数理解析研究所講究録
1165 巻 2000 年 95-109

An Optimal Cost Allocation Rule in General Equilibrium Models

by Kazuya Kamiya
Faculty of Economics
University of Tokyo

Abstract

In this paper, we consider general equilibrium models with public utilities which produce public goods or private goods. In the models, the case of increasing returns is not a priori excluded. The products of the public utilities are allocated to the consumers according to rules that depend on information communicated to the public utilities. We present a cost allocation rule for which equilibrium allocations are always Pareto optimal. Moreover, the message spaces of the mechanisms are of finite dimension.

1 Introduction

It is now widely recognized that, in economies with nonconvex production sets, marginal cost pricing does not necessarily leads to a Pareto optimal allocation. Recently, in a class of economies with nonconvex production sets, Kamiya [1995] and Moriguchi [1996] presented decentralized mechanisms (nonlinear pricing rules) which determine Pareto optimal allocations. However, the production sets in their models are somewhat restrictive. In Moriguchi [1996], nonconvexity is solely caused from fixed costs, although joint production is allowed. In Kamiya [1995], each firm produces single output, although wide classes of nonconvexities are allowed. The main purpose of this paper is to present a pricing rule for firms with general nonconvex production sets, which determines Pareto optimal allocations.

On the other hand, in economies with public goods in which the case of increasing returns is not a priori excluded, Kaneko [1977] and Mas-Colell and Silvestre [1989] presented cost allocation rules which determine Pareto optimal allocations. We also show that our rule is applicable to their models, i.e., the equilibrium allocations are always Pareto optimal if the firms adopt our rule. Moreover, the message space of our model is of finite dimension, although Kaneko, Mas-Colell, and Silvestre directly used cost functions which may not be parametrized by a finite number of parameters.

It is now well known that marginal cost pricing just meets the first order necessary condition for Pareto optimality. Then it is natural to investigate the following two problems:

1. What additional conditions guarantee Pareto optimality of marginal cost pricing equlilibria?
2. What pricing rules lead the economy to Pareto optimal allocations?

Dierker [1986] and Quinzii [1991] investigated the first problem. They showed that marginal cost pricing equilibria are Pareto optimal if the elasticities of demand function and of the cost function satisfy conditions which guarantee that social indifference curve, at an equilibrium, does not "cut inside" the production set.

The second problem is rather involved. Since marginal cost pricing rule is a linear pricing rule and it is the first order necessary condition for Pareto optimality, then any linear pricing rules, except marginal cost pricing, satisfy even the first order condition for Pareto optimality. Thus any linear pricing rules do not lead the economy to Pareto optimal allocations. In order to overcome the difficulty, Kamiya [1995] and Moriguchi [1996] investigated nonlinear pricing rules. They showed that if the firms follow their nonlinear pricing rules, then the equilibrium allocations are always Pareto optimal. However, their pricing rules are only applicable to some special cases. The main purpose of this paper is to extend their approach to general cases.

It is worthwhile noting that the equilibria in our model have similar properties as Roemer and Silvestre [1993]'s proportional solutions. However, unlike our approach, their model does not have an explicit mechanism to achieve the solution.

The paper is organized as follows. First, in section 2, we present our model and functions used for our mechanisms. Section 3 is devoted to an economy with public goods. We present a mechanism for the economy and prove the Pareto optimality of equilibria. Moreover, in the case of decreasing returns, we prove the existence of equilibria. In section 4, we present a mechanism for an economy with private goods and prove the Pareto optimality of equilibria. The existence of equilibria is proved in the case of decreasing returns.

2 The Basic Model

We consider an economy with $\ell_1 + \ell_2$ goods, where $\ell_1 \geq 1$ and $\ell_2 \geq 1$. The first ℓ_1 goods, called P goods, are produced by firms with nonconvex production sets and the other ℓ_2 goods, called C goods, are not produced. There are n firms and the j-th firm produces the $n(j-1)+1$-th, $\cdots$, $n(j)$-th goods, where $\sum_{j=1}^{n} n(j) = \ell_1$, $n(j) \geq 1$, $j = 1, \cdots, n$, and $n(0) = 0$. That is joint production is allowed and each P good is produced just by one firm. Each firm produces its outputs using C goods, *i.e.*, the j-th firm has an input requirement set correspondence $\eta_j : R_+^{n(j)} \to R_+^{\ell_2}$. That is, for a given output vector $y_j \in R_+^{n(j)}$, $\eta_j(y_j)$ is a set of inputs (vectors of C goods) from which the firm can produce y_j. There are $m \geq 2$ consumers and the i-th consumer has the consumption set $R_+^{\ell_1+\ell_2}$, an initial endowment $\omega_i \in R^{\ell_1+\ell_2}$, and a utility function $u_i : R_+^{\ell_1+\ell_2} \to R$, $i = 1, \cdots, m$.

The notations are as follows. For a vector $z \in R^{\ell_1+\ell_2}$, $z^p \equiv (z_1, \cdots, z_{\ell_1})$ and $z^c \equiv (z_{\ell_1+1}, \cdots, z_{\ell_1+\ell_2})$. For $e \in R^{\ell_2}$ and $f \in R^{\ell_2}$, $e \cdot f$ denotes their inner product. For a set B in the Euclidean space, int B denotes the interior of B in the space.

We use the following assumptions.

Assumption A1: (i) For $i = 1, \cdots, m$, u_i is strictly increasing, *i.e.*, $u_i(x_i) > u_i(x_i')$ for $x_{ik} \geq x_{ik}'$, $k = 1, \cdots, \ell_1 + \ell_2$, with at least one strict inequality. (ii) For $i = 1, \cdots, m$, $\omega_{ik} = 0$, $k = 1, \cdots, \ell_1$, and $\omega_{ik} > 0$, $k = \ell_1 + 1, \cdots, \ell_1 + \ell_2$.

Assumption A2: For $j = 1, \cdots, n$, (i) η_j is a continuous correspondence for $y_j \in R_+^{n(j)} \backslash \{0\}$ and is nonempty convex valued for $y_j \in R_+^{n(j)}$, (ii) $0 \in \eta_j(0)$, and (iii) for y_j, $y_j' \in R_+^{n(j)}$ such that $y_{jk} \geq y_{jk}'$ for $k = n(j-1)+1, \cdots, n(j)$, $\eta_j(y_{jk}) \subset \eta_j(y_{jk}')$.

