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Effect of Quasi-holes on the Structure of Odd-mass Nuclei

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It is well known that the Tamm-Dancoff approximation (TDA) is less adequate than the random phase approximation (RPA) for the treatment of the quadrupole and octupole collective vibration in even-even nuclei since the latter includes the ground state correlation.[1] Although in previous theories RPA has been employed for the description of the even-even core, essentially only the TDA scheme has been used for the odd nucleus since quasi-hole, or the backward scattering effect for an unpaired nucleon, was not taken into account. In the present paper, it is shown that a description including this effect explains some important features of odd nuclei not predicted by previous theories.[2,3]

We introduce the Hamiltonian and the coupled equations of motion of quasi-particles and quasi-holes. The Hamiltonian is

$$H = H_{\mathrm{BCS}} + (H_{22} + H_{40} + H_{31} + \mathrm{c.c.}) \tag{1}$$

where the operators have been ordered in normal form, *i.e.*, all the creation operators α^{+}_{jm} are placed to the left of the destruction operators α_{j-m}. The indices n_1 and n_2 of $H_{n_1 n_2}$ refer to the numbers of α^{+}_{jm} and α_{j-m}, respectively. The equation of motion becomes[4]

$$\begin{aligned}[][H, \alpha^{+}_{jm}] = E_j \alpha^{+}_{jm} &+ \sum_{j'} K_{jj'} (\lambda\mu j'm' \mid jm) \alpha^{+}_{j'm'} \{Q^{+}_{\lambda\mu} + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \\ &+ \sum_{j'} M_{jj'} (\lambda\mu j'm' \mid jm) (-)^{j'-m'} \alpha_{j'-m'} \{Q^{+}_{\lambda\mu} + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \end{aligned} \tag{2a}$$

$$\begin{aligned}[][H, (-)^{j-m}\alpha_{j-m}] = \\ - E_j (-)^{j-m} \alpha_{j-m} &- \sum_{j'} K_{jj'} (\lambda\mu j'm' \mid jm) (-)^{j'-m'} \alpha_{j'-m'} \{Q^{+}_{\lambda\mu} + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \\ &+ \sum M_{jj'} (\lambda\mu j'm' \mid jm) \alpha^{+}_{j'm'} \{Q^{+}_{\lambda\mu} + (-)^{\lambda-\mu} Q_{\lambda-\mu}\} \end{aligned} \tag{2b}$$

where[5] $K_{jj'} \equiv -S'^{-1/2}_{\lambda} (2\lambda + 1/2j + 1)^{1/2} < j' \| q_\lambda \| j > v_{jj'}$

$M_{jj'} \equiv -S'^{-1/2}_{\lambda} (2\lambda + 1/2j + 1)^{1/2} < j' \| q_\lambda \| j > u_{jj'}$

Here E_j is energy of a quasi-particle with spin j and $Q^{+}_{\lambda\mu}(Q_{\lambda-\mu})$ represents the creation (destruction) operator of a phonon of multipole order λ, μ for the even-even core. It is assumed that $Q^{+}_{\lambda\mu}$ and $Q_{\lambda-\mu}$ commute with α^{+}_{jm} and α_{j-m}, also that $Q_{\lambda\mu}|0> = 0$. The physical meaning of the second assumption is: the ground state $|0>$ of the even-even core in an odd nucleus is the same as the ground state of its even-even neighbor. $S'^{-1/2}_{\lambda}$, the strength of the phonon-quasi-particle and phonon-quasi-hole interactions, is calculated in the scheme of RPA (hereafter denoted $S'^{-1/2}_{\lambda}$(e-e)).[2] In the present cal-

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culation, the BCS equations were solved for two major shells employing level energies which are almost the same as those found from Mottelson, Nilsson, and Prior's orbits ($\delta=0$).[6] The strengths of the pairing and quadrupole forces were determined to fit the energy gaps obtained from the even-odd mass difference and the energies of the first 2^+ states in neighboring even-even nuclei, respectively. For one quasi-particle and hole (in the two major shells) with zero, one, and two phonons, the equations of motion are linearized by employing the following relations[7]

$$\langle jm|[H, \alpha^+_{jm}]|N, JM\rangle \simeq (E^0_j - N\hbar\omega_\lambda)\langle jm|\alpha^+_{jm}|N, JM\rangle$$

$$\langle jm|\{Q^+_{2\mu}+(-)^{-\mu}Q_{2-\mu}\}\alpha^+_{j'm'}|N, JM\rangle$$
$$= \sum_{N',J'M'} \langle jm|\alpha^+_{j'm'}|N', J'M'\rangle \langle N', J'M'|\{Q^+_{2\mu}+(-)^{-\mu}Q_{2-\mu}\}|N, JM\rangle$$

Here $|jm\rangle$, E^0_j, and $|N, JM\rangle$ represent a state with spin j in odd nucleus, its energy eigenvalue, and the N-phonon state with spin J, respectively. In the second equation the last term on the right hand side is calculated in the same manner as for an even-even nucleus.

