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Note

Anomalous Coupling States in Medium-weight Nuclei with Pairing plus Quadrupole Forces*

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In this lecture we shall try to explain the so-called anomalous coupling states on the basis of the BCS method and RPA — Random Phase Approximation. Most of the discussion is based on work done in collaboration with M. Sano — Muraoka of Osaka University.

First of all, we shall review an historical story on the anomalous coupling states in order to make the point clear.

As is well known, nuclear shell model predicts low-lying $9/2^+$ and $1/2^-$ states for nuclei with proton or neutron numbers just below 50($41 < Z, N < 49$). However, experimental data show the existence of many low-lying $7/2^+$ states, even the $7/2^+$ ground states. Isomeric E3 transitions between the $7/2^+$ and $1/2^-$ states have also been observed. They are all strongly retarded. From this fact, M. Goldhaber and A.

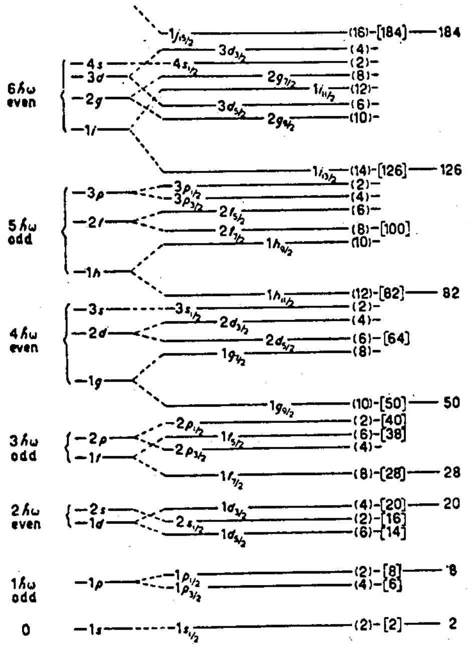

Fig. 1. Nuclear shell model orbit energies (for proton).

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* This lecture was also presented at Brookhaven National Laboratory and at Oak Ridge National Laboratory.

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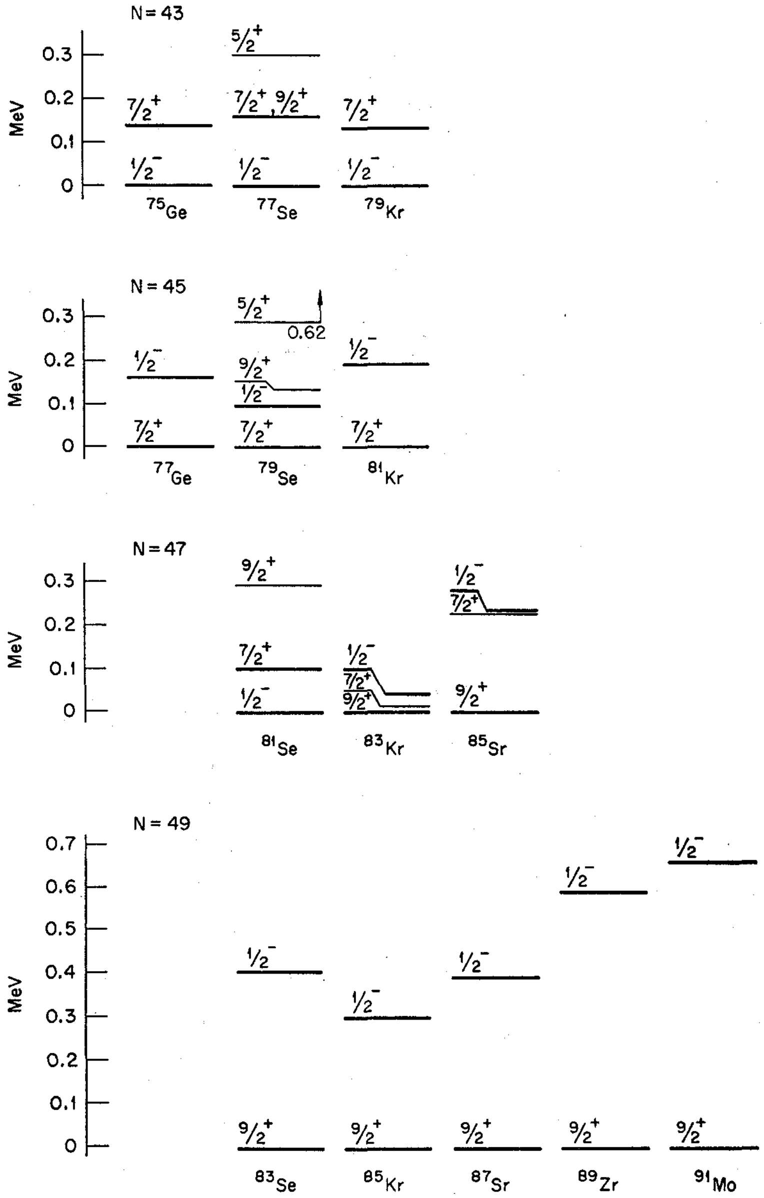