Assumption A1-(i) is standard in general equilibrium theory. Assumption A1-(ii) says that each consumer is endowed with all C goods and do not have any P goods. In Assumption A2-(i), it is assumed that the input requirement set $\eta_j(y_j)$ is convex for each $y_j \epsilon R_+^{n(j)}$ and Assumption A2-(iii) is the standard monotonicity condition. Notice that in Assumption A2, no convexity is assumed in the output space.

For $j = 1, \cdots, n$, we define the cost function $C_j : R_+^{n(j)} \times R_+^{\ell_2}(\ni (y_j, q)) \to R_+$ and the demand correspondence (for inputs) $\varphi_j : R_+^{n(j)} \times R_+^{\ell_2}(\ni (y_j, q)) \to R_+^{\ell_2}$ as follows, where $q \in R_+^{\ell_2}$ is a price vector of C goods.

$$C_j(y_j, q) \equiv \min\{q \cdot s_j | s_j \in \eta_j(y_j)\}$$

$$\varphi_j(y_j, q) \equiv \arg\ \min\{q \cdot s_j | s_j \in \eta_j(y_j)\}.$$

By Assumption A2, C_j and φ_j are well defined.

By Berge's maximum theorem and Assumption A2, the following lemma holds.

Lemma 2.1: Under Assumption A2, for $j = 1, \cdots, n$, (i) $C_j(0, q) = 0$ for all $q \in R_+^{\ell_2}$, (ii) C_j is increasing, *i.e.*, for any given $q \in R_+^{\ell_2} \backslash \{0\}$, $C_j(y_j, q) \geq C_j(y_j', q)$ if

$y_{jk} \geq y'_{jk}$, $k = 1, \cdots, n(j)$, (iii) C_j is continuous in $q \in R_+^{\ell_2}$ and $y_j \in R_+^{n(j)} \backslash \{0\}$, (iv) C_j is homogeneous of degree one in $q \in R_+^{\ell_2}$, *i.e.*, $C_j(y_j, \lambda q) = \lambda C_j(y_j, q)$ for $\lambda > 0$, $(v)\varphi_j$ is upper hemi-continuous in $q \in R_+^{\ell_2}$ and $y_j \in R_+^{\ell(j)} \backslash \{0\}$, and (vi) φ_j is homogeneous of degree 0 in $q \in R_+^{\ell_2}$, *i.e.*, $\varphi_j(y_j, \lambda q) = \varphi_j(y_j, q)$ for $\lambda > 0$.

In Lemma 2.1, C_j can be discontinuous at $y_j = 0$, *i.e.*, fixed cost is allowed.
Next, we define functions which will be used for the definition of our cost allocation rule.

Definition 2.1: Functions $\beta_{jk} : R_+ \times R_{++}^{\ell_2} \times R_{++}^{n(j)} (\ni (y_{jk}, q, \bar{v}_j)) \to R$, $k = n(j-1)+1, \cdots, n(j)$, are said to be cost allocation functions if, for given $q \in R_{++}^{\ell_2}$ and $\bar{v}_j \in R_{++}^{n(j)}$,

(i) $\beta_{jk}(y_{jk};\, q,\, \bar{v}_j)$, a function of y_{jk}, is expressed by a finite number of parameter,
(ii) $\sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(\bar{v}_{jk};\, q,\, \bar{v}_j) = C_j(\bar{v}_j, q)$
(iii) if $y_{jk} \geq y'_{jk}$ then $\beta_{jk}(y_{jk};\, q,\, \bar{v}_j) \geq \beta_{jk}(y'_{jk};\, q,\, \bar{v}_j)$,
(iv)$\sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(y_{jk};\, q,\, \bar{v}_j) \leq C_j(y_j, q)$ for all $y_j \in R_+^{n(j)}$

Definition 2.2: Functions $\beta_{jk}^c : R_+ \times R_{++}^{\ell_2} \times R_{++}^{n(j)} (\ni (y_{jk}, q, \bar{v}_j)) \to R$, $k = n(j-1)+1, \cdots, n(j)$, are said to be concave cost allocation functions, if, for given $q \in R_{++}^{\ell_2}$ and $\bar{v}_j \in R_{++}^{n(j)}$, (i), (ii), (iii), (iv), and the following condition are satisfied.

$(v)\beta_{jk}$ is concave in $y_{jk} \in R_+$.

Under the following assumption, we can construct cost allocation functions.

Assumption A3: For $j = 1, \cdots, n$, C_j is C^2 with respect to $y_j \in R_{++}^{n(j)}$ and is strictly increasing, *i.e.*, for any given $q \in R_+^{\ell_2} \backslash \{0\}$, $C_j(y_j, q) > C_j(y'_j, q)$ for $y_j, y'_j \in R_+^{n(j)}$ such that $y_{jk} \geq y_{jk'}$, $k = n(j-1)+1, \cdots, n(j)$, with at least one strict inequality.

Theorem 2.1: Under Assumptions A2 and A3, there exist concave cost allocation functions.

Proof: See Appendix.

3 An Economy with Public Goods

In this section, P goods are considered as public goods.[1] Using the cost allocation function β_{jk}, we construct our cost allocation rule by an iterative process as follows. (Note that the concave cost allocation function β^c_{jk} will be used in Section 4.) The j-th firm chooses real numbers $\alpha_{ik} \geq 0$, $i = 1, \cdots, m$, $k = 1, \cdots, \ell_1$, such that $\sum_{i=1}^{m} \alpha_{ik} = 1$. Each consumer first reports the demand for the k-th P good (public good) for $k = 1, \cdots, \ell_1$; later, we discuss how the first report is determined. Let $\bar{x}_{ik}$ denote the $i-$th consumer's demand for the k-th P good. If the i-th consumer's new demand for the $k - th$ P good is x_{ik}, then he pays $\beta_{jk}(x_{ik};\, \bar{x}_{ik},\, q)\alpha_{ik}$.[2]

Note that if $\bar{x}_{ik} = x_{ik} = x_{i'k} = \bar{x}_{i'k}$ for all i, $i' = 1, \cdots, m$, and all $k = 1, \cdots, \ell_1$, then by Definition 2.1 (ii) the total outlay is equal to the cost. That is if all consumers demand the same amount of public goods, then the firms break even.

