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THEORETICAL STUDY

OF

STOCHASTIC FLUCTUATIONS IN AN AT-POWER REACTOR

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Abstract

A theoretical study is made on stochastic fluctuations in power reactors. The theory is demonstrated by treating a specific but widely existing example of non-boiling liquid--cooled and -moderated reactor operated with either natural convection or forced circulation.

In Chapter I, attention is first directed to the thermodynamical analysis of the elementary transport processes of the energy released by fissions through the fuel and coolant. A set of equations for the lumped reactor system are derived for the state variables, i.e., coolant flow-speed, coolant temperature and fuel temperature. A noise source bringing about fluctuations of the coolant flow-speed is assumed to be the most influential among all the noise sources. This idea is taken into the stochastic model by using the Langevin method.

As a result of the above treatment, a stochastic description is given for the transport processes of nuclear, thermal and hydraulic quantities by deriving the Markoffian master equation and subsequently the moment equations. The theory is examined by reference to the neutron noise spectrum for the two kinds of the coolant flow pattern. It is shown that the theory can describe quantitatively, at least in part, the actual observed noise, in particular the anomalous growth of neutron noise spectra at lower frequencies.

In Chapter II, the model is extended to three different reactor systems: (a) where there exists a relaxation process

corresponding to the effect of buoyant flow; (b) where a control or fuel element vibrates randomly, due to coolant flow-rate fluctuations; (c) where there are fluctuations in the inlet temperature with a non-white spectrum.

The noise spectra are derived for various state quantities with use made of the Langevin procedure. The theory is illustrated by referring chiefly to the neutron noise spectra, and comparing with the results of observations. It is shown that the noise sources in question contribute significantly to the spectra, as compared with a low frequency component due to an inherent noise source in the coolant flow. In particular, a strong resonance peak of the spectra arises from the coupling between the random mechanical vibrations and the coolant flow-rate fluctuations.

In Chapter III, numerical calculations have been made of both noise spectra and relative standard deviations for fluctuations in various quantities, such as neutron number, coolant temperature and coolant flow-speed. The calculations are based on a stochastic model of Chap. I and carried out for the case of natural convection cooling, at various values of reactor power up to 100 kW. Some of the results are compared with experiment. It is shown that the low-frequency fluctuations, caused by coolant flow-speed fluctuations, become significant at increased power levels, and above several kW, the fluctuations in flow are visibly reflected in those of neutron number.

Contents

Nomenclature

$\mathbf{A}(\boldsymbol{\alpha})$	Drift vector
$\boldsymbol{\alpha}$	State vector
b	Relaxation constant of coolant temperature fluctuations
C	Random variable for the number of precursors
C_0	Steady-state value of the number of precursors
c_v^c	Heat capacity of coolant at constant volume
c_p^c	Heat capacity of coolant at constant pressure
c_v^F	Heat capacity of fuel
$C_{Ni}(\omega)$	Coherence function for the spectrum $P_{Ni}(\omega)$
$C_{NM}(\omega)$	Coherence function for the spectrum $P_{NM}(\omega)$
$C_{NQ}(\omega)$	Coherence function for the spectrum $P_{NQ}(\omega)$
$2\mathbf{D}$	Diffusion matrix
$2D_{ii}$	Noise source amplitude for inlet temperature
$2D_{MM}$	Noise source amplitude for coolant temperature
$2D_{NN}$	Noise source arising chiefly from nuclear fission
$2D_{QQ}$	Noise source bringing about coolant flow-rate fluctuations
$2D_{XX}$	Noise source bringing about random mechanical vibrations
d	Coefficient representing the contribution of coolant flow-rate fluctuations to coolant temperature fluctuations
$\mathbf{E}$	Unit matrix
F	Random variable for temperature of fuel
f	Coefficient representing the contribution of random mechanical vibrations to coolant flow-rate fluctuations
$\mathbf{G}(i\omega)$	Green function matrix
$\mathfrak{g}$	Gravity constant
g	Coefficient representing the contribution of coolant flow-rate fluctuations to random mechanical vibrations
h_0	Heat transfer coefficient

h_t Heat transfer coefficient, $h_t = h_0 / C_v^F V_F \rho_F$

$I(\omega)$ Input noise source spectrum

ℓ Neutron generation time

M Random variable for temperature of coolant

N Random variable for the number of neutrons

N_0 Steady-state value of the number of neutrons

P Reactor power

$P(x,t)$ Pressure

P_c Average pressure in coolant

$\mathbf{P}(\omega)$ Noise spectrum matrix

$P_{FF}(\omega)$ Noise spectrum for fuel temperature

$P_{ii}(\omega)$ Noise spectrum for inlet temperature

$P_{MM}(\omega)$ Noise spectrum for moderator-coolant temperature

$P_{Ni}(\omega)$ Cross noise spectrum of neutron number-inlet temperature fluctuations

$P_{NM}(\omega)$ Cross noise spectrum of neutron number-coolant temperature fluctuations

$P_{NN}(\omega)$ Noise spectrum for the number of neutrons

$P_{NQ}(\omega)$ Cross noise spectrum of neutron number-coolant flow-speed fluctuations

$P_{QQ}(\omega)$ Noise spectrum for coolant flow-speed

p Coefficient representing the contribution of inlet temperature fluctuations to coolant temperature fluctuations

Q Random variable for flow-speed of coolant

$\mathfrak{q}$ Heat current

q Heat energy released by one fission

S Neutron source strength

S_c Moderator-coolant flow area

$T(x,t)$ Absolute temperature

T_1 Random variable for inlet temperature

$T(i\omega)$	Transfer function
$T_0(i\omega)$	Zero-power-reactor transfer function
V_c	Moderator-coolant volume
V_F	Fuel volume
$\mathbf{v}(\mathbf{x},t)$	Velocity
v_c	Average flow-speed of coolant
v_2	Coolant outlet flow-speed
v_1	Coolant inlet flow-speed
v_c^0	Steady-state value of coolant mean flow-speed
v_2^0	Steady-state value of coolant outflow-speed
$X(t)$	Displacement of randomly vibrating element
$\mathbf{X}_c$	Average external force exerted on a unit mass of coolant
x_B	Coefficient arising from buoyancy
$\boldsymbol{\alpha}$	Deviation of $\mathbf{a}$ from steady-state value $\mathbf{a}_0$
α	Adjustable parameter to express relative standard deviation of coolant flow fluctuations for $x_B=0$
α_i	Relative standard deviation of inlet temperature fluctuat
α_m	Coefficient of cubic expansion
α_θ	Temperature coefficient of reactivity
β	Delayed neutron fraction
γ	Ratio of density of coolant outflow to that of the inflow
η	Ratio of heat capacity of fuel to that of coolant
θ_1	Inlet temperature
θ_2	Outlet temperature
θ_c	Mean temperature of coolant, $T_c=273+\theta_c$
θ_F	Mean temperature of fuel, $T_F=273+\theta_F$
θ_1^0	Steady-state value of inlet temperature
θ_2^0	Steady-state value of coolant outlet temperature
θ_c^0	Steady-state value of coolant mean temperature
θ_F^0	Steady-state value of fuel mean temperature

κ	Compressibility
$\mathbf{\Lambda}$	Relaxation matrix
Λ_0	Relaxation constant, $(=\Lambda_Q+\Lambda_x)$
Λ_a	Absorption rate, $v\Sigma_a$
Λ_c	Non-fission removal rate, $v\Sigma_c$
Λ_f	Fission rate, $v\Sigma_f$
Λ_i	Relaxation constant of inlet temperature fluctuations
Λ_Q	Relaxation constant of coolant flow-rate fluctuations
Λ_x	Relaxation constant of random mechanical vibrations
λ	Delayed neutron precursor decay constant
μ_1	Coefficient representing the contribution of fuel temperature fluctuations to fission rate
μ_2	Coefficient representing the contribution of coolant temperature fluctuations to fission rate
μ_3	Temperature reactivity coefficient of fuel
μ_4	Temperature reactivity coefficient of coolant
ν_0	Number of prompt neutrons emitted per fission
ν_1	Number of precursors emitted per fission
$\tilde{\boldsymbol{\xi}}(t)$	Random driving force vector
$\tilde{\xi}_i(t)$	Random driving force of inlet temperature fluctuations
$\tilde{\xi}_M(t)$	Random driving force of coolant temperature fluctuations
$\tilde{\xi}_Q(t)$	Random driving force of coolant flow-rate fluctuations
$\tilde{\xi}_x(t)$	Random driving force of random mechanical vibrations
ρ	Reactivity, $(\bar{k}-1)/\bar{k}$
$\rho(x,t)$	Density
ρ_0	Steady-state value of the mean density of coolant
ρ_c	Average density of coolant
ρ_F	Average density of fuel
σ^2	Variance

Introduction

On account of the stochastic nature of neutron chain processes, the number of neutrons present in a reactor fluctuates. The fluctuations of neutron population in a reactor at zero-power and their space- and time-correlation have been studied to full extent in theory as well as by experiments(1). The zero-power theories, however, are applicable only to a reactor operated at very low power-levels where thermal and hydraulic effects add only a small contribution to the reactivity.

One of the recent problems in reactor noise analysis is, therefore, to develop a theory of stochastic fluctuations in power reactors operated at higher power levels, as well as to investigate the fluctuations experimentally. The present paper is a theoretical attempt in this direction.

Until now, noise spectrum measurements have been performed in many liquid-cooled and -moderated reactors operated at higher power-levels by observing fluctuations in the neutron population, the fission gamma-rays, the coolant temperature and the coolant flow-rate(2)-(12). The experimental results show unusual frequency components of the neutron noise spectra, which can hardly be explained by the zero-power theory; for example, an anomalous growth in the lower frequency region, the appearance of resonance peaks at specific frequencies and so on. The measurements also give the following pieces of information on these components. The lower frequency component increases in its amplitude with rising power-level[2][3]-[6][8][11], and the shapes of these unusual components change completely according to the type of the coolant flow, i.e., natural convective cooling or forced circulation[6][9][12]. The experimental results further show that there is a strong similarity between the noise spectral shapes of the neutron fluctuations and of the coolant temperature fluctuations[4][9].

From these results, it is considered as follows. Neutron fluctuations are modulated by internal reactivity fluctuations induced mainly by fluctuations of the coolant temperature through the temperature coefficient of reactivity. And the temperature fluctuations, probably, arise from the statistical nature of such transport phenomena as heat transfer from fuel to coolant, convective heat currents created by the coolant mass flow, steam void generation in the BWR and so on. We note here that these phenomena are particular to flowing fluid or closely related to the property thereof.

Now, there have been in the past a number of attempts directed toward developing a theory of reactor noise[13]-[24]. Above all, Nomura's noise analysis with the BWR is a study of high interest[24]. He showed that the dominant noise source is the random generation of steam voids and that the neutron noise spectrum is determined in large part by the behavior of the voids in the two-phase flow. His theoretical results well describe the power spectrum of neutron fluctuations in the JPDR. For other stochastic models of a reactor, the stochastic equations of the transport processes of heat energy have been given intuitively for the temperatures of either the whole reactor system[(13)-(18)] or solely of the fuel and coolant[(19)(20)(22)*]. But the equations for the above two kinds of temperature are of such form that they cannot be derived without the assumption of little or no variations in the coolant flow-rate. This derivation will be given in Eq. (1.26)'. Thus their stochastic models for an at-power reactor are described, principally, for the heat conduction phenomena in the fuel, i.e., the heat energy production by fission and the heat transfer to the coolant, which are also the noise sources constituting a characteristic feature of these models. Consequently, it follows that the theories in question are applicable only to a reactor operated at low power-levels where the coolant flows so quietly as to have little influence upon the reactivity.

The principal purpose of the present paper is therefore to develop a theory of power reactor noise applicable to at-power reactors, such as for example a liquid-cooled and -moderated reactor operated at any level from very low power to full power with natural convection or forced circulation without coolant boiling. A reactor of this type is introduced into the theory as a power reactor model.

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* In Refs. (21) and (23), a theory of power reactor noise is developed without the explicit use of stochastic equations for state variables.

The theory will be presented in Chapter I.

The basic concept adopted in the model is as follows[25]. The coolant flowing through a reactor core is usually accompanied by random variations in the flow-rate, which are responsible for fluctuations in the coolant temperature, and thereby in the internal reactivity, by virture of the temperature coefficient of reactivity. In order to ascertain how satisfactorily the present model can account for the anomalous growth of the noise spectra at lower frequencies, a semi-quantitative analysis has been made in the frequency domain for the power spectral density of the neutron fluctuations in a reactor operating under either natural convection or forced circulation, by comparing with the observations reported by many investigators[2]-[12].

In Sec. 4 of Chap. I, this analysis will be described.

Now, a convincing number of experimental results obtained on noise spectra, chiefly related to neutron fluctuations, point toward the existence of some intrinsic noise sources and/or reactivity feedbacks besides those taken up in Chap. I, which play a role in present-day power reactors. For example, Yamada and Kage[6] have shown that a resonance peak observed in the spectra on the Hitachi Training Reactor (HTR) was due to the reactivity change caused by random vibrations of the control rods, induced by coolant-flow fluctuations. A similar resonance peak was also observed at the Oak Ridge Research Reactor (ORR) by Stephenson et al.[26] and Robinson[7]. Earlier, Boardman[27] had pointed out that in the Dounreay Fast Reactor (DFR) only a minor portion of the power fluctuations were due to the inlet coolant temperature fluctuations, and that there was a large low-frequency component in the reactor--power noise that was independent of the inlet coolant temperature instability. Experimental results obtained in other reactors have been reviewed by Thie[28].

Recently, Kosály and Williams[23] have made an attempt to deal with the neutron fluctuations induced by the inlet temperature fluctuations and the random mechanical vibrations of a control rod. Their model of a reactor is based on the results of Robinson[7], who derived a distributed-parameter model to describe the dynamic behavior of a system with the aid of one-dimensional temperature equations for the fuel and coolant, where heat energy is produced by fission and transferred from the fuel to the coolant by conduction. For the inlet temperature fluctuations, the assumption of white-noise is taken, and for the reactivity changes due to the fluctuations, the functions given by Robinson are used. On the other hand, the reactivity changes due to the random mechanical vibrations are expressed in terms of the Lorentzian type of noise spectrum, which has been derived from the theory of random vibrations, making use of the work of Williams[29] and Reavis[30]. This form of spectrum which is the same as expressed by Eq. (2.38), is applied only for angular frequencies near a resonance peak, as will be seen later. It has been pointed out that the strong ascendance of the spectra towards decreasing frequencies at the left hand side of the peaks cannot be understood without presuming other possible noise sources.

On the basis of a stochastic model of Chap. I, in which a noise source briging about fluctuations of the coolant flow-speed had been taken into account, the present author studied theoretically an effect of possible noise sources in non-boiling liquid reactors on stochastic fluctuations in various state quantities[31]. The model is extended to three different reactor systems: (a) where there exists a relaxation process corresponding to the effect of buoyant flow; (b) where a control or fuel element vibrates randomly, due to coolant flow-rate fluctuations; (c) where there are fluctuations in the inlet temperature with a non-white spectrum.

The theory and its application will be presented in Chapter II.

Now the theoretical treatment and analysis in Chapters I and II for power reactor noise has brought out many problems that require further investigation.

Some examples will be given in Sec. 5 of Chap. II.

In order to ascertain how quantitatively the present stochastic model can describe the experiments, numerical analysis was made of both noise spectra and relative standard deviations for stochastic fluctuations in a natural convection non-boiling light-water reactor[32]. Some of the results are compared with the observations[4][5][8][33][34].

The detail of the analysis will be described in Chapter III.

The present analysis was treated the case of natural convection cooling, but it is possible to apply our stochastic model to the case of forced circulation. When more experimental data are available, it should be very interesting to compare with such data the numerical calculations based on our stochastic model.

Recent interest in power noise investigations stems from the need for a gaining a better understanding of the stochastic fluctuations in an at-power reactor in order to assemble information on the reactor kinetic parameters and/or to detect anomalous behavior of reactor components. Actually, a number of experimental studies have been performed with various types of reactor at many different power levels[35]. One of the problems that have been drawing attention in the field of reactor noise analysis is therefore to formulate simple theoretical models of at-power reactors that provide results agreeing acceptably well with the observed data. Such a problem is left future investigation.

CHAPTER I

Theory of Stochastic Fluctuations in an At-power Reactor

§ 1. Introduction

In order to develop a theory of power reactor noise, there appear to be two fundamental problems which require examination. The first is to investigate the noise sources, namely the random events occurring in the thermal and hydraulic transport processes of heat energy released by fissions. The second is to establish a set of stochastic equations that describe thermodynamically the state of the coolant and fuel.

The principal aims of the present chapter is therefore to obtain solutions to the two fundamental problems, for the purpose of developing a theoretical model of a reactor applicable to at-power reactors, such as for example a liquid--cooled and -moderated reactor operated at any level from very low power to full power with natural convection or forced circulation without coolant boiling. A reactor of this type is introduced into the theory as a power reactor model.

First of all, we shall study the mean behavior of this reactor system by deriving the transport equations for mass, momentum and energy. In Sec. 2.1, a set of equations that express thermodynamically the non-equilibrium states of the coolant and fuel are introduced in the local forms with the aid of the theory of non-equilibrium thermodynamics(36)(37).

In Sec. 2.2, these equations are transformed into equations of an equivalent lumped system and finally reduced to a set of equations for the three state variables, i.e., temperature and flow-velocity of the coolant, and temperature of the fuel.

This reductive derivation is performed by making use of the physical simplification of the present reactor model. We note here that the set of equations are connected with the conservation laws for the energy in the fuel and the momentum and energy in the coolant, including the conservation law of the mass, and further that every coefficient is expressed strictly in terms of the system parameters of this reactor model. Therefore it is clear that the set of equations obtained finally are soundly founded.

In Sec. 2.3, the set of equations are transformed into the dimensionless forms convenient for stochastic description, and the resulting equations are examined by observing the dynamical behavior of their linearized forms. Now, in order to discuss the thermal and hydraulic noise sources, we shall derive the set of stochastic equations by replacing the state variables with the corresponding stochastic state variables. Then it is easy to see that each term of the stochastic equations arises from the random occurring event and further expresses its rate of occurrence.

In Sec. 2.4, a number of noise sources are discussed in reference to a representative modern reactor, by classifying them into three kinds. The first is the well-known nuclear noise sources arising from the branching processes within the neutron chain reactions(1). The second is the thermic internal noise sources due to the above-mentioned random events occurring in the transport processes of the energy. The third is the external noise sources of a kind that cannot be predicted explicitly from the set of stochastic equations, such as for example, a noise source bringing about fluctuations of the coolant flow-rate and the random vibrations of control or fuel plates.

The above-mentioned noise source bringing about flow fluctuations, which seems to an inherent noise source in fluid flow, is considered to be the most influential noise source for all the fluctuations considered as a whole in the present reactor model. This idea is taken into the model as follows. Assuming that this noise source has a white-noise spectrum, and using the stochastic equation for the coolant flow-rate as the Langevin equation, we can relate the amplitude of the auto-correlation for this noise source to a variance of fluctuations of the coolant flow-rate with use made of the Einstein relation. The variance is easily estimated by the standard deviation of the fluctuations, which appears in the theory as a phenomenological parameter.

Now, the general formalism of the stochastic theory has been presented in detail(38)(39). A straightforward application of this formalism to power reactor noise has been made by using the Markoffian master equation method and/or the Langevin method(13)-(23). Among these, Saito's works are representative of the practical application of the methods.

In the present paper as well, the Markoffian formalism is used in a similar, but slightly more compact, fashion. The present stochastic processes, however, are described in such manner as to make its physical nature as clear as possible, by giving explicit forms to such quantities as transition probability, relaxation matrix and diffusion matrix.

In Sec. 3.1, the transport processes of nuclear, thermal and hydraulic quantities are written in an explicit form of the Smoluchowski equation (Markoffian master equation), so that the moment equations are derived. By assuming a quasi--linear nature for the processes, we obtain in Sec. 3.2 the spectral density matrix as well as the first- and second-moment equations.

Section 4 is devoted to illustrative examination of the theory and model. An example is given for a power reactor operating in either natural convection or forced circulation. A discussion is presented on the neutron noise spectrum by giving attention to its frequency dependence in the lower frequency region, such as its slope and amplitude. The theoretical results are compared with the actually observed spectra measured in at-power reactors[(2)-(8)(10)-(12)].

§ 2. Transport Processes of Heat Energy in a Reactor

2.1. Mass, Momentum and Energy Transport Equations

In the present model, the coolant is considered to be a single component system, in which reactions such as void generation do not occur. The mass density $\rho(\mathbf{x},t)$ obeys the equation of continuity:

$$\frac{\partial}{\partial t}\rho + \nabla(\rho \mathbf{v}) = 0 \quad , \tag{1.1}$$

where $\mathbf{v}(\mathbf{x},t)$ is the velocity of the coolant. The momentum is carried only by mass flow, and produced by the volume force $\int_V \rho \mathbf{X} d\mathbf{x}$ and by the surface force $\int_S \boldsymbol{\sigma} d\mathbf{S}$, where $\mathbf{X}$ denotes the external force exerted on a unit mass such as buoyancy and gravity, while $\boldsymbol{\sigma}$ represents the stress tensor exerted on a unit area. In what follows, we shall assume that viscous flow has negligible influence on the state of the coolant, so that we have $\boldsymbol{\sigma} = -P\mathbf{1}$, where P and $\mathbf{1}$ are pressure and unit dyadic respectively. Then the equation of motion can be written in the form

$$\rho\frac{\partial}{\partial t}\mathbf{v} + \rho\mathbf{v}\cdot\nabla\mathbf{v} = \rho\mathbf{X} - \nabla P \ . \tag{1.2}$$

This is a Navier-Stokes equation involving no viscosity.

Next, we obtain an expression of the thermodynamical state of a coolant that is exchanging heat and matter with the surroundings. The expression is written in the form of the balance equation for the internal energy, involving the heat and diffusion flow. However, with help of the phenomenological laws between the thermodynamic fluxes and forces, we can write down an alternative form of the energy transport equation, using the Maxwell and Gibbs relations(36)(37). In a system in question, we shall take the following equations for the absolute temperature $T(\mathbf{x},t)$ and the pressure $P(\mathbf{x},t)$, i.e.,

$$\rho C_v \frac{\partial}{\partial t} T + \rho C_v \mathbf{v} \cdot \nabla T = - \frac{\alpha_m}{\kappa} T \, \nabla \cdot \mathbf{v} - \nabla \cdot \mathbf{q} \,, \tag{1.3}$$

$$\rho C_v \frac{\partial}{\partial t} P + \rho C_v \mathbf{v} \cdot \nabla P = - \frac{1}{\kappa} \rho C_p \nabla \cdot \mathbf{v} - \frac{\alpha_m}{\kappa} \nabla \cdot \mathbf{q} \,, \tag{1.4}$$

where C_v and C_p are the heat capacity of the coolant at a constant volume and that at a constant pressure respectively, while κ is the compressibility and α_m the coefficient of cubic expansion. On the right-hand side of the above equations, heat expansion and heat flow are taken into account.

The symbol $\mathbf{q}$ represents the second law heat flux flowing out of the system by heat conduction. The linear phenomenological equation for $\mathbf{q}$ is

$$\mathbf{q} = -L \nabla \ln T \,, \tag{1.5}$$

using $\nabla \ln T$ for the force, and the heat transfer coefficient L for the phenomenological coefficient. Regarding two neighboring systems as homogeneous, we obtain

$$\nabla \ln T = \frac{\Delta T}{T} \,, \quad \Delta T = T_F - T_c > 0 \,, \tag{1.6}$$

where T_F and T_C are the uniform temperatures in fuel and coolant respectively. Hence the heat flux is given by

$$\mathbf{q} = -\left(\frac{L}{T}\right)(T_F - T_c) = -h(T_F - T_c) \,, \tag{1.7}$$

where $h=L/T$ is the heat transfer coefficient commonly used.

For the state of the fuel, in which nuclear fission and heat conduction take place, we can easily obtain for the absolute temperature the equation

$$\rho_F C_v^F \frac{\partial}{\partial t} T = - \nabla \mathbf{q}_F + q \Sigma_f^F \phi \,, \tag{1.8}$$

where ρ_F is the fuel density, C_v^F the fuel heat capacity, q the fission energy, Σ_f^F the macroscopic fission cross section and ϕ the local neutron flux. For the two homogeneous systems considered in Eq. (1.6), it follows that the heat flux $\mathbf{q}_F$ is equal to $-\mathbf{q}$ defined by Eq. (1.7).

For the reactor noise at higher power, we must consider a multi-component system such as a steam-water mixture associated with phase change. The state equations will be derived in local forms for the state variables, i.e., mass density of each chemical component, its flow velocity, temperature and pressure. These equations become more complicated, but they are indispensable for the stochastic description of this reactor noise. In particular, the kinetic equations for density and pressure will play an essential role, because we are dealing with high pressure and boiling phenomena.