Fig. 2. Nuclear levels in anomalous region (neutron levels).

Sunyar concluded that these anomalous $7/2^+$ states probably resulted from the coupling of several $g_{9/2}$ particles.[1] The first try of explanation was done by B. H. Flowers in the scheme of $j-j$ coupling.[2] His calculation based on the configuration $(g_{9/2})^3$ has shown that the $7/2^+$ states never go down below the $9/2^+$ states for the values of the

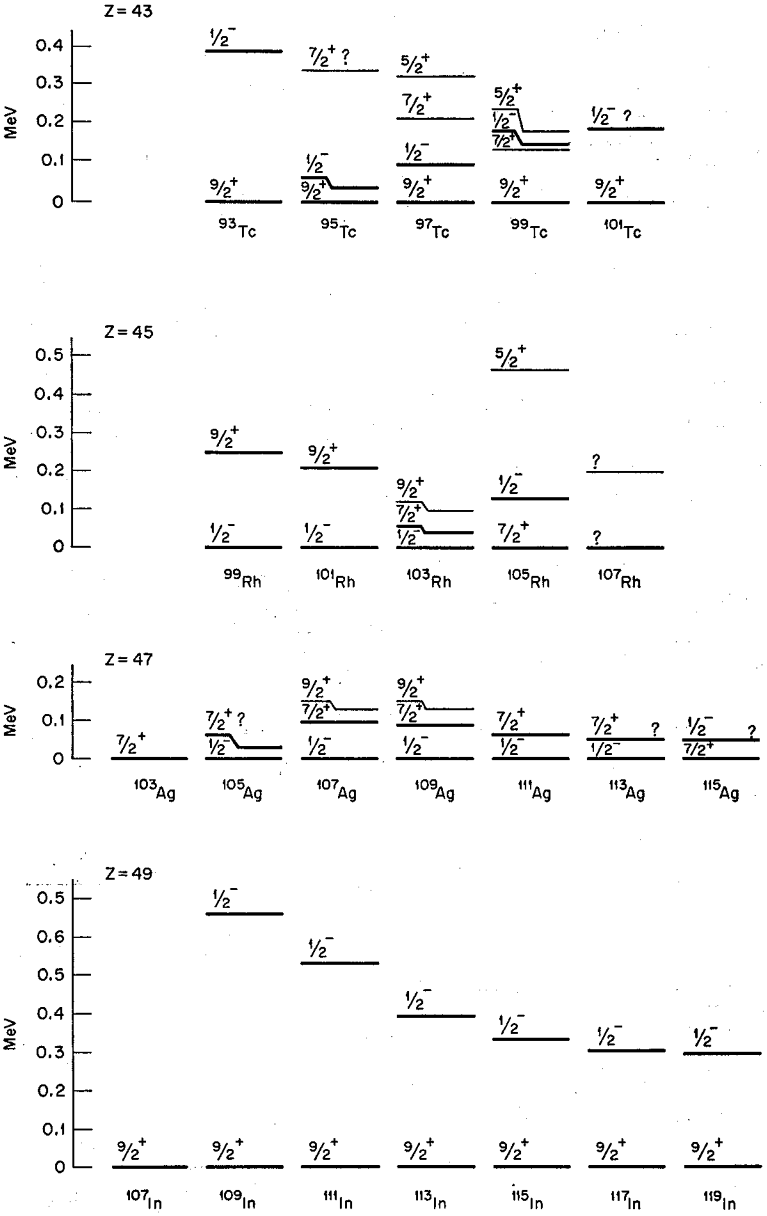

Fig. 3. Nuclear levels in anomalous region (proton levels).

