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INEXTENDABLE SOLUTIONS OF HYPERBOLIC MONGE-AMPÈRE EQUATIONS.

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1. Hyperbolic Monge-Ampère equations.

Consider a Monge-Ampère equation

$$A+Bz_{xx}+2Cz_{xy}+Dz_{yy}+E(z_{xx}z_{yy}-z_{xy}^{2})=0, \tag{1}$$

where coefficients A, B, C, D, and E are fixed smooth functions that depend on x, y, z, p, and q. Here x and y are independent variables,

$$z=z(x,y)$$

is an unknown function, and we use Monge's notations

$$p=z_{x}, \qquad q=z_{y}.$$

Let $\mathbb{R}^5$ be the space of parameters x, y, p, q, and z. The linear differential form

$$\omega_0 = dz - pdx - qdy$$

defines the standard contact structure on $\mathbb{R}^5$. The effective differential 2-form

$$\omega = Adx\wedge dy + Bdp\wedge dy + C(dx\wedge dp + dq\wedge dy) + Ddx\wedge dq + Edp\wedge dq$$

on $\mathbb{R}^5$ is associated with the left part of the equation (1) in the obvious way (see [1]). The pfaffian

$$Pf(\omega) = -C^2 + BD - AE$$

of this form up to it's sign coincides with the characteristic discriminant of the equation (1). Suppose the equation (1) is hyperbolic, i.e.,

$$Pf(\omega) < 0.$$

Put

$$\lambda_j = (-1)^{3-j}\sqrt{-Pf(\omega)},$$

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where $j = 1, 2$.

Definition 1.1. *The* characteristic bundle H_j *of the equation (1) is the linear subbundle of the tangent bundle* $T\mathbb{R}^5$ *whose fiber* $H_j(m)$ *at a point* $m \in \mathbb{R}^5$ *is defined by the equality*

$$(2) \qquad H_j(m) = \left\{\xi \in T_m\mathbb{R}^5 : \xi \lrcorner \omega_0(m) = 0,\ \xi \lrcorner (\omega(m) - \lambda_j d\omega_0(m)) = 0\right\},$$

$j = 1, 2$ *(cf.* [2]*)*.

2. Multivalued solutions.

Definition 2.1. *An immersion*

$$(3) \qquad \sigma : S \longrightarrow \mathbb{R}^5$$

of a two-dimensional manifold S *is a* multivalued solution *of the equation (1) if the equations*

$$\sigma^*(\omega_0) = 0, \qquad \sigma^*(\omega) = 0$$

are through.

According to the Frobenius theorem for any point $r \in S$ of the solution (3) there exists the unique maximal one-dimensional integral submanifold

$$(4) \qquad \gamma_{jr} : Z_{jr} \longrightarrow S$$

of the characteristic subbundle (2) that passes through the point r, i.e., $r \in \gamma_{jr}(Z_{jr})$, and

$$(\sigma \circ \gamma_{jr})_*(T_w Z_{jr}) \subset H_j(\sigma \circ \gamma_{jr}(w))$$

for all $w \in Z_{jr}$.

Definition 2.2. *The submanifold (4) is called the* characteristic *of the equation (1) that lies on the solution (3), passes through the point* $r \in S$*, and belongs to the* j*-th family,* $j = 1, 2$.

3. Cauchy problem

Consider an initial value for the equation (1), i.e., an immersion

$$(5) \qquad l : Z \longrightarrow \mathbb{R}^5,$$

$Z = (a, b)$, $-\infty \leq a \leq b \leq +\infty$, such that

$$l^*(\omega_0) = 0.$$

This immersion is called an *initial curve* to the equation (1) if it is *free*, i.e.,

$$\dot{l}(t) \notin H_j(l(t))$$

for $j = 1, 2$ and $a < t < b$.

Definition 3.1. *The multivalued solution (3) of the equation (1) is called a* solution of the Cauchy problem *(5) if there exists an imbedding*

$$L : Z \longrightarrow S$$

such that

$$l = \sigma \circ L.$$

This solution (σ, L) is called determined *if for any point $r \in S$, characteristic γ_{jr}, and the initial curve (5) the intersection*

$$L(Z) \cap \gamma_{jr}(Z_{jr})$$

consists of exactly one point for $j = 1, 2$.

Let (σ, L) and $(\tilde{\sigma}, \tilde{L})$ be two arbitrary determined multivalued solutions of the Cauchy problem (1), (5).

Definition 3.2. *A determined solution $(\tilde{\sigma}, \tilde{L})$ of the problem (1), (5) is called* inextendable *if for any determined solution (σ, L) of this problem there exists an imbedding*

$$\varphi : S \longrightarrow \tilde{S}$$

such that

$$\tilde{\sigma} \circ \varphi = \sigma, \qquad \varphi \circ L = \tilde{L}.$$

Theorem 3.1. *Let the equation (1) be hyperbolic and it's coefficients and the initial curve (5) be smooth. Then up to parametrization there exists a unique smooth inextendable solution $(\tilde{\sigma}, \tilde{L})$ of the Cauchy problem (1), (5).*

Proof. See [3]. ■

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