2.2. Transport Equations of a Lumped System

In this section, we shall simplify the state equations obtained in the previous section into a set of equations for an equivalent lumped system. Regarding the coolant as a homogeneous system and carring out the integral $V_c^{-1}\int_{V_c} d\mathbf{x}$ over the coolant volume V_c, we obtain from Eq. (1.1),

$$\frac{d}{dt}\rho_c + \frac{1}{V_c}\int_S (\rho\mathbf{v})\cdot d\mathbf{S} = 0 \quad , \tag{1.9}$$

and from Eq. (1.2), (See Appendix I)

$$\rho_c \frac{d}{dt}\mathbf{v}_c = \frac{1}{V_c}\mathbf{v}_c\cdot\int_S (\rho\mathbf{v})d\mathbf{S} - \frac{1}{V_c}\int_S (\rho\mathbf{v}\mathbf{v})\cdot d\mathbf{S} + \rho_c\mathbf{X}_c + \frac{1}{V_c}\int_S (-P)\mathbf{1}\cdot d\mathbf{S} \; . \tag{1.10}$$

Here the average density ρ_c and velocity $\mathbf{v}_c$ are defined by

$$\rho_c(t) = \frac{1}{V_c}\int_{V_c} \rho(\mathbf{x},t)\,d\mathbf{x}\ ,\quad \mathbf{v}_c(t) = \frac{1}{\rho_c V_c}\int_{V_c} \rho(\mathbf{x},t)\mathbf{v}(\mathbf{x},t)\,d\mathbf{x}\ . \tag{1.11}$$

The average external force is

$$\mathbf{X}_c = \frac{1}{\rho_c V_c}\int_{V_c} \rho\mathbf{X}\,d\mathbf{x}\ , \tag{1.12}$$

which can be written in terms of the buoyancy force, dependent on the average temperature T_c of the coolant, and of the gravity force:

$$\begin{aligned}\rho_c\mathbf{X}_c &= (\rho_w - \rho_c)\mathbf{g} \\ &= \left[\rho_w - \rho_w\left\{1 - \frac{\alpha_m}{3}(T_c - T_w)\right\}^3\right]\mathbf{g}\ ,\end{aligned} \tag{1.12'}$$

where T_w and ρ_w are the temperature and density outside the system respectively, and $\mathbf{g}$ the gravity constant. Developing the right-hand side with respect to $\alpha_m(T_c-T_w)$ and retaining only the linear term, we have

$$\rho_c\mathbf{X}_c = \alpha_m(T_c - T_w)\rho_w\mathbf{g}\ . \tag{1.13}$$

In the same manner, we have from Eq. (1.3)

$$\begin{aligned}C_v^c\rho_c\frac{d}{dt}T_c &= \frac{C_v^c}{V_c}\left\{-\int_S T\rho\mathbf{v}\cdot d\mathbf{S} + T_c\int_S \rho\mathbf{v}\cdot d\mathbf{S}\right\} \\ &\quad - \frac{\alpha_m}{\kappa}(\nabla\cdot\mathbf{v})_0 T_c - \frac{1}{V_c}\int_S \mathbf{q}\cdot d\mathbf{S}\ ,\end{aligned} \tag{1.14}$$

and from Eq. (1.4)

$$\begin{aligned}C_v^c\rho_c\frac{d}{dt}P_c &= \frac{C_v^c}{V_c}\left\{-\int_S \rho P\mathbf{v}\cdot d\mathbf{S} + P_c\int_S \rho\mathbf{v}\cdot d\mathbf{S}\right\} \\ &\quad - \frac{C_p^c}{\kappa}\rho_c(\nabla\cdot\mathbf{v})_0 - \frac{\alpha_m}{\kappa V_c}\int_S \mathbf{q}\cdot d\mathbf{S}\ ,\end{aligned} \tag{1.15}$$

where the average temperature T_c and pressure P_c are defined by

$$T_c = \frac{1}{\rho_c V_c}\int_{V_c} \rho(\mathbf{x},t)T(\mathbf{x},t)\,d\mathbf{x}\ ,\quad P_c = \frac{1}{\rho_c V_c}\int_{V_c} \rho(\mathbf{x},t)P(\mathbf{x},t)\,d\mathbf{x}\ , \tag{1.16}$$

where again, C_v^c and C_p^c are the heat capacities of coolant at constant volume and at constant pressure respectively.

The second term on the right-hand side of Eqs. (1.14) and (1.15) is a result of the following approximate procedure. Using Eqs. (1.2) and (1.4) in a stationary state, we have

$$(\nabla\cdot\mathbf{V}) = \frac{-\alpha_m(\nabla\cdot\mathbf{q}) - C_v\kappa\rho_c^2\,\mathbf{V}\cdot\mathbf{X}}{C_p\rho_c\{1-\frac{C_v}{C_p}\kappa\rho_c\mathbf{V}^2\}} \approx -\frac{\alpha_m}{C_p\rho_c}(\nabla\cdot\mathbf{q})\ , \qquad (1.17)$$

and from Eq. (1.7)

$$(\nabla\cdot\mathbf{V})_0 \simeq \frac{1}{V_c}\int_{V_c}(\nabla\cdot\mathbf{V})d\mathbf{X} = \frac{\dot{\alpha}_m}{C_p\rho_c}\frac{1}{V_c}h_0(T_F-T_c)\ (>0)\ , \qquad (1.18)$$

where h_0 is the heat transfer coefficient and S_h the heat transfer area, i.e., $h_0 = hS_h$.

Thus we have obtained the four equations that describe almost exactly the thermodynamical state of the coolant. Further, for the model reactor operated at low power level, the fluctuations of density and pressure are not as important as those of temperature and flow-rate in determining the dynamical state of the coolant, because there is neither boiling nor high pressure. Therefore, we assume that in Eqs. (1.10) and (1.14), the density $\rho_c(t)$ can be replaced by the steady-state value ρ_0 and that Eq. (1.15) can be written in the form of a steady-state equation:

$$\frac{C_v^c}{V_c}\int_s P_0\rho_0\mathbf{V}_0\cdot d\mathbf{S} = -\frac{C_p^c}{\kappa}\rho_0(\nabla\cdot\mathbf{V})_0 - \frac{\alpha_m}{\kappa V_c}\int_s \mathbf{q}\cdot d\mathbf{S}\ , \qquad (1.19)$$

whose explicit expression can be obtained by using Eqs. (1.7) and (1.18).

Then we have the following equations to represent the thermodynamical state of the coolant.

$$\rho_0 \frac{d}{dt} \mathbf{V}_c = \frac{1}{V_c} \left\{ \mathbf{V}_c \cdot \int_S \rho \mathbf{V} d\mathbf{S} - \int_S \rho \mathbf{V} \mathbf{V} \cdot d\mathbf{S} \right\}$$

$$+ \alpha_m (\theta_c - \theta_W) \rho_W \mathbf{g} + \frac{1}{V_c} \int_S (-P_0) \mathbf{1} \cdot d\mathbf{S} \quad , \qquad (1.20)$$

$$c_V^c \rho_0 \frac{d}{dt} \theta_c = \frac{c_V^c}{V_c} \left\{ \theta_c \int_S \rho \mathbf{V} \cdot d\mathbf{S} - \int_S \theta \rho \mathbf{V} \cdot d\mathbf{S} \right\}$$

$$- \frac{\alpha_m}{K} (\nabla \cdot \mathbf{V})_0 (273 + \theta_c) + \frac{1}{V_c} h_0 (\theta_F - \theta_c) \quad , \qquad (1.21)$$

where we have used Eqs. (1.7), (1.13) and the relation $T(°K) = 273 + \theta(°C)$.

We now apply these equations to a coolant channel. The coolant flow can be regarded as a one-dimensional flow along the fuel element. Therefore the surface integral of a flux $\mathbf{J}$ can be written in terms of inflow J_1, outflow J_2 and flow area S_c, i.e.,

$$\int_S \mathbf{J} d\mathbf{S} = (J_2 - J_1) S_c \quad . \qquad (1.22)$$

Under the assumptions of

$$V_c(t) = \frac{1}{2} \{ V_1 + V_2(t) \} \; , \quad V_1 \sim \text{const.} \quad , \qquad (1.23)$$

$$\theta_c(t) = \frac{1}{2} \{ \theta_1 + \theta_2(t) \} \; , \quad \theta_1 \sim \text{const.} \; , \qquad (1.24)$$

the state equations of the coolant become

$$\frac{1}{2} \rho_0 \frac{d}{dt} V_2 = \frac{S_c}{V_c} \left[\frac{1}{2} (V_1 + V_2)(\rho_2 V_2 - \rho_1 V_1) - (\rho_2 V_2^2 - \rho_1 V_1^2) \right]$$

$$+ \alpha_m (\theta_c - \theta_W) \rho_W g + \frac{1}{V_c} \int_S (-P_0) \mathbf{1} \cdot d\mathbf{S} \quad , \qquad (1.25)$$

$$c_V^c \rho_0 \frac{d}{dt} \theta_c = \frac{c_V^c S_c}{V_c} \left[\theta_c (\rho_2 V_2 - \rho_1 V_1) - \{ (2\theta_c - \theta_1) \rho_2 V_2 - \theta_1 \rho_1 V_1 \} \right]$$

$$- \frac{\alpha_m}{K} (\nabla \cdot \mathbf{V})_0 (273 + \theta_c) + \frac{1}{V_c} h_0 (\theta_F - \theta_c) \; . \qquad (1.26)^*$$

---

* It may be worth while to compare Eq. (1.26) with the

corresponding equation in Refs. (19), (20) and (22). Assuming that there is little or no variation in the mass flow-rate of the coolant, i.e., $\rho_1 v_1 = \rho_2 v_2 = \rho_0 v_0$ (constant value), and that the thermal expansion of the coolant can be neglected, i.e., $\alpha_m=0$, Eq. (1.26) becomes

$$C_m \frac{d}{dt}\theta_c = -\frac{2v_0 C_m}{L_c}(\theta_c - \theta_1) + h_0(\theta_F - \theta_c) \quad , \qquad (1.26)'$$

where $C_m = C_v^c \rho_0 V_c$ is the total heat capacity of the coolant, and $L_c = V_c/S_c$ the length of a coolant channel.

This equation, however, remains different from the corresponding one in Refs. (19), (20) and (22), in which the coefficient of the first term on the right-hand side has been represented by $C_m v_0/L_c$, and the inlet temperature θ_1 has not been described.

These equations are the conservation laws of momentum and energy including the conservation of mass flux.

At the end of this section, we shall examine Eq. (1.8), for the fuel, in a similar manner. Carrying out the volume integral $V_F^{-1}\int_{V_F} d\mathbf{x}$ over the fuel volume V_F, we obtain

$$\rho_F C_v^F \frac{d}{dt} T_F = -\frac{1}{V_F}\int_S \mathbf{q}\cdot d\mathbf{S} + \frac{1}{V_F}\, \varepsilon \int_{V_F} \Sigma_f^F \phi \, d\mathbf{x} \quad , \qquad (1.27)$$

where the average temperature T_F is

$$T_F(t) = \frac{1}{V_F}\int_{V_F} T(\mathbf{x}, t)\, d\mathbf{x} \quad . \qquad (1.28)$$

The fission energy production can be written in the form

$$\frac{1}{V_F}\, \varepsilon \int_{V_F} \Sigma_f^F \phi \, d\mathbf{x} = \frac{1}{V_F}\, \varepsilon v \Sigma_f N(t) \quad , \qquad (1.29)$$

where v, Σ_f and $N(t)$ are the thermal neutron speed, the macroscopic fission cross-section and the total number of neutrons present in the core. Then the law of energy conservation in the fuel is expressed by

$$\rho_F C_v^F \frac{d}{dt}\theta_F = -\frac{1}{V_F} h_0(\theta_F - \theta_c) + \frac{1}{V_F} q\nu\Sigma_f N(t) \,. \qquad (1.30)$$

which has the same form as used in the Refs. (19), (20) and (22).

As mentioned above, we have now been able to derive in definite form the set of equations for the three state variables in concise expressions of the conservation law. Every coefficient is expressed strictly in terms of the system parameters of the present reactor model. We note here that these equations are based on the various simplifications of the model.

2.3. Properties of Transport Equations

We now rewrite the set of equations in dimensionless form convenient for the stochastic description of energy and momentum. Firstly we replace with extensive values the intensive quantities used in the set of equations, taking heat energy released by one fission as an energy unit, to obtain

$$\left.\begin{aligned} F(t) &= \frac{1}{q}c_v^F \rho_F V_F \theta_F(t) \,, & M(t) &= \frac{1}{q}c_v^C \rho_0 V_c \theta_c(t) \\ Q(t) &= \frac{1}{q}c_v^c S_c \theta_2^0 \rho_2 v_2(t) \,, & Q_1 &= \frac{1}{q}c_v^c S_c \theta_1 \rho_1 v_1 \\ T_1 &= \frac{1}{q}c_v^c \rho_0 V_c \theta_1 \,, & T_w &= \frac{1}{q}c_v^c \rho_0 V_c \theta_w \end{aligned}\right\} (1.31)$$

where θ_2^0 is a steady-state value of $\theta_2(t)$.

Secondly we rewrite Eqs. (1.30), (1.26) and (1.25) in the form

$$\frac{d}{dt}F(t) = v\Sigma_f N(t) - h_t(F-\eta M) \quad , \tag{1.32}$$

$$\frac{d}{dt}M(t) = M(d_2Q-d_1Q_1)-\left\{d_2Q(2M-T_1)-d_1Q_1T_1\right\}-(E_0+eM)$$
$$+h_t(F-\eta M) \quad , \tag{1.33}$$

$$\frac{d}{dt}Q(t) = \frac{1}{d_2}(\gamma d_1Q_1+d_2Q)(d_2Q-d_1Q_1)-\frac{2}{d_2}(d_2^2Q^2-\gamma d_1^2Q_1^2)$$
$$+ x_B(M-T_w) + Q_p \quad , \tag{1.34}$$

where a number of coefficient are newly defined as follows:

$$\left.\begin{aligned}
h_t &= \frac{1}{c_v^F v_F \rho_F}h_0 \quad , \quad \eta = \frac{c_v^F \rho_F v_F}{c_v^c \rho_0 v_c} \\
d_1 &= \frac{q}{c_v^c \rho_0 v_c \theta_1} \quad , \quad d_2 = \frac{q}{c_v^c \rho_0 v_c \theta_2^0} \\
E_0 &= \frac{\alpha_m}{\kappa}(\nabla\cdot\mathbf{V})_0\frac{273V_c}{q} \quad , \quad e = \frac{\alpha_m}{\kappa}(\nabla\cdot\mathbf{V})_0\frac{1}{c_v^c \rho_0} \\
x_B &= \frac{2}{\rho_0^2 v_c}\alpha_m \rho_w g s_c \theta_2^0 \rho_2 \quad , \quad \gamma = \frac{\rho_2}{\rho_1} \simeq 1-\alpha_m(\theta_2^0-\theta_1) \\
Q_p &= \frac{2}{\rho_0 v_c q}c_v^c s_c \theta_2^0 \rho_2 \int_s (-P_0)\mathbf{1}\cdot d\mathbf{s}
\end{aligned}\right\} \tag{1.35}$$

We further investigate the kinetic behavior of the state variables about the small deviations around the steady-state. The deviations are defined by

$$\left.\begin{aligned}
\widetilde{N}(t) &= N(t) - N_0 \quad , \quad \widetilde{F}(t) = F(t) - F_0 \\
\widetilde{M}(t) &= M(t) - M_0 \quad , \quad \widetilde{Q}(t) = Q(t) - Q_0
\end{aligned}\right\} , \tag{1.36}$$

where N_0, F_0, M_0 and Q_0 are the steady state values of the variables. Taking the linear approximation, it follows that

$$\frac{d}{dt}\tilde{F} = \Lambda_{f0}\tilde{N} - (h_t - \mu_1 N_0)\tilde{F} + (h_t\eta + \mu_2 N_0)\tilde{M} \quad , \tag{1.37}$$

$$\frac{d}{dt}\tilde{M} = h_t\tilde{F} - (b + h_t\eta)\tilde{M} - d\tilde{Q} \quad , \tag{1.38}$$

$$\frac{d}{dt}\tilde{Q} = x_B\tilde{M} - \Lambda_Q\tilde{Q} \quad , \tag{1.39}$$

where the fission rate Λ_f dependent on temperature is expanded in the form

$$\Lambda_f \simeq \Lambda_{f0} + \mu_1\tilde{F} + \mu_2\tilde{M} \quad , \tag{1.40}$$

while b, d and Λ_Q are defined by

$$b = 2d_2Q_0 + e \quad , \quad d = d_2(M_0 - T_1) \quad , \quad \Lambda_Q = (3-\gamma)d_2Q_0 \tag{1.41}$$

Thus, a set of linearized equations (1.37)-(1.39) have been derived definitely together with the various relaxation constants. These constants are specified completely by the many system parameters of the present reactor model, and are connected with the physical events related to the transport processes of heat energy. For instance, h_t is the heat transfer coefficient between the fuel and coolant, b and Λ_Q strongly depend on the coolant flow-rate, x_B arises from the buoyancy effect and d from the convective mass flow. Such knowledge of the relaxation constants will be very helpful in understanding the time-dependent behavior of the set of kinetic equations (1.37)-(1.39).

We should here take note of the coupling between the deviations of the coolant temperature and flow-rate, which is expressed by the two coefficient d and x_B. For at-power reactors in natural convection operation, this coupling plays an important role in the stability of reactor dynamics. Evidently, this coupling effect will be observed in experiments of power reactor noise.

2.4. Noise Sources

Now let us discuss the noise sources characteristic of stochastic fluctuations in the present reactor model. In the present paper, noise sources will be classified into the following three kinds. First is the nuclear noise sources, which arise from the neutron branching processes. These have been well described by the zero-power theory and its related experiment[1].

Second is the thermic internal noise sources, which are due to the physical events relevant to the transport processes of heat energy released by fissions. In order to derive this kind of noise source, we shall transform the set of equations (1.32)-(1.34) into a set of stochastic equations by replacing the state variables used therein the corresponding stochastic state variables. It will then be easily seen that each term of the stochastic equations corresponds to the random occurring physical event and its rate of occurrence, as has been mentioned in the previous section. All of these events are related to the transport processes of the energy. On account of the statistical nature of these random events, it is assumed that all of them occur at random so as to increase or decrease the related stochastic variables by one energy unit, being of magnitude q. The rates of occurrence of these events can, therefore, be determined by the individual terms of the set of stochastic equations (1.32)-(1.34).

Third is the external noise sources which are those of a kind that can not be derived from the set of stochastic equations. As actual examples, we can mention such noise sources as bring about fluctuations of some physical quantities in the coolant, i.e., flow-rate, inlet temperature, inlet flow-rate and so on. But their statistical nature is at present almost unknown. As other examples, we can give the random mechanical vibrations of control rods or fuel plates.

Now, some of these external noise sources appear to play an essential role in the present stochastic model, i.e., a liquid-cooled and -moderated at-power reactor operated with either natural convection or forced circulation. From among these noise sources we shall choose one that is predominant and which would appear to affect the fluctuations as a whole more significantly than the other noise sources, for instance, prompt neutron generation by fission at very low power level[1] and steam void generation in the BWR[24].

Relevant phenomena are considered as follows. Fluctuations of the coolant flow-rate are invariably present at any power--level in any at-power reactor of the present type. This is naturally expected from the fact that liquid flow through a narrow channel can scarcely be free from flow-rate fluctuations. These fluctuations of flow-rate, namely of heat removal rate, cause temperature fluctuations of the coolant and consequently lead to significant reactivity input through the negative temperature coefficient. Accordingly, a predominant noise source in the present stochastic model is assumed to be that which causes fluctuations of the coolant flow. This noise source can be considered inherent in the statistical nature of turbulent flow and hence its complete description should be almost impossible even with the theory of turbulence, and far less with the present transport theory.

Nevertheless, the phenomenon can be easily taken into account in the present stochastic model by using the following Langevin equation for fluctuations of the coolant flow-rate,

$$\frac{d}{dt}\,\widetilde{Q}(t) = -\Lambda_Q \widetilde{Q}(t) + \widetilde{\xi_Q}(t) \quad . \tag{1.42}$$

This corresponds to Eq. (1.39), except that the external force x_B is neglected for the sake of simplicity. We will not inquire into the origin of the random driving force $\widetilde{\xi}_Q(t)$, but we assume that its correlation time is infinitely short, namely

$$\langle \widetilde{\xi_Q}(t_1)\widetilde{\xi_Q}(t_2) \rangle = 2D_Q'\delta(t_1-t_2) \quad . \tag{1.43}$$

Further, we introduce the Einstein relation

$$2D_Q' = 2\Lambda_Q \langle \widetilde{Q}^2 \rangle \quad . \tag{1.44}$$

This variance of $\widetilde{Q}$ can be related to a standard deviation α expressed in percentage by using the definition (1.31) for $Q(t)$:

$$\frac{\sqrt{\langle \widetilde{Q}^2 \rangle}}{Q_0} \times 100 = \frac{\sqrt{\langle \widetilde{v}_2^2 \rangle}}{v_2^0} \times 100 = \alpha \quad . \tag{1.45}$$

Consequently we obtain the amplitude of the correlation function for the random driving force $\widetilde{\xi}_Q(t)$:

$$2D_Q' = 2\Lambda_Q(\alpha Q_0 \times 10^{-2})^2 \quad , \tag{1.46}$$

where α is introduced as parameter in the present model.

We here add some remarks on the relaxation constant Λ_Q and the relative standard deviation α for the following two cases. In the case of natural convection, the expression (1.41) for Λ_Q will hold approximately, since there is little influence from the exterior of the system. Accordingly, α is the only parameter in this case. In the case of forced circulation, the mean mass flow of coolant is kept controlled, but the mass flow itself is largely disturbed. Therefore, the fluctuations of the coolant flow are determined by physical conditions prevailing, not inside, but outside the system, so that the relaxation constant Λ_Q differs from the expression (1.41). In this case we consider both Λ_Q and α to be parameters.

§ 3. Stochastic Fluctuations in an At-power Reactor

3.1. Markoffian Description

In this section, a stochastic formulation is made on the assumption that the present random processes are Markoffian. First, we shall define a state vector $\boldsymbol{\alpha}$, whose components are the following random variables, i.e., the number of neutrons N, the number of precursors C, the fuel temperature F, the coolant temperature M and the coolant flow-rate Q. We note that the latter three variables defined by Eq. (1.31) will be regarded as discrete values describing the number of units of heat energy where each unit is of magnitude q. Consequently, a state vector is defined by

$$\alpha = \begin{pmatrix} N \\ C \\ F \\ M \\ Q \end{pmatrix} \quad . \tag{1.47}$$

Next, we shall define the probability $w_{\alpha'\alpha}\Delta t$ of a transition from a state α' to another state α in time Δt by any given event. A number of random events have already been discussed in Sec. 2.4. The nuclear and thermic internal events are defined fully in terms of their kind and statistical nature. In order to introduce into the present Markoffian formalism the inherent event of fluctuations in the coolant flow-rate, we denote the rate of occurrence of this event by $f(Q)$, and assume that this $f(Q)$ contributes little to the relaxation of flow-rate fluctuations, but greatly to the noise sources, as has been mentioned in Sec. 2.4. All of the random events are presented in Table 1.1. The neutron removal rate Λ_c includes both capture and leakage, and also depends on the fuel and coolant temperatures, like fission rate Λ_f in the expression (1.40); $P_f(\nu)$ is the probability of ν_0 prompt neutrons, and ν_1 the precursors born in a fission.

Consequently, the transition probability $w_{\alpha'\alpha}\Delta t$ is defined as follows:

$$\begin{aligned} w_{\alpha'\alpha}\Delta t = \Big[& \Lambda_C N'\delta_{N',N+1} + S\delta_{N',N-1} + \sum_{\nu_0\nu_1} \Lambda_f N' P_f(\nu)\delta_{N',N-\nu_0+1} \times \\ & \delta_{C',C-\nu_1}\delta_{F',F-1} + \lambda C'\delta_{N',N-1}\delta_{C',C-1} + h_t(F'-\eta M') \times \\ & \delta_{F',F+1}\delta_{M',M-1} + \left\{(E_0+eM')+d_2Q'(2M'-T_1)+M'd_1Q_1\right\} \times \\ & \delta_{M',M+1} + \left\{d_1Q_1T_1+M'd_2Q'\right\}\delta_{M',M-1} + \left\{x_B(M'-T_w)+Q_p+\right. \end{aligned}$$

$$2\gamma\frac{d_1^2}{d_2}Q_1^2+\frac{1}{d_2}(\gamma d_1Q_1+d_2Q')d_2Q'+f(Q')\Big\}\delta_{Q',Q-1}+$$

$$\Big\{2d_2Q'^2+\frac{1}{d_2}(\gamma d_1Q_1+d_2Q')d_1Q_1\Big\}\delta_{Q',Q+1}\Big]\Delta t \quad , \qquad (1.48)$$

where $\delta_{i,j}$ is the Kronecker delta; and when there is no variation of a random variable in an event, the corresponding Kronecker delta is omitted, e.g., $\Lambda_C(M')N'\delta_{N',N+1}\delta_{C',C}\delta_{F',F}\delta_{M',M}\delta_{Q',Q}$ is written $\Lambda_C(M')N'\delta_{N',N+1}$.