range parameter less than 2.0. Here the range parameter $\alpha \equiv (a/a_0)^2$ is the ratio of the range of the residual force (Gaussian potential) and to that of the wave function (Harmonic Oscillator wave function). On the other hand, for the parameter values larger than 1.4, the $5/2^+$ states are lower than the $7/2^+$ states. Thus the $7/2^+$ states can never

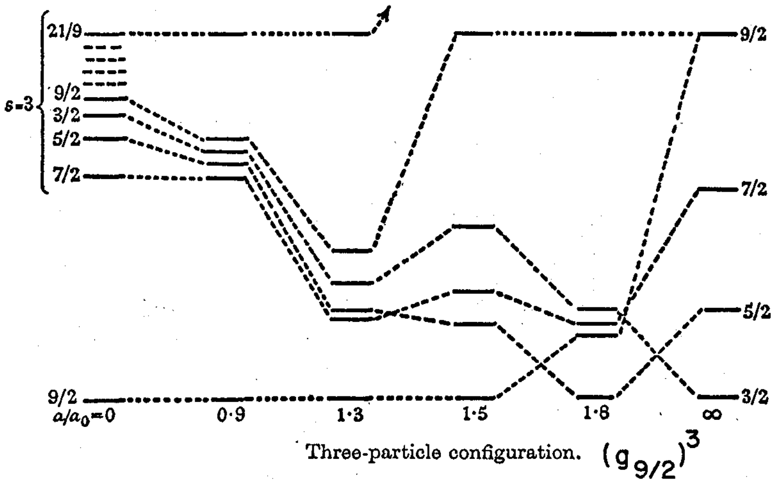

Fig. 4. Anomalous levels predicted by *j*–*j* coupling model.

be the ground state. This situation is rather independent of the detailed characteristics of the residual interaction such as its radial dependence, *etc.*, as long as it is attractive. The result was confirmed by I. Talmi³⁾ and is also similar to those obtained by D. Kurath⁴⁾ for the configurations $(d_{5/2})^3$ and $(f_{7/2})^3$. Though there is some difficulty, pointed out by Flowers, the anomalous states have long been described as multiparticle configurations: $(g_{9/2})^{3,5,7}$, $(f_{7/2})^{3,5}$, and $(d_{5/2})^3$.

Several attempts⁵⁾ have also been made to lower the levels of spin $J=j-1$ below the levels of spin $J=j$ by postulating stable nuclear deformation although there is no systematic evidence for deformation in the region $40 \lesssim A \lesssim 120$. The role of weak surface coupling in the configuration $(f_{7/2})^3$ has been treated with use of the perturbation theory by K. W. Ford and C. Levinson.⁶⁾ They concluded that interparticle interaction might be more important than interaction with weak surface vibrations (with "zero thickness" — because they employed "closure approximation").

Calculations based on the BCS method⁷⁾ and RPA — Random Phase Approximation⁸⁾ — have explained successfully general features of many medium-weight nuclei. However, it is easily seen as pointed out by L. S. Kisslinger and R. A. Sorensen that such a calculation for the coupling of one $g_{9/2}$ quasi-particle to phonons of quadrupole type is inadequate to interpret the $7/2^+$ and $5/2^+$ states.⁹⁾

Now, it is interesting to note that these earlier efforts which have been mostly confined to explain the level energies, have not been successful in lowering the $7/2^+$ states.

In elucidating the nature of the anomalous coupling $7/2^+$ and $5/2^+$ states, it may be pertinent to consider the following new information.¹⁰⁾

1. For nuclei with $N=40+1$, low-lying anomalous coupling states (the 0.0135 MeV $5/2^+$ state of ^{73}Ge, the $5/2^+$ ground state of ^{75}Se, and the $7/2^+$ ground state of ^{77}Kr) are observed. There is also some evidence which suggests existence of the $9/2^+$, $7/2^+$, and $5/2^+$ triplet in these nuclei [$9/2^+$ (ground state)$-5/2^+$(0.0135)$-7/2^+$(0.067) in ^{73}Ge]. Even for nuclei with $Z=35$ and 33, the $5/2^+$ and $9/2^+$ doublets [$5/2^+$,(0.131 MeV)$-9/2^+$(0.107 MeV) in $^{77}_{35}$Br], [$5/2^+$(0.632 MeV)$-9/2^+$(0.475 MeV) in $^{77}_{33}$As], and [$5/2^+$(0.401 MeV)$-9/2^+$(0.304 MeV) in $^{75}_{33}$As] are observed.