The consumers maximize their utilities subject to their budget constraints, *i.e.*, for $i = 1, \cdots, m$,

$$\begin{aligned} &\max \quad u_i(x) \\ &s.t. \ \sum_{j=1}^{n} \sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(x_{ik};\, \bar{x}_{ik},\, q)\alpha_{ik} + q \cdot x_i^c \leq q \cdot \omega_i^c, \\ &x \in R_+^{\ell_1+\ell_2}. \end{aligned}$$

If the i-th consumer's demand for the k-th P good, $\hat{x}_{ik}$, is not equal to $\bar{x}_{ik}$, $\hat{x}_{ik}$ will be $\bar{x}_{ik}$ in the next iteration. Note that $\bar{x}_{ik}$ in the first iteration can be chosen arbitrarily.

Definition 3.1: An $m\ell_1 + m(\ell_1 + \ell_2) + \ell_1 + \ell_2$ tuple $((\alpha^*_{ik}),\, (x^*_i),\, (y^*_j),\, q^*) \in S^{m\ell_1} \times R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1} \times R_+^{\ell_2}$ is said to be an equilibrium if the following conditions hold, where S^m is the $(m-1)$-dimensional unit simplex and $S^{m\ell_1}$ is the ℓ_1 times product of S^m.

(i) For all $i = 1, \cdots, m$,

---

[1] It is worthwhile comparing our model with Kaneko [1977]'s model and Mas-Colell and Silvestre [1989]'s model. In Mas-Colell and Silvestre's model, the number of firms is one, *i.e.*, $n = l$, and the number of inputs is one, *i.e.*, $\ell_2 = l$. In Kaneko's model, joint production is not allowed, *i.e.*, $n(j) = l$ for all $j = l, \cdots, n$, and the number of inputs is one, *i.e.*, $\ell_2 = l$. Of course, our model has several firms and several inputs, and allows for joint production.

[2] Unlike our allocation rule, Kaneko [1977] and Mas-Colell and Silvestre [1989] constructed cost allocation rules directly using the cost functions.

$x_i^* \in \arg\ \max\{u_i(x_i) | \sum j = 1^n \sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(x_{ik};\ x_{ik}^*,\ q^*)\alpha_{ik}^* + q^* \cdot x_i^c \leq q^* \cdot \omega_i^c,$ $x_i \in R_+^{\ell_1+\ell_2}\}$,

(ii) $\sum_{j=1}^n s_j^* + \sum_{i=1}^m x_i^{*c} \leq \sum_{i=1}^m \omega_i^c$ for some $s_j^* \in \varphi_j(y_j^*,\ q^*)$,

(iii) $y_{jk}^* = x_{ik}^*$ for all $k = n(j-1)+1, \cdots, n(j)$ and all $j = 1, \cdots, n$.

(iv) $x_{ik}^* > 0$ for all $i = 1, \cdots, m$ and all $k = 1, \cdots, \ell_1$.

In the above definition, (i) is standard, and (ii) and (iii) are the market clearing condition for C goods and P goods, respectively, (iv) simply says the demands for P goods are positive. Note that, by (i), $x_{ik}^* = \bar{x}_{ik}$, $i = 1, \cdots, m$, $k = 1, \cdots, \ell_1$ hold in equilibria.

The following definitions are also standard.

Definition 3.2: An $m(\ell_1+\ell_2)+\ell_1$ tuple $((x_i), (y_j)) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ is said to be a feasible allocation if $\sum_{j=1}^n s_j + \sum_{i=1}^m x_i^c \leq \sum_{i=1}^m \omega_i^c$ for some $s_j \in \eta_j(y_j)$, and $x_{ik} = y_{jk}$ for all $k = n(j-1)+1, \cdots, n(j)$ and all $j = 1, \cdots, n$.

Definition 3.3: An $m(\ell_1+\ell_2)+\ell_1$ tuple $((x_i), (y_j)) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ is said to be a Pareto optimal allocation if it is a feasible allocation and there is no feasible allocation $((x_i'), (y_j')) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ such that $u_i(x_i') \geq u_i(x_i)$ for all $i = 1, \cdots, m$ with at least one strict inequality.

Theorem 3.1: Under Assumptions A1 and A2, if an $m\ell_1 + m(\ell_1+\ell_2) + \ell_1 + \ell_2$ tuple $((\alpha_{ik}^*), (x_i^*), (y_j^*), q^*) \in S^{m\ell_1} \times R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1} \times S^{\ell_2}$ is an equilibrium, then $((x_i^*), (y_j^*))$ is a Pareto optimal allocation.

Proof: Suppose $((x_i'), (y_j')) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ is a feasible allocation satisfying $u_i(x_i') \geq u_i(x_i^*)$ for all $i = 1, \cdots, m$ with at least one strict inequality. Then, by A1-(i),

$$\sum_{i=1}^m (\sum_{j=1}^n \sum_{k=n(j)+1}^{n(j)} \beta_{jk}(x_{ik}';\ x_{ik}^*,\ q^*)\alpha_{ik}^* + q^* \cdot x_i'^c) > \sum_{i=1}^m q^* \cdot \omega_i^c.$$

Since $((x_i'), (y_j'))$ is feasible,

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(y_{jk}';\ x_{ik}^*,\ q^*)\alpha_{ik}^* - q^* \cdot \sum_{j=1}^n s_j' > 0 \text{ for some } s_j' \in \eta_j(y_j').$$

This contradicts $\sum_{i=1}^m \alpha_{ik}^* = 1$, $k = 1, \cdots, \ell_1$, and

$\sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(y'_{jk};\, x^*_{ik},\, q^*) \leq C_j(y'_j,\, q^*) \leq q^* \cdot s'_j$. Q.E.D.

As in Mas-Colell and Silvestre [1989], we prove the existence of equilibria only in the case of decreasing returns, *i.e.*, the case of convex cost functions. In the case of increasing returns, although the existence of equilibria is not automatically excluded, the curvatures of indifference curves and of β_{jk} will be essential for the existence.

We use the following assumptions.

Assumption A4: For $j = 1, \cdots, n$, (i) $\eta_j : R_+^{n(j)} \to R_+^{\ell_2}$ has a convex graph, (ii) for any sequence $\{y_j^t\}(y_j^t \in R_+^{n(j)}, \lim_{t\to\infty} \|y_j^t\| = +\infty)$, $\lim_{t\to\infty} \inf\{\|s_j\| | s_j \in \eta_j(y_j^t)\} = \infty$, where $\|\cdot\|$ denotes the Euclidean norm and (iii) C_j is differentiable with respect to $y_j \in R_{++}^{n(j)}$ and

$$\lim_{\epsilon \downarrow 0} \frac{C_j(y_{j1}\,, \cdots, y_{jk} + \epsilon\,, \cdots, y_{jn(j)}) - C_j(y_j)}{\epsilon}$$

exists for $y_j \in R_+^{n(j)}$ such that $y_{jk} = 0$.