Following the theory of random processes(38)(39), the distribution function $P(\alpha,t)$ of a state α at time t is written in the form of a Markoffian master equation, or Smoluchowski consistency condition:

$$P(\alpha,t+\Delta t) = \sum_{\alpha'} P(\alpha',t)P(\alpha'|\alpha,\Delta t) \quad , \qquad (1.49)$$

where $P(\alpha'|\alpha,\Delta t)$ is the probability of a transition from a state α' to a state α in time Δt. We assume that for small Δt this equation can be expanded into form

$$P(\alpha'|\alpha,\Delta t) = \delta(\alpha-\alpha')(1-\Gamma_\alpha\Delta t) + \Delta t\, W_{\alpha'\alpha} \quad , \qquad (1.50)$$

where $\Gamma_\alpha\Delta t$ is the total transition probability out of α in time Δt

$$\Gamma_\alpha\Delta t = \sum_{\alpha'} W_{\alpha\alpha'}\Delta t$$

$$= \Big\{\Lambda_C N + S + \Lambda_f N + \lambda C + h_t(F-\eta M) + (E_0+eM) +$$

$$d_2Q(2M-T_1) + Md_1Q_1 + d_1Q_1T_1 + Md_2Q + x_B(M-T_w) +$$

$$Q_P + 2\gamma\frac{d_1^2}{d_2}Q_1^2 + (\gamma d_1Q_1+d_2Q)Q + f(Q) + 2d_2Q^2 +$$

$$\frac{1}{d_2}(\gamma d_1Q_1+d_2Q)d_1Q_1\Big\}\Delta t \quad . \qquad (1.51)$$

We can easily find that the transition probability (1.50) satisfies the normalization and initial conditions. The Markoffian master equation (1.49) can be written in the form

$$\frac{\partial}{\partial t}P(\alpha,t) = \sum_{\alpha'} P(\alpha',t)W_{\alpha'\alpha} - \Gamma_\alpha P(\alpha,t) \quad . \qquad (1.52)$$

The first term on the right-hand side represents the rate of transitions from all states α' to the state α, and the second the total of the transitions out of the state α.

Making use of the master equation (1.52), we can obtain useful moment equations including the effect of non-linear feedback, by introducing the n-th moment $\mathbf{D}_n$ of the transition probability (1.48) in the form

$$n!\,\mathbf{D}_n(\alpha')\Delta t = \sum_{\alpha} W_{\alpha'\alpha}(\alpha-\alpha')^n\Delta t \quad . \qquad (1.53)$$

It then follows that

$$\frac{\partial}{\partial t}\langle\alpha\rangle = \langle\mathbf{A}(\alpha)\rangle \quad , \qquad (1.54)$$

$$\frac{\partial}{\partial t}\langle\alpha^2\rangle = 2\langle\alpha\mathbf{A}(\alpha)\rangle + 2\langle\mathbf{D}(\alpha)\rangle \quad , \qquad (1.55)$$

$$\frac{\partial}{\partial t}\langle\alpha^3\rangle = 3\langle\alpha^2\mathbf{A}(\alpha)\rangle + 6\langle\alpha\mathbf{D}(\alpha)\rangle + 6\langle\mathbf{D}_3(\alpha)\rangle, \qquad (1.56)$$

where we use the drift vector $\mathbf{A}(\alpha)$ as an abbreviation for $\mathbf{D}_1(\alpha)$ and the diffusion matrix $\mathbf{D}(\alpha)$ for $\mathbf{D}_2(\alpha)$. From the expression (1.48), we obtain, respectively

$$\mathbf{A}(\alpha) = \begin{pmatrix} -\Lambda_C N + (\overline{\nu_0}-1)\Lambda_f N + \lambda C + S \\ \overline{\nu_1}\Lambda_f N - \lambda C \\ \Lambda_f N - h_t(F-\eta M) \\ M(d_2Q-d_1Q_1) - \left\{d_2Q(2M-T_1) - d_1Q_1T_1\right\} - (E_0+eM) + h_t(F-\eta M) \\ \frac{1}{d_2}(\gamma d_1Q_1+d_2Q)(d_2Q-d_1Q_1) - \frac{2}{d_2}(d_2^2Q^2-\gamma d_1^2Q_1^2) + x_B(M-T_w) + \\ +Q_p + f(Q) \end{pmatrix} \qquad (1.57)$$

$$2\,\mathbf{D}(\mathbf{Q})=\begin{pmatrix} 2D_N & \overline{\nu_1(\nu_0-1)}\Lambda_f N-\lambda C & \overline{(\nu_0-1)}\,\Lambda_f N & 0 & 0 \\ * & \overline{\nu_1^2}\,\Lambda_f N+\lambda C & \overline{\nu_1}\Lambda_f N & 0 & 0 \\ * & * & \Lambda_f N+h_t(F-\eta M) & -h_t(F-\eta M) & 0 \\ * & * & * & 2D_M & 0 \\ * & * & * & * & 2D_Q \end{pmatrix} \quad (1.58)$$

where * signifies a symmetrical element on account of the cyclic property of $2\mathbf{D}_2$, and

$$2D_N = \Lambda_C N + \overline{(\nu_0-1)^2}\Lambda_f N + \lambda C + S \;, \qquad (1.59)$$

$$2D_M = M(d_2Q+d_1Q_1)+\left\{d_2Q(2M-T_1)+d_1Q_1T_1\right\}+(E_0+eM)+h_t(F-\eta M), \quad (1.60)$$

$$2D_Q = \frac{1}{d_2}(\gamma d_1Q_1+d_2Q)(d_2Q+d_1Q_1)+\frac{2}{d_2}(d_2^2Q^2+\gamma d_1^2Q_1^2)+x_B(M-T_w)+$$

$$+Q_p+f(Q) \;. \qquad (1.61)$$

We note here that the matrix (1.58) is also the autocorrelation of the Langevin noise source[38][39], and that f(Q) in Eq. (1.61) can be written from Eq. (1.46) in the form

$$f(Q) = 2\Lambda_Q(\alpha Q_0 \times 10^{-2})^2 \;. \qquad (1.62)$$

The moment $\mathbf{D}_n$ for $n \geq 3$ can be obtained in the same manner, but it is then necessary to assume some further conditions in respect of predominant noise source f(Q).

Generally speaking, these moment equations may be analyzed by expanding $\mathbf{D}_n(\mathbf{Q})$ for deviations from the steady state values, and by adopting certain approximations to deal with what is known as the hierarchy[21][38] for the higher moments. However, it is difficult to consider that such a higher order effect should make any significant contribution to fluctuations. The effect, possibly, would become a small correction at most, and more probably negligible, because of the quasi-linear nature of most noise problems. Therefore we assume that the stochastic processes in the present theory can be treated as quasi-linear.

3.2. Quasi-Linear Processes and Noise Spectrum

Let us now define the absorption and fission rates dependent on the fuel and coolant temperature by

$$\left.\begin{aligned} \Lambda_a &= \Lambda_c + \Lambda_f \simeq \Lambda_{a0}(\theta_F^\circ, \theta_c^\circ)\{1-\beta_3\widetilde{\theta}_F - \beta_4\widetilde{\theta}_C\} \\ \Lambda_f &\simeq \Lambda_{f0}(\theta_F^\circ, \theta_c^\circ)\{1-\beta_1\widetilde{\theta}_F - \beta_2\widetilde{\theta}_C\} \end{aligned}\right\}, \tag{1.63}$$

where $\widetilde{\theta}_F$ and $\widetilde{\theta}_c$ are small deviations from the respective steady state values, and β_i $(i=1,2,3,4)$ should be determined by a large number of material parameters of the system. The total multiplication rate and the temperature coefficient α_θ are written in the form

$$-\Lambda_a + \bar{\nu}_0 \Lambda_f = \frac{1}{\ell}\{(\rho-\beta)-\alpha_1\widetilde{\theta}_F - \alpha_2\widetilde{\theta}_C\}, \tag{1.64}$$

with

$$\alpha_1 = -\beta_3(1-\rho)+\beta_1(1-\beta), \quad \alpha_2 = -\beta_4(1-\rho)+\beta_2(1-\beta), \tag{1.65}$$

and

$$\alpha_\theta = -\alpha_1 - \alpha_2 \qquad [(\delta k/k)/\text{deg.}], \tag{1.66}$$

where the customary kinetic parameters are used, i.e., neutron generation time ℓ, reactivity ρ and delayed neutron fraction β.

Furthermore, we rewrite Λ_f and $-\Lambda_a+\bar{\nu}_0\Lambda_f$ in the form

$$\Lambda_f = \frac{1}{\bar{\nu}_1}\frac{\beta}{\ell} + \mu_1\widetilde{F} + \mu_2\widetilde{M} \quad , \tag{1.67}$$

$$-\Lambda_a + \bar{\nu}_0\Lambda_f = \frac{\rho - \beta}{\ell} + \mu_3\widetilde{F} + \mu_4\widetilde{M} \quad , \tag{1.68}$$

using the definitions of Eq. (1.31), with

$$\left.\begin{aligned} \mu_1 &= -\frac{1}{\bar{\nu}\ell}\frac{q}{c_v^F\rho^F V^F}\beta_1 \quad , \quad \mu_2 = -\frac{1}{\bar{\nu}\ell}\frac{q}{c_v^C\rho_0 V_c}\beta_2 \\ \mu_3 &= -\frac{1}{\ell}\frac{q}{c_v^F\rho^F V^F}\alpha_1 \quad , \quad \mu_4 = -\frac{1}{\ell}\frac{q}{c_v^C\rho_0 V_c}\alpha_2 \end{aligned}\right\} . \tag{1.69}$$

Now we shall expand into powers of $\boldsymbol{\alpha} = \mathbf{a} - \mathbf{a}_0$ the drift vector $\mathbf{A}(\mathbf{a})$ in Eq. (1.57) and the diffusion matrix $2\mathbf{D}(\mathbf{a})$ in Eq. (1.58), and retain only the non-vanishing terms of lowest order. Using Eqs. (1.37)-(1.39) with the results (1.67) and (1.68), we obtain

$$\mathbf{A}(\mathbf{a}) \simeq \mathbf{A}(\mathbf{a}_0) - \boldsymbol{\Lambda}\boldsymbol{\alpha} \quad , \tag{1.70}$$

$$2\mathbf{D}(\mathbf{a}) \simeq 2\mathbf{D}(\mathbf{a}_0) = 2\mathbf{D} \quad . \tag{1.71}$$

The steady-state values $\mathbf{a}_0$ should be chosen so that

$$\mathbf{A}(\mathbf{a}_0) = 0 \quad , \tag{1.72}$$

where we have employed the mass conservation law for the steady state, i.e., $d_2Q_0 = d_1Q_1$, and have supposed that $f(Q_0)$ is far less effective for the equation of mean regression.

The relaxation matrix $\boldsymbol{\Lambda}$ leads to

$$\boldsymbol{\Lambda} = \begin{pmatrix} -\frac{\rho-\beta}{\ell} & -\lambda & -\mu_3 N_0 & -\mu_4 N_0 & 0 \\ -\frac{\beta}{\ell} & \lambda & -\bar{\nu}_1\mu_1 N_0 & -\bar{\nu}_1\mu_2 N_0 & 0 \\ -\Lambda_{f0} & 0 & h_t - \mu_1 N_0 & -h_t\eta - \mu_2 N_0 & 0 \\ 0 & 0 & -h_t & b + h_t\eta & d \\ 0 & 0 & 0 & -x_B & \Lambda_Q \end{pmatrix} \tag{1.73}$$

Making use of Eq. (1.72), we obtain the diffusion matrix $2\mathbb{D}$ by

$$2\mathbb{D} = \begin{pmatrix} 2D_{NN} & (\overline{\nu_1(\nu_0-1)}+\overline{\nu_1})\Lambda_{f0}N_0 & (\overline{\nu_0}-1)\Lambda_{f0}N_0 & 0 & 0 \\ * & (\overline{\nu_1^2}+\overline{\nu_1})\Lambda_{f0}N_0 & \overline{\nu_1}\Lambda_{f0}N_0 & 0 & 0 \\ * & * & 2\Lambda_{f0}N_0 & -\Lambda_{f0}N_0 & 0 \\ * & * & * & 2D_{MM} & 0 \\ * & * & * & * & 2D_{QQ} \end{pmatrix} \quad (1.74)$$

with the definition

$$2D_{NN} = \overline{\nu_0(\nu_0-1)}\,\Lambda_{f0}N_0 + 2\left(-\frac{\rho-\beta}{\ell}\right)N_0 \ , \qquad (1.75)$$

$$2D_{MM} = 2\left\{\Lambda_{f0}N_0 + d_2Q_0(M_0+T_1)\right\} \ , \qquad (1.76)$$

$$2D_{QQ} = (6+2\gamma)d_2Q_0^2 + 2\Lambda_Q(aQ_0\times 10^{-2})^2 \ , \qquad (1.77)$$

where we replace $f(Q)$ in Eq. (1.61) by Eq. (1.62), which is assumed to be the predominant noise source in this model.

Consequently, the equations of the first moments and the variances become

$$\frac{d}{dt}\langle\alpha\rangle = -\Lambda\langle\alpha\rangle \ , \qquad (1.78)$$

$$\frac{d}{dt}\langle\alpha\,\alpha\rangle = 2\mathbb{D} - \Lambda\langle\alpha\alpha\rangle - \langle\alpha\alpha\rangle\Lambda^{+} \ , \qquad (1.79)$$

where Λ^{+} is the transpose of Λ. The steady state moments $\langle\alpha\alpha\rangle$ can be obtained by solving the stationary form of Eq. (1.79), which is the concrete expression of the Einstein relation.

For the case of linear regression laws as in Eq. (1.78), we can easily find the correlation function matrix and the spectral density matrix with use made of the matrixes (1.73) and (1.74)[38][39]. We write here the spectral density matrix

$$P(\omega) = 2G(i\omega)\, 2D\, G(-i\omega)^{+} , \tag{1.80}$$

where the Green function matrix $G(i\omega)$ is given by

$$G(i\omega) = (i\omega E + \Lambda)^{-1} , \tag{1.81}$$

together with the unit matrix E.

§ 4. General Features and Neutron Noise Spectra

In what follows, we shall discuss the general features of the present stochastic theory and model, referring to the noise spectrum of neutron fluctuations. In particular, we take note of the frequency dependence of the noise spectrum in the lower frequency region, namely its slope and amplitude. And we compare these dependences with the corresponding ones of actually observed noise spectra. A comparison of this kind will permit simple verification of the validity of the model and theory.

Let us first consider the general features of the noise spectra. From Eq. (1.80), the (r,s) component is

$$P_{rs}(i\omega) = 2\sum_{i,j=1}^{5} G_{ri}(i\omega)(2D_{ij})G_{js}(-i\omega)^{+} . \tag{1.82}$$

In this model, there are two predominant noise sources, one being the neutron generation by fission constituting a nuclear noise source, the other being the thermodynamical noise source existing in the mass flow of coolant, causing flow fluctuations. The former has been already elucidated clearly by the zero power theory, and the latter is assumed in the present theory.

Therefore we take the following simplified spectrum based on the definitions (1.75) and (1.77):

$$P_{rs}(i\omega) \simeq 4D_{NN}G_{r1}(i\omega)G_{s1}(-i\omega)+4D_{QQ}G_{r5}(i\omega)G_{s5}(-i\omega) \quad . \tag{1.83}$$

The autocorrelation spectrum $P_{NN}(i\omega)$ of neutron noise is

$$P_{NN}(i\omega) \simeq 4D_{NN}\left|G_{11}(i\omega)\right|^2\left\{1+\left|\frac{G_{15}(i\omega)}{G_{11}(i\omega)}\right|^2\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} \quad . \tag{1.84}$$

The Green function $G_{11}(i\omega)$ is the transfer function with the notation $T(i\omega)$, which includes the various feedback reactivity mechanisms. Moreover we define the input noise source $I(i\omega)$ by

$$I(i\omega) \simeq 4D_{NN}\left\{1+\left|\frac{G_{15}(i\omega)}{G_{11}(i\omega)}\right|^2\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} \quad . \tag{1.85}$$

4.1. Neutron Noise Spectra in Natural Convection Cooling

As a typical example of the present model, i.e., a liquid--cooled and -moderated at-power reactor, let us consider the case of natural convection cooling with light-water. In such a reactor, the reactivity feedback largely depends on the variations in the coolant temperature, so that

$$\mu_1 \approx 0 \quad , \quad \mu_2 \approx 0 \quad \text{and} \quad \mu_3 \approx 0 \quad . \tag{1.86}$$

From Eq. (1.81), the transfer function is given by

$$T(s) = \frac{\dfrac{1}{s+\dfrac{-\rho+\beta}{\ell}}}{1-\dfrac{1}{s+\dfrac{-\rho+\beta}{\ell}}\left\{\dfrac{\lambda}{s+\lambda}\dfrac{\beta}{\ell}+\mu_4 N_0\dfrac{\Lambda_{f0}h_t(s+\Lambda_Q)}{(s+\Lambda_Q)F(s)+x_B d(s+h_t)}\right\}} \quad , \tag{1.87}$$

with

$$F(s) = (s+h_t)(s+b+h_t\eta)-h_t^2\eta \quad . \tag{1.88}$$

The transfer function is shown in Fig. 1.1 together with the noise sources as random driving forces. The physical meaning of this block diagram are clear from Eqs. (1.37)-(1.39) and from the Langevin viewpoint.

The input noise source is written in the form

$$I(i\omega) = 4D_{NN}\left\{1+(\mu_4 N_0 d)^2\left|\frac{i\omega+h_t}{(i\omega+\Lambda_Q)F(i\omega)+x_B d(i\omega+h_t)}\right|^2\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\}. \quad (1.89)$$

Assuming $\eta \ll 1$ from the definition (1.35), we take the approximation

$$F(i\omega) \simeq (i\omega+h_t)(i\omega+b) \quad , \quad (1.90)$$

and neglecting the effects of heat expansion and buoyancy from the definitions (1.35) and (1.41), i.e.,

$$\Lambda_Q \simeq b \simeq 2d_2 Q_0 \quad \text{and} \quad x_B = 0 \quad , \quad (1.91)$$

the input noise source can be expressed in the form

$$I(i\omega) \approx 4D_{NN}\left\{1+\frac{(\mu_4 N_0 d)^2}{(\omega^2+b^2)^2}\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} . \quad (1.92)$$

In the case of $\omega < b$, we obtain

$$I(i\omega) \approx 4D_{NN}\left[1+\frac{N_0}{\ell}\,\frac{\overline{\nu}}{\overline{\nu_0(\nu_0-1)}}\left\{\alpha_2(\theta_2^{\circ}-\theta_1)\right\}^2\frac{\alpha^2 10^{-4}}{2b}\right], \quad (1.93)$$

where we have used the definitions (1.31), (1.41), (1.69), (1.75) and (1.77). The second term between the brackets can be easily estimated by making use of the following values in common use:

$$\left.\begin{array}{ll}\bar{\nu} = 2.6 \quad , & \overline{\nu_0(\nu_0-1)} = 5.3 \\ \ell = 10^{-4}\ [\text{sec}] \quad , & \alpha_2 = 10^{-4}\ \ [(\delta k/k)/{}^{\circ}\text{C}]\end{array}\right\} , \quad (1.94)$$

and the predicted values at 100 kW then become

$$\left.\begin{aligned} & N_0 = 10^{12} \ \text{[neutrons]} \\ & \theta_c^0 - \theta_1 = 2 \ [^\circ C] \ , \quad b = 0.1 \ [sec^{-1}] \end{aligned}\right\} . \tag{1.95}$$

Here, b is determined by the definitions (1.31), (1.35) and (1.41):

$$b = 2d_2Q_0 = 2\left\{(V_c/S_c)/\mathcal{V}_2^0\right\}^{-1} , \tag{1.96}$$

while with use made of the parameters(40),

$$\frac{V_c}{S_c} = 60 \ [cm] \ , \quad \mathcal{V}_2^0 = 3 \ [cm/sec] \ . \tag{1.97}$$

Consequently, the input noise source (1.93) reduces to

$$I(i\omega) \approx 4D_{NN}\left\{1 + 10^5 \times \alpha^2\right\} \qquad \text{for} \quad \omega \ll 0.1 \ . \tag{1.98}$$

Now, $2D_{QQ}$ has been expressed in terms of the standard deviation α of fluctuations in coolant flow-rate. From Eqs. (1.77) and (1.91), it is

$$\begin{aligned} 2D_{QQ} &= \left\{8d_2 + 2\times 10^{-4}\alpha^2 b\right\}Q_0^2 \\ &\doteqdot 2\times 10^{-4}\alpha^2 b Q_0^2 \qquad \text{for} \quad \alpha \gg 2\times 10^2\sqrt{\frac{d_2}{b}} \quad [\%] \end{aligned} \tag{1.99}$$

which has been used in Eqs. (1.93) and (1.98).

To estimate d_2 defined by (1.35), if we adopt

$$\left.\begin{aligned} & C_v^c \rho_0 V_c = 2.80\times 10^2 [kw/^\circ C] \ , \quad \theta_2^0 = 36.0 \ [^\circ C] \\ & q = 3.23\times 10^{-14} \ [kw\text{-}sec] \end{aligned}\right\} , \tag{1.100}$$

the condition requisite to permit use of Eq. (1.99) as $2D_{QQ}$ is

$$\alpha \gg 1.1\times 10^{-6} \ [\%] \ . \tag{1.101}$$

This clearly shows how effective the noise source for turbulent flow is to the present reactor model, as has been mentioned for the external noise source in Sec. 2.4. Supposing that the standard deviation α is equal to only 1 %, $2D_{QQ}$ is completely dominated by the noise source for turbulent flow, and thereby the input noise source (1.98) shows an increase of amplitude beyond comparison with the nuclear noise source. This increment is the same in the neutron noise spectrum $P_{NN}(i\omega)$.

In the case of $\omega > b$, we easily obtain the frequency dependence of the input noise source in the form

$$I(i\omega) \approx 4D_{NN}\left\{1+\omega^{-4}(\mu_4 N_0 d)^2\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} , \qquad (1.102)$$

and the noise spectrum $P_{NN}(i\omega)$ also has the same dependence, since the transfer function of a low power reactor is almost constant in a frequency region for which $\lambda < \omega < -(\rho-\beta)/\ell$.

Now, neutron noise spectra have been measured in light--water-cooled and -moderated reactors under natural convection at various power-levels up to 100 kW, e.g., HTR(6), KUR(4)(5), and TTR1(8). In comparing the experimental observations with the theoretical results (1.98) and (1.102), it is seen that the present stochastic model not only has correct qualitative features, but also reveals semi-quantitative agreement with the experiments.

In passing, let us consider another stochastic model for a power reactor, which contains no contributions from fluctuations in the coolant flow-rate, i.e., a reactor model such as could be described by the usual equations for neutrons and precursors, Eq. (1.30) for the fuel temperature and Eq. (1.26)' for the coolant temperature, together with the related noise sources. This means that the inherent noise sources of the power reactor arise solely from the elementary events relevant to the coolant temperature fluctuations, principally, heat transfer from the fuel to the coolant. The input noise source, therefore, can in this case be written in the form

$$I(i\omega) \simeq 4D_{NN}\left\{1+\left|\frac{G_{14}(i\omega)}{G_{11}(i\omega)}\right|^2\left(\frac{2D_{MM}}{2D_{NN}}\right)\right\}$$

$$\simeq 4D_{NN}\left\{1+(\mu_4 N_0)^2\left|\frac{(i\omega+\Lambda_Q)(i\omega+h_t)}{(i\omega+\Lambda_Q)F(i\omega)+x_B d(i\omega+h_t)}\right|^2\left(\frac{2D_{MM}}{2D_{NN}}\right)\right\}$$

$$\approx 4D_{NN}\left\{1+\frac{(\mu_4 N_0)^2}{\omega^2+b^2}\left(\frac{2D_{MM}}{2D_{NN}}\right)\right\} \quad . \qquad (1.103)$$

Consequently, it follows that

$$I(i\omega) \approx 4D_{NN}\left\{1+5.9\times10^{-6}\right\} \quad \text{for} \quad \omega\ll 0.1 \quad , \qquad (1.104)$$

$$\approx 4D_{NN}\left\{1+\omega^{-2}(\mu_4 N_0)^2\left(\frac{2D_{MM}}{2D_{NN}}\right)\right\} \quad \text{for} \quad \omega\gg 0.1 \quad , \qquad (1.105)$$

where we have used the definitions (1.31), (1.35) and (1.76), and the above-mentioned values (1.94), (1.95) and (1.100). These results evidently disagree with the actual observed noise spectra[(4)-(6)(8)]. Most of the theoretical studies for power reactor noise, however, have been discussed on a stochastic model of this kind[(19)(20)(22)] or else on an even simpler one[(13)-(18)].

4.2. Neutron Noise Spectra in Forced Circulation Cooling

As an another example of this reactor model, we shall consider the case of forced circulation. Attention is first directed to the relaxation constant Λ_Q of fluctuations in the coolant flow, which will differ from the definition (1.41), as noted in Sec. 2.4. Accordingly, it is presumed as follows.

There are three types of relaxation phenomena. First is the slow relaxation due to the regression of fluctuations through the whole recirculation flow. This kind of fluctuations, even if the amplitude is very small, appear to have a marked effect on the other fluctuations in the reactor system, on account of the large coolant flow-rate. In the present study, this phenomenon is expressed with the Langevin equation (1.42) and estimated in Eq. (1.46) in terms of two phenomenological parameters, α and Λ_Q. The second phenomenon is the fast relaxation due to the local fluctuations of turbulent flow, which we shall not discuss in this chapter. The third is the oscillatory relaxation which arises from the random mechanical vibrations of the control rod[3][6], fuel plates and other structural components. This will be discussed in a next chapter.