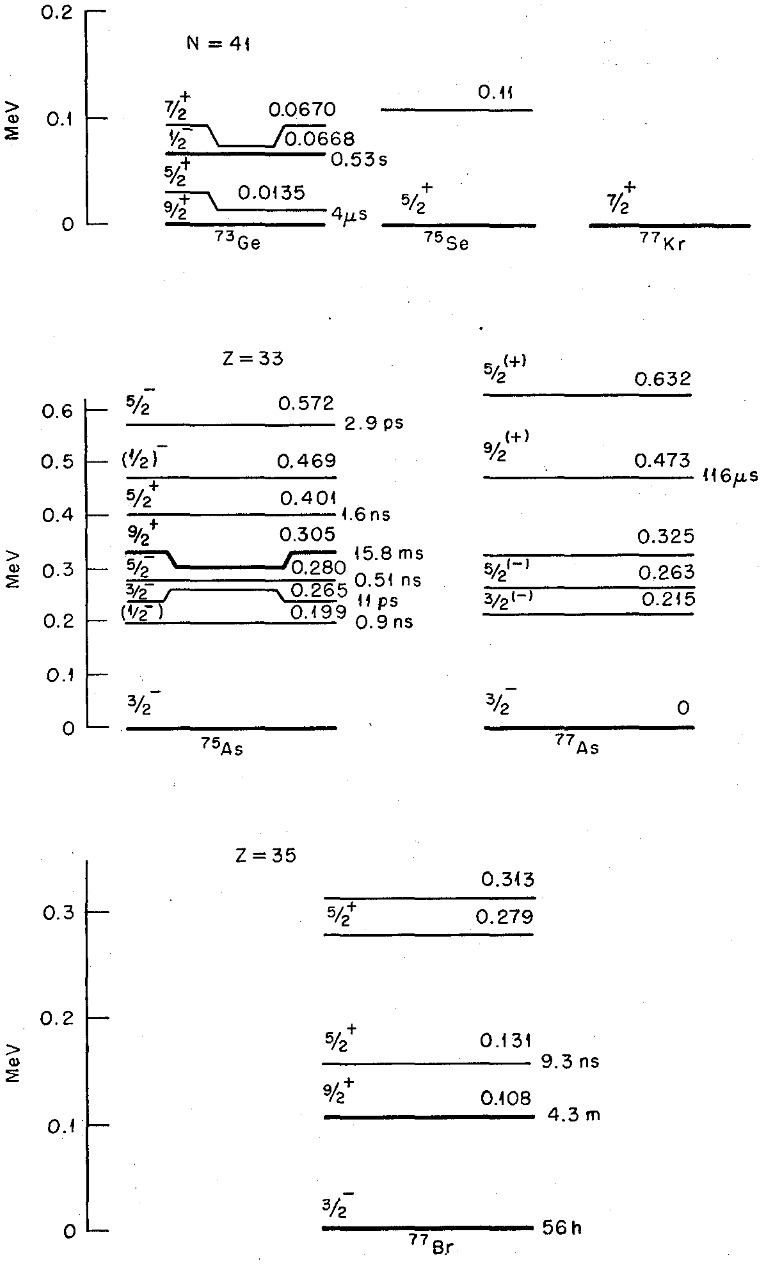

Fig. 5. New anomalous states. They are not expected from previous theories.

Table I. Comparison of E2 Transitions of Anomalous Coupling States and Phonon Transitions in Neighboring Even-Even Nuclei.