Assumption A4-(i) says that the technologies do not exhibit increasing returns. Assumption A4-(ii) means that the technologies are proper.

For a condition for η_j which guarantees the differentiability of C_j, see Fujiwara [1985].

Assumption A5: For $i = 1, \cdots, m$, (i) u_i is a continuous, quasi-concave function for $i = 1, \cdots, m$, (ii) for any $x_i \in R_{++}^{\ell_1+\ell_2}$ and $x'_i \in \partial R_+^{\ell_1+\ell_2}$, $u_i(x_i) > u_i(x'_i)$ holds, where $\partial R_+^{\ell_1+\ell_2}$ denotes the boundary of $R_+^{\ell_1+\ell_2}$.

By Assumption A4, β_{jk} can be linear. Namely, for arbitrary chosen $\xi_{jk} \geq 0$ such that $\sum_{k=n(j-1)+1}^{n(j)} \xi_{jk} = 1$,

(3.1) $\beta_{jk}(y_{jk};\, q,\, \bar{v}_j) = \frac{\partial C_j(\bar{v}_j\,, q)}{\partial y_{jk}}(y_{jk} - \bar{v}_{jk}) + \xi_{jk} C_j(\bar{v}_j,\, q)$

satisfies all the conditions for β_{jk}. Note that if $\bar{v}_{jk} = 0$, then $\frac{\partial C_j(\bar{v}_j\,, q)}{\partial y_{jk}}$ denotes the right differential coefficient.

Theorem 3.2: Under Assumptions A1, A2, A4 and A5, if β_{jk} has the form (3.1), then there exists an equilibrium.

Proof: See Appendix.

Remark 3.1: As in Kamiya [1995], it is easy to introduce competitive firms, which produce C goods using P goods and C goods, in our model. The competitive firms maximize their profits for a given price vector of C goods and the given cost allocation rule for P goods. The profits are distributed in proportion to the share holdings of the consumers. In such a economy, the existence and the Pareto optimality of equilibria are established using the same proof as of Theorems 3.1 and 3.2.

4. An Economy with Private Goods

In this section, the P goods are considered as private goods.[3] The cost allocation rules (nonlinear prices of P goods) are defined using β_j^c's. First, we consider the following process. The i-th consumer first reports their demands for P goods, $\bar{x}_i^p = (\bar{x}_{i1} \cdots, \bar{x}_{i\ell_1})$; as in Section 3, the first report is chosen arbitrarily. According to the report, the firms set an individualized (nonlinear) price schedule as follows. For $\bar{x}_{ik} > 0$, the i-th consumer's outlay for x_{ik}, $k = n(j-1)+1, \cdots, n(j)$, is

$$\zeta_{ik}(x_{ik}, \bar{v}_j, \bar{x}_{ik}, q) \equiv \beta_{jk}^c(x_{ik}\tfrac{\bar{v}_{jk}}{\bar{x}_{ik}};\, q, \bar{v}_j)\tfrac{\bar{x}_{ik}}{\bar{v}_{jk}}$$

where $\bar{v}_j \equiv (\bar{v}_{jn(j-1)+1}, \cdots, \bar{v}_{jn(j)}) \equiv (\sum_{i=1}^m \bar{x}_{in(j-1)+1}, \cdots, \sum_{i=1}^m \bar{x}_{in(j)})$.

Note that if $\bar{x}_{ik} = x_{ik}$ for all $i = 1, \cdots, m$, and all $k = 1, \cdots, \ell_1$, the total outlay is equal to the cost.

The consumers maximize their utilities subject to their budget constraints, *i.e.*, for $i = 1, \cdots, n$,

$$\begin{aligned} &\max \quad u_i(x) \\ &s.t. \ \sum_{j=1}^n \sum_{k=n(j-1)+1}^{n(j)} \zeta_{jk}(x_{ik}, \bar{v}_j, \bar{x}_{ik}, q) + q \cdot x_i^c \le q \cdot \omega_i^c, \ \ x \in R_+^{\ell_1+\ell_2}. \end{aligned}$$

[3] Below, we compare our model with Kamiya [1995]'s model and Moriguchi [1996]'s model. In Kamiya's model, joint production is not allowed, *i.e.*, $n(j) = l$ for all $j = l, \cdots, n$. In Moriguchi's model, although joint production is allowed, increasing returns is solely caused from fixed costs. Of course, our model allows for joint production and for general types of increasing returns.

If the i-th consumer's demand for the $k-th$ P good, $\hat{x}_{ik}$, is not equal to $\bar{x}_{ik}$, $\hat{x}_{ik}$ will be $\bar{x}_{ik}$ in the next iteration. Note that $\bar{x}_{ik}$ in the first iteration can be chosen arbitrarily.

Definition 4.1: An $m(\ell_1+\ell_2)+\ell_1+\ell_2$ tuple $((x_i^*), (y_j^*), q^*) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1} \times R_+^{\ell_2}$ is said to be an equilibrium if the following conditions hold.

(i) For all $i = 1, \cdots, m$,

$x_i^* \in \arg\ \max\{u_i(x_i) | \sum_{j=1}^{n} \sum_{k=n(j-1)+1}^{n(j)} \zeta_{jk}(x_{ik}, \bar{v}_j^*, x_{ik}^*, q^*) + q^* \cdot x_i^c \leq q^* \cdot \omega_i^c,\ x_i \in R_+^{\ell_1+\ell_2}\}$,

where $\bar{v}_{jk}^* = \sum_{i=1}^{m} x_{ik}^*$, $j = 1, \cdots, n$, $k = n(j-1)+1, \cdots, n(j)$,
(ii) $\sum_{j=1}^{n} s_j^* + \sum_{i=1}^{n} x_i^{*c} \leq \sum_{i=1}^{n} \omega_i^c$ for some $s_j^* \in \eta_j(y_j^*)$,
(iii) $y_{jk}^* = \sum_{i=1}^{m} x_{ik}^*$ for all $j = 1, \cdots, n$, $k = n(j-1)+1, \cdots, n(j)$
(iv) $x_{ik}^* > 0$ for all $i = 1, \cdots, m$ and all $k = 1, \cdots, \ell_1$.