For the time being, we shall only outline roughly the frequency dependence of the input noise source, as well as the noise spectrum. In the case in question, the flow-speed of the coolant is in general fast enough to consider, on account of (1.96), that

$$\Lambda_Q \ll b \propto V_2^0 \quad . \tag{1.106}$$

Therefore, the input noise source (1.89) can be written in the form

$$I(i\omega) \simeq 4D_{NN}\left\{1+\left(\frac{\mu_4 N_0 d}{\Lambda_Q b}\right)^2 \left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} \quad \text{for } \omega \ll \Lambda_Q \quad , \qquad (1.107)$$

$$\simeq 4D_{NN}\left\{1+ \omega^{-2}\left(\frac{\mu_4 N_0 d}{b}\right)^2 \left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} \quad \text{for } \Lambda_Q \ll \omega \ll b , \qquad (1.108)$$

$$\simeq 4D_{NN}\left\{1+ \omega^{-4}(\mu_4 N_0 d)^2\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} \quad \text{for } \omega \gg b \quad , \qquad (1.109)$$

where we have taken the approximation (1.90) and neglected the effects of thermal expansion and buoyancy. From these expressions we may state as follows. If the flow-rate is not very high, i.e., $b \gtrsim \Lambda_Q$, the input noise source will take the form of the frequency dependence (1.107) and (1.109), which has a close resemblance to the case of natural convection. On the other hand, for sufficiently large flow-speed, it will be expressed by (1.107), (1.108) and $I(i\omega) \simeq 4D_{NN}$ for $\omega \gg b$. Therefore, its frequency dependence is ω^{-2}, which can be termed characteristic of the case of forced circulation.

Now, let us try to compare these two cases with the neutron noise spectra observed in at-power reactors of forced circulation cooling with light-water. The former case, probably, should correspond to the spectra in HTR[3][6] (where the flow speed is about 16 cm/sec with forced circulation and 2 cm/sec at 100 kW with natural convection), except for the resonance peaks actually observed due to control rod vibration. The latter case should be applicable to the spectra from JMTR[11] (flow speed about 7 m/sec with forced circulation), KUR[10] (about 10 m/sec with forced circulation and 3.7 cm/sec at 100 kW with natural convection[40]), ORR (see Fig. 6 in Ref. (7) or Figs. 1 and 4 in Ref. (23)) and MURR (see Fig. 10 in Ref. (12)). In this latter case, the above-mentioned second and/or third relaxation phenomena are quite conspicuous.

We shall here compare our results with the power spectral density measurements conducted on the Sodium Reactor Experiments (SRE)[2]. In such a liquid-sodium-cooled reactor, hydraulic flow fluctuations cause variations in reactivity by varying the the amplitudes and gradients of fluctuations of the coolant temperature. Hence, the present stochastic reactor model should be applicable to the SRE. The shape of the input noise spectrum given by Eqs. (1.107), (1.108) and $I(i\omega) \simeq 4D_{NN}$ for $\omega \gg b$, corresponds to the driving function noise spectra (Fig. 4 in Ref. (2)). And the frequency dependence of the neutron noise spectrum taken in the form ω^{-2}, agrees with the observed spectra (Figs. 6 and 8 in same).

It is important to note that the standard deviation of fluctuations for turbulent flow need not nearly be as large as the case of natural convection. The reason is as follows. The flow speed, i.e., Q_0, is very large, so that the noise source $2D_Q'$ in Eq. (1.46) becomes extremely large. If we take the same order of magnitude for the input noise source as the case of natural convection, the standard deviation α will become a very small value.

§ 5. Concluding Remarks

In what precedes, we have fulfilled the two principal aims stated in the beginning:

(1) Of determining the noise sources occurring in the transport processes of heat energy

(2) Establishment of a set of system equations for the thermal and hydraulic state variables

Here it should be emphasized that the present thermodynamical investigation, though it has been made on the mean behavior, provides us with a reliable basis for the study of power reactor noise. This has permitted us to proceed to a stochastic description of the random processes.

The present theory has been proved to be fairly reliable upon comparison with actually-observed neutron noise spectra. In the case of natural convection, assuming that there are a few percent of fluctuations in the coolant flow-rate, the theory gives a semi-quantitative agreement with the noise spectra measured in reactors, e.g. KUR[4][5], HTR[6] and TTR1[8]. In the case of forced circulation, with much lower proportion of fluctuations, the theory can explain in a satisfactory manner the noise spectra in HTR[3][6], JMTR[11], KUR[10], ORR[7], MURR[12] and SRE[2]. Consequently, we conclude that the theory is widely applicable to the present type of at-power reactors.

Table 1.1 Elementary events in an at-power reactor

Elementary Event	Its rate	Net Number by event
Neutron Removal	$\Lambda_C N$	-1 neutron
Neutron Source	S	1 neutron
Fission	$\Lambda_f N$	$(\nu_0 - 1)$ neutrons ν_1 precursors 1 energy in fuel
Decay	λC	-1 precursor 1 neutron
Heat Transfer	$h_t(F - \eta M)$	-1 energy in fuel 1 energy in coolant
Heat Expansion	$E_0 + eM$	-1 energy in coolant
Energy Inflow	$d_1 Q_1 T_1$	1 energy in coolant
Energy Outflow	$d_2 Q(2M - T_1)$	-1 energy in coolant
Mass Inflow	$Md_1 Q_1$	-1 energy in coolant
Mass Outflow	$Md_2 Q$	1 energy in coolant
Buoyancy	$x_B(M - T_w)$	1 momentum of coolant outflow
Pressure	Q_p	1 momentum of coolant outflow
Momentum Inflow	$\frac{2}{d_2} d_1^2 Q_1^2$	1 momentum of coolant outflow
Momentum Outflow	$2d_2 Q^2$	-1 momentum of coolant outflow
Mass Inflow	$\frac{d_1}{d_2}(\Upsilon d_1 Q_1 + d_2 Q)Q_1$	-1 momentum of coolant outflow
Mass Outflow	$(\Upsilon d_1 Q_1 + d_2 Q)Q$	1 momentum of coolant outflow
Unknown event	$f(Q)$	momentum of coolant outflow

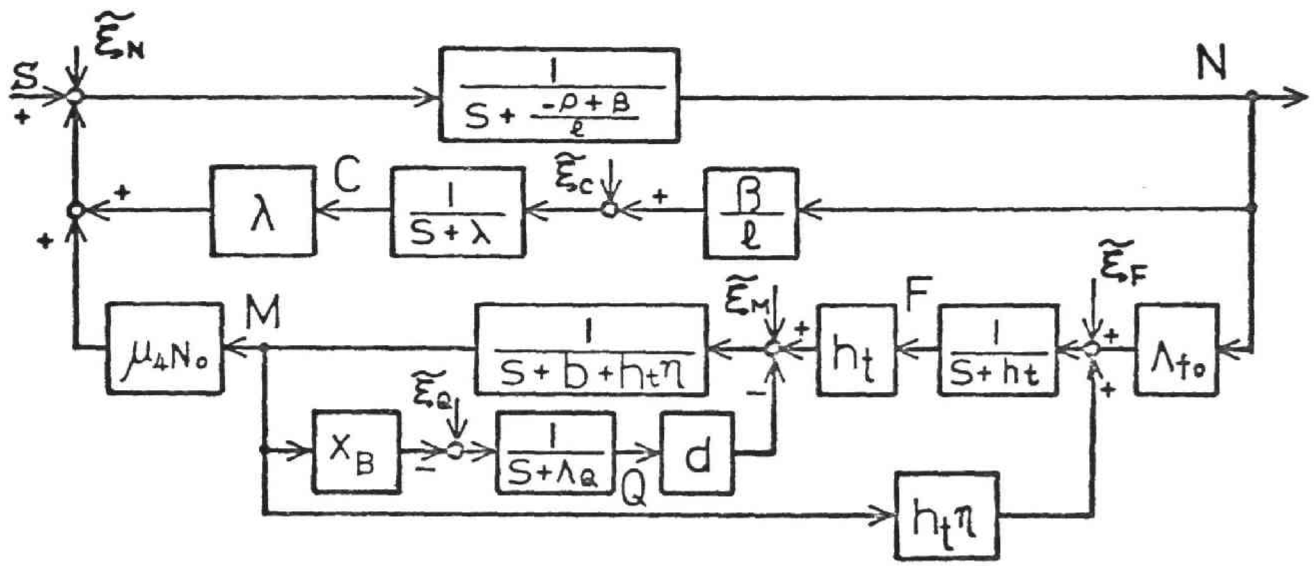

Fig. 1.1 Transfer function block diagram and noise sources

CHAPTER II

Effect on Stochastic Fluctuations of Possible Noise Sources in Non-Boiling Liquid Reactors

§ 1. Introduction

In the present chapter, we shall treat stochastic fluctuations in a system at power, where there are, chiefly, three kinds of noise sources, i.e., the statistical nature of neutron chain reaction, the inherent noise source in fluid flow as in Chap. I, and noise arising from: The effect of buoyancy in the coolant flow*, the random vibrations of a control or fuel element, and the inlet temperature fluctuations**.

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* In Chap. I, this effect is first incorporated in the model, but then discarded from consideration further on in the same chapter when is undertaken.

** The inlet flow fluctuations have already been studied in Chap. I. In the case of forced circulation, the coolant flow of a core will fluctuate in one block, roughly in a single phase, owing to the very high flow-speed. It then follows that the fluctuations in question can be expressed in terms of Eq. (1.42), embodying a parameter Λ_Q. On the other hand, in the case of natural convection cooling, we have discussed fluctuations in the outlet flow-rate, and thereby assumed the fluctuations in question to be white noise.

A theoretical treatment is applied, making use of the Langevin method[38][41], on the basis of the stochastic model presented in Chap. I: A set of linearized equations (1.37)-(1.39) embodying thermodynamic and hydraulic state variables are adopted as the Langevin equations, which contain source terms to represent random driving forces. The random forces have white noise spectra, whose amplitudes are given by the corresponding components of the matrix (1.74). We shall furthermore take the linear Langevin equations of first order, considered to govern variables representing the displacement of the randomly vibrating element and the inlet temperature. In the present paper, these equations are taken as starting point for further analysis. Explicit expressions are derived for the noise spectra associated with the above-mentioned variables. Examples of the spectra thus determined are compared with the results of observation.

In Sec. 2, some simplifications on the model presented in Chap. I are considered, and also, the effect of buoyancy acting on the coolant flow is studied. In Sec. 3, we shall consider a system where random vibrations are induced in the control or fuel element by fluctuations in the coolant flow-rate. In Sec. 4, we shall study a system characterized by inlet temperature fluctuations, which have been assumed in Chap. I to be white-noise. Sec. 5 is devoted to a short summary and a brief discussion on the theoretical treatment of at-power reactor noise.

§ 2. Buoyancy Effect in Coolant Flow

2.1. Simplified Model Reactor

Let us define the Markoffian random processes discussed in Chap. I with use made of the Langevin equation:

$$\frac{d}{dt}\tilde{\boldsymbol{\alpha}}(t) = -\boldsymbol{\Lambda}\tilde{\boldsymbol{\alpha}}(t) + \tilde{\boldsymbol{\xi}}(t) , \qquad (2.1)$$

where $\tilde{\boldsymbol{\alpha}}(t)$ represents the fluctuations in the set of state variables defined by (1.47), referred to the respective steady--state values, and $\boldsymbol{\Lambda}$ is the relaxation matrix defined by (1.73). The random driving forces $\tilde{\boldsymbol{\xi}}(t)$ are governed by the relationships

$$\langle \tilde{\boldsymbol{\xi}}(t) \rangle = 0 , \qquad (2.2)$$

$$\langle \boldsymbol{\xi}(t)\boldsymbol{\xi}(u) \rangle = 2\mathbf{D}\,\delta(t-u) , \qquad (2.3)$$

where $2\mathbf{D}$ is the diffusion matrix given by (1.74). By Langevin treatment of the linear Markoff processes[38][41], we obtain the noise spectrum matrix

$$\mathbf{P}(\omega) = 2(i\omega\mathbf{E} + \boldsymbol{\Lambda})^{-1}2\mathbf{D}(-i\omega\mathbf{E} + \boldsymbol{\Lambda}^{+})^{-1}, \qquad (2.4)$$

where $\boldsymbol{\Lambda}^{+}$ is the transpose matrix of $\boldsymbol{\Lambda}$, and $\mathbf{E}$ the unit matrix.

Let us now discuss the noise spectrum relevant to the simplified model reactor (SMR) based on three assumptions:
(a) that the strongly influential noise sources affecting the fluctuations as a whole are the neutron generation by fission and the noise source bringing about fluctuations in the coolant flow-rate, i.e., in the matrix (1.74),

$$\text{all } D_{ij} = 0 \text{ but } D_{11} = D_{NN} \neq 0 \text{ and } D_{55} = D_{QQ} \neq 0 \,; \qquad (2.5)$$

(b) that the reactivity feedback largely depends on the variations in the coolant temperature through the changes of the total multiplication rate of neutrons, i.e., in Eqs. (1.67) and (1.68),

$$\mu_1 = \mu_2 = \mu_3 = 0 \text{ and } \mu_4 \neq 0 \; ; \tag{2.6}$$

and (c) that the ratio η of the total specific heat between that of the fuel and that of the coolant is very much smaller than unity, so that Eq. (1.88) reduces to

$$F(s) = (s+h_t)(s+b) \; . \tag{2.7}$$

2.2 Discussion

For the neutron fluctuations, the noise spectrum can be written in the form

$$P_{NN}(\omega) = |T(i\omega)|^2 I(\omega) \; , \tag{2.8}$$

where the transfer function $T(i\omega)$ is given by the expression (1.87) combined with Eq. (2.7), and the input noise source

$$I(\omega) = 2(2D_{NN})\left\{1+ \frac{(d\mu_4 N_0)^2}{\{\omega^2-(b\Lambda_Q+x_B d)\}^2+\omega^2(b+\Lambda_Q)^2}\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} \; . \tag{2.9}$$

This function clearly reproduces the resonance-like structure observed on the spectrum, which is due to coupling between the fluctuations of the coolant temperature and those of the coolant flow-rate brought about by the effect of buoyancy.
In fact, for $x_B=0$, the input noise source reduces to

$$I(\omega) = 2(2D_{NN})\left\{1+(\mu_4 N_0)^2 \frac{d^2}{\omega^2+b^2}\,\frac{1}{\omega^2+\Lambda_Q^2}\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right\} \; , \tag{2.10}$$

which means that the hydraulic flow fluctuations governed by the decay constant Λ_Q cause variations in reactivity by varying with a decay constant b the amplitudes and gradients of the fluctuations in the coolant temperature. A further discussion on input noise source of the form (2.10) has been given in Chapter I.

Now, buoyancy brings upon a power reactor a negative-feedback effect, since an increase in the coolant temperature due to an increase in the energy transferred from the fuel leads to an increase of the coolant flow-rate, which in turn decreases the coolant temperature, as is seen from Eqs. (1.38) and (1.39). Hence it should be of interest to examine in some detail this feedback due to buoyancy. Applying the expressions (1.91) and (1.96) to $b\Lambda_Q$, and (1.31), (1.35) and (1.41) to $x_B d$, we obtain

$$b\Lambda_Q = \left\{\frac{2\mathcal{V}_2^0}{V_c/S_c}\right\}^2 \propto (\mathcal{V}_2^0)^2 \quad , \tag{2.11}$$

$$x_B d \simeq 2a_m \vartheta \frac{S_c}{V_c}(\Theta_c^0 - \Theta_1) \propto (\Theta_c^0 - \Theta_1) \ . \tag{2.12}$$

It follows from this that the feedback effect due to buoyancy appears distinctly in the neutron noise spectra of reactors that are operated with natural convection cooling at full power, on account of the relatively slow coolant flow combined with a large temperature difference between the moderator-coolant and the inlet flow.

We next discuss the noise spectrum $P_{QQ}(\omega)$ of the coolant flow-rate fluctuations, which can be written for the SMR in the form

$$P_{QQ}(\omega) = 2(2D_{QQ})\frac{\omega^2+b^2}{\{\omega^2-(b\Lambda_Q+x_Bd)\}^2+\omega^2(b+\Lambda_Q)^2}$$

$$\times\left\{1+|T_0(i\omega)|^2\frac{(h_t\Lambda_{f0}x_B)^2}{(\omega^2+h_t^2)(\omega^2+b^2)}\left(\frac{2D_{NN}}{2D_{QQ}}\right)\right\}, \qquad (2.13)$$

with the zero-power-reactor transfer function

$$T_0(s) = \frac{1}{s-\frac{\rho-\beta}{\ell}-\frac{\beta}{\ell}\frac{\lambda}{s+\lambda}} \quad . \qquad (2.14)$$

Here we have let $\mu_4=0$ on the supposition that variations in the reactivity emanating from coolant temperature fluctuations are far less influential on fluctuations in the coolant flow--rate. This noise spectrum also reveals a resonance-like structure induced by buoyancy. For $x_B=0$, Eq. (2.13) reduces to

$$P_{QQ}(\omega) = 2(2D_{QQ})\frac{1}{\omega^2+\Lambda_Q^2} \quad , \qquad (2.15)$$

which follows directly from Eq. (1.42) combined with the relation (1.43).

Other noise spectra, if necessary, can be easily obtained from the matrix (2.4) on the SMR, with consideration given to the influence of buoyancy effect. As examples, the auto--correlation noise spectrum $P_{MM}(\omega)$ of the coolant temperature fluctuations and the cross-correlation noise spectrum $P_{NM}(\omega)$ between the neutron and coolant-temperature fluctuations are given in the Appendix II. All the spectra mentioned above have been calculated numerically as function of power-level. The results of this calculations will be shown in a next chapter.

§ 3. Random Mechanical Vibrations

3.1. Stochastic Model

We shall adopt

$$\frac{d}{dt}\widetilde{Q}(t) = -\Lambda_Q \widetilde{Q}(t) + f\widetilde{X}(t) + \widetilde{\xi_Q}(t) , \qquad (2.16)$$

$$\frac{d}{dt}\widetilde{X}(t) = -\Lambda_x \widetilde{X}(t) - g\widetilde{Q}(t) + \widetilde{\xi_x}(t) , \qquad (2.17)$$

as the phenomenological Langevin equations for the coolant flow-rate fluctuations $\widetilde{Q}(t)$ and the displacement $\widetilde{X}(t)$ of the element. Here $\widetilde{\xi_Q}(t)$ and $\widetilde{\xi_x}(t)$ are the random driving forces, Λ_Q and Λ_x the relaxation constants, and f and g $(fg > 0)$ the coefficients representing the coupling between the flow-rate fluctuations and the random mechanical vibrations. These two equations can be written in the form of a second order differential equation in $\widetilde{Q}(t)$:

$$\frac{d^2}{dt^2}\widetilde{Q}(t) + \Lambda_0 \frac{d}{dt}\widetilde{Q}(t) + \omega_0^2 \widetilde{Q}(t) = \widetilde{\xi_0}(t) , \qquad (2.18)$$

with

$$\Lambda_0 = \Lambda_Q + \Lambda_x , \qquad (2.19)$$

$$\omega_0^2 = \Lambda_Q \Lambda_x + fg , \qquad (2.20)$$

$$\widetilde{\xi_0}(t) = \Lambda_x \widetilde{\xi_Q} + f\widetilde{\xi_x} + \frac{d}{dt}\widetilde{\xi_Q} . \qquad (2.21)$$

This is essentially an equation representing the damped harmonic oscillator, and agrees in form with what follows from an inspection of the noise spectra of coolant flow-rate fluctuations observed in the HTR(6), in which it was found that resonance peak was due to the random vibrations of a control rod.

We now discuss the random driving forces in Eqs. (2.16) and (2.17), which are required to satisfy the conditions

$$<\widetilde{\xi_Q}(t)\widetilde{\xi_Q}(u)> \;=\; 2D_{QQ}\delta(t-u) \;, \tag{2.22}$$

$$<\widetilde{\xi_Q}(t)\widetilde{\xi_x}(u)> \;=\; 0 \;, \tag{2.23}$$

$$<\widetilde{\xi_x}(t)\widetilde{\xi_x}(u)> \;=\; 2D_{xx}\delta(t-u) \;. \tag{2.24}$$

With the help of the Einstein relation(38)(41), it then follows that

$$<(\widetilde{Q})^2> \;=\; \frac{1}{2}\frac{(\Lambda_x^2+\omega_0^2)(2D_{QQ}) + f^2(2D_{xx})}{\Lambda_0\omega_0^2} \;, \tag{2.25}$$

$$<(\widetilde{X})^2> \;=\; \frac{1}{2}\frac{(\Lambda_Q^2+\omega_0^2)(2D_{xx}) + g^2(2D_{QQ})}{\Lambda_0\omega_0^2} \;. \tag{2.26}$$

For convenience, we take a case where the coupling does not exist, i.e., $f=g=0$. Then the variances become

$$<(\widetilde{Q})^2> \;=\; \frac{1}{2\Lambda_Q}(2D_{QQ}) \;, \tag{2.27}$$

$$<(\widetilde{X})^2> \;=\; \frac{1}{2\Lambda_x}(2D_{xx}) \;. \tag{2.28}$$

The noise source $2D_{QQ}$, if necessary, can be evaluated with a standard deviation of fluctuations in the coolant flow-rate, as in Eqs. (1.45) and (1.46), while $2D_{xx}$ is related to the mean amplitude of random vibrations of a control or fuel element.

3.2. Stochastic Formulation

Let us describe the random processes of the present system by using the Langevin equation as in Eq. (2.1). The stochastic model is the same as employed in Chapter I, except for the existence this time of the above-mentioned coupling condition. Accordingly, the state vector $\tilde{\alpha}$ consists of a set of random variables, i.e.,

$$\tilde{\alpha}^{+} = (\tilde{N},\tilde{C},\tilde{F},\tilde{M},\tilde{Q},\tilde{X}) \ , \tag{2.29}$$

and the relaxation matrix Λ can be written in the form

$$\Lambda = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & \Lambda_0 & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & d & 0 \\ \hline 0 & 0 & 0 & -x_B & \Lambda_Q & -f \\ 0 & 0 & 0 & 0 & \Lambda_x & g \end{array}\right) , \tag{2.30}$$

where Λ_0 is the matrix given by the upper left-hand 4X4 submatrix of (1.73). The amplitude $2\mathbf{D}_0$ of the correlation function for $\tilde{\xi}(t)$ now becomes

$$2\mathbf{D} = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & 2\mathbf{D}_0 & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 2D_{QQ} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2D_{xx} \end{array}\right) , \tag{2.31}$$

where $2\mathbf{D}_0$ is the matrix defined by the upper left-hand 4X4 submatrix of (1.74).

3.3. Analysis

In order to illustrate the influence of the random mechanical vibrations on the fluctuations as a whole we shall take the noise spectra associated with the neutron fluctuations and the coolant flow-rate fluctuations in the SMR. The neutron noise spectrum can be expressed in terms of both the transfer function $T(i\omega)$ and the input noise source $I(i\omega)$:

$$T(\;i\omega) = \left. \frac{\dfrac{1}{s+\dfrac{-\rho+\beta}{\ell}}}{1-\dfrac{1}{s+\dfrac{-\rho+\beta}{\ell}}\left\{\dfrac{\beta}{\ell}\dfrac{\lambda}{s+\lambda}+\dfrac{\mu_4 N_0 h_t \Lambda_{f0}}{(s+h_t)(s+b)+\dfrac{x_B d(s+h_t)(s+\Lambda_x)}{s^2+\Lambda_0 s+\omega_0^2}}\right\}} \right|_{s=i\omega}, \quad (2.32)$$

$$I(\;i\omega) = 2(2D_{NN})\left[1+\frac{d^2(\mu_4 N_0)^2}{\left|(s^2+\Lambda_0 s+\omega_0^2)(s+b)+x_B d(s+\Lambda_x)\right|^2}\right.$$

$$\left.\left. \times \left\{|s+\Lambda_x|^2\left(\frac{2D_{QQ}}{2D_{NN}}\right)+f^2\left(\frac{2D_{xx}}{2D_{NN}}\right)\right\}\right]\right|_{s=i\omega}. \quad (2.33)$$

Note that for $x_B=0$ Eq. (2.33) becomes

$$I(\omega) = 2(2D_{NN})\left[1+\frac{d^2(\mu_4 N_0)^2}{\left\{(\omega_0^2-\omega^2)^2+\omega^2\Lambda_0^2\right\}(\omega^2+b^2)}\left\{(\omega^2+\Lambda_x^2)\left(\frac{2D_{QQ}}{2D_{NN}}\right)+f^2\left(\frac{2D_{xx}}{2D_{NN}}\right)\right\}\right]. \quad (2.34)$$

Thus the neutron noise spectrum has the resonance component dominated by a single angular frequency ω_0 and damping constant Λ_0. The transfer function (2.32) reflects the effect of the coupling phenomenon, by which the changes in coolant flow-rate due to the random vibrations of the element immediately affect the displacement of an element randomly vibrating from its mean position. This effect acts as temperature reactivity feedback, as can be seen from Eq. (1.87) and Fig. 1.1.

On the other hand, the input noise source (2.33), and also (2.34), exhibit a certain resonance peak. Let us now consider two special cases.

(i) In the underdamped case ($\omega_0 \gg \Lambda_0$), the function reaches a maximum at $\omega=\omega_0$, whose magnitude depends on both the standard deviation of the coolant flow-rate fluctuations and the mean amplitude of the random mechanical vibrations, by virtue of the relations (2.27) and (2.28). The sharpness $\Delta\omega_0$ of a resonance, namely the half width of this maximum, in the expression (2.34) is

$$\Delta\omega_0 \simeq \Lambda_0 (1-\frac{\Lambda_0}{2\omega_0}) \ , \qquad (2.35)$$

which shows that the half width is roughly described by the two relaxation constants Λ_Q and Λ_x, in other words, by the regression of fluctuations through the recirculation flow as a whole (cf. Sec. 4.2 of Chapter I) and the damping characteristic of random vibrations of the structural elements. This statement should be valid in approximation at the resonance frequency ω_0 in view of the relation (2.20). In particular, if Λ_Q has a flow-speed dependence like that in the expression (1.41), Λ_Q increases with the flow-speed, so that ω_0 shifts toward higher frequencies. This behavior has been observed experimentally in the ORR by means of power-spectral-density measurements (see Figs. 1 and 2 in Ref. (26) and Fig. 6 in Ref. (7)).

