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B, gray	0.150	0.154
C, brown	0.158	
D, white	0.094	
clay powder (higher grade)	0.138	
〃 (lower grade)	0.208	

* These are the average values obtained from two samples.

** These are the values obtained after precipitating $Fe(OH)_3$ and $Al(OH)_3$.

The brown or gray-black color of the rock are the deeper, the more the iron content. The whitest and high grade paper clay contains very small quantities of iron. By the proper selection of the rock the color of paper clay will be improved.

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13. Study on High Dielectric Constant Ceramics. (XV)

Coupled Vibration in Electrostrictive Vibrators

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Theoretical analysis of coupled vibration, which can be seen in a mechanical vibrator having more than two modes of vibrations, was considered in previous report (K. Abe, T. Tanaka and K. Uo: *Jour. of the Denki Hyoron*, **39**, No. 12 (1951), 2.), and it was concluded that such theory agrees quite well with the experimental result in the case of $BaTiO_3$ ceramic vibrator having the shape of rectangular plate. The same manner can be applied in the treatment of hollow cylinder or circular disc or cylinder.

Consider a thin hollow cylinder having axial length a and radius r, then the resonant angular frequencies ($r.\ a.\ f.$) are given by the next formula when no coupling effect exists between the two vibrations:

$$w_a^2=\frac{\pi^2 E}{a^2\rho}, \qquad w_r^2=\frac{E}{r^2\rho}. \tag{1}$$

If coupling effect is considered, $r.\ a.\ f.$ become as follows, substituting $p=a/\pi r$:

$$\begin{aligned} w_1^2&=\frac{E}{r^2\rho}\cdot\frac{(p^2+1)-\sqrt{(p^2+1)^2-4p^2(1-\mu^2)}}{2p^2(1-\mu^2)}=\frac{E}{r^2\rho}u_1(p^2,\mu)\\ w_2^2&=\frac{E}{r^2\rho}\cdot\frac{(p^2+1)+\sqrt{(p^2+1)^2-4p^2(1-\mu^2)}}{2p^2(1-\mu^2)}=\frac{E}{r^2\rho}u_2(p^2,\mu)\end{aligned} \tag{2}$$

where μ is the coupling coefficient. Compared with Love's same result about thin hollow cylinder (A. E. H. Love: "Mathematical Theory of Elasticity". Chap. XXIV, p. 546, 4th. ed. 1927), it is concluded that μ is equal to Poisson's ratio σ.

As to the vibration of circular disc or cylider, the same manner can be applied. When no coupling exists, the *r. a. f.* become

$$w_a^2=\frac{\pi^2 E}{a^2\rho}, \qquad w_r^2=\frac{\zeta^2 E}{r^2\rho}\cdot\frac{1}{1-\sigma^2}, \tag{3}$$

where ζ is the roots of the equation which contains Bessel functions, and takes a value of 2.03 when $\sigma=0.27$. When considered the coupling, the *r. a. f.* must be described as follows:

$$w_1^2=\frac{\pi^2 E}{b^2\rho}\ u_1(p^2,\eta), \qquad w_2^2=\frac{\pi^2 E}{b^2\rho}\ u_2(p^2,\eta), \tag{4}$$

where $p^2=a^2/b^2=a^2\zeta^2/\pi^2r^2(1-\mu^2)$

and η is three dimentional coupling coefficient.

1) When $a/r\to o$, we get

$$w_1=w_r,\ w_2^2=\frac{\pi^2 E}{a^2\rho}\cdot\frac{1}{1-\eta^2}. \tag{5}$$

w_2 represents the *r. a. f.* of thickness vibration of circular disc, and so it must be the same representation with the next formula

$$w_t^2=\frac{\pi^2}{t^2}\cdot\frac{\lambda+2\mu'}{\rho}=\frac{\pi^2 E}{t^2\rho}\cdot\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}, \tag{6}$$

where λ and μ' are Lamé's elastic constants. From the above two equations, we deduce the equation

$$\eta^2=\frac{2\sigma^2}{1-\sigma}=\frac{2\mu^2}{1-\mu}. \tag{7}$$

2) When $a/r\longrightarrow\infty$,

$$w_1=w_2,\ w_2^2=\frac{\pi^2 E}{b^2\rho}\cdot\frac{1}{1-\eta^2}=\frac{\zeta^2 E}{r^2\rho}\cdot\frac{1}{(1+\mu)^2(1-2\mu)}, \tag{8}$$

w_2 represents the *r. a. f.* of extentional radial vibration of infinitely long cylinder. From Airey's theory (J. R. Airey: *Arch. d. Mathem. u. Phys.* 3. 20 (1913), 294.), the *r. a. f.* of the same vibration is represented by

$$w^2=\frac{\xi^2 E}{r^2\rho}\cdot\frac{1-\sigma}{(1+\sigma)(1-2\sigma)}, \tag{9}$$

where ξ is the roots of the equation which contains Bessel functions. From the above two equations, using the relation $\mu=\sigma$, we obtain the equation

$$\xi^2=\frac{\zeta^2}{1-\mu^2}. \tag{10}$$

Such relations could be verified by many experimental results employing $BaTiO_3$ ceramic vibrators.

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14. Study on High Dielectric Constant Ceramics. (XVI)

Langevin Type $BaTiO_3$ Ceramic Vibrator

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A Langevin type vibrator using $BaTiO_3$ ceramics as the electrostrictive