Definition 4.2: An $m(\ell_1+\ell_2)+\ell_1$ tuple $((x_i), (y_j)) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ is said to be a feasible allocation if $\sum_{j=1}^{n} s_j + \sum_{i=1}^{m} x_i^c \leq \sum_{i=1}^{m} \omega_i^c$ for some $s_j \in \eta_j(y_j)$, $\sum_{i=1}^{m} x_{ik} \leq y_{ik}$ for all $k = n(j-1)+1, \cdots, n(j)$ and all $j = 1, \cdots, n$.

Definition 4.3: An $m(\ell_1+\ell_2)+\ell_1$ tuple $((x_i), (y_j)) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ is said to be a Pareto optimal allocation if it is a feasible allocation and there is no feasible allocation $((x_i'), (y_j')) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ such that $u_i(x_i') \geq u_i(x_i)$ for all $i = 1, \cdots, m$ with at least one strict inequality.

Theorem 4.1: Under Assumptions A1 and A2, if an $m(\ell_1+\ell_2)+\ell_1$ tuple $((x_i^*), (y_j^*), q^*) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1} \times S^{\ell_2}$ is an equilibrium, then $((x_i^*), (y_j^*))$ is a Pareto optimal allocation.

Proof: Suppose $((x_i'), (y_j')) \in R_+^{m(\ell_1+\ell_2)} \times R_+^{\ell_1}$ is a feasible allocation satisfying $u_i(x_i') \geq u_i(x_i^*)$ for all $i = 1, \cdots, m$ with at least one strict inequality. Then, by A1-(i),

$$\sum_{i=1}^{m} (\sum_{j=1}^{n} \sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}^c(x_{ik}' \frac{\bar{v}_{jk}^*}{x_{ik}^*}\ ;\ q^*, \bar{v}_j^*) \frac{x_{ik}^*}{\bar{v}_{jk}^*} + q^* \cdot x_i^{*c}) > \sum_{i=1}^{m} q^* \cdot \omega_i^c.$$

Since $((x_i'), (y_j'))$ is feasible,

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=n(j-1)+1}^{n(j)}\beta_{jk}^{c}(x_{ik}'\tfrac{\bar{v}_{jk}^{*}}{x_{ik}^{*}}\ ;\ q^{*},\ \bar{v}_{j}^{*})\tfrac{x_{ik}^{*}}{\bar{v}_{jk}^{*}}-q^{*}\cdot\sum_{j=1}^{n}s_{j}'>0 \text{ for some } s_{j}'\in\eta_{j}(y_{j}').$$

Since $\sum_{j=1}^{n}\sum_{k=n(j-1)+1}^{n(j)}\beta_{jk}^{c}(y_{jk}';\ q^{*},\ \bar{v}_{j}^{*})\leq C_{j}(y_{j}',\ q^{*})\leq q^{*}\cdot s_{j}'$

$$\sum_{j=1}^{n}\sum_{k=n(j-1)+1}^{n(j)}[\sum_{i=1}^{m}\beta_{jk}^{c}(x_{ik}'\tfrac{\bar{v}_{jk}^{*}}{x_{ik}^{*}}\ ;\ q^{*},\ \bar{v}_{j}^{*})\tfrac{x_{ik}^{*}}{\bar{v}_{jk}^{*}}-\beta_{jk}^{c}(y_{jk}';\ q^{*},\ \bar{v}_{j}^{*})]>0.$$

Since $((x_{i}'),\ (y_{j}'))$ is feasible and β_{jk}^{c} is increasing in the first argument,

$$\sum_{j=1}^{n}\sum_{k=n(j-1)+1}^{n(j)}[\sum_{i=1}^{m}\beta_{jk}^{c}(x_{ik}'\tfrac{\bar{v}_{jk}^{*}}{x_{ik}^{*}}\ ;\ q^{*},\ \bar{v}_{j}^{*})\tfrac{x_{ik}^{*}}{\bar{v}_{jk}^{*}}-\beta_{jk}^{c}(\sum_{i=1}^{m}x_{ik}';\ q^{*},\ \bar{v}_{j}^{*})]>0. \tag{1}$$

By $\sum_{i=1}^{m}(x_{ik}'\frac{\bar{v}_{jk}^{*}}{x_{ik}^{*}})\frac{x_{ik}^{*}}{\bar{v}_{jk}^{*}}=\sum_{i=1}^{m}x_{ik}'$ and the concavity of β_{jk}^{c} in the first argument, the left hand side of (1) must be nonpositive. This contradicts (1). *Q.E.D.*

As in Section 3, we prove the existence of equilibria only in the case of decreasing returns. We use

$$\beta_{jk}^{c}(y_{jk};\ q,\ \bar{v}_{j})=\tfrac{\partial C_{j}(\bar{v}_{j},q)}{\partial y_{jk}}(y_{jk}-\bar{v}_{jk})+\xi_{jk}C_{j}(\bar{v}_{j},q).$$

In this case, if $\bar{v}_{jk}>0$,

$$\zeta_{ik}(x_{ik},\ \bar{v}_{j},\ \bar{x}_{ik},\ q)=\tfrac{\partial C_{j}(\bar{v}_{j}\ ,q)}{\partial y_{jk}}x_{ik}+(\xi_{jk}C_{j}(\bar{v}_{j},q)-\tfrac{\partial C_{j}(\bar{v}_{j}\ ,q)}{\partial y_{jk}}\bar{v}_{jk})\tfrac{\bar{x}_{ik}}{\bar{v}_{jk}}$$

holds, and if $\bar{v}_{jk}=0$, we define

$\zeta_{ik}(x_{ik},\ \bar{v}_{j},\ 0,\ q)=\frac{\partial C_{j}(\bar{v}_{j},q)}{\partial y_{jk}}x_{ik}$, where $\frac{\partial C_{j}(\bar{v}_{j},q)}{\partial y_{jk}}$ denotes the right differential coefficient.

Theorem 4.2: Under Assumptions A1, A2, A4, and A5, if ζ_{ik} has the above linear form, then there exists an equilibrium.

Proof: See Appendix.

Remark 4.1: As in Remark 3.1, it is easy to introduce competitive firms in our model.

Appendix

Proof of Theorem 2.1: The strategy of the construction of β^c_{jk} is as follows. First, we find quadratic functions which locally satisfy the conditions for β^c_{jk}'s. Then we extend the functions to the domain R_+ using linear functions.