Let us now turn our attention to the frequency dependence of the input noise source of the form (2.33). In this instance, we shall take the example of forced circulation cooling, in which the value of b, defined by Eq. (1.96), becomes very large, the flow-speed being in general very fast. For the case in which the random mechanical vibrations significantly contribute to the input noise source ($2D_{xx} \gg 2D_{QQ}$), we obtain

$$\frac{I(\omega)}{2(2D_{NN})} \simeq \frac{d^2(\mu_4 N_0)^2}{\{(\omega_0^2-\omega^2)^2+\omega^2\Lambda_0^2\}(\omega^2+b^2)} f^2\left(\frac{2D_{xx}}{2D_{NN}}\right), \qquad (2.36)$$

which leads to

$$\frac{I(\omega)}{2(2D_{NN})} \simeq \frac{d^2(\mu_4 N_0)^2}{\omega_0^4 b^2} f^2\left(\frac{2D_{xx}}{2D_{NN}}\right) \qquad \text{for } \omega < \omega_0 , \qquad (2.37)$$

$$\simeq \frac{1}{\{(\omega_0-\omega)^2+(\Lambda_0/2)^2\}} \frac{d^2(\mu_4 N_0)^2}{\omega_0^2 b^2} f^2\left(\frac{2D_{xx}}{2D_{NN}}\right) \quad \text{for } \omega \simeq \omega_0 , \qquad (2.38)$$

$$\simeq \omega^{-4} \frac{d^2(\mu_4 N_0)^2}{b^2} f^2\left(\frac{2D_{xx}}{2D_{NN}}\right) \qquad \text{for } \omega_0 < \omega < b, \qquad (2.39)$$

$$\simeq \omega^{-6} d^2(\mu_4 N_0)^2 f^2\left(\frac{2D_{xx}}{2D_{NN}}\right) \qquad \text{for } b < \omega . \qquad (2.40)$$

These results will be later compared with observations.

(ii) In the strongly overdamped case ($\Lambda_0 \gg \omega_0$), e.g., where the flow-speed of the coolant is extremely slow, or where a randomly vibrating structural element is rapidly restored to its original place (namely $\Lambda_Q \ll \Lambda_x$, $fg \approx 0$), the input noise source (2.33) is reduced to

$$\frac{I(\omega)}{2(2D_{NN})} \simeq 1 + \left|\frac{d\mu_4 N_0}{(i\omega+\Lambda_Q)(i\omega+b)+x_B d}\right|^2 \left(\frac{2D_{QQ}}{2D_{NN}}\right) . \qquad (2.41)$$

This is the input noise source given by Eq. (2.9), which has been discussed in detail in Chapter I and in the previous section.

Let us finally consider the noise spectrum associated with fluctuations in the coolant flow-rate:

$$P_{QQ}(\omega) = 2(2D_{QQ})\left|\frac{\omega^2+\Lambda_x^2}{(\omega_0^2-\omega^2)^2+\omega^2\Lambda_0^2}\right|^2\left\{1+\frac{f^2}{\omega^2+\Lambda_x^2}\left(\frac{2D_{xx}}{2D_{QQ}}\right)\right\}, \quad (2.42)$$

where we have put $\mu_4=0$ and $x_B=0$, as in Eqs. (2.13) and (2.15). We shall now discuss two particular cases:

(i) The underdamped case, where the spectrum exhibits a resonance peak, and whose frequency and half-width are the same as in $P_{NN}(\omega)$. In a system where a control or fuel element should vibrate significantly on account of a large quantity of coolant flow, Eq. (2.42) leads to

$$P_{QQ}(\omega) \simeq \frac{1}{(\omega_0-\omega)^2+(\Lambda_0/2)^2}\frac{f^2}{\omega_0^2}(2D_{xx}) \quad \text{for } \omega\simeq\omega_0 . \quad (2.43)$$

(ii) The strongly overdamped case, when Eq. (2.42) is reduced to the form of Eq. (2.15).

Other noise spectra, if necessary, can be easily obtained from the component of the matrix (2.4) in combination with the matrixes (2.30) and (2.31).

3.4. Discussion

In actually operating reactors, the random mechanical vibrations of a structural element should probably exert an effect, not upon the whole coolant flow, but upon only a portion thereof. This portion will then obey the Langevin equation of the form taken by Eq. (2.18), while the remaining portion of the coolant flow will be described by Eq. (1.42). Accordingly, it follows that stochastic fluctuations in such a system are subjected to the coolant temperature fluctuations induced by the two kinds of fluctuations in the coolant flow-rate: the one arising from the noise source $2D_{xx}$ and the other from the $2D_{QQ}$ given by Eq. (1.77) together with Eq. (1.42).

As an pertinent example, we shall take up the neutron noise spectrum for the case of forced circulation cooling, which has two different types of frequency component: one of resonance-like structure given by the input noise source (2.36) with the transfer function (2.32), and the other of the form taken by the expressions (1.107), (1.108) and (1.109) together with (1.87). This result, in particular the predicted slope of the spectrum, is qualitatively in agreement with the observations obtained on the ORR (see Figs. 1, 2 and 3 in Ref. (26) and Figs. 6, 12 and 13 in Ref. (7)).

Considerations along similar lines can be pursued in respect of the noise spectrum of the coolant flow-rate fluctuations to derive the components represented by the expressions (2.15) and (2.43). This result should, in principle, correspond to the spectra from the HTR (see Fig. 7 in Ref. (6)).

As an another example of forced circulation cooling, we shall consider the case where the flow-rate is not very high, but where, nevertheless, a structural element vibrates randomly. We have, in this case,

$$b \gtrsim \Lambda_Q \sim \Lambda_0 \quad \text{and} \quad 2D_{xx} \gtrsim 2D_{QQ} , \qquad (2.44)$$

so that the input noise source (2.34) may be written in the form

$$\frac{I(\omega)}{2(2D_{NN})} \simeq \frac{d^2(\mu_4 N_0)^2}{\omega_0^4 b^2}\left\{\Lambda_x^2\left(\frac{2D_{QQ}}{2D_{NN}}\right) + f^2\left(\frac{2D_{xx}}{2D_{NN}}\right)\right\} \text{ for } \omega<\Lambda_0, \quad (2.45)$$

$$\simeq \frac{d^2(\mu_4 N_0)^2}{\omega_0^4}\left\{\left(\frac{2D_{QQ}}{2D_{NN}}\right) + \omega^{-2}f^2\left(\frac{2D_{xx}}{2D_{NN}}\right)\right\} \text{ for } \Lambda_0<\omega<\omega_0, \quad (2.46)$$

$$\simeq \frac{1}{(\omega_0-\omega)^2+(\Lambda_0/2)^2}\,\frac{d^2(\mu_4 N_0)^2}{\omega_0^6}\left\{\omega_0^2\left(\frac{2D_{QQ}}{2D_{NN}}\right)+f^2\left(\frac{2D_{xx}}{2D_{NN}}\right)\right\} \text{ for } \omega\simeq\omega_0, \quad (2.47)$$

$$\simeq \omega^{-4}d^2(\mu_4 N_0)^2\left\{\left(\frac{2D_{QQ}}{2D_{NN}}\right)+\omega^{-2}f^2\left(\frac{2D_{xx}}{2D_{NN}}\right)\right\} \text{ for } \omega>\omega_0\,. \quad (2.48)$$

These results should be applicable directly to the observed neutron noise spectra at 50 Watts on the HTR, in which the coolant flow-speed is 16 cm/sec at any power level in forced circulation, (see Figs. 4 and 6 in Ref. (6)). Introducing the low-frequency component in the expressions (1.107)-(1.109) into the present case, the above results can be compared with the neutron noise spectra observed on the HTR at various power levels (see Figs. 1, 5 and 8 in Ref. (6)).

§ 4. Inlet Temperature Fluctuations

4.1. Stochastic Model

We consider a system subjected to fluctuations in the inlet temperature of the coolant on the basis of the model adopted in Chapter I. We now assume the Langevin equation for the fluctuations:

$$\frac{d}{dt}\widetilde{T_1}(t) = -\Lambda_i\widetilde{T_1}(t) + \widetilde{\xi_i}(t) \ , \qquad (2.49)$$

where $\widetilde{T_1}(t)$ corresponds to the fluctuations in the inlet temperature $\theta_1(t)$ in reference to the definition (1.31), and Λ_i is a relaxation constant regarded as a phenomenological parameter. The random driving force $\widetilde{\xi_i}(t)$ has the property

$$<\widetilde{\xi_i}(t)> = 0 \ , \qquad (2.50)$$

$$<\widetilde{\xi_i}(t)\widetilde{\xi_i}(u)> = 2D_{ii}\delta(t-u) \ . \qquad (2.51)$$

The amplitude $2D_{ii}$ can be easily expressed in terms of a standard deviation α_i of the fluctuations as in Eq. (1.46):

$$2D_{ii} = 2\Lambda_i(\alpha_i T_1 \times 10^{-2})^2 \ . \qquad (2.52)$$

Next it is necessary to regard the constants θ_1 in Eqs. (1.24) and (1.26) and T_1 in Eq. (1.33) as variables. The linearized equation (1.38) for the coolant temperature is therefore rewritten in the Langevin form:

$$\frac{d}{dt}\widetilde{M}(t) = h_t\widetilde{F}(t)-(b+h_t\eta)\widetilde{M}(t)-d\widetilde{Q}(t)+p\widetilde{T_1}(t)+\widetilde{\xi_M}(t) \ , \qquad (2.53)$$

where the coefficient p is newly defined by

$$p = d_2Q_0 + d_1Q_1 > 0 \ , \qquad (2.54)$$

and the random driving force $\widetilde{\xi_M}(t)$ has the same properties as $\widetilde{\xi_i}(t)$, except that the amplitude $2D_{MM}$ is as defined by Eq. (1.76). The linearized equations for the other variables, namely Eqs. (1.37) and (1.39), are valid without modification.

4.2. Noise Spectra

We now define the whole random processes of the present system by using the Langevin equation of the form represented by Eq. (2.1). The fluctuations $\tilde{\boldsymbol{\alpha}}$ from the respective steady-state values are given by

$$\tilde{\boldsymbol{\alpha}}^{+} = (\tilde{N}, \tilde{C}, \tilde{F}, \tilde{M}, \tilde{Q}, \tilde{T}_1) \;, \tag{2.55}$$

The decay matrix is written in the form

$$\boldsymbol{\Lambda} = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & \boldsymbol{\Lambda}_0 & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & d & -p \\ \hline 0 & 0 & 0 & -x_B & \Lambda_Q & 0 \\ 0 & 0 & 0 & 0 & 0 & \Lambda_i \end{array} \right), \tag{2.56}$$

where we have used Eqs. (2.49) and (2.53) together with Eq. (1.78) for the other variables. And the diffusion matrix, defined in Eq. (2.3), becomes

$$2\mathbf{D} = \left(\begin{array}{c|cc} 2\mathbf{D}_0 & & 0 \\ \hline 0 & 2D_{QQ} & 0 \\ & 0 & 2D_{ii} \end{array} \right), \tag{2.57}$$

whose element can be seen from Eqs. (2.3) and (1.74) together with Eq. (2.5). Thus the noise spectrum matrix $\mathbf{P}(\omega)$ of the form given by Eq. (2.4) is completely determined.

4.3. Analysis and Discussion

(1) Noise Spectrum for Neutron Number

Let us consider first the neutron noise spectrum for the

SMR. From the (1,1) component of the matrix $\mathbb{P}(\omega)$, we obtain the desired spectrum: the transfer function T(s) is of the same form as Eq. (1.87) with Eq. (2.7) for F(s), and the input noise source takes the form

$$I(\omega) = 2(2D_{NN})\left[1 + \frac{d^2(\mu_4 N_0)^2}{(b\Lambda_Q + x_B d - \omega^2)^2 + (b+\Lambda_Q)^2\omega^2}\left(\frac{2D_{QQ}}{2D_{NN}}\right)\right.$$

$$\left. \times \left\{1 + \frac{p^2}{d^2}\,\frac{\omega^2+\Lambda_Q^2}{\omega^2+\Lambda_i^2}\left(\frac{2D_{ii}}{2D_{QQ}}\right)\right\}\right] . \qquad (2.58)$$

This is identical with the expression (2.9) for the case $2D_{ii}=0$ or $p=0$, and with the corresponding Eqs. (1.89) and (1.107)-(1.109) for both the above cases and $x_B=0$.

In what follows, therefore, we shall confine our attention to an analysis of the quantity written between the curly brackets in Eq. (2.58). This quantity is the ratio of frequency component between that arising from the inlet temperature fluctuations and that from the flow-rate fluctuations. The contribution of this quantity to the neutron noise spectrum can be easily estimated according to whether the ratio is much greater than unity or not so much. Now the quantity in question becomes

$$1 + \frac{p^2}{d^2}\,\frac{\Lambda_Q^2}{\Lambda_i^2}\left(\frac{2D_{ii}}{2D_{QQ}}\right) \qquad \text{for } \omega \ll \Lambda_Q \text{ and } \Lambda_i , \qquad (2.59)$$

$$1 + \frac{p^2}{d^2}\left(\frac{2D_{ii}}{2D_{QQ}}\right) \qquad \text{for } \omega \gg \Lambda_Q \text{ and } \Lambda_i . \qquad (2.60)$$

Thus a difference between those two quantities arises from the value of Λ_Q^2/Λ_i^2. The expression (2.59) can be rewritten in the form

$$1 + 4\frac{\Lambda_Q}{\Lambda_i}\left(\frac{\alpha_i}{\alpha}\right)^2\left(\frac{\theta_1^0}{\theta_c^0 - \theta_1^0}\right)^2 \quad , \tag{2.61}$$

where we have used the expressions (1.41), (1.77), (2.51) and (2.54) for d, $2D_{QQ}$, $2D_{ii}$, and p respectively together with the definitions (1.31) and (1.35). This result shows how sensitively the inlet temperature fluctuations influence the neutron noise spectrum.

For example, in the case of natural convection cooling, we can make use of the parameters presented in Eqs. (1.95) and (1.100) to obtain the result

$$1 + \left(\frac{32\alpha_i}{\alpha}\right)^2 \frac{\Lambda_Q}{\Lambda_i} \quad . \tag{2.62}$$

Assuming $\Lambda_Q \simeq \Lambda_i$, the condition of α_i for obtaining a value of the expression (2.62) much greater than unity is

$$\alpha_i \gg \frac{1}{32}\alpha \quad . \tag{2.63}$$

This is an expected result.

(2) Noise Spectrum for Inlet Temperature

Let us now consider the power spectral density $P_{ii}(\omega)$ of the inlet temperature fluctuations and the cross power spectral density $P_{Ni}(\omega)$ between the neutron and inlet-temperature fluctuations. From the (6,6) and (1,6) components of the matrix $\mathbf{P}(\omega)$, we obtain for the SMR

$$P_{ii}(\omega) = 2(2D_{ii})(\omega^2 + \Lambda_i^2)^{-1} \quad , \tag{2.64}$$

$$P_{Ni}(\omega) = 2(2D_{ii})(\omega^2 + \Lambda_i^2)^{-1} p \left\{ \frac{(i\omega + \Lambda_Q)}{(i\omega+\Lambda_Q)(i\omega+b)+x_B d} \right\}$$

$$\times \mu_4 N_0 T(i\omega) \quad , \qquad (2.65)$$

with the transfer function (1.87) for T(s). Neither of these spectra contain components arising from the noise sources $2D_{NN}$ and $2D_{QQ}$.

From the spectra (2.58), (2.64) and (2.65), we may state as follows. There is a correlation between the neutron and inlet-temperature fluctuations, but it would be expected to be small because of the presence of the noise sources $2D_{QQ}$ and $2D_{NN}$. This can be seen clearly if we consider the coherence function

$$C_{Ni}(\omega) = \frac{|P_{Ni}(\omega)|}{\sqrt{P_{NN}(\omega)}\sqrt{P_{ii}(\omega)}} \qquad (2.66)$$

$$= \left\{ 1 + \frac{(b\Lambda_Q + x_B d - \omega^2)^2 + (b+\Lambda_Q)^2\omega^2}{(p\mu_4 N_0)^2} \frac{\omega^2+\Lambda_i^2}{\omega^2+\Lambda_Q^2}\left(\frac{2D_{NN}}{2D_{ii}}\right) + \frac{d^2(\omega^2+\Lambda_i^2)}{p^2(\omega^2+\Lambda_Q^2)}\left(\frac{2D_{QQ}}{2D_{ii}}\right) \right\}^{-\frac{1}{2}}$$

$$\leq 1 \ . \qquad (2.67)$$

(i) In the lower frequency region where $I(\omega) \gg 2(2D_{NN})$, the function can be written in the form

$$C_{Ni}(\omega) \simeq \frac{1}{\sqrt{1 + \frac{d^2}{p^2}\frac{\omega^2+\Lambda_i^2}{\omega^2+\Lambda_Q^2}\left(\frac{2D_{QQ}}{2D_{ii}}\right)}} \ . \qquad (2.68)$$

(ii) In the higher frequency region where $I(\omega) \simeq 2(2D_{NN})$, we have

$$C_{Ni}(\omega) \simeq \omega^{-2} p |\mu_4 N_0| \sqrt{\left(\frac{2D_{ii}}{2D_{NN}}\right)} \ll 1 \ . \qquad (2.69)$$

It follows from these results that the coherence is significantly different from unity when an inherent noise source in fluid flow constitutes a possible noise source in non-boiling liquid reactors.

Now, it has been reported by Boardman[27] that in the reactor noise measurements on the DFR, only a few of the power fluctuations are due to the inlet coolant temperature noise, and that in particular, there is a large low-frequency component in reactor-power noise which is not due to the inlet coolant temperature as measured by the plenum thermocouple. Our results, though qualitative, are compatible with his observations.

(3) Noise Spectrum for Coolant Flow-rate

We shall here draw attention to the coherence function $C_{NQ}(\omega)$ for fluctuations in the neutron number and the coolant flow-rate. For the SMR, the noise spectrum $P_{QQ}(\omega)$ for the coolant flow-rate fluctuations and the cross noise spectrum $P_{NQ}(\omega)$ between the neutron number and coolant flow-rate fluctuations are given by

$$P_{QQ}(\omega) = 2(2D_{QQ})(\omega^2 + \Lambda_Q^2)^{-1}, \tag{2.70}$$

$$P_{NQ}(\omega) = 2(2D_{QQ})T(i\omega)(-d\mu_4 N_0)(i\omega+b)^{-1}(\omega^2 + \Lambda_Q^2)^{-2}, \tag{2.71}$$

which have been obtained from the (5,5) and (1,5) components of the matrix $\mathbb{P}(\omega)$, with $x_B=0$ for simplicity. The cross noise spectrum $P_{NQ}(\omega)$ consists of two kinds of amplitude response function to the random driving force $\widetilde{\xi_Q}$:

$$(-i\omega + \Lambda_Q) \ , \tag{2.72}$$

and

$$T(i\omega)(-d\mu_4 N_0)(i\omega+b)(i\omega+\Lambda_Q) \ . \tag{2.73}$$

The former corresponds to that of the coolant flow-rate and the latter to that of the neutron number, and thus the physical meaning of $P_{NQ}(\omega)$ becomes clear.

Now, the coherence function in question is written in the form

$$C_{NQ}(\omega) = \frac{|P_{NQ}(\omega)|}{\sqrt{P_{NN}(\omega)}\sqrt{P_{QQ}(\omega)}}$$

$$= \left\{1+\left(\frac{2D_{NN}}{2D_{QQ}}\right)\frac{(\omega^2+\Lambda_Q^2)(\omega^2+b^2)}{d^2(\mu_4 N_0)^2} + \left(\frac{2D_{ii}}{2D_{QQ}}\right)\frac{p^2(\omega^2+\Lambda_Q^2)}{d^2(\omega^2+\Lambda_i^2)}\right\}^{-\frac{1}{2}} , \tag{2.74}$$

where we have used Eqs. (1.87) and (2.58) for $T(s)$ and $I(\omega)$ respectively with $x_B=0$. A discussion on the frequency dependence of $C_{NQ}(\omega)$ can be made in the same manner as presented in Eq. (2.67).

It follows from our results for $C_{Ni}(\omega)$ and $C_{NQ}(\omega)$ that it may be possible to derive the major noise source in the present type of power reactor by observing experimentally the values of $C_{Ni}(\omega)$ and $C_{NQ}(\omega)$ in an appropriate frequency region. If we have the result

$$C_{Ni}(\omega) < C_{NQ}(\omega) \ , \tag{2.75}$$

it would be expected that the coolant flow-rate fluctuations contribute more significantly to the system under consideration than the inlet temperature fluctuations.

Recently the coherence functions $C_{Ni}(\omega)$ and $C_{NQ}(\omega)$ have been determined by Batch and Klickman[42] in the core of the Enrico Fermi reactor at three power levels : 0.05 Mwt, 56 Mwt and 88 Mwt. They concluded that two major noise sources $2D_{QQ}$ and $2D_{ii}$ are present independently of each other, and that the inlet temperature fluctuations influence reactor power more than the coolant flow-rate fluctuations.

§ 5 Short Summary and Some Remarks

On the basis of the stochastic model presented in Chapter I, we have studied the frequency responses of a system to the random driving forces of: (a) buoyancy effect, (b) random mechanical vibrations and (c) inlet temperature fluctuations. These responses were all considered over the whole range of frequency through combination with the low-frequency component due to an inherent noise source in the fluid flow. Thus we have been able to compare satisfactorily these theoretical results with the observations. An illustrative analysis has been made, principally for the neutron noise spectra, but other noise spectra, if necessary, can be readily obtained and illustrated.

Now the theoretical treatment in the previous and present chapters for a study of at-power reactor noise has brought out many problems that require further investigation. To give some examples:

(i) Noise sources strongly influencing the entire fluctuations

A parameter has been introduced into the model in the form of a relative standard deviation of fluctuations in the coolant flow-rate, but the theoretical estimation of its magnitude remains to be undertaken. Another question is the number of effective noise sources existing in a power reactor. Thie[28] has pointed out that there are many kinds of intrinsic noise sources in various types of at-power reactors.

(ii) The choice of a set of random variables

A set of random variables required to describe the kinetic behavior of a reactor system can be chosen with the aid of the theory of non-equilibrium thermodynamics, as has been done in Chapter I. These variables will constitute a set of variables, sufficient to permit a Markoffian description to be formulated. In practice, however, we shall deal with fewer variables in order to simplify the model in so far as possible. Attention should first be directed to the problem of how we can systematically carry out this reduction of variables. Next, we must consider a random variable that assumes a number of different values in a system, e.g., there may be a number of different kinds of coolant-flow speed in a core.

(iii) Local fluctuations of temperature and flow-rate

We have hitherto dealt with the fluctuations in a lumped system. In actual noise experiments, however, local fluctuations in the state quantities are usually observed. The relation between the actually observed quantities and the theoretical values has not yet been made adequately clear.

(iv) Description of nonlinear processes

As a general expression of nonlinear feedback processes, a convolution form has been employed. In the previous and present chapters, the feedback processes have been expressed in terms of linearized equations of first order. It should be of interest to discuss this problem together with the correlation function of random driving force.

CHAPTER III

Analysis of Stochastic Fluctuations in a Natural Convection Non-boiling Light-water Reactor

§ 1. Introduction

In the present chapter, we have calculated numerically for the stochastic model of Chapter I, both the noise spectra and the variances of stochastic fluctuations in neutron number, fuel temperature, coolant temperature and coolant flow-speed. The calculations have been made for various levels of reactor power for the case of natural convection cooling, using a plausible set of parameters for a typical light-water reactor, for example, the Kyoto University Reactor (KUR)[5]. The results have been illustrated in terms of an analytical expression derived for the simplified model reactor (SMR), which has already been obtained in Chapter II on the basis of the model of Chapter I. Some of the results are compared with the observations of Utsuro et al.[5] and Utsuro[4] in the KUR, and Nomura[8] in the TTR1. It will be seen that our model is valid for explaining in a satisfactory manner the experimental results obtained by Yamada[3] and Yamada et al.[6] in the HTR, though the resonance like behavior of the noise spectra is not taken into consideration.

In Section 2, a number of model parameters used here are shown. And we shall choose the value of the parameter α (%), which is introduced in the present model as an adjustable parameter to express a relative standard deviation of fluctuations in the coolant flow-rate for the case of $x_B=0$ (see Sec. 2.4 of Chap. I).

Numerical calculations of the noise spectra have been made for a delayed critical system ($\rho=0$) for five different values of the reactor power P from 10 W to 100 kW. For these calculations, we have adopted an analytical expression of the spectrum obtained from an element of the noise spectrum matrix

$$\mathbf{P}(\omega) = 2(i\omega\mathbf{E} + \mathbf{\Lambda})^{-1} 2\mathbf{D} (-i\omega\mathbf{E} + \mathbf{\Lambda}^{+})^{-1}, \qquad (3.1)$$

where $\mathbf{\Lambda}$ and $2\mathbf{D}$ are given by the expressions (1.73) and (1.74) respectively, and $\mathbf{E}$ is the unit matrix. The results of these calculations will be shown and discussed in detail in Section 3.

