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2. Study on Surface Electricity. (XVIII)*

On the Q-values of Interfaces

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Two methods can be devised in measuring the Q-value of a resonance circuit containing interfacial phase, the one from the sharpness of the resonance curve and the other from the principle of the so-called Q-meter.

The results obtained by the former method at mercury—N.H_2SO_4 aq. interface at 2,500 cps. at various amplitudes of vibration are tabulated. The apparent values obtained increase as the amplitude decreases. As this circuit contains periodically changing capacitance, the inductance value at resonance changes with it, which produces an effect as if the resonance curve is broadened by the same amount. Hence, it is clear that it influences on the apparent Q-value, when the periodical capacity change takes place, as in the case of U-effect II.

The Q-value is very low in our measurement, which is the result of low reactance and high resistance values in our experiments.

INTRODUCTION

Although the interfacial double layer is approximately equivalent to a perfect condenser without leakage, we must take account of the mechanical loss as well as the solution resistance in making a resonance circuit with inductive load, as was described in the preceding article.[2] It is a well-known fact that a resonance circuit has a characteristic value, called "Quality factor" (abbreviated "Q-value"), which is defined by the ratio of the reactance and resistance components of the circuit and is the reciprocal of the tangent of the loss angle.

There are two principles in estimating this value, the one from the sharpness of the resonance curve and the other from the ratio of the resonant voltage and the applied voltage. Both of them can be used for our measurent at interfaces, and we shall make descriptions on them here.

1. DEFINITION OF Q-VALUE

It was proposed in the preceding article.[2] that the equivalent circuit of an interfacial

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phase system connected with an inductive load was a network composed of the average double layer capacity K, load inductance L, the total equivalent resistance R_t and the electromotive force of U-effect II ($\dot{V}=E_d\cdot(\Delta\dot{C}/K)$) in series. The total equivalent resistance was defined to be a combination of the solution reistance R_0 and the load resistance R in series.[1] However, in considering the mechanism of U-effect II, we must take into account the mechanical work of vibration, which will be treated in section 3.

The Q-value of this circuit is defined by the following formula, i.e.

$$Q=\omega L_r/R_t=1/\omega KR_t, \tag{1}$$

because the following equation applies to resonance,

$$\omega L_r=1/\omega K.$$

If we can measure Q-value by some independent methods, we can estimate the value of R_t and hence that of R_0, from eq. (1) and the value of L_r or K.

2. PRINCIPLES OF Q-MEASUREMENTS

(1) From Resonance Curves

It is an established fact that Q-value is a measure of sharpness of the resonance curve. We can, therefore, measure this value by the graphical analysis of the resonance curve. In the following derivation, the same notations are used as in the preceding papers.[1,2]

The equation of current of U-effect II was given, in the articles quoted, by

$$\dot{I}=\frac{\dot{V}}{\sqrt{R_t^2+(\omega L-1/\omega K)^2}}e^{-j\varphi}, \tag{2}$$

where

$$\varphi=\mathrm{tg}^{-1}\frac{\omega L-1/\omega K}{R_t}.$$

The values of I, when the value of load inductance L is changed continuously, show a curve as is shown schematically in Fig. 1. This is the so-called "resonance curve", and the reactance change is linear and passes zero at resonance point. This curve is approximately symmetrical near resonance point, and when we put $\dot{I}=\dot{I}_r$, $\dot{I}_1$ and $\dot{I}_2$ for the values of L equal L_r, L_1 and L_2, as are shown in the same figure, the following equation applies,

$$2L_r=L_1+L_2,$$

when $I_1=I_2$. This relation is not used in this section.

Substituting these values of $\dot{I}$ and L in the above eq. (2), we get a series of

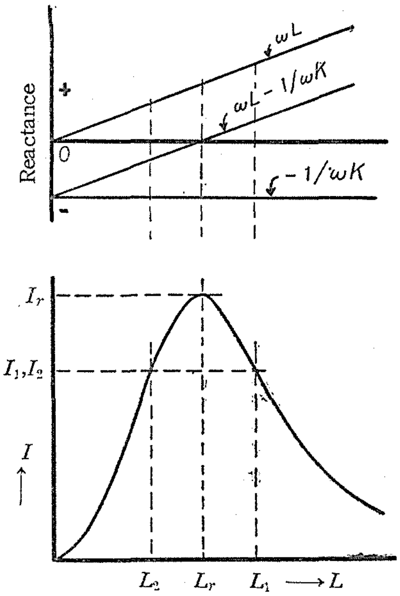

Fig. 1. A schematic resonance curve, which is a plotting of current when inductance value is changed. The upper diagram shows the variation of reactance ($\omega L - 1/\omega K$)

equations as follows:

$$\dot{I}_r = \dot{V}/R_t, \tag{3}$$

$$\dot{I}_i = \frac{\dot{V}}{R_t\sqrt{1+\frac{1}{R_t^2}(\omega L_i - 1/\omega K)^2}}\, e^{-j\varphi_i} \tag{4}$$

and

$$\varphi_i = \mathrm{tg}^{-1}\frac{\omega L_i - 1/\omega K}{R_t}, \ (i=1,2). \tag{4'}$$