Let a be a negative real number satisfying

$$a < -\frac{1}{2}\sum_{k=n(j-1)+1}^{n(j)}\sum_{k'=n(j-1)+1}^{n(j)}\left|\frac{\partial^2 C_j(\bar{v}_j, q)}{\partial y_{jk}\partial y_{jk'}}\right|.$$

Let $b_{jk} \equiv \frac{\partial C_j(\bar{v}_j, q)}{\partial y_{jk}} - 2a\bar{v}_{jk}$, $k = n(j-1)+1, \cdots, n(j)$

and
$c_{jk} \equiv \frac{1}{n(j)}C_j(\bar{v}_j, q) - a\bar{v}^2_{jk} - b_{jk}\bar{v}_{jk}$, $k = n(j-1)+1, \cdots, n(j)$.

Then we define

$$\tau_{jk}(y_{jk}; q, \bar{v}_j) \equiv ay^2_{jk} + b_{jk}y_{jk} + c_{jk},\ k = n(j-1)+1, \cdots, n(j).$$

It is easy to check that $\tau'_{jk}s$ locally satisfy the conditions for β^c_{jk}'s, i.e., there exist intervals $[\underline{y}_{jk}, \bar{y}_{jk}] \subset [0, \infty]$, $k = n(j-1)+1, \cdots, n(j)$, such that $\bar{v}_{jk} \in (\underline{y}_{jk}, \bar{y}_{jk})$ and $\tau'_{jk}s$ satisfy the conditions for β^c_{jk}'s in $\prod_{k=n(j-1)+1}^{n(j)}[\underline{y}_{jk}, \bar{y}_{jk}]$. Note that condition (iv) can be verified using the second order Taylor formula of $C_j(\cdot, q)$ around $\bar{v}_j$. Then we extend $\tau'_{jk}s$ to R_+.

Let $A \equiv \{y_j \in R^{n(j)}_+ | y_{jk} \le \bar{y}_{jk}$ for all $k = n(j-1)+1, \cdots, n(j)$, and $y_{jk} \le \underline{y}_{jk}$ for some $k\}$. For $k = n(j-1)+1, \cdots, n(j)$, let

$$\sigma^\alpha_{jk}(y_{jk}; \bar{v}_j, q) \equiv \begin{cases} \alpha(y_{jk} - \underline{y}_{jk}) - \tau_{jk}(\underline{y}_{jk}; \bar{v}_j, q) \text{ if } 0 \le y_{jk} \le \underline{y}_{jk} \\ \tau_{jk}(\underline{y}_{jk}; \bar{v}_j, q) \text{ if } \underline{y}_{jk} \le y_{jk} \le \bar{y}_{jk} \end{cases}$$

Below, we show that there exists $\alpha \in R_+$ such that $\alpha \ge \frac{\partial\tau_{jk}(\underline{y}_{jk}; \bar{v}_j, q)}{\partial y_{jk}}$ for all k and

$\sum_{k=n(j-1)+1}^{n(j)} \sigma^\alpha_{jk}(y_{jk}; \bar{v}_j, q) \le C_j(y_j, q)$ for all $y_j \in A$.

Suppose the contrary. Then we can find sequences $\{\alpha^t\}$ and $\{y^t_j\}(y^t_j \in A)$ such that $\lim_{t\to\infty}\alpha^t = \infty$ and $\sum_{k=n(j-1)+1}^{n(j)} \sigma^{\alpha^t}_{jk}(y^t_{jk}; \bar{v}_j, q) > C(y^t_j, q)$ for all t. Since A is compact, $\{t\}$ has a subsequence $\{t'\}$ such that $\lim_{t'\to\infty} y^{t'}_{jk} = y^*_{jk} \in A$ for all $k = n(j -$

1) + 1, $\cdots$, $n(j)$. If $y_j^* \neq \underline{y}_j = (\underline{y}_{jn(j-1)+1}, \cdots, \underline{y}_{jn(j)})$, by the construction of σ_{jk}^{α}, $\lim_{t'\to\infty} \sum_{k=n(j-1)+1}^{n(j)} \sigma_{jk}^{\alpha^{t'}}(y_{jk}^{t'};\ \bar{v}_j,\ q) = -\infty$. This is a contradiction. If $y_j^* = \underline{y}_j$, $\lim_{t'\to\infty} \sum_{k=n(j-1)+1}^{n(j)} \sigma_{jk}^{\alpha^{t'}}(y_{jk}^{t'};\ \bar{v}_j,\ q) \geq C(\underline{y}_j,\ q)$ holds. This contradicts $\lim_{t'\to\infty} \sum_{k=n(j-1)+1}^{n(j)} \sigma_{jk}^{\alpha^{t'}}(y_{jk}^{t'};\bar{v}_j,q) \leq \sum_{k=n(j-1)+1}^{n(j)} \tau_{jk}(\underline{y}_{jk};\ \bar{v}_j,\ q) < C(\underline{y}_j,\ q)$. Thus we can choose an α satisfying the above conditions.

Finally, for all $k = n(j-1)+1, \cdots, n(j)$, let

$$\beta_{jk}^{c}(y_{jk};\bar{v}_j,q) \equiv \begin{cases} \tau_{jk}(\bar{y}_{jk};\bar{v}_j,q) & \text{if } y_{jk} \geq \bar{y}_{jk} \\ \tau_{jk}(y_{jk};\bar{v}_j,q) & \text{if } \underline{y}_{jk} \leq\ y_{jk} \leq \bar{y}_{jk} \\ \alpha(y_{jk}-\underline{y}_{jk}) + \tau_{jk}(\underline{y}_{jk};\bar{v}_j,q) & \text{if } 0 \leq\ y_{jk} \leq \underline{y}_{jk} \end{cases}$$

By the construction, β_{jk}^{c}'s obviously satisfy $(i)-(v)$. *Q.E.D.*

Proof of Theorem 3.2: Let $S^{\ell_2} \equiv \{q \in R_+^{\ell_2} | \sum_{k=1} q_k = 1\}$. We use S^{ℓ_2} as the domain of the price vector of the C goods.

The attainable consumption set is defined as follows:

$Q \equiv \{(x_1,\ \cdots,\ x_m) \in R_+^{m(\ell_1+\ell_2)} |$ there exist $y_j \in R_+^{n(j)}$, $j = 1, \cdots, n$, such that $\sum_{i=1}^{m} x_i^c + \sum_{j=1}^{n} s_j \leq \sum_{i=1}^{m} \omega_i^c$ for some $s_j \in \eta_j(y_j)$ and, for all $j = 1, \cdots, n$, $y_{jk} = x_{ik}$, $i = 1, \cdots, m$, $k = n(j-1)+1, \cdots, n(j)\}$.