Section 4 is devoted to the discussion of the variances and the relative standard deviations obtained numerically. The calculations have been performed for a delayed critical system ($\rho=0$) from P=1 W to P=100 kW, and for a subcritical system from reactivity ρ = -10 ¢ (P$\simeq$10 mW) to $\rho = -10^{-6}$ ¢ (P$\simeq$100 kW), by using the algebraical equation

$$2\mathbf{D} = \mathbf{\Lambda}\langle\alpha\alpha\rangle + \langle\alpha\alpha\rangle\mathbf{\Lambda}^{+} \quad . \qquad (3.2)$$

§ 2. Model Parameters and Steady-state Values

Numerical calculations were performed with use made of a number of nuclear and material constants and model parameters, whose values are presented in Tables 3.1 and 3.2. Our values of those parameters were chosen to be applicable to a typical light-water reactor, such as for example the KUR[5], and it has been assumed that the values are independent of variations of the reactor power level P (kW).

The present calculations were made, furthermore, on the assumption of the following two relations:

$$\upsilon_c^0 = 1.16P^{0.25} \ \text{(cm/sec)} , \qquad (3.3)$$

empilically obtained from measurements of the coolant flow--speed υ_c^0 at various power levels on the KUR[40]; and

$$h_0 = 11.7(\Theta_F^0 - \Theta_c^0)^{0.33} \ (\text{kW}\,{}^\circ\text{C}^{-1}) , \qquad (3.4)$$

for all power levels. Such a relation has been employed in the case of heat transfer by natural convection[43]. The coefficient has been so determined that the heat transfer coefficient h_0 becomes 20 kW$^\circ$C^{-1} when the temperature difference $\Theta_F^0 - \Theta_c^0$ is 5 °C at P=100 kW. Here the value of h_0 has been computed from the stationary form of Eq. (1.30) for the applicable fuel temperature, i.e.,

$$h_0 = P/(\Theta_F^0 - \Theta_c^0) . \qquad (3.5)$$

Let us now express the steady-state values of the state quantities as a function of P. To determine the number of neutrons present in a reactor, we find that in a delayed critical system it is

$$N_0 = \frac{1}{q\,\Lambda_{f0}}P = \frac{\bar{\nu}\ell}{q}P \ , \tag{3.6}$$

while in a subcritical system, we obtain from Eq. (1.72)

$$N_0 = \frac{\ell}{-\rho}S \ , \tag{3.7}$$

as a function of the reactivity ρ, from which we can readily determine the reactor power by making use of Eq. (3.6). For the number of delayed neutron precursors, we have

$$C_0 = \frac{\bar{\nu}_1}{\lambda}\frac{P}{q} \quad . \tag{3.8}$$

From the stationary form of Eq. (1.26) and the relation (1.18), the coolant temperature can be written in the form

$$\theta_c^{\circ} = \frac{\theta_1 + \dfrac{P}{2S_c C_v^c \rho_c V_c^0}\left\{1 - \dfrac{273\,\alpha_m^2}{K\rho_c C_v^c}\right\}}{1 + \dfrac{P}{2S_c C_v^c \rho_c V_c^0}\,\dfrac{\alpha_m^2}{K\rho_c C_v^c}} \quad (^{\circ}\mathrm{C}) \ , \tag{3.9}$$

together with the relation (3.3) for V_c^0, so that the coolant outlet temperature becomes

$$\theta_2^0 = 2\,\theta_c^0 - \theta_1 \quad (^{\circ}\mathrm{C}) \ , \tag{3.10}$$

in view of the relation (1.24). An expression for the fuel temperature is obtained from the relation (3.4) and (3.5), i.e.,

$$\theta_F^0 = \theta_c^0 + \left(\frac{P}{11.7}\right)^{0.75} \quad (^{\circ}\mathrm{C}) \ . \tag{3.11}$$

In Fig. 3.1, the steady-state values of state quantities obtained above are plotted as a function of P.

Using the relations (3.3)-(3.11), an analytical expression was obtained as a function of P or ρ for such quantities as the thermo-hydraulic variables defined by Eq. (1.31), the relaxation constants by Eqs. (1.35), (1.41) and (1.69), and consequently each element of the matrices (1.73) and (1.74) for $\boldsymbol{\Lambda}$ and $2\mathbf{D}$ respectively. In Fig. 3.2, typical relaxation constants are shown for various values of reactor power.

In order to calculate numerically the noise spectra and the variances, we need to know the value of the parameter α defined by the expression (1.45). Then we have computed the neutron noise spectra for different values of α at P=100 kW and $\rho=0$. The value of α was chosen such that a reasonably good fit with the experimental results of Utsuro et al.[5], and Nomura[8] was obtained. The result was that

$$\begin{aligned} \alpha &= 1\ \% \quad \text{for TTR1} \\ &= 4\ \% \quad \text{for KUR} \quad . \end{aligned} \tag{3.12}$$

The values of α, chosen here, were adopted for our calculations made for various power levels up to 100 kW. This treatment, though undoubtedly quite crude, has, nevertheless, revealed many interesting features of our stochastic model, as will be seen later.

§ 3. Noise Spectra

3.1. Noise Spectra for Neutron Number

In Fig. 3.3, we have compared the calculated neutron noise spectra with the results of measurements performed in the KUR[5] and TTR1[8]. We have also shown the theoretical noise spectra for the two different cases: (a) when power level is 1 kW, and (b) when it is 4 kW, of which 1 kW is produced as a result of nuclear fissions and 3 kW is due to the other reactions, such as for example, fission-product gamma heating. The results of this experiment may be confirmed indirectly by observations in the KUR on the fission-product decay heat energy*. But whether this should hold true in the present case remains to be proved experimentally.

The transfer function and the input noise source are plotted in Fig. 3.4 for five different values of P. Corresponding analytical expressions applicable to the SMR are represented by Eqs. (1.87) and (2.9), respectively, together with Eq. (2.7) for F(s).

With the foregoing choice of values for the parameter α, it is seen that the general configuration of the calculated spectra roughly agree with the corresponding experimental results in respect of both break frequency and slope of the

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* Recently, measurements of fission-product decay heat have been performed in the KUR by observing the coolant mean flow--speed 50 hours after shutdown upon 100 hours of operation at 5 MW[40]. The decay heat energy was about 10 kW. This experiment was undertaken later than when the noise spectra shown in Fig. 3.3 were obtained, at which time the full power of the KUR was 1 MW. The decay heat corresponding to this smaller power may hence be estimated to have been about 2 kW.

low-frequency component. It may be judged from this that the low-frequency fluctuations result from the coolant flow-rate fluctuations: its characteristic relaxation constants depend on the mean flow-speed of the coolant as indicated by Eq. (2.11), and its amplitude on the deviations of the flow speed from the mean value as given by Eqs. (1.44) and (1.77). The higher the power level, the faster becomes the flow speed, which increases the low-frequency component. This relation is shown in Fig. 3.4. Such behavior was also observed by Yamada[3] and Yamada et al.[6] in the HTR.

We shall now present some remarks on the experimental and theoretical neutron noise spectra. First, there is the possibility that the low frequency fluctuations in the neutron number arise from a combination of several kinds of reactivity change caused by corresponding local fluctuations in the coolant temperature, which, in turn is generated by variations in the flow-speed of the coolant. Such local fluctuations should be considered to have their inherent relaxation constant and noise source. The neutron noise spectra would thereby acquire a complicated structure. In fact, as shown in Fig. 3.3, there is a significant difference in the experimental results obtained from the KUR at P=100 kW for the two kinds of core configuration embodying either four water plugs (triangles in Fig. 3.3) or none (open circles in same).

Secondly, it is seen from Figs. 3.3 and 3.4 that the calculations reveal an obtuse peak in the neighborhood of ω= 0.2 rad/sec at higher power levels. This peak is due to the effect of buoyancy in the coolant flow, as discussed in Sec. 2.2 of Chap. II.

Thirdly, the origin of the low-frequency fluctuations is not always dependent on reactor power level. For example, in a reactor at low power level, and carrying a large fission-product inventory, the coolant flow will be caused, chiefly, by the decay heat energy, so that the low-frequency behavior would not be coupled directly to power level, as a case that was observed in the FNR[9].

Finally, we note here that, in the frequency region above 0.1 rad/sec, the shape of the calculated noise spectra is roughly the same as given by the zero power reactor transfer function for power levels below a few kW. Hence it follows that, at such high frequencies and low power levels, the zero power reactor noise theory can be validly applied to describe the noise spectrum in question.

3.2. Noise Spectra for Coolant Temperature

Figure 3.5 shows the noise spectra associated with the coolant temperature fluctuations, whose analytical expression has been given by Eq. (A3) of Appendix II for the SMR. The general shape of the spectra is almost independent of reactor power for the power levels shown. It is determined, mainly, by a frequency component generated by the coolant flow-rate fluctuations. Hence it is possible to write the spectra in the simpler form

$$P_{MM}(\omega) \simeq \frac{d^2}{\{\omega^2-(b\Lambda_Q+x_B d)\}^2+\omega^2(b+\Lambda_Q)^2} 2(2D_{QQ}) \quad . \qquad (3.13)$$

The characteristic relaxation constants are, therefore, Λ_Q and b. The frequency dependence is ω^{-4}. These characteristics are the same as those of the low-frequency component in the neutron noise spectra. It follows then that the neutron fluctuations at lower frequencies are caused by fluctuations in the coolant temperature through the temperature coefficient of reactivity.

The theoretical results are compared with the coolant--temperature noise spectra observed on the KUR at a power level of 100 kW[4]. Fairly good agreement is obtained on the whole: the characteristic features revealed from our calculations as expressed in terms of the relaxation constants and the frequency dependence, is in conformity with the observations, as can be seen from Fig. 3.5. A similar agreement with experiment is also obtained in the case of measurements performed in an out-of-pile natural convection heat transfer loop by Nishihara[33] The present results, however, disagree with those observed in the FNR at low power levels from 1 kW to 5 kW, reported by Lehto et al.[9]. Their experiments show a frequency dependence related to ω^{-2}. We have no explanation for this difference in observed characteristics.

3.3. Noise Spectra for Coolant Flow-rate

Figure 3.6 gives the noise spectra obtained for fluctuations in the coolant flow-rate. Its analytical form has been given by Eq. (2.13) on the SMR. For the power levels shown, the nuclear noise source $2D_{NN}$ is scarcely responsible for the spectra in question, so that we can rewrite Eq. (2.13) in the form

$$P_{QQ}(\omega) \simeq 2(2D_{QQ}) \frac{\omega^2+b^2}{\{\omega^2-(b\Lambda_Q+x_B d)\}^2 + \omega^2(b+\Lambda_Q)^2} \, . \qquad (3.14)$$

The coupling effect due to buoyancy is reflected in the curves: a resonance structure appears in the vicinity of ω=0.1 rad/sec for higher power levels. The spectra have a frequency dependence related to ω^{-2} at higher frequencies. This dependence has been observed also in the HTR[6], though, in addition, a resonance peak has been found at $\omega \simeq 8.2$ rad/sec.

3.4. Cross Noise Spectra for Neutron Number and Coolant Temperature

(1) Magnitude

The magnitude of the cross noise spectra $P_{NM}(\omega)$ between the coolant temperature fluctuations and the neutron number fluctuations is shown in Fig. 3.7. An analytical expression of this $P_{NM}(\omega)$ has been given for the SMR by Eq. (A4) of Appendix II. Our calculations show that the spectra for the power levels shown can be expressed in the simpler form

$$P_{NM}(\omega) \simeq 2(2D_{QQ})\frac{\mu_4 N_0 d^2 T_0(i\omega)}{\left\{\omega^2-(b\Lambda_Q+x_B d)\right\}^2+(b+\Lambda_Q)^2\omega^2} \ . \qquad (3.15)$$

This simplification consists in assuming that a frequency component due to the nuclear noise source $2D_{NN}$ contributes little to the spectra $P_{NM}(\omega)$, and that for higher frequencies above 0.01 rad/sec the shape of the transfer function T(s) is nearly the same as the zero power transfer function $T_0(s)$ (see Fig. 3.4).

From the spectrum (3.15) as well as from Eq. (A4) of Appendix II combined with the above assumptions, it follows that small deviations of the coolant flow-rate are responsible for fluctuations in the coolant temperature and the neutron number through the response functions, respectively, of

$$d\left\{(-i\omega+\Lambda_Q)(-i\omega+b)+x_B d\right\}^{-1} \ , \qquad (3.16)$$

and

$$T_0(i\omega)\mu_4 N_0 d\left\{(i\omega+\Lambda_Q)(i\omega+b)+x_B d\right\}^{-1} \ . \qquad (3.17)$$

These response functions have already been taken up in the discussion of the spectra $P_{NN}(\omega)$ and $P_{MM}(\omega)$, so the physical meaning of the results of our calculations is clear.

On comparing the P=100 kW curve with the experimental points, it is seen that for $\omega \lesssim 1$ rad/sec a reasonably good agreement is obtained on the whole, but for $\omega > 1$ rad/sec the coincidence is not so good. This difference may arise from local fluctuations in the coolant temperature, as has been discussed in Sec. 3.1.

(2) Coherence

An interesting behavior of $P_{NM}(\omega)$ is noted for the coherence function of the form

$$C_{NM}(\omega) = \frac{|P_{NM}(\omega)|}{\sqrt{P_{NN}(\omega)}\sqrt{P_{MM}(\omega)}} \quad . \tag{3.18}$$

The results of our calculations for $C_{NM}(\omega)$ are shown in Fig. 3.8. For higher power levels, on account of the inherent noise source in the coolant flow-rate fluctuations, the greatest coherence occurs between the neutron number fluctuations and the coolant temperature fluctuations, which is 1.0 at lower frequencies. For lower power levels, another kind of coherence appears in the higher frequency region, though this is very weak. This is due to the random generation of neutrons by nuclear fission.

On the SMR, the coherence function in question can be written in the form

$$C_{NM}(\omega) = \frac{1}{\sqrt{1+\epsilon^{-1}\left\{\omega^4+(b^2+\Lambda_Q^2-2x_B d)\omega^2+(b\Lambda_Q+x_B d)^2\right\}}} , \tag{3.19}$$

$$\epsilon = (d\,\mu_4 N_0)^2\left(\frac{2D_{QQ}}{2D_{NN}}\right) \quad . \tag{3.20}$$

In writing Eq. (3.19) we have used Eqs. (2.8), (3.13) and (3.15) for $P_{NN}(\omega)$, $P_{MM}(\omega)$ and $P_{NM}(\omega)$ respectively, on the supposition that we can replace $T(i\omega)$ by $T_0(i\omega)$. In the lower frequency region, $C_{NM}(\omega)$ has the form given by

$$C_{NM}(\omega) = \frac{1}{\sqrt{1+\epsilon^{-1}(b\Lambda_Q+x_B d)^2}} \quad . \tag{3.21}$$

In particular, for higher power levels, this is nearly equal to 1.0, because ϵ becomes very large. At higher frequencies, Eq. (3.19) reduces to

$$C_{NM}(\omega) = \epsilon^{\frac{1}{2}}\omega^{-2} . \qquad (3.22)$$

These results are reflected on the curves of Fig. 3.8. A break frequency ω_B can be easily obtained from Eqs. (3.21) and (3.22):

$$\omega_B \simeq \sqrt[4]{\epsilon+(b\Lambda_Q+x_B d)^2} . \qquad (3.23)$$

It follows from this that the coherence in question is dependent not only on the relaxation constants, but also, greatly on the relative magnitude of the noise source $2D_{QQ}$.

(3) Phase

In what follows, we shall deal with the ratio between the imaginary and real parts of $P_{NM}(\omega)$, which corresponds to the phase of the complex function $P_{NM}(\omega)$. In Fig. 3.9 we have plotted the phase response of $P_{NM}(\omega)$. From an inspection of the curves, one very important result emerges: for higher power levels, the general behavior of the ratio is nearly the same as given by the zero power transfer function. Namely, using Eq. (3.15), we have

$$\frac{\mathrm{Im}P_{NM}(\omega)}{\mathrm{Re}P_{NM}(\omega)} = \frac{\mathrm{Im}T_0(i\omega)}{\mathrm{Re}T_0(i\omega)}$$

$$= -\frac{\omega\left(1+\frac{\beta}{\ell}\frac{\lambda}{\omega^2+\lambda^2}\right)}{-\frac{\rho}{\ell}+\frac{\beta}{\ell}\frac{\omega^2}{\omega^2+\lambda^2}} . \qquad (3.24)$$

However, as reactor power decreases, there is increasing deviation from Eq. (3.24). An oscillatory behavior of the ratio is seen at higher frequencies, which is due to the contribution of the noise source $2D_{NN}$ to the spectrum $P_{NM}(\omega)$.

§ 4. Relative Standard Deviations

4.1. Fluctuations in a Delayed Critical System

Using Eq. (3.2), we have calculated the variances and the relative standard deviations of fluctuations in state quantities as a function of P for two different values of α=1 % and 4 %. In Fig. 3.10, we have plotted the relative standard deviations which have been expressed in percentage. Also shown in Fig. 3.10 are the experimental results of the neutron noise measurements performed in the KUR by Oka et al. (34). It will be seen that the curve for α=1 % has a fair resemblance with the experimental results, while for α=4 % a reasonably good fit was obtained with the observed neutron noise spectra of the KUR at P=100 kW. We note here that the experimental point at 10 W assumes a value of about 0.22 % which is quite close to the value of 0.30 % calculated for the neutron fluctuations not in a critical system, but in a sub-critical system, as will be seen from Fig. 3.12. It follows from this that there is a possibility that the experimental data for P=10 W were obtained in subcritical state. This surmise is supported by the fact that in the KUR there is a considerable contribution of the neutron source resulting from the (γ,n) reactions in the D_2O column[44].

At higher power levels, fluctuations in the coolant flow speed are somewhat moderated by a feedback effect due to buoyancy. We should note here that the quantity α (%) which was introduced in theoretical derivation in the form of an adjustable parameter plays an important role in determining the amplitude of a noise source of the coolant flow speed fluctuations for the case of $x_B=0$, as seen from Eqs. (1.42)-(1.46). Therefore, the parameter α corresponds to the relative standard deviation of the fluctuations when we consider the case $x_B=0$ in our stochastic model.

At higher power levels, the flow speed fluctuations, while their relative amplitude is only a few percent, have a dominant effect on fluctuations in the other state quantities shown, while at lower power levels a noise source due to the random generation of prompt neutrons by nuclear fissions contributes significantly to the fluctuations in the neutron number and the fuel temperature. This result is as expected.

In Fig. 3.11, we have shown how influential are the nuclear noise sources and the thermo-hydraulic noise sources on the neutron fluctuations. Making use of Eq. (3.2) for the variance σ^2 of the fluctuations in question, two kinds of the variance — σ_n^2 and σ_t^2 — can be defined by

$$\sigma_n^2 = \langle (N-N_0)^2 \rangle_n = \sum_{i,j=1}^{2} f_{ij}(2\mathbf{D})_{ij} , \qquad (3.25)$$

$$\sigma_t^2 = \langle (N-N_0)^2 \rangle_t = \sum_{i,j=3}^{5} f_{ij}(2\mathbf{D})_{ij} , \qquad (3.26)$$

so that we have

$$\sigma^2 = \langle (N-N_0)^2 \rangle = \sigma_n^2 + \sigma_t^2 \quad , \tag{3.27}$$

where f_{ij} is a quantity composed of all elements of the matrix Λ. In other words, σ_n^2 and σ_t^2 correspond to the amplitude responses of the neutron fluctuations to the nuclear and thermo-hydraulic noise sources respectively, and in the particular case of the SMR, to $2D_{NN}$ and $2D_{QQ}$ respectively. We have plotted in Fig. 3.11 the ratios σ_n^2/σ^2 and σ_t^2/σ^2. It is seen from the curves in this figure, as well as in Fig. 3.4, that as P increases, low frequency fluctuations become more and more dominant, and thereby the neutron fluctuations undergo a marked variation in the wave shapes in the vicinity of P=1 kW for α=1 % and of P=200 W for α=4 %. This behavior can be expected to be substantiated by experiment.

We shall here compare our calculations with the observations of Nishihara[33], who has carried out coolant temperature fluctuations measurements in a natural convection heat transfer loop decoupled from neutronics. In his work, the relative standard deviation is 0.11 % (the root mean square amplitude is 0.04 °C) and 0.15 % (0.05 °C) for an electric power of 200 W and 300 W respectively. It is estimated that the KUR reactor power of 100 kW is equivalent to an electric power level somewhere between 200 W and 300 W. In the present study, for P=100 kW, it is 0.067 % (0.022 °C) for α=4 % and 0.017 % (0.0056 °C) for α=1 %. Judging from the crudeness of our calculations where the lumped parameter model was used, this difference can probably be attributed to the local temperature fluctuations which have been measured by him.

4.2. Fluctuations in a Subcritical System

In Fig. 3.12, the relative standard deviations of fluctuations in state quantities are shown as a function of the reactivity ρ ($\rho<0$). In the same figure, we have also shown the results calculated for the neutron fluctuations with the zero-power reactor-noise theory. Its analytical expression is given by

$$\frac{\sqrt{\sigma^2}}{N_0} \times 100 = \frac{1}{\sqrt{S}} \sqrt{\frac{\left\{\overline{\nu_0(\nu_0-1)}\,\Lambda_{f0} + \frac{-\rho+\beta}{\ell}\right\}\left(1+\frac{-\rho}{\lambda\ell} + \Upsilon_c\right)}{2\left(1+\frac{1}{\lambda}\frac{-\rho+\beta}{\ell}\right)}} \times 100 \ (\%), \tag{3.28}$$

where

$$\Upsilon_c = \frac{\beta\Lambda_{f0}\left\{-\bar{\nu}+\beta\bar{\nu}+3\beta\overline{\nu(\nu-1)}\right\}}{\overline{\nu_0(\nu_0-1)}\,\Lambda_{f0}+2\left(\frac{-\rho+\beta}{\ell}\right)} \lesssim -\beta \ . \tag{3.29}$$

For the very subcritical state, the curves for the neutron fluctuations are the same as that given by zero-power reactor--noise theory. But as the core goes critical and the power increases, the departure from theory accentuates due to feedback from the coolant temperature fluctuations. For $|\rho| \lesssim 3\times10^{-2}$ ¢, the neutron fluctuations are largely perturbed by fluctuations in the coolant flow-rate through the temperature coefficient of reactivity. Such a behavior is similar to that presented in Fig. 3.10.

For the fuel temperature, our calculated values decrease for large absolute values of ρ. This result is different from that in Fig. 3.10. The reason is as follows. The variance is related to the corresponding noise spectrum through

$$\langle (F-F_0)^2 \rangle = \frac{1}{\pi} \int_0^{\infty} P_{FF}(\omega)\, d\omega \ . \tag{3.30}$$

On the SMR, the spectrum $P_{FF}(\omega)$ for the fuel temperature is given by

$$P_{FF}(\omega) = 2(2D_{NN}) \left| T(\omega) \right|^2 \Lambda_{f0}^2 (\omega^2 + h_t^2)^{-1} \ , \tag{3.31}$$

where we have neglected the thermo-hydraulic contribution of the noise source $2D_{QQ}$, since our attention is focused on very low power levels below 10 W. With decreasing values of P, the transfer function $T(i\omega)$ tends to show a behavior similar to the zero power reactor transfer function $T_0(i\omega)$, as shown in Fig. 3.4. Therefore, in a delayed critical system, the amplitude of $T(i\omega)$, and hence of $P_{FF}(\omega)$, becomes very large for small values of ω. This is reflected in the curves of Fig. 3.10. On the other hand, in a subcritical system, the amplitude decreases with diminishing ρ, so that we have a result as shown in Fig. 3.12.

The above-mentioned behavior is discernible, though only faintly, in Figs. 3.10 and 3.12 for the coolant temperature fluctuations at lower power levels. For the coolant flow-rate, our calculated results are nearly the same as shown in Fig. 3.10.

§ 5. Concluding Remarks

The main conclusion of the present chapter is that our stochastic model can be considered to account reasonably well for the experimental facts regarding random fluctuations in natural convection light-water reactors operating at various power levels up to 100 kW. An adjustable parameter α introduced here plays a significant and quantitative role in determining the noise spectra and the variances. Our calculations suggest that we require more experimental data on fluctuations in the temperature and the flow-speed of the coolant.

The present study has treated the case of natural convection cooling, but it is possible to apply our model to the case of forced circulation, for which, however, we require a proper choice of the values of the two adjustable parameters Λ_Q and α for different values of flow-speed of the coolant. When more experimental data are available, it should be very interesting to compare with such data the numerical calculations based on our stochastic model, particularly in respect of phase response and coherence of the neutron number—coolant temperature cross noise spectra.

In the present paper, we have adopted the lumped parameter model based on both thermodynamical analysis of the mean behavior of a system and phenomenological considerations on the thermo-hydraulic noise sources. We might point out here that such a theoretical approach should help in understanding the experimental data and in suggesting new experiments on random fluctuations in various types of reactor, such as BWR and PWR. . . .