E2 transitions of AC states					Phonon transitions	
Nucleus	I_i	I_f	E_γ(MeV)	M^2/M^2_{sp}	Nucleus	M^2/M^2_{sp}
$^{73}Ge_{41}$	$5/2^+$	$9/2^+$	0.0135	$50\sim120$*	^{72}Ge, ^{74}Ge	120, 170
$^{75}_{33}As$	$5/2^+$	$9/2^+$	0.097	$\gtrsim100$	^{74}Ge	170
$^{77}_{35}Br$	$5/2^+$	$9/2^+$	0.024	$\gtrsim100$	^{76}Se, ^{78}Kr	220, 170
$^{99}_{43}Tc$	$5/2^+$	$9/2^+$	0.181	60	^{98}Mo, ^{100}Ru	100, 200

* The value 10 in Ref. 11 should be read 100.

2. Some E2 transitions between the anomalous coupling states and the $9/2^+$ states are strongly enhanced. The enhancements of the transitions are comparable to those of phonon transitions in neighboring even-even nuclei (Table I).

3. Observed static quadrupole moments of some anomalous coupling states (the $5/2^+$ ground state of $^{75}Se_{41}$ and the $7/2^+$ ground state of $^{79}Se_{45}$) are almost ten times as large as those expected from the nuclear shell model.

Table II. M1 Transitions of Anomalous Coupling States.

Nucleus	I_i	I_f	E_γ (MeV)	M^2/M^2_{sp}
$^{73}Ge_{41}$	$7/2^+$	$9/2^+$	0.067	0.31
$^{83}_{47}Kr$	$7/2^+$	$9/2^+$	0.009	0.16
$^{99}_{43}Tc$	$5/2^+$	$7/2^+$	0.041	0.06

4. Some observed M1 transitions between the $9/2^+$ and $7/2^+$ states are rather weakly hindered (Table II).

How is one to understand these features?

A natural framework for discussion of the striking E2-enhancement of these anomalous coupling states is found, however, in the concept of the phonon coupling, though it was thought to be inadequate to explain their level energies.

How is one to lower the $7/2^+$ states in the scheme of phonon coupling? Now it is important to note that all earlier efforts have made use of only one shell model orbit having the same parity as that of the anomalous states. It seemed important, therefore, to investigate the problem by taking account of several major shells. Such an investigation has been carried out by H. Ikegami and M. Sano.[11] In the present calculation, the BCS equations were solved by using slightly modified Mottelson, Nilsson, and Prior's orbit energies ($\delta=0$). It was assumed that $G_n=G_p$, the strength of the pairing force, is the same (about two-thirds of Kisslinger and Sorensen's value)[9] for all major shells. The calculated half-energy gap Δ fits well with those obtained from the even-odd mass difference, even for closed shell ±1 nucleon nuclei.

The strength of the quadrupole force, $X_n=X_p=X_{np}$, was determined in order to fit energies of the first 2^+ states in even-even nuclei. Then, the strength of the phonon-quasi-particle interaction $S'^{-1/2}$ was calculated.[12]

$$S'_\lambda=\frac{\partial}{\partial(\hbar\omega_\lambda)}S_\lambda$$

$$S'_\lambda\equiv\sum_{j'j}\frac{(E_{j'}+E_j)<j'\|i^\lambda Y_\lambda w_\lambda\|j>(U_{j'}U_j+V_{j'}V_j)^2}{(E_{j'}+E_j)^2-(\hbar\omega_\lambda)^2},\quad w_\lambda\equiv\{(m\omega_0/\hbar)^{\frac{1}{2}}r\}^\lambda$$

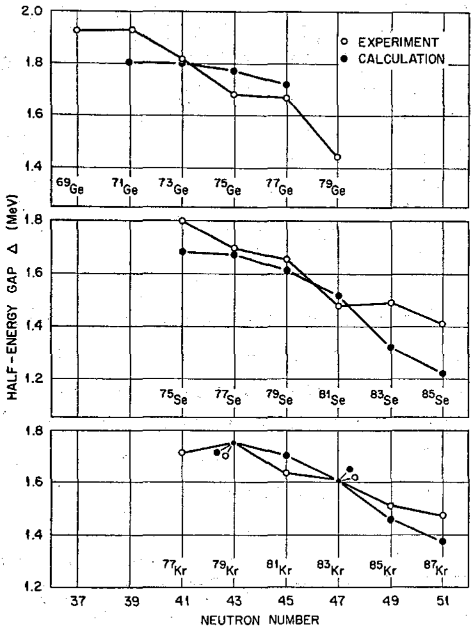

Fig. 6. Observed and calculated energy gap (neutron).