Now, we take the values of L_1 and L_2 so as to satisfy the condition,

$$I_1 = I_2 = I_r/\sqrt{2}\,. \tag{5}$$

As the reactance values at these values of I are of the same moduli and of opposite signs, as are clear from Fig. 1, we get

$$\omega L_1 - 1/\omega K = 1/\omega K - \omega L_2.$$

Hence,

$$1/\omega K = \frac{\omega(L_1+L_2)}{2}. \tag{5'}$$

Using eq. (5), we obtain from eqs. (3) and (4)

$$\sqrt{2}=\sqrt{1+\frac{1}{R_t^2}(\omega L_1-1/\omega K)^2},$$

and therefore

$$R_t=\omega L_1-1/\omega K. \tag{6}$$

From eqs. (5), (5′) and (6) we get

$$Q=\frac{L_1+L_2}{L_1-L_2}. \tag{7}$$

This relation gives the method for calculating Q-value from the values of L_1 and L_2. We can, accordingly, calculate the equivalent interfacial series resistance from the definition of Q-value (eq. (1)) and graphical analysis of the resonance curve, and hence estimate the loss factor of the interface from the difference of this value and the calculated value of the solution resistance.

(2) From Voltage Ratio (Q-meter)

In this method an alternating voltage is fed from some outer source, and the ratio of the resonant and injected voltages is measured.

Consider an a.c. circuit as is shown by the diagram in Fig. 2, where the symbols

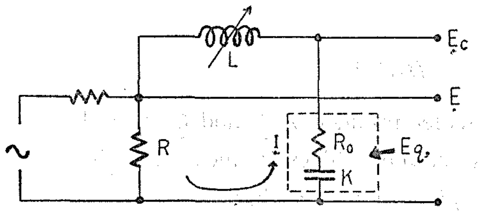

Fig. 2. Circuit diagram of Q-meter method.
E_q. : Equivalent circuit of interfacial phase.

have the same significances as in the above treatise. An alternating voltage is fed from outer source across the load resistance R. It is clear from simple calculation of a.c. theory that the current of this circuit is given by

$$\dot{I}=\frac{\dot{E}}{R_t+j\omega L+1/j\omega K},$$

where $\dot{E}$ is the applied alternating voltage. The voltage occurring at the two ends of the interfacial phase system ($\dot{E}_c$) is given by

$$\dot{E}_c=\dot{I}\cdot(R_0+1/j\omega K).$$

Substitution of the above formula of $\dot{I}$ into this equation gives

$$\dot{E}_c=\frac{\dot{E}(R_0+1/j\omega K)}{j\omega L+(R_t+1/j\omega K)}\ .$$

Now, we define a complex quantity $\dot{Q}$ by

$$\dot{Q}=\dot{E}_c/\dot{E}\ ,$$

and we get

$$\dot{Q}=\frac{R_0+j(-1/\omega K)}{j(\omega L)+[R_t+j(-1/\omega K)]}=\frac{\dot{A}}{\dot{B}x+\dot{C}}\ ,$$

where

$$\dot{A}=R_0+j(-1/\omega K),$$

$$\dot{B}=j,$$

$$\dot{C}=R_t+j(-1/\omega K),$$

and

$$x=\omega L,\ (\textit{variable.})$$

The modulus of $\dot{Q}$ has its maximum value

$$Q_{max}=\frac{AB}{\sqrt{B^2C^2-\widehat{B\cdot C}^2}}\ ,$$

when

$$x=-\ \widehat{B\cdot C}/B^2,$$

where $\widehat{B\cdot C}$ means the scaler product of $\dot{B}$ and $\dot{C}$, and A,B and C the moduli of each complex quantities, respectively. These values are easily calculated by substitutions of respective quantities given above, giving

$$Q_{max}=\frac{1}{R_t^2\omega K}\sqrt{R_0^2\omega^2K^2+1}\ , \tag{8}$$

when

$$\omega L=1/\omega K. \tag{9}$$

Eq. (9) is just the resonance condition and if

$$R_0^2\omega^2K^2\ll 1 \tag{10}$$

applies, eq. (9) gives the Q-value of this circuit.

3. MEASUREMENT AND RESULTS

The resonance curves in the same authors' preceding paper[2] make possible the estimation of Q-values of the circuit containing mercury—N.H_2SO_4 aq. interface by eq. (7). We must, however, note that the amplitude of capacitance change induced by the mechanical vibration of capillary element influences the measured values of L_1 and L_2, contrary to the case of the resonance point measurements. Table 1 gives

Table 1. Apparent Q-values (Q') for different amplitudes of capacity change, which are shown by GO, with resonant inductance values (L_r) and current (I_r). frequency=2,500 cps. R=500Ω.