Table 3.1 Nuclear and material constants

Constants	Values	Constants	Values
$\overline{\nu}$	2.59	α_m ($°C^{-1}$)	1.00×10^{-4}
$\overline{\nu(\nu-1)}$	5.31	$c_v^c \simeq c_p^c$ (cal g^{-1} $°C^{-1}$)	1.00
β	7.55×10^{-3}	κ (atm^{-1})	2.00×10^{-5}
λ (sec^{-1})	7.70×10^{-2}	ρ_c ($g\ cm^{-3}$)	1.00
q (Mev)	200		

Table 3.2 Model parameters

Parameters	Values	Reference Eqs.
ℓ (sec^{-1})	1.00×10^{-4}	(1.64)
S (sec^{-1})	10^6	(1.57) & (3.6)
α_1 ($\delta k/k/°C$)	5.65×10^{-7}	(1.64)
α_2 ($\delta k/k/°C$)	1.13×10^{-4}	(1.64)
β_1 ($°C^{-1}$)	1.77×10^{-5}	(1.63)
β_2 ($°C^{-1}$)	1.30×10^{-4}	(1.63)
θ_1 ($°C$)	30.0	(1.24)
$V_F \rho_F c_v^F$ (Kcal $°C^{-1}$)	18.7	(1.8) & (1.27)
V_c (ℓ)	67.9	(1.9)
S_c (cm^2)	1.05×10^3	(1.22)

$(\alpha_\theta = -\alpha_1 - \alpha_2)$

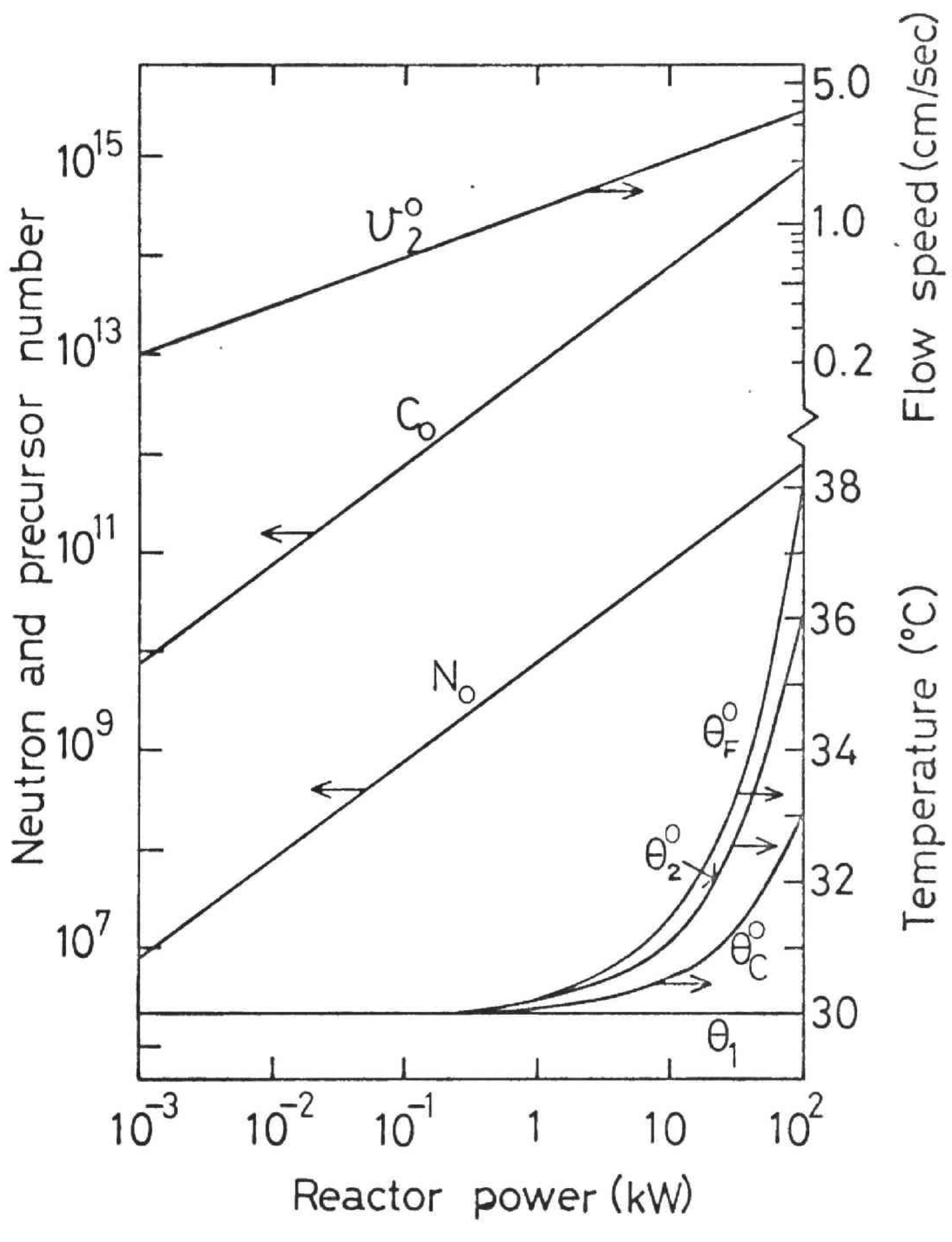


Fig. 3.1 Reactor power dependence of steady-state values o quantities shown

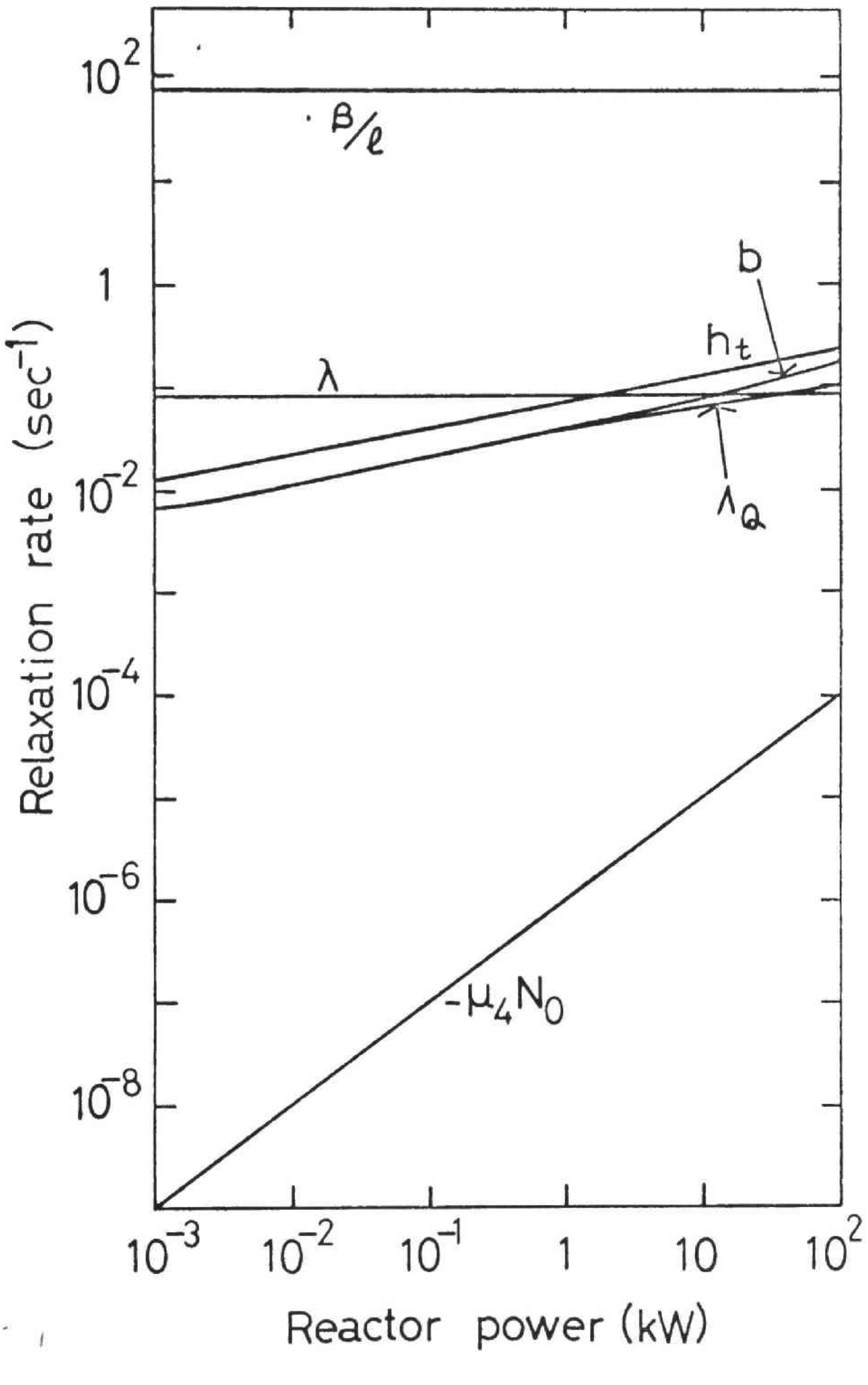


Fig. 3.2 Reactor power dependence of typical relaxation constants

Fig. 3.3 Comparison of theoretical neutron noise spectra with experimental results (Refs. 5 and 8) (normalized at ω = 20 rad/sec)

——— calculated for α = 4 %

- - - - - for α = 1 %

—·—·—· with the theory of zero power reactor noise

o : 100 kw in KUR($\alpha_\theta \cong -1\times10^{-4}\delta k/k/°C$)

Δ : 100 kw in KUR($\alpha_\theta \cong +1\times10^{-5}\delta k/k/°C$)

● : 1 kw in KUR($\alpha_\theta \cong -1\times10^{-4}\delta k/k/°C$)

▲ : 1 kw in KUR($\alpha_\theta \cong +1\times10^{-5}\delta k/k/°C$)

□ : 100 kw in TTR ($\alpha_\theta = -1.28\times10^{-4}\delta k/k/°C$)

■ : 10 kw in TTR ($\alpha_\theta = -1.28\times10^{-4}\delta k/k/°C$)

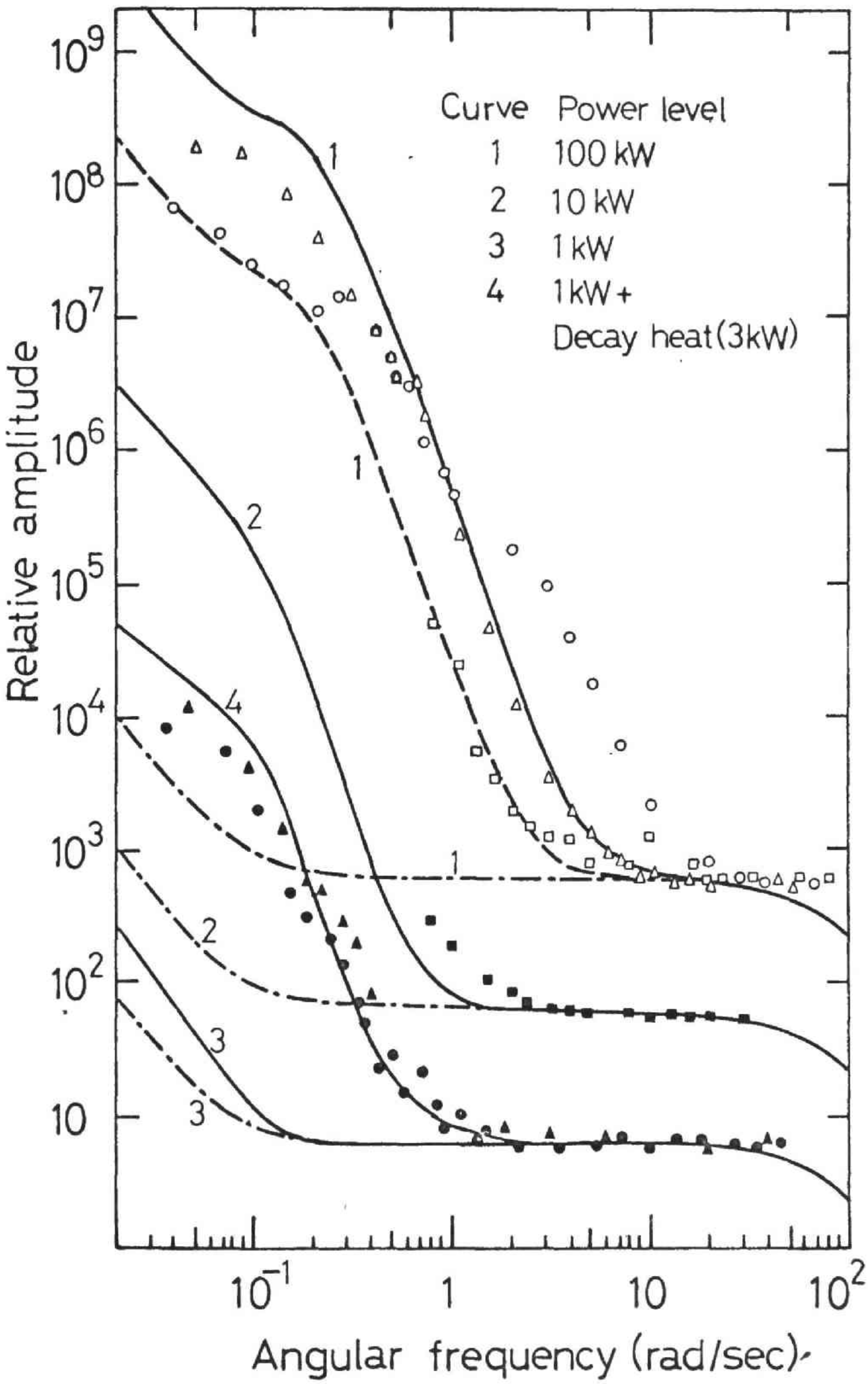
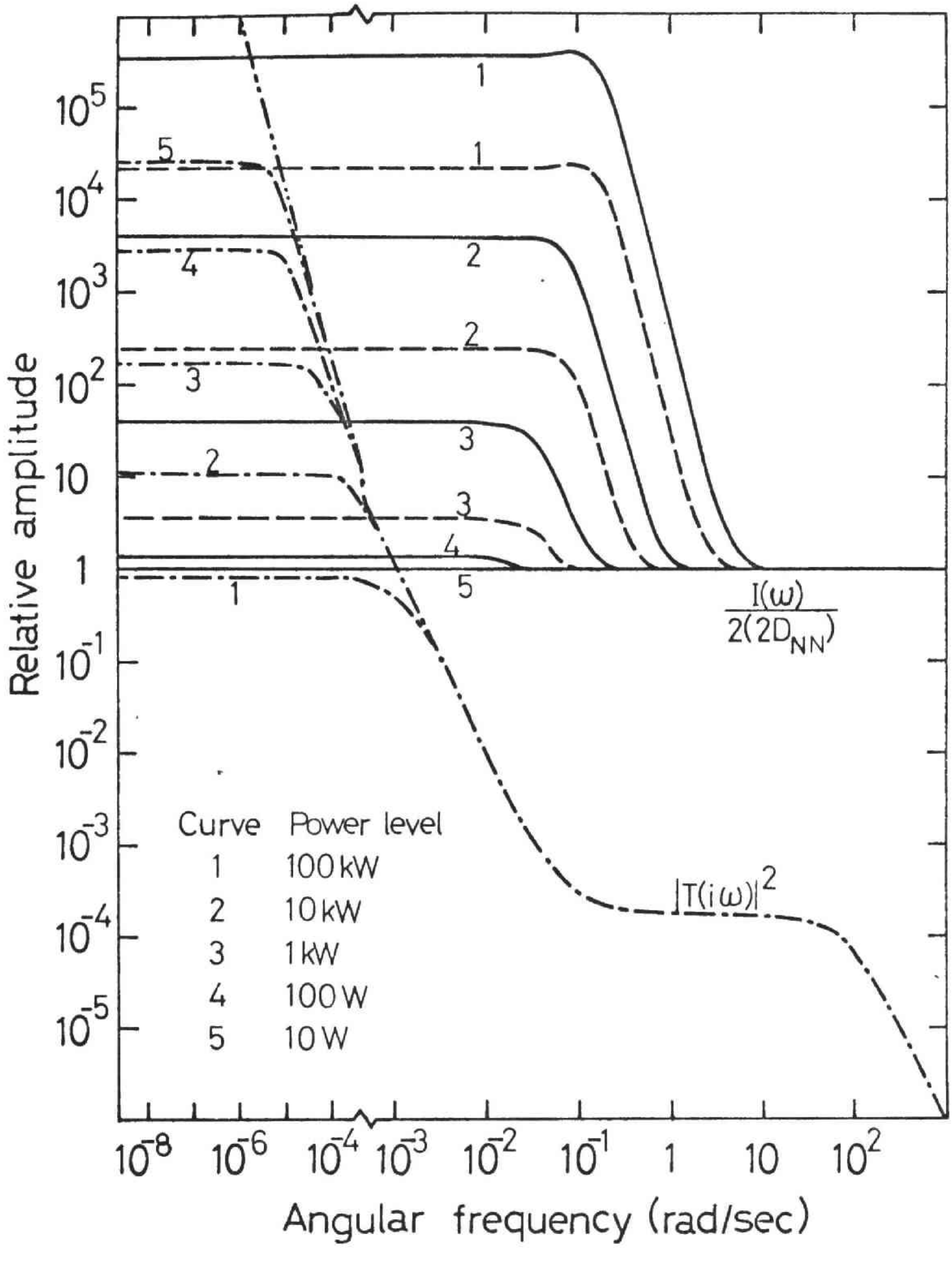

Curve Power level
1 100 kW
2 10 kW
3 1 kW
4 1 kW +
Decay heat (3kW)
Relative amplitude
Angular frequency (rad/sec)
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
10^{-1}
1
10
10^2

Fig. 3.9



———— calculated for α = 4 %

- - - - - for α = 1 %

—··—··— with the theory of zero power reactor noise

Fig. 3.4 Transfer functions and input noise source spectra for values of power levels shown

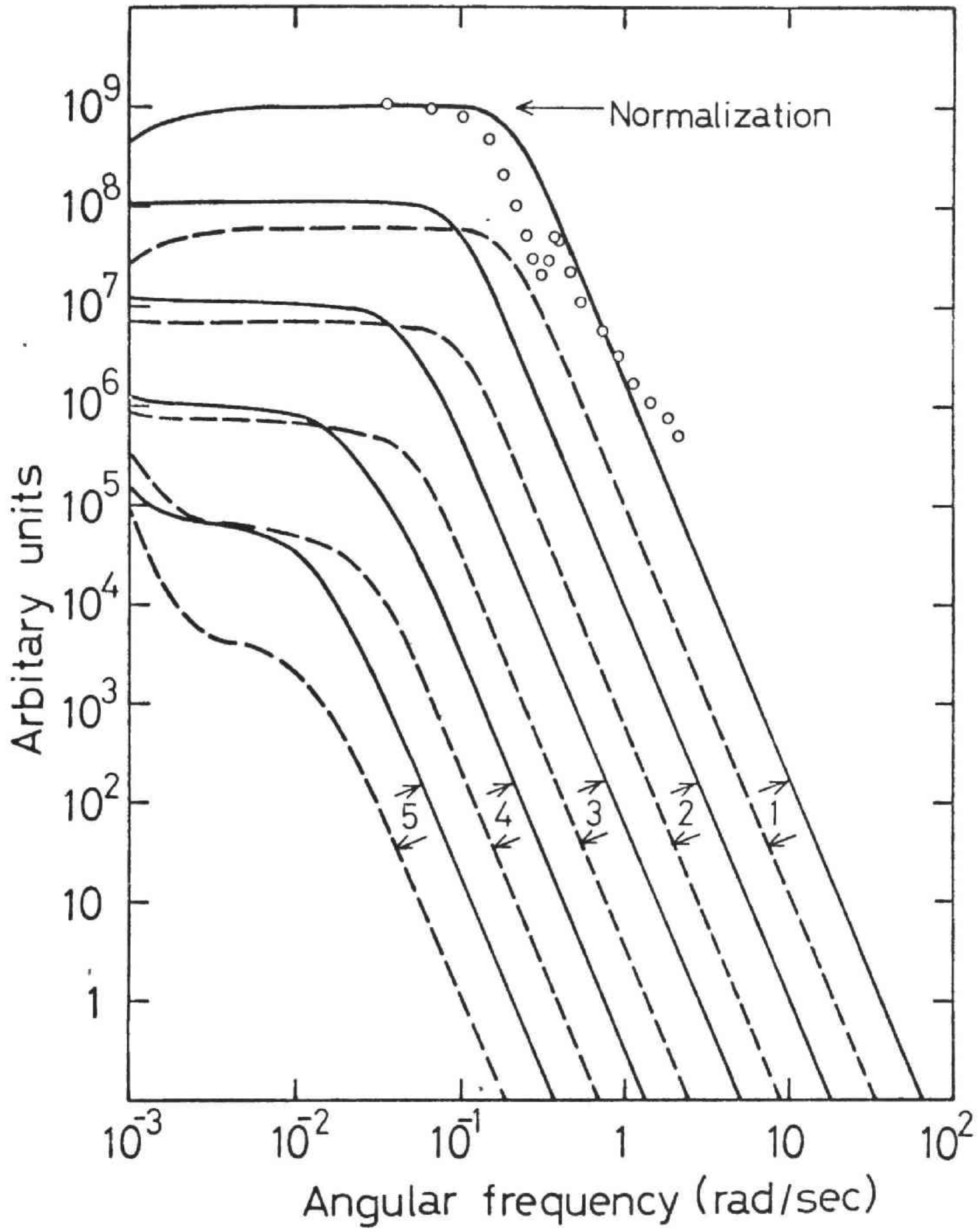


Fig. 3.5 Noise spectra of coolant temperature fluctuations The solid and dashed curves have been calculated fc the same values of power level P and parameter α as in Fig. 3.4. The open circles are experimental points (Ref. 4).

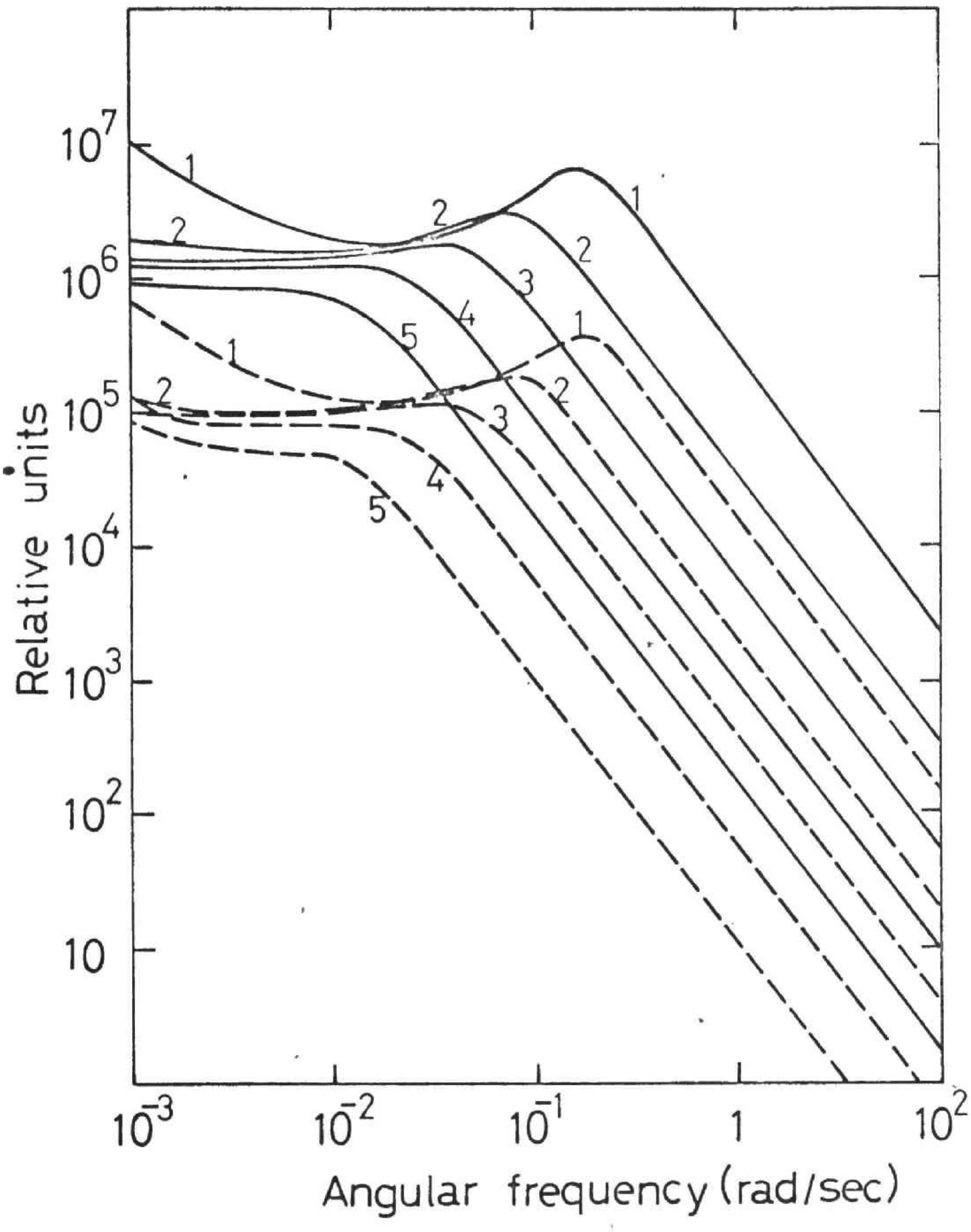


Fig. 3.6 Theoretical noise spectra of coolant flow-speed fluctuations for the same values of P and α as in Fig. 3.4

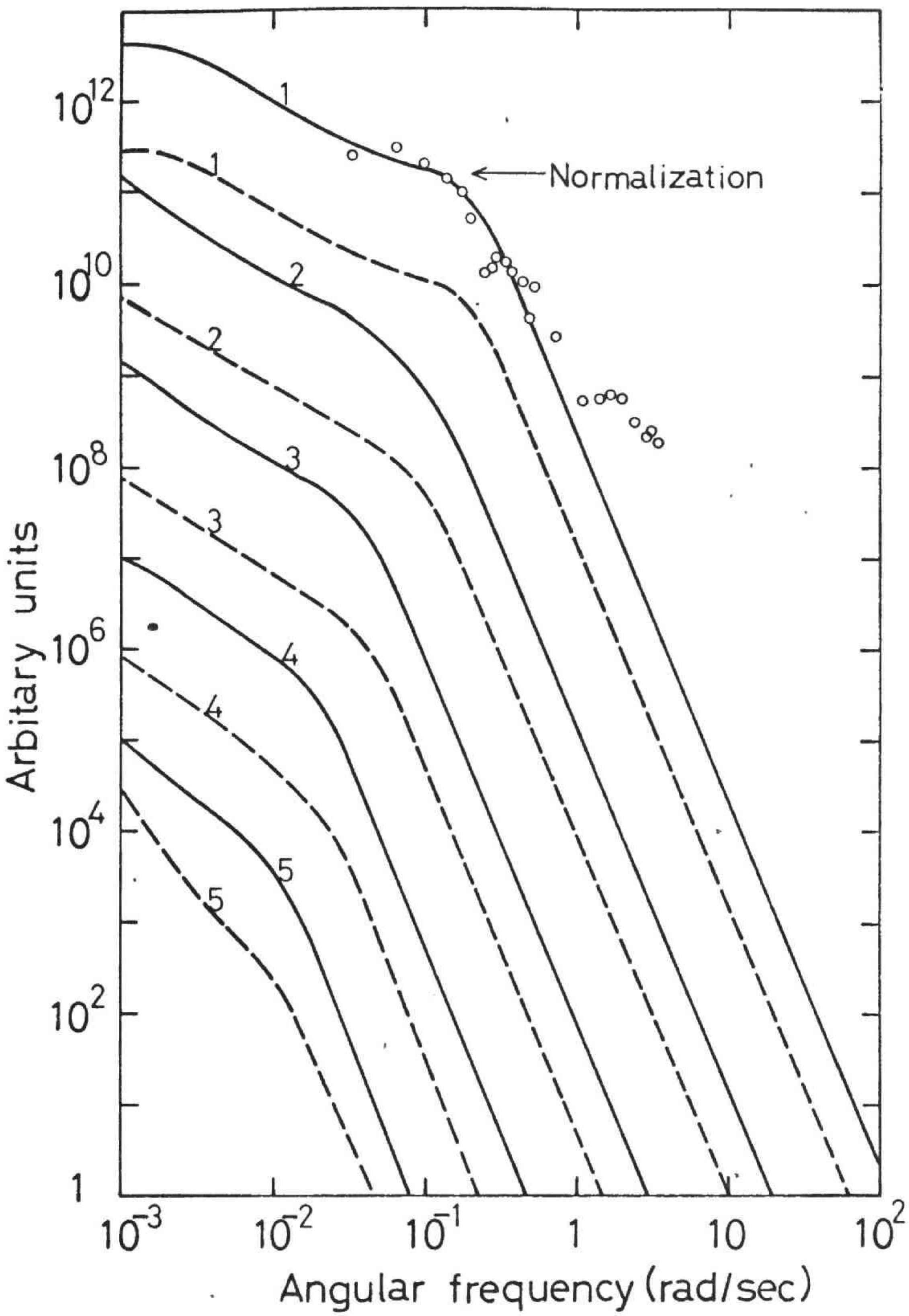


Fig. 3.7 Magnitude of cross noise spectra of neutron number--coolant temperature fluctuations
The solid and dashed curves have been calculated for the same values of P and α as in Fig. 3.4. The open circles are experimental points (Ref. 4).