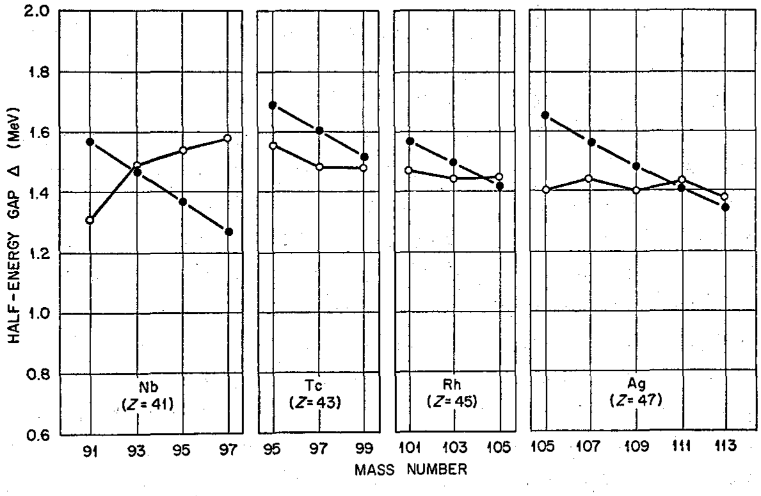

Fig. 7. Observed and calculated energy gap (proton).

In one quasi-particle states with zero, one, and two phonons, the phonon-quasi-particle interaction Hamiltonian was diagonalized.

$$<j'N'I'J|H_{13}|jNIJ> = \delta_{j',j}\delta_{N',N}\delta_{I',I}(N\hbar\omega_\lambda + E_j) + k^\lambda_{j',j}A(j'N'I';\ jNI;\ J\lambda)$$

$$k^\lambda_{j',j} \equiv <j'\|i^\lambda w_\lambda Y_\lambda\|j>(U_{j'}U_j - V_{j'}V_j)S'^{-\frac{1}{2}}$$

$$A(j'N'I';\ jNI;\ J\lambda) = (-)^{j'+I+J+\lambda}(2\lambda+1)^{\frac{1}{2}}W(j'I'jI;\ J\lambda)$$

$$\times[\delta_{N',N+1}<N'I'\|Q^+_\lambda\|NI> + \delta_{N',N-1}<N'I'\|Q_\lambda\|NI>]$$