GO	L_r	I_r	Q'	$1/Q'$
100	142mh.	104.2	2.01	0.498
20	142	68.0	2.27	0.440
10	142	33.9	2.61	0.383
5	142	8.5	3.36	0.298
3	142	2.5	3.83	0.261

the apparent Q-values (Q') for different amplitudes of the capacity change, which are shown here by the oscillator output (GO). It is noticed that Q' increases as GO decreases.

This is due to the fact, in the authors' opinion, that the capacitance change of the resonance circuit acts as a sort of frequency modulation[1] and, in addition, this modulating frequency is just the same with the oscillating frequency in our case.

The variation of L_r by that of the capacitance (ΔC) is given by the following derivation, i.e.

$$L_r = 1/\omega^2 K$$

$$\therefore\ \Delta L_r = -L_r(\Delta C/K).$$

Now, we assume for simplicity that the resonance curve is modulated with the same total shape by an amount ΔL_r at every instance. This is shown in enlarged scale for

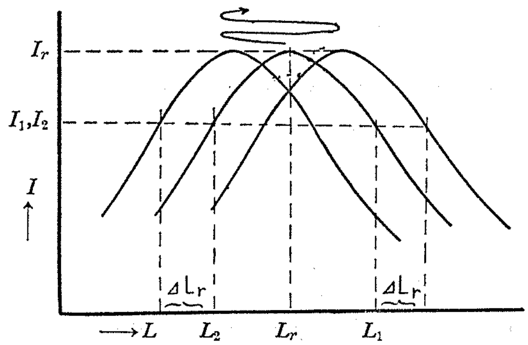

Fig. 3. Modulation of resonance curve by capacity change.

L in Fig. 3. The apparent Q-value (Q') calculated from eq. (7) applied to this broadened resonance curve is evidently given by the following equation,

$$\frac{1}{Q'}=\frac{L_1+|\varDelta L_r|-L_2+|\varDelta L_r|}{L_1+L_2}=\frac{1}{Q}+\frac{|\varDelta L_r|}{L_r}=\frac{1}{Q}+\frac{\varDelta C}{K},$$

because

$$L_1+L_2\doteqdot 2L_r.$$

This shows that $1/Q'$ increases with $\varDelta C$ and the true Q-value must be obtained by extrapolating $1/Q'$ versus $\varDelta C$ curve to $\varDelta C \to 0$.

Fig. 4 shows this plotting for the above case, where I_r is the reading of the valve

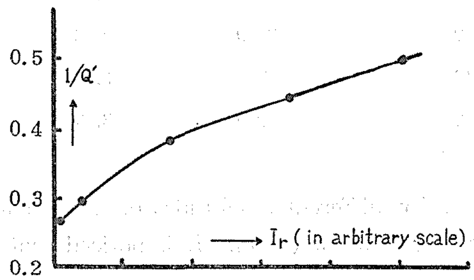

Fig. 4. Reciprocal of apparent Q-value ($1/Q'$) *versus* amplitude of periodical capacity change.

voltmeter at resonance, indicating the resonant current. It is clear from the mechanism of the a.c. generation of U-effect II, that the current of the circuit at constant load is proportional to the amplitude of the capacitance change.[3)]

This example gives very small Q-value compared with the ordinary Q-values found in electrostatic condensers (about 4 in our case). The reactance value of the interface is approximately

$$\omega L_r=2\pi\cdot 2{,}500\cdot 142\cdot 10^{-3}\sim 2{,}230(\Omega),$$

and therefore

$$R_t=\omega L_r/Q\sim 558(\Omega).$$

This is very near the load resistance value (500Ω) and we must use much lower resistance for load in accurate estimation of the interfacial Q-values.

CONCLUSION

The circuit-theoretical treatise of a C-L-R circuit is not complete before both of the resonance point and Q-value are determined. We have described the principle of latter measurement for the interfacial system by two different means.

The observations were very unsatisfactory in their accuracy, as the interfacial reactance was very low as compared with the ordinary electrostatic condensers.

The Q-meter method is given only in principle here, and the data obtained by this method are not given here, because they are also very low, for the same reason as above. This method is intimately related with the resonance method without the use of U-effect II, to be published in the next article.

The authors wish to express their gratitude to Prof. I. Tachi for his continued interest and encouragement.

REFERENCES AND NOTE

(1) Watanabe, Tsuji, Nishizawa and Ueda. This Bulletin, **31**, 249 (1953).
(2) Watanabe, Tsuji, Nishizawa and Ueda, *ibid.*, **32**, 12 (1954).
(3) Eq. (3) reads

$$\underset{\cdot}{I}_r=\underset{\cdot}{V}/R_t,$$

where

$$\underset{\cdot}{V}=E_a\cdot(\varDelta \underset{\cdot}{C}/K).$$

It is evident from these relations that I_r is proportional to the amplitude of mechanical vibration, because the last mentioned quantity is proportional to $\varDelta C$.