1.0+18.0/23 GO TO 10
0021 , IF(L>18.0) GO TO 10
0022 , L=L-18.0/23 GO TO 20
0023 , GO TO 399
0024 , 20 GA1>2.0*PA1*(SP-SN)/G
0025 , GB1=0.5*W+5976
0026 , GLA=5976.0
0027 , GO TO 999
0028 , 30 GA2>0.75*PA1*(G1+G2)
0029 , GO=0.0
0030 , GC2<0.0,W=F+12.0*GE/(-C10*GNG)
0031 , GL=GA2*W*B2+5*W*SP+{2.0*G+3.0*F}*SN
0032 , GO TO 400
0033 , 40 GA3>3.0*PA1*(SP-SN)/G
0034 , GB3>5.0*PA1/({2.0*G+3})
0035 , GL3=3.0*PA1*(SP-SN)/G
0036 , GD3=GC3*GS*GD3*SN
0037 , GO=0.0
0038 , GH3<2.5*W*W+(-6.0*GE+E)/(3.0*G*GE+3)
0039 , GL=GA3+3.0*GE/E-(G1+3*GH3)*GO
0040 , 999 GOTO 999
0041 , RETURN
0042 , END  
AGS03500
```

```

0001 , FUNCTION FORM(ALPHAP,P)
0002 , FORM FACTOR OF HELIUM-3
0003 , P=1.331*V2
0004 , FORM=>SHRT ALPHAP=(PA1+(ALPHA+ALPHA+P*P))
0005 , RETURN
0006 , END  
AGS03510
```

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0001 , SUBROUTINE SIMPSN(P,A+B,DEL,IMAX,S11,S,N,IER)
0002 , DIMENSION P(1:IMAX)
0003 , S11=0.0
0004 , E=0.0
0005 , NQ=0
0006 , KMAX=A
0007 , TMAX=(B-A)/DEL
0008 , 10 I=NQ+1
0009 , 100 IER=0
0010 , 200 I=1+IMAX
0011 , 210 IF(I>IMAX) GOTO 10
0012 , 220 IF(I>A) GOTO 20
0013 , 230 IF(I>B) GOTO 100
0014 , 240 IF(I>TMAX) GOTO 210
0015 , 250 IF(I>KMAX) GOTO 220
0016 , 260 IF(I>E) GOTO 230
0017 , 270 IF(I>NQ) GOTO 240
0018 , 280 IF(I>B) GOTO 260
0019 , 290 IF(I>A) GOTO 270
0020 , 300 IF(I>E) GOTO 280
0021 , 310 IF(I>NQ) GOTO 290
0022 , 320 IF(I>B) GOTO 300
0023 , 330 IF(I>A) GOTO 310
0024 , 340 IF(I>E) GOTO 320
0025 , 350 IF(I>NQ) GOTO 330
0026 , 360 CALL NPSCIN(P,A+B,DEL,IMAX,S11,S,N,IER)
0027 , 370 IER=IER+1
0028 , 380 GOTO(40,50,50,40,3,IER)
0029 , 40 , 50 IF(ABS(S11-ABS(DEL*S))>28+28
0030 , 50 RETURN
0031 , 30 IF(IER<0) RETURN
0032 , 40 RETURN
0033 , 50 RETURN  
AGS03580
```

```

0001 , 100000.0
0002 , 100000.0
0003 , 100000.0
0004 , 100000.0
0005 , 100000.0
0006 , 100000.0
0007 , 100000.0
0008 , 100000.0
0009 , 100000.0
0010 , 100000.0
0011 , 100000.0
0012 , 100000.0
0013 , 100000.0
0014 , 100000.0
0015 , 100000.0
0016 , 100000.0
0017 , 100000.0
0018 , 100000.0
0019 , 100000.0
0020 , 100000.0
0021 , 100000.0
0022 , 100000.0
0023 , 100000.0
0024 , 100000.0
0025 , 100000.0
0026 , 100000.0
0027 , 100000.0
0028 , 100000.0
0029 , 100000.0
0030 , 100000.0
0031 , 100000.0
0032 , 100000.0
0033 , 100000.0
0034 , 100000.0
0035 , 100000.0
0036 , 100000.0
0037 , 100000.0
0038 , 100000.0
0039 , 100000.0
0040 , 100000.0
0041 , 100000.0
0042 , 100000.0  
AGS04090
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