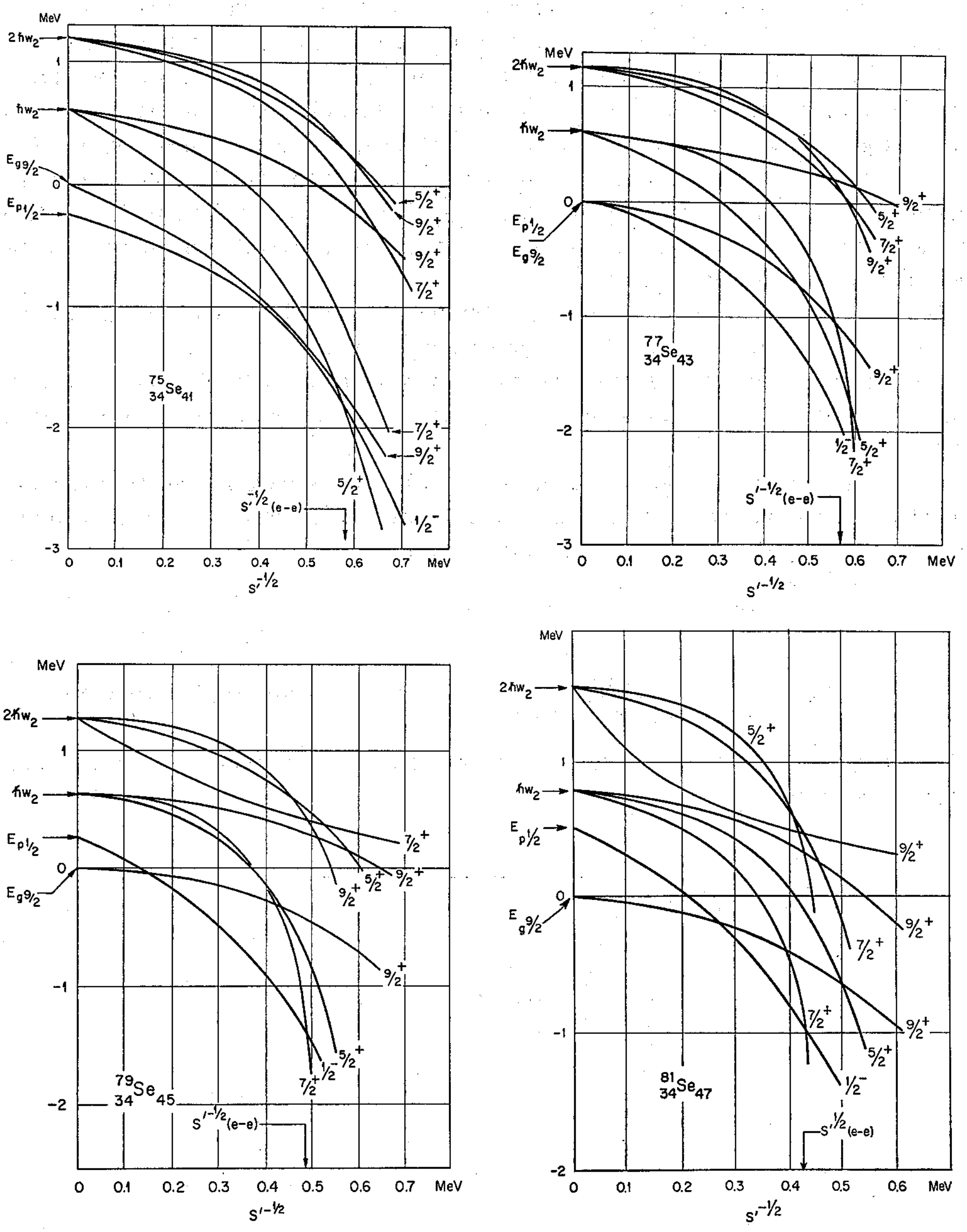

Fig. 8. Predicted low-lying levels of $9/2^+$, $7/2^+$, $5/2^+$, and $1/2^-$.

Calculations were carried out for the $9/2^+$, $7/2^+$, $5/2^+$, and $1/2^-$ states in 25 nuclei (Ge, Se, Kr, Y, Nb, Tc, Rh, and Ag). Figure 8 shows predicted levels (with positive parity) in Se isotopes compared with available experimental results. Most of the $7/2^+$ and $9/2^+$ levels except the lowest $9/2^+$ levels are, however, probably missing in the (*d*, *p*) studies.

The features evident from the present calculation are the following:

a. As is seen in Fig. 8, the anomalous coupling state can be the ground state even for N=41 nucleus (see above evidence 1). For these low-lying anomalous coupling states, one quasi-particle components in the wave functions are usually small.

b. In some cases, in the wave functions of the $7/2^+$ and $5/2^+$ states the one quasi-particle components and the one quasi-particle plus one phonon components are both fairly large and thus the phonon contribution in the quadrupole moments dominates. A large phonon contribution in gamma transitions between the $9/2^+$ states and the anomalous coupling states especially of $5/2^+$ is predicted. A large phonon contribution in the excitations by the inelastic scattering of charged particles is also predicted for some nuclei (see the above evidences 2 and 3).

c. Weakly hindered M1 transitions between the $9/2^+$, $7/2^+$, and $5/2^+$ states (especially between the $9/2^+$ and $7/2^+$) are predicted (evidence 4).

d. Spectroscopic factor of (*d*, *p*) reactions with final states of anomalous coupling increases with neutron number. The trend is in agreement with experimental results.[13)]

e. The present calculation explains the well known systematic feature of the hindered E3 transitions between the $7/2^+$ and $1/2^-$ states. Effective charge of octupole was calculated with use of the perturbation theory.

$$e_{eff} \equiv e_{intr} + e_{coll}, \qquad e_{intr} = 1(\text{for proton}) \\ \qquad\qquad\qquad\qquad = 0(\text{for Neutron})$$

$$e_{coll} \equiv \frac{S^p}{S'} \cdot \frac{2}{\hbar\omega} \cdot \frac{(\hbar\omega)^2}{(\hbar\omega)^2 - (E_i - E_f)^2}, \quad S_p;\ S\text{-for proton configuration only}$$

The strength of the octupole force was determined in the same manner as in the quadrupole force case.