The compactness of Q follows from Assumptions A2-(i) and A4-(ii). Moreover, the nonemptyness of Q follows from Assumption A1-(ii) and A2-(ii). Thus there exists $b \in R_{++}$ such that $Q \subset int[0,\ b]^{m(\ell_1+\ell_2)}$. Let $\hat{X}_i \equiv [0,\ b]^{\ell_1+\ell_2}$, $i = 1, \cdots, m$.

We choose an arbitrary $h_j \in \eta_j(b, \cdots, b)$. Let $\hat{\eta}_j(y_j) \equiv \eta_j(y_j) \cap \{s_j \in R_+^{\ell_2} | s_{jk} \leq h_{jk}$, $k = 1, \cdots, \ell_2\}$ for $y_j \in [0,\ b]^{n(j)}$ and $\hat{\varphi}_j(y_j,\ q) \equiv \arg\ \min\{q \cdot s_j | s_j \in \hat{\eta}_j(y_j)\}$ for $y_j \in [0,\ b]^{n(j)}$ and $q \in S^{\ell_2}$. By Assumption A2-(iii), $\hat{\eta}_j$ is nonempty, convex, compact valued and, by Assumptions A2 and A4 $\hat{\varphi}_j$ is an upper hemi-continuous correspondence with a nonempty, convex, compact value for $y_j \in [0,\ b]^{n(j)}$ and $q \in S^{\ell_2}$.

Let $\bar{x}_i^p \equiv (\bar{x}_{i1}, \cdots, \bar{x}_{i\ell_1})$. Let the correspondence $F \equiv (F_1,\ F_2,\ F_3,\ F_4) : S^{m\ell_1} \times \Pi_{i=1}^{m} \hat{X}_i \times [0,\ b]^{m\ell_1} \times S^{\ell_2} \to S^{m\ell_1} \times \Pi_{i=1}^{m} \hat{X}_i \times [0,\ b]^{m\ell_1} \times S^{\ell_2}$ be

$F_1((\alpha_{ik}), (x_i), (\bar{x}_i^p), q) \equiv \left(\left\{ \frac{\alpha_{ik} + \max\{0, x_{ik} - \frac{1}{m}\sum_{i=1}^{m} x_{ik}\}}{1+\sum_{i=1}^{m} \max\{0, x_{ik} - \frac{1}{m}\sum_{i=1}^{m} x_{ik}\}} \right\}\right)$

$F_2((\alpha_{ik}),\ (x_i),\ (\bar{x}_i^p),\ q) \equiv (\arg\ \max_{x_i \in \hat{X}_i} \{u_i(x_i) | \sum_{j=1}^{n} \sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(x_{ik},\ \bar{x}_{ik},\ q)\alpha_{ik} + q \cdot x_i^c \leq q \cdot \omega_i^c\})$,

$F_3((\alpha_{ik}),\ (x_i),\ (\bar{x}_i^p),\ q) \equiv (\{x_i^p\})$,

$F_4((\alpha_{ik}),\ (x_i),\ (\bar{x}_i^p),\ q) \equiv \arg\ \max_{r \in S^{\ell_2}}\{r \cdot (\sum_{j=1}^n \hat{\varphi}_j(\frac{1}{m}\sum_{i=1}^m x_{ik},\ q) \ + \sum_{i=1}^m x_i^c - \sum_{i=1}^m \omega_i^c\}$.

F is obviously an upper hemi-continuous, convex valued correspondence; the convex valuedness of F_2 follows from the linearity of β_{jk}. Note that, by $q \in S^{\ell_2}$ and $\omega_{ik} > 0$ for $i = 1, \cdots, m$ and $k = \ell_1 + 1, \cdots, \ell_1 + \ell_2$, $q \cdot \omega_i^c > 0$ always holds and thus F_2 is upper hemi-continuous. Thus, by Kakutani's fixed point theorem, F has a fixed point $((\alpha_{ik}^*),\ (x_i^*),\ (\bar{x}_i^{*p}),\ q^*)$. Below, we show that $((\alpha_{ik}^*),\ (x_i^*),\ (y_j^*),\ q^*)$ is an equilibrium, where $y_{jk}^* \equiv \frac{1}{m}\sum_{i=1}^m x_{ik}^*$, $k = n(j-1)+1, \cdots, n(j)$, $j = 1, \cdots, n$.

Since

$$\alpha_{ik}^* = \frac{\alpha_{ik}^* + \max\{0\ ,x_{ik}^* - \frac{1}{m}\sum_{i=1}^m x_{ik}^*\}}{1+\sum_{i=1}^m \max\{0\ ,x_{ik}^* - \frac{1}{m}\sum_{i=1}^m x_{ik}^*\}}$$

holds, then

$$\sum_{i=1}^m \max\{0,\ x_{ik}^* - \frac{1}{m}\sum_{i=1}^m x_{ik}^*\} = \max\{0,\ x_{ik}^* - \frac{1}{m}\sum_{i=1}^m x_{ik}^*\}$$

holds. Thus $(m-1)\sum_{i=1}^m \max\{0,\ x_{ik}^* - \frac{1}{m_i}\sum_{i=1}^m x_{ik}^*\} = 0$ holds so that $x_{ik}^* - \frac{1}{m_i}\sum_{i=1}^m x_{ik}^* \le 0$ holds for all $i = 1, \cdots, m$. If there exists i' such that $x_{i'k}^* - \frac{1}{m_i}\sum_{i=1}^m x_{ik}^* < 0$, then $x_{i'k}^* + \sum_{i \neq i'} x_{ik}^* < m \cdot \frac{1}{m_i}\sum_{i=1}^m x_{ik}^* = \sum_{i=1}^m x_{ik}^*$ holds. This is a contradiction. Thus $x_{ik}^* = \frac{1}{m_i}\sum_{i=1}^m x_{ik}^* = y_{jk}^*$ holds for all $i = 1, \cdots, m$, so that (iii) in Definition 3.1 holds.