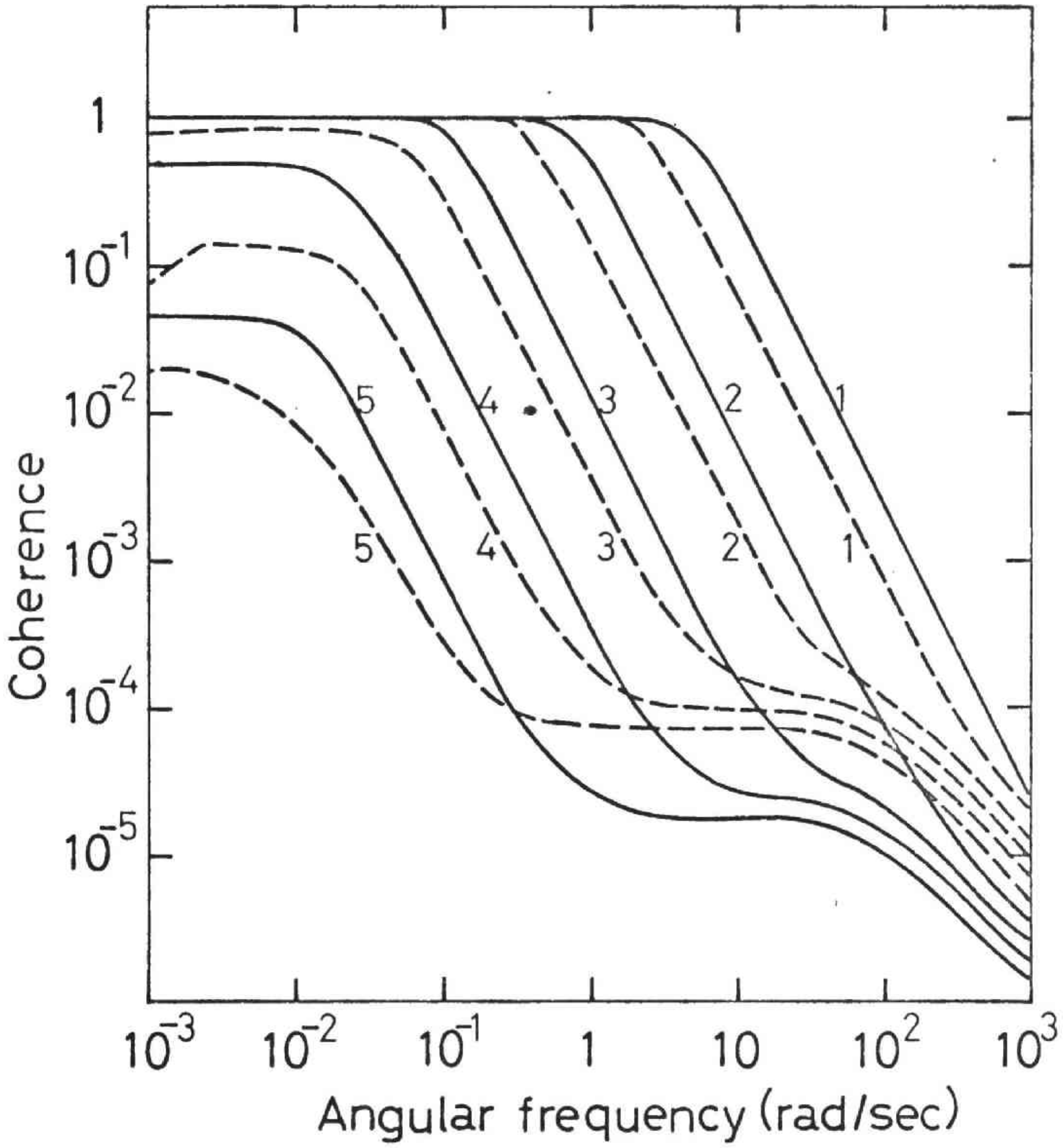
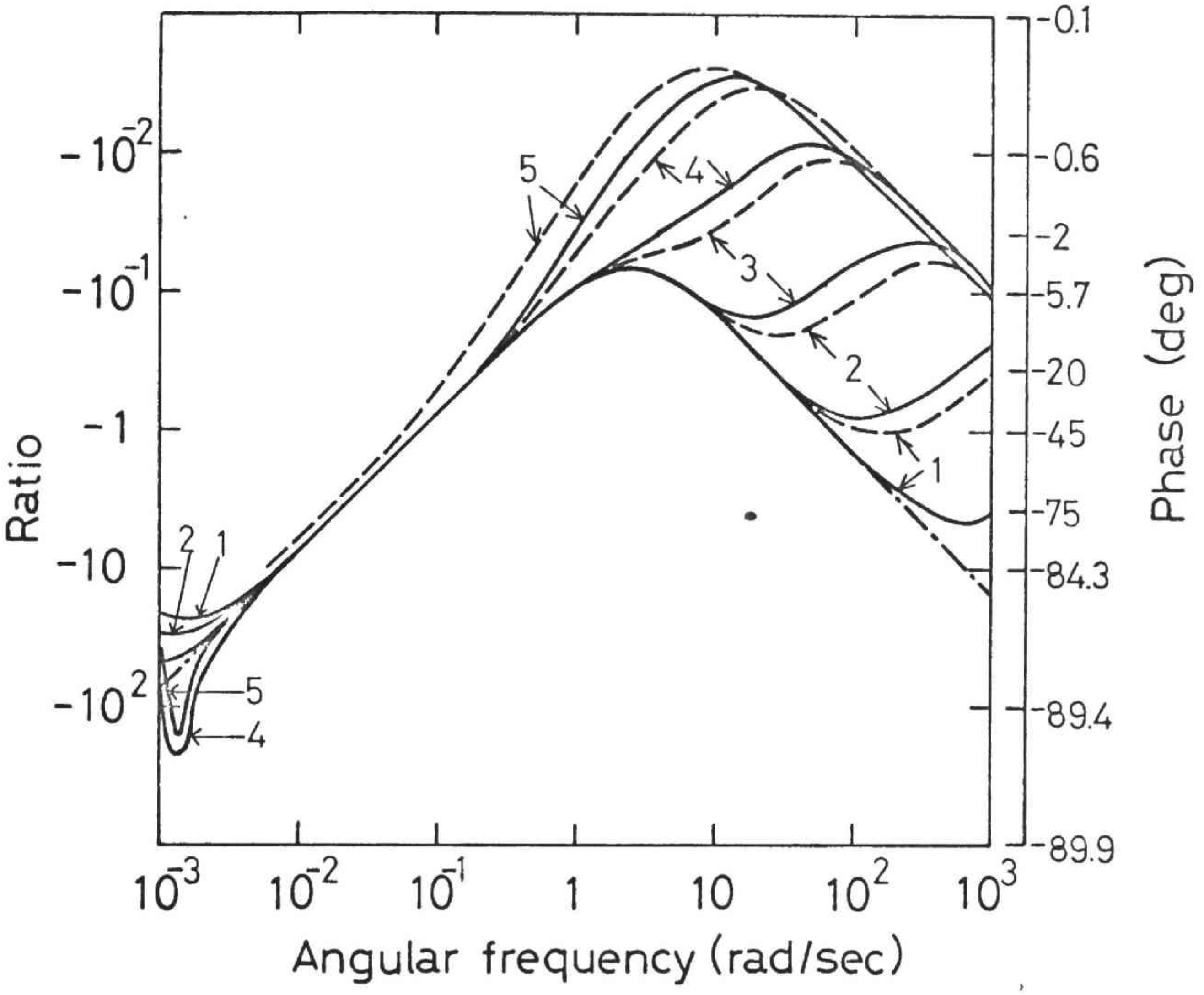


Fig. 3.8 Coherence of neutron number-coolant temperature cross power spectra for the same values of P and α as in Fig. 3.4



—·—·—·— phase response of the zero power transfer function

Fig. 3.9 Ratio of the imaginary part to the real part of the neutron number-coolant temperature cross power spectra for the same values of P and α as in Fig. 3.4
The dashed curves in the lower frequency region ($\omega < 10^{-2}$ rad/sec) have not been shown, because the behavior is similar to that of the solid curves.

Fig. 3.10 Relative standard deviations of fluctuations of state quantities as function of reactor power level in a delayed critical system
The open circles denote experimental results of neutron noise measurements (Ref. 34).

Curve A : Number of neutrons

B : Temperature of the fuel

C : Temperature of the coolant

D : Flow-speed of the coolant

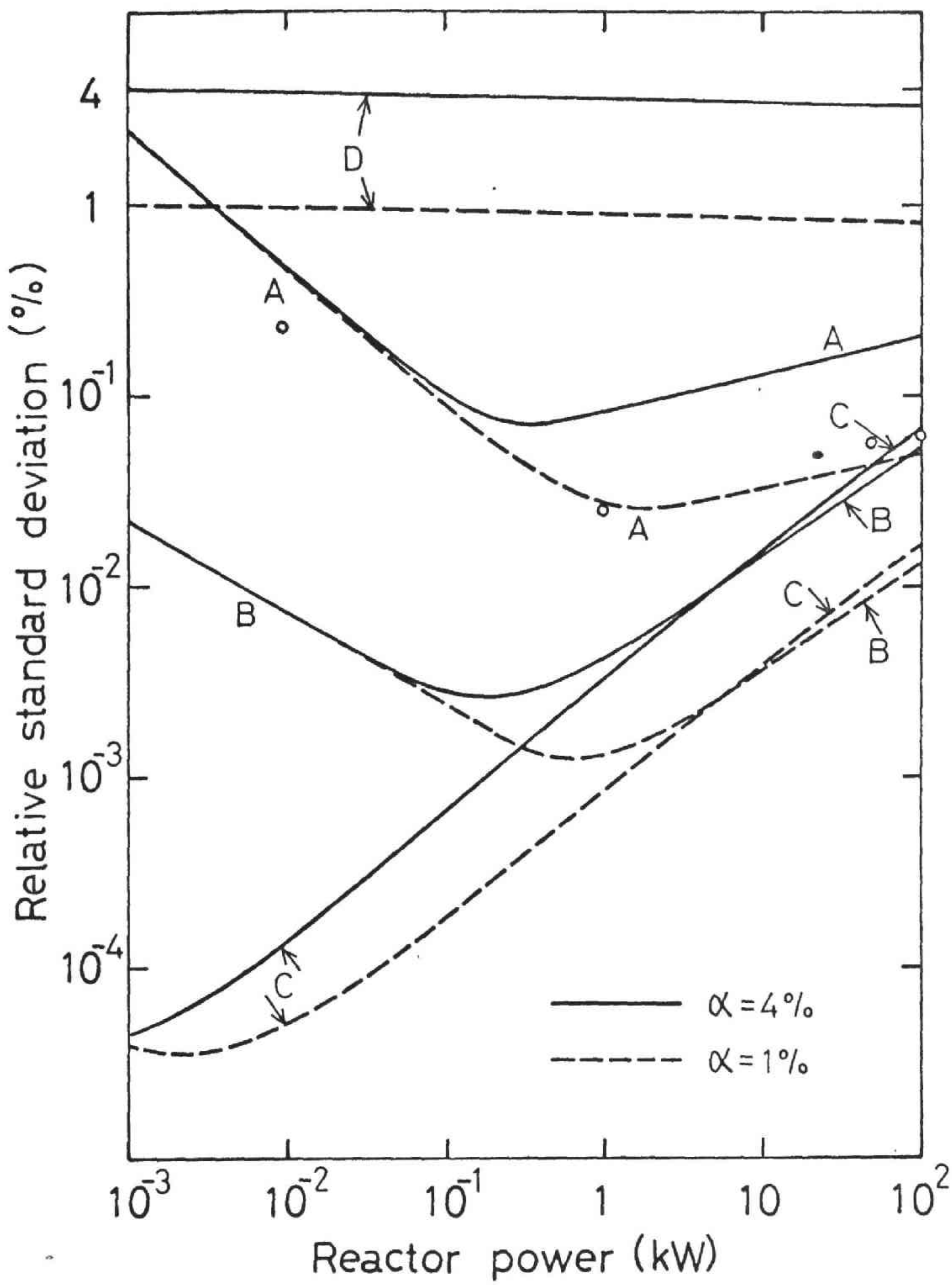

4
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
Relative standard deviation (%)
10⁻³
10⁻²
10⁻¹
1
10
10²
Reactor power (kW)
A
B
C
D
α = 4%
α = 1%

Fig. 3.10

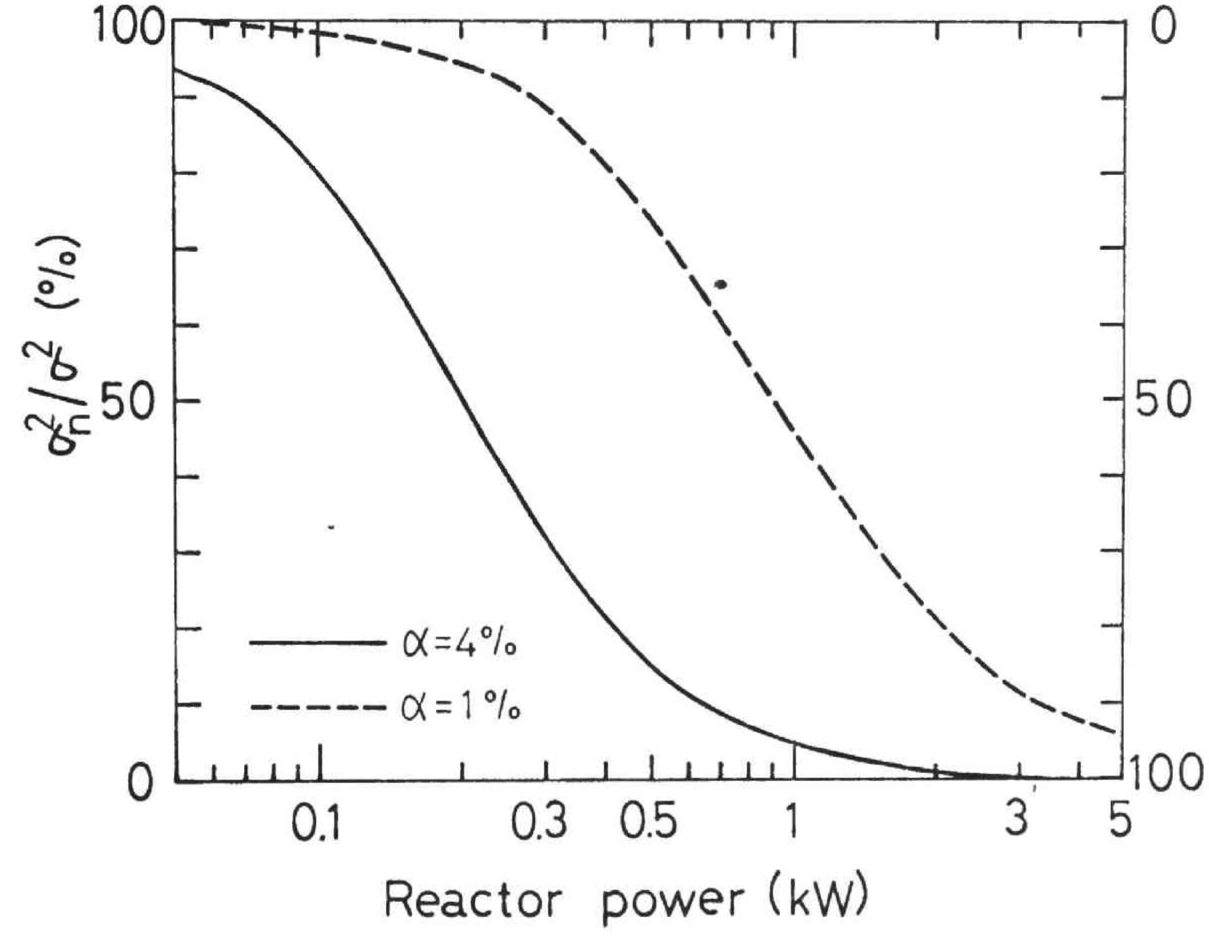
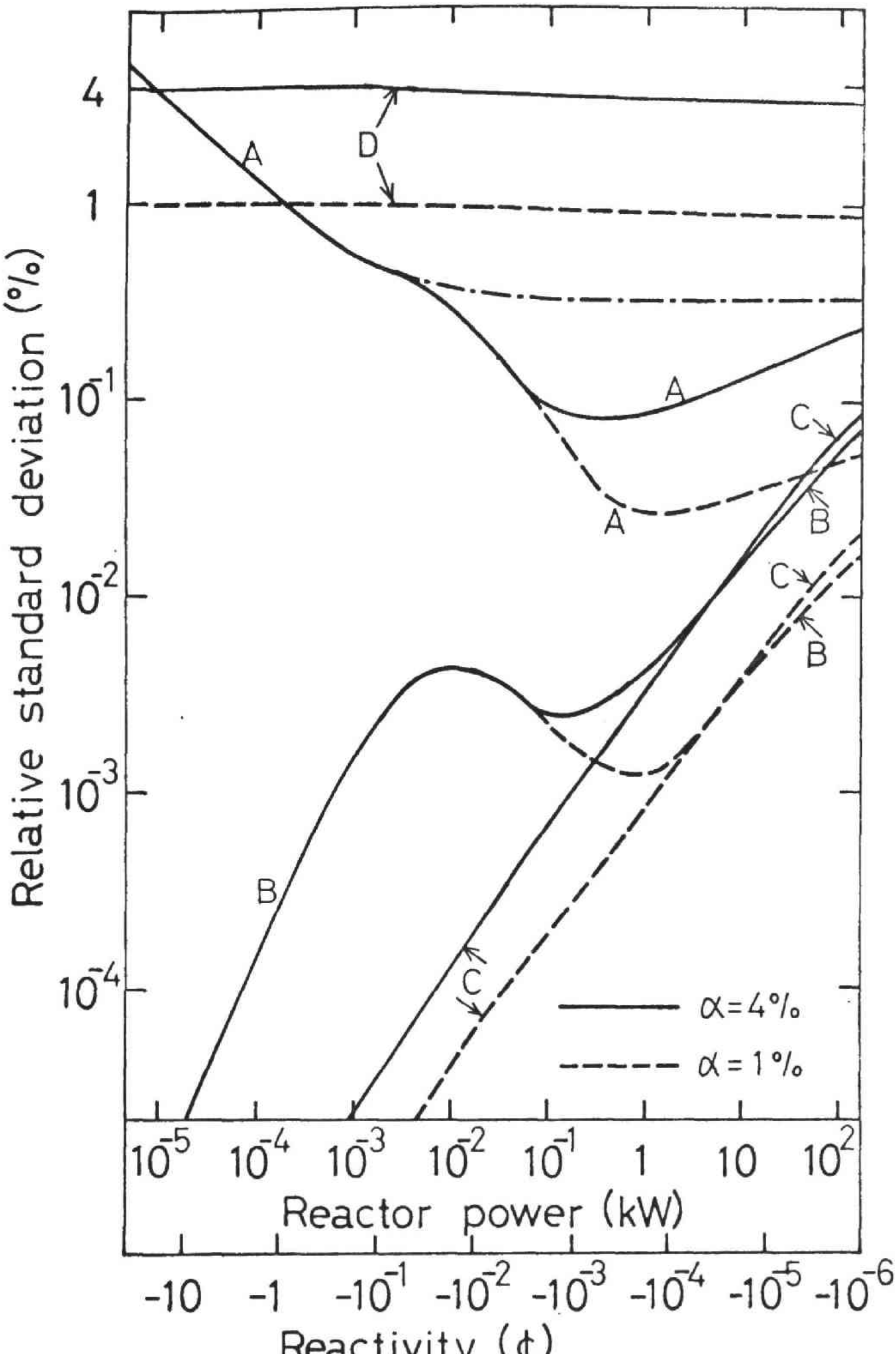


Fig. 3.11 σ_n^2/σ^2 and σ_t^2/σ^2 versus reactor power
σ^2 is the variance of the neutron fluctuations arising from all the noise sources, σ_n^2 from the nuclear and σ_t^2 from the thermal-hydraulic sources.



—·—·—·— calculated with the theory of zero power reactor noise

Fig. 3.12 Relative standard deviations of fluctuations of state quantities as function of reactivity

The curves A, B, C and D represent the same quantities as in Fig. 3.10.

Appendix I

Equation (1.10) is obtained as follows.

Combining Eqs. (1.1) and (1.2), the momentum conservation is rewritten in the form

$$\frac{\partial}{\partial t}(\rho \mathbf{V}) + \nabla\cdot(\rho \mathbf{V}\mathbf{V}) = \rho \mathbf{X} - \nabla\cdot P\mathbf{1} \tag{A1}$$

Carrying out the volume integral, we have

$$\frac{d}{dt}(\rho_c \mathbf{V}_c) + \frac{1}{V_c}\int_S (\rho \mathbf{V}\mathbf{V})\cdot d\mathbf{S} = \rho_c \mathbf{X}_c - \frac{1}{V_c}\int_S (-P\mathbf{1})\cdot d\mathbf{S} \tag{A2}$$

which leads to Eq. (1.10) with the aid of Eq. (1.9). The Eqs. (1.14) and (1.15) are similarly obtained using Eq. (1.1).

Appendix II

The noise spectra $P_{MM}(\omega)$ and $P_{NM}(\omega)$ for the SMR:

$$P_{MM}(\omega) = 2\left|\frac{1}{(s+\Lambda_Q)(s+b)\left\{1-\mu_4 N_0 \frac{h_t\Lambda_{f0}}{(s+b)(s+h_t)}T_0(s)\right\}+\chi_B d}\right|^2$$

$$\times\left\{d^2(2D_{QQ}) + h_t^2\Lambda_{f0}^2\left|T_0(s)\frac{s+\Lambda_Q}{s+h_t}\right|^2(2D_{NN})\right\}\Bigg|_{s=i\omega} \tag{A3}$$

$$P_{NM}(\omega) = 2|T(s)|^2\left|\frac{1}{(s+\Lambda_Q)(s+b)+\chi_B d}\right|^2\Big\{\mu_4 N_0 d^2\, T_0(s^*)^{-1}(2D_{QQ})$$

$$+ h_t\Lambda_Q\frac{s^*+\Lambda_Q}{s^*+h_t}\{(s+\Lambda_Q)(s+b)+\chi_B d\}(2D_{NN})\Big\}\Bigg|_{s=i\omega} \tag{A4}$$

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