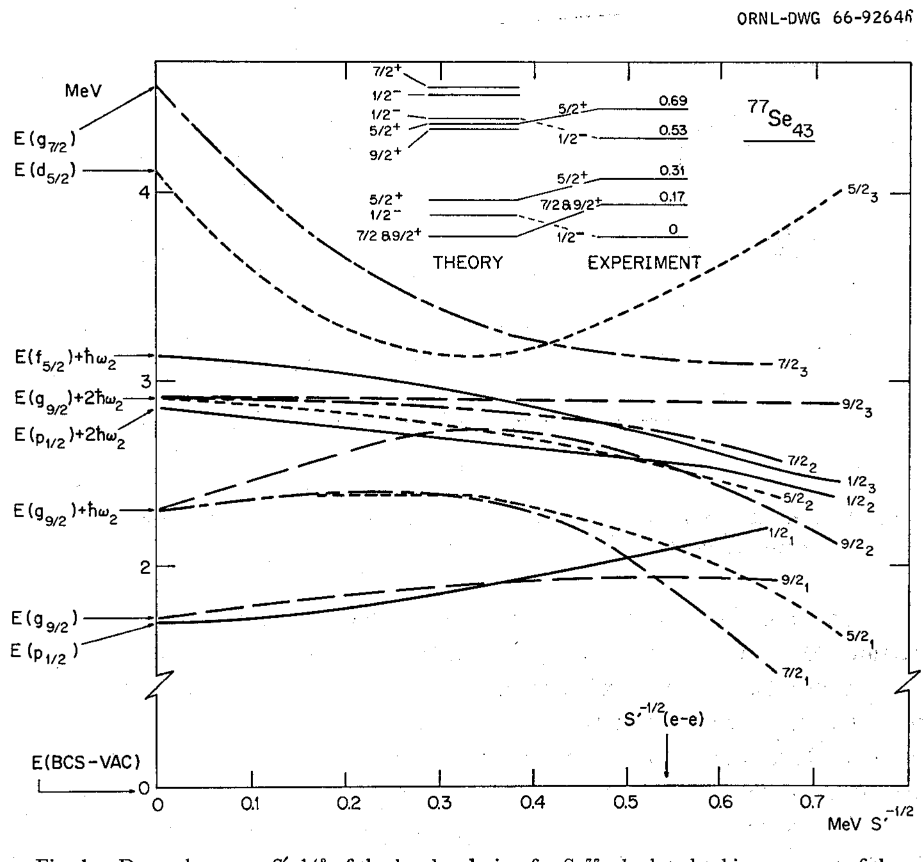

Fig. 1. Dependence on $S_2'^{-1/2}$ of the level ordering for Se^{77} calculated taking account of the quasi-hole effect. The value of $S_2'^{-1/2}$ $(e\text{–}e)$ is estimated from the 2^+ level energies of Se^{76} and Se^{78}.

Figure 1 shows the predicted level ordering in Se^{77}_{43} as a function of the strength $S'^{-1/2}$. Figure 2 shows the level ordering when the quasi-hole effect is omitted.[8] The striking features evident from comparison of the two figures are:

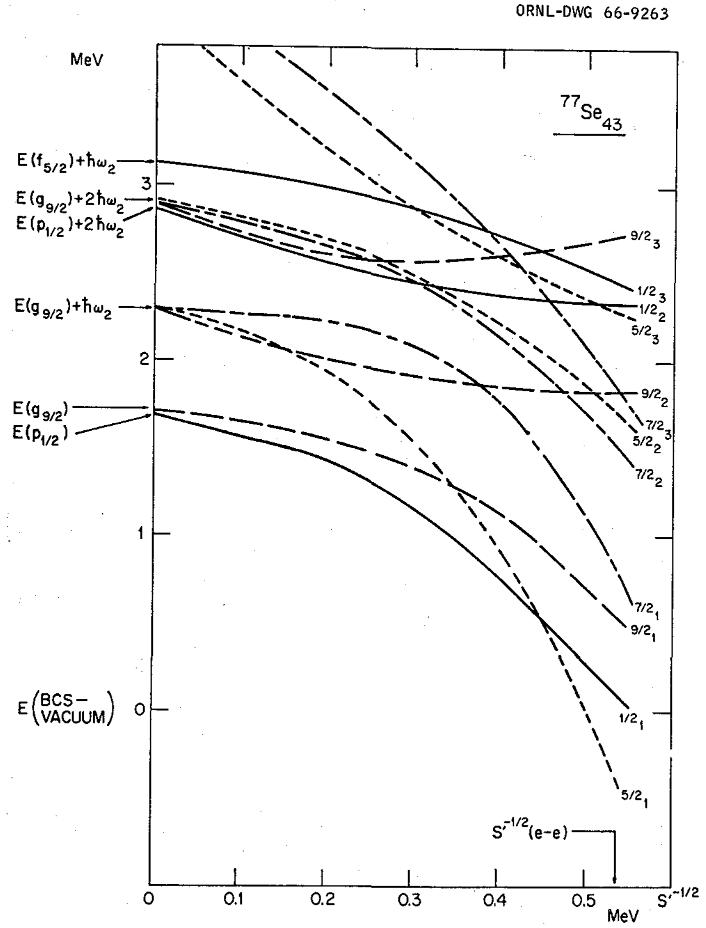

Fig. 2

Fig. 2. Dependence on $S_2'^{-1/2}$ of the level ordering for Se^{77} calculated neglecting the quasi-hole effect.

i) Levels of quasi-particle near the Fermi level are pushed up because of their interaction ($M_{j'j}$ term in Eq. 2) with the quasi-holes.

ii) Levels having the same spin and parity lie fairly close together in Fig. 1 in contrast with their separated positions in Fig. 2. The close spacing of Fig. 1 is consistent with observation, close to the ground state of odd nuclei with strong vibrational nature, of a number of doublets having the same spin and parity.[9]

iii) The pushing-up effect, due to the quasi-hole, on the so called anomalous-coupling $7/2^+$ and $5/2^+$ levels is weaker than for normal states. This is another reason for the low-lying position of the anomalous-coupling states in addition to our previous explanation.[6]

iv) Though the effect of the quasi-hole on level energies of odd nuclei, particularly on level energies of quasi-particles lying near the Fermi level, is very remarkable, it changes transition probabilities, multipole moments, spectroscopic factors, and so on by less than 10% from values obtained by the usual calculations.

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(2) S. Yoshida, *Nucl. Phys.*, **38**, 380 (1962).

(3) L. S. Kisslinger and R. A. Sorensen, *Rev. Mod. Phys.*, **35**, 853 (1963).

(4) In the RPA scheme, one may also add the following terms:

$$\sum_{j'j_1j_1'} L_{jj':j_1j_1'}(\lambda\mu j'm'|jm)\alpha^{+}_{jm}\{\psi_{j_1j_1'}(-)^{\lambda-\mu}Q_{\lambda-\mu}+\varphi_{j_1j_1'}Q^{+}_{\lambda\mu}\}$$

and

$$\sum_{j'j_1j_1'} L_{jj':j_1j_1'}(\lambda\mu j'm'|jm)(-)^{j'-m'}\alpha_{j'-m'}\{\psi_{j_1j_1'}(-)^{\lambda-\mu}Q_{\lambda-\mu}+\varphi_{j_1j_1'}Q^{+}_{\lambda\mu}\}$$

$$L_{jj':j_1j_1'}\equiv-\chi_\lambda(2\lambda+1/2j+1)^{1/2}<j_1'\|q_\lambda\|j'><j\|q_\lambda\|j_1>v_{j_1'j'}u_{jj_1}W(jj_1j'j_1':\lambda\lambda)$$

to the right hand side of Eqs. 2a and 2b, respectively. They are numerically smaller than the terms including $K_{jj'}$ and $M_{jj'}$. Here, χ_λ represents a strength of the two body interaction of 2^λ-pole. Explicit expressions for $\varphi_{jj'}$ and $\psi_{jj'}$ can be seen in Ref. 2.

(5) Here

$$q_\lambda\equiv i^\lambda w_\lambda Y_\lambda,\ w_\lambda\equiv[(m\omega_0/\hbar)^{1/2}r]^\lambda$$
$$u_{j'j}\equiv U_{j'}V_j+V_{j'}U_{j'},\ v_{j'j}\equiv U_{j'}U_j-V_{j'}V_j$$

where r is the radial co-ordinate of nucleon, m being its mass. Harmonic oscillator wave functions, with angular frequency ω_0, are used for single-particle wave functions. U_j and V_j mean the probability of orbit j being empty and occupied, respectively.

(6) It is indispensable to take accounts of several major shells for an explanation of the anomalous-coupling states on the basis of the BCS method and RPA. H. Ikegami and M. Sano, *Phys. Lett.*, **21**, 323 (1966).

(7) Half the solutions of the equations of motion, whose energies tend to those of a quasihole with zero, one, and two phonons in the limit of $S_2'^{-1/2}=0$ are unphysical and must be rejected as in the RPA for even-even nuclei.

(8) The result shown in Fig. 2 is essentially almost the same with those presented in Ref. 6.

(9) For examples, low-lying levels having the same spin and parity are:
The ground and 0.265 MeV states; $3/2^-$ in As^{75}.
The 0.249 and 0.439 MeV states; $5/2^-$ in Se^{77}.
The 0.131 and 0.279 MeV states; $5/2^+$ in Br^{77}.
The ground and 0.261 MeV states; $3/2^-$ in Br^{79}, *etc.*