By $\bar{x}_i^{*p} = x_i^{*p}$ for $i = 1, \cdots, m$, local nonsatiation of u_i, and $q^* \cdot \hat{\varphi}_j(y_j^*,\ q^*) = C_j(y_j^*,\ q^*)$,

$$\begin{aligned} q^* \cdot (\textstyle\sum_{j=1}^n \hat{\varphi}_j(y_j^*,\ q^*) + \sum_{i=1}^m x_i^{*c} - \sum_{i=1}^m \omega_i^c) \\ = \textstyle\sum_{j=1}^n C_j(y_j^*,\ q^*) + \sum_{i=1}^m q^* \cdot (x_i^* - \omega_i^c) \\ = \textstyle\sum_{j=1}^n C_j(y_j^*,\ q^*) + \sum_{j=1}^n \sum_{k=n(j-1)+1}^{n(j)} \beta_{jk}(x_{ik}^*,\ x_{ik}^*,\ q^*) \\ = 0 \end{aligned}$$

holds. Thus, by $q^* \in F_4((\alpha_{ik}^*),\ (x_i^*),\ (\bar{x}_i^{*p}),\ q^*)$, it is easy to verify (ii) in Definition 3.1.

(i) in Definition 3.1 can be proved by $x_i^* \in F_2((\alpha_{ik}^*),\ (x_i^*),\ (\bar{x}_i^{*p}),\ q^*)$ and the standard argument. (See, for example, Debreu [1959, Chapter 5].) By $q^* \cdot \omega_i^c > 0$ and Assumption A5 (ii), it is easy to show (iv) in Definition 3.1. *Q.E.D.*

Proof of Theorem 4.2: The attainable consumption set is defined as follows:

$Q' \equiv \{(x_1, \cdots, x_m) \in R_+^{\ell_1+\ell_2} |$ there exist $y_j \in R_+^{n(j)}$, $j = 1, \cdots, n$, such that $\sum_{i=1}^m x_i^c + \sum_{j=1}^n s_j \leq \sum_{i=1}^m \omega_i^c$ for some $s_j \in \eta_j(y_j)$ and $y_{jk} = \sum_{i=1}^m x_{ik}$, $j = 1, \cdots, n$, $k = n(j-1)+1, \cdots, n(j)\}$.

As in the proof of Theorem 3.2, the compactness and the nonemptyness of Q' follow from Assumptions A2-(i)(ii) and A4-(ii). Thus there exists $d \in R_{++}$ such that $Q' \subset int[0, d]^{m(\ell_1+\ell_2)}$. Let $\hat{X}_i' \equiv [0, d]^{\ell_1+\ell_2}$, $i = 1, \cdots, m$.

We choose an arbitrary $h_j' \in \eta_j(md, \cdots, md)$. Let $\hat{\eta}_j' \equiv \eta_j(y_j) \cap \{s_j \in R_+^{\ell_2} | s_{jk} \leq h_{jk}'$, $k = 1, \cdots, \ell_2\}$ for $y_j \in [0, md]^{n(j)}$ and $\hat{\varphi}_j'(y_j, q) \equiv \arg\min\{q \cdot s_j | s_j \in \hat{\eta}_j'(y_j)\}$ for $y_j \in [0, md]^{n(j)}$ and $q \in S^{\ell_2}$. As in the proof of Theorem 3.2, $\hat{\varphi}_j$ is an upper hemi-continuous correspondence with nonempty, convex, compact values for $y_j \in [0, md]^{n(j)}$ and $q \in S^{\ell_2}$.

Let the correspondence $F' \equiv (F_1', F_2', F_3') : \Pi_{i=1}^m \hat{X}_i' \times [0,d]^{m\ell_1} \times S^{\ell_2} \to \Pi_{i=1}^m \hat{X}_i' \times [0, d]^{m\ell_1} \times S^{\ell_2}$ be

$F_1'((x_i), (\bar{x}_i^p), q) \equiv (\arg\max_{x_i \in \hat{X}_i'}\{u_i(x_i) | \sum_{j=1}^n \sum_{k=n(j-1)+1}^{n(j)} \zeta_{jk}(x_{ik}, \bar{v}_j, \bar{x}_{ik}, q) + q \cdot x_i^c \leq q \cdot \omega_i^c\})$,

$F_2'((x_i), (\bar{x}_i^p), q) \equiv (\{x_i^p\})$,

$F_3'((x_i), (\bar{x}_i^p), q) \equiv \arg\max_{r \in S^{\ell_2}}\{r \cdot (\sum_{j=1}^n \hat{\varphi}_j'(y_j, q) + \sum_{i=1}^m x_i^c - \sum_{i=1}^m \omega_i^c)\}$,

where $y_{jk} = \sum_{i=1}^m x_{ik}$. F is obviously an upper hemi-continuous, convex valued correspondence. Thus, by Kakutani's fixed point theorem, F has a fixed point $((x_i^*), (\bar{x}_i^{*p}), q^*)$. Let $y_{jk}^* = \sum_{i=1}^m x_{ik}^*$. Then, using the same argument as in the proof of Theorem 3.2, we can show that $((x_i^*), (y_j^*), q^*)$ is an equilibrium. *Q.E.D.*

References

Brown, D.J., 1991, Equilibrium Analysis with Nonconvex Technologies, in Handbook of Mathematical Economics, Vol.4, W. Hildenbrand and H. Sonnenschein eds., (North Holland).

Cornet, B., 1988, General Equilibrium Theory and Increasing Returns: Presentation, Journal of Mathematical Economics 17, 103-118.

Debreu, G., 1959, Theory of Value (Wiley, New York).

Dierker, E., 1986, When Does Marginal Cost Pricing Lead to Pareto Efficiency, Journal of Economics, Suppl. 5, 41-66.

Fujiwara, O., 1985, A Note on Differentiability of Global Optimum Values, Mathematics of Operations Research 10, 612-618.

Kamiya, K., 1995, Optimal Public Utility Pricing: A General Equilibrium Analysis, Journal of Economic Theory 66, 548-72.

Kaneko, M., 1977, The Ratio Equilibrium and a Voting Game in a Public Goods Economy, Journal of Economic Theory 16, 123-136.

Mas-Colell, A. and J. Silvestre, 1989, Cost Share Equilibria: A Lindahl Approach, Journal of Economic Theory 47, 239-256.

Moriguchi, C., 1996, Two Part Marginal Cost Pricing in a General Equilibrium Model, Journal of Mathematical Economics 26, 363-85.

Quinzii, M., Efficiency of Marginal Cost Pricing Equilibria, in "Equilibrium and Dynamics, Essays in Honor of David Gale", M. Mujumder, Ed.,MacMillan, London.

Roemer, J.E. and J. Silvestre, 1993, The Proportional Solution for Economies with both Private and Public Ownership, Journal of Economic Theory 59, 426-44.