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Author(s)	Okada, Akira
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Social Development Promoted by Cooperation: A Simple Game Model[1]

Akira Okada

Institute of Economic Research, Kyoto University, Kyoto 606-8501, Japan

Abstract

This paper presents a simple game theoretic model of development with population growth, based on the idea that the engine of development is cooperation organized by self-interested individuals. The development level of a society as well as the population affects the possibility of an organization of cooperation. While the monotone convergence of development holds under the full-cooperation hypothesis, the society develops by repeating growth and decline through the creation of new organizations. Long-run development is determined both by the "fundamentals" of the society and by the institutional conditions on costs for enforcing cooperation.

1. Introduction

This paper presents a simple game theoretic model of social development with the idea that the engine of development is cooperative actions of individuals.[2] A society is regarded as an intermixture of conflict and cooperation where the maximization of individual utilities may deviate from cooperation. Various kinds of cooperative actions by

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[1] This is a shorter version of a working paper by Okada (1998). The working paper is available upon request.

[2] It is beyond the scope of the present paper to discuss the content of development itself. Rather, it is simply assumed that there exists some appropriate measure of development such as a stock of social overhead capital and environmental improvements, etc. For a detailed discussion about the concept of development, see Sen (1988).

individuals is needed for development. For example, economic development can be promoted by the success of market, state and community in economic systems (Hayami, 1997). Observed failures, however, in these institutions are caused by non-cooperative actions often termed as moral hazard, rent-seeking and free-riding. Many developing economies suffer from these problems.[3]

We adopt a dynamic model of the n-person prisoners' dilemma game played by non-overlapping generations to examine the role of cooperation in social development. Since the individual rationality prescribes non-cooperation in the prisoners' dilemma game, some suitable mechanism for cooperation is necessary for development. The model incorporates the voluntary creation of an organization to enforce cooperative actions on participants.[4] Individuals attempt to create an organization of cooperation. If the organization is successfully formed, it can enhance development. In turn, the development may affect individuals' incentive to create the organization. By a generation game model with population growth, we consider this kind of dynamic interrelation among individuals, organization and society.

The paper is organized as follows. Section 2 introduces a dynamic model of the n-person prisoners' dilemma game with non-overlapping generations. Section 3 incorporates a multi-stage game model of organization formation into the dynamic model of development. The equilibrium size of an organization is analyzed. Section 4 investigates dynamic patterns of development through organizations. Section 5 has the conclusion. All proofs are given in Okada (1998).

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[3] For a recent work to study the problem of divergence in development by endogenous growth models, see Lucas (1988).

[4] Schumpeter (1934) embodied economic development as "the carrying out of new combinations" implemented by entrepreneurs, and included the creation of a new organization in the list of them.

2. A dynamic prisoner's dilemma with non-overlapping generations

Let $N_t = \{1, ..., n_t\}$ be the set of players in generation t (= 1, 2, ...). Each player i in each generation has two possible actions: a_i = C (cooperation), D (defection). The model includes a state variable k_t, called the *development variable*, representing the level of development in generation t. Examples of such a state variable are the stock of social overhead capital as public goods and various kinds of environmental indicators. The domain of the development variable k_t is R_+, the set of nonnegative real numbers.

A payoff for player i in generation t depends upon both the development variable and the actions selected by all players including himself. Following the standard model of the n-person prisoners' dilemma game, it is assumed that all players i have the same payoff functions:

$$f(k_t, a_i, h_{-i}), \quad k_t \in R_+, \quad a_i = C, D, \quad h_{-i} = 0, 1, ..., n_t - 1 \qquad (2.1)$$

where h_{-i} is the number of all players, except i, selecting C. Note that the current generation has no concern about the welfare of future generations.

Assumption 2.1. The payoff function (2.1) for player i satisfies: for all k_t in R_+ and all h = 0, 1, ..., n_t-1, (1) $f(k_t, D, h) > f(k_t, C, h)$, (2) $f(k_t, C, n_t-1) > f(k_t, D, 0)$, (3) $f(k_t, a_i, h)$ is increasing in k_t and in h for each a_i = C, D.

These properties imply that given any level of development k_t, society in every generation can be described as an n-person prisoners' dilemma game and that development is "good" to all players. Specifically, the action combination (D, ..., D) is a unique Nash equilibrium point of the game, but it is Pareto-inferior to the action combination (C, ..., C). If all players jointly select C, each of them is better off than at the Nash equilibrium point.

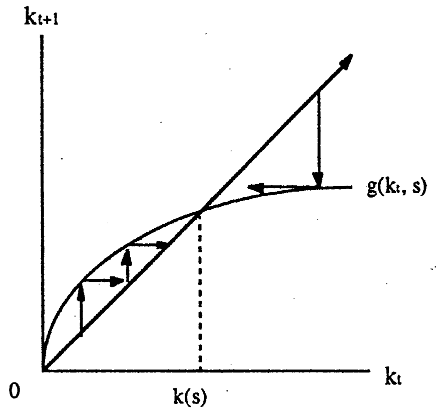
The model has two dynamical systems that describe social development and population growth, respectively. First, the dynamic system of development is given by

$$k_{t+1} = g(k_t, s_t), \quad t = 1, 2, \dots \tag{2.2}$$

where $s_t = 0, 1, \dots, n_t$ is the number of all players in generation t selecting C.

Assumption 2.2. (1) $g(k_t, s_t)$ is increasing in k_t and in s_t, and continuous in k_t, (2) for every $s_t = 0, 1, \dots, n_t$, the equation $k = g(k, s_t)$ has a unique fixed point, denoted by $k(s_t)$, such that $k < g(k, s_t)$ and $k > g(k, s_t)$, respectively, if $0 \leq k < k(s_t)$ and $k > k(s_t)$, respectively, and (3) $0 < g(k_t, 0) < k_t$ for all k_t ($\neq 0$), and $g(0, 0) = 0$.

Given the number s_t of cooperators, $g(k, s_t)$ is an increasing function of k and intersects with the 45-degree line uniquely at $k(s_t)$. The graph of $g(k, s_t)$ intersects the 45-degree line from above to below as k increases (see Figure 2.1). As well-known, this property implies that $k(s_t)$ is the globally stable fixed point of dynamical system (2.2). The fixed point $k(s_t)$ is called the *potential function of development* since it describes the potential level of development that the society can achieve in the long run, depending upon the number of cooperators. From Property (1), the transition function $g(k, s_t)$ shifts upward