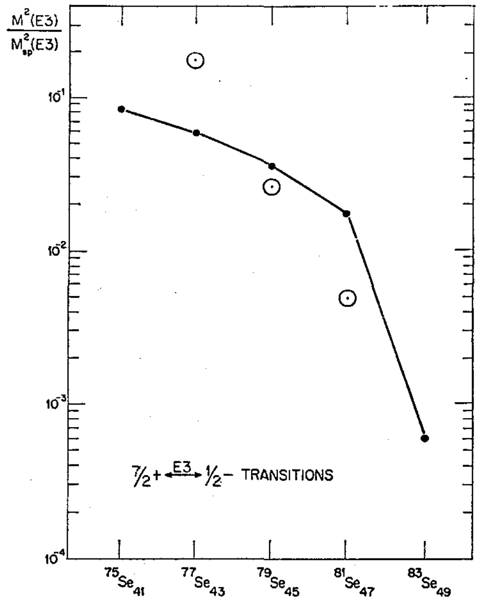

Figure 9 Predicted and observed E3 transions between the $7/2^+$ and $1/2^-$ states. Open and closed circles represent experimental and predicted values respectively.

It is of value to note about lowering the anomalous coupling state energies. In the anomalous region of nuclei with Z or N=43, 45, and 47, the Fermi level crosses the $g_{9/2}$ orbit energy. And, thus, the pairing force supresses the effect of the quadrupole force on the $9/2^+$ level because of the factor $(U_iU_j-V_iV_j)$ included in the phonon-quasi-particle interaction Hamiltonian.$^{14)}$ This is just the reason for the occurrence of the low-lying $7/2^+$ and $5/2^+$ states. Since the $g_{9/2}$ orbit is isolated from those having the same parity, the situation becomes seemingly drastic and yielded the name "Anomalous State". Similar remarks apply to the configurations $(f_{7/2})^{3,5}$ and $(d_{5/2})^3$ where the $7/2^-$ and $5/2^-$ and $5/2^+$ and $3/2^+$ levels sometimes appear to be inverted.

As seen in Fig. 8, we have some problem to improve the relative position of the anomalous $7/2^+$ and $5/2^+$ states. We found that the backward scattering effect (or quasi hole state effect) lifts up the $9/2^+$ state but doesn't improve the relative position of the anomalous $7/2^+$ and $5/2^+$ states. We are convinced that overlap of the $d_{5/2}$ and $g_{9/2}$ wave functions in our calculation might be too large. A refined calculation is now going on.

Though there remains some room for making refined arguments, essential features of the AC states are well predicted on the basis of BCS and RPA.

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REFERENCES

(1) M. Goldhaber and A. Sunyar, *Phys. Rev.,* **83**, 906 (1951).
(2) B. H. Flowers, *Proc. Roy. Soc.,* **215**, 398 (1952).
(3) I. Talmi, *Phys. Rev.,* **82**, 101 (1951) and *Helv. Phys. Acta.* 25, 185 (1952).
(4) D. Kurath, *Phys. Rev.,* **80**, 98 (1950).
(5) For instance, A. Bohr and B. R. Mottelson, *Dan, Mat. Fys. Medd..* **27**, No. 16 (1953).
(6) K. W. Ford and C. Levinson, *Phys. Rev.,* **100**, 1 (1955).
(7) For example, J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.,* **108**, 1175 (1957).
(8) K. Sawada, *Phys. Rev.,* **106**, 372 (1957).
(9) L. S. Kisslinger and R. A. Sorrensen, *Rev. Mod. Phys.,* **35**, 853 (1963).
(10) W. B. Ewbank. M. J. Martin, S. .C Pancholi, K. Way and H. Ikegami, *Nucl. Data* **B**, A= 70, 71, 72, 73, 74, 75, 76 (1966).
(11) H. Ikegami and M. Sano, *Phys. Lett.* **21**, 323 (1966). . .
(12) S. Yoshida, *Nucl. Phys.,* **38**, 380 (1962).
(13) E. K. Lin, *Phys. Rev.,* **139**, B340 (1965).
(14) For instance, H. Ikegami and T. Udagawa, *Phys. Rev.,* **133**, B1388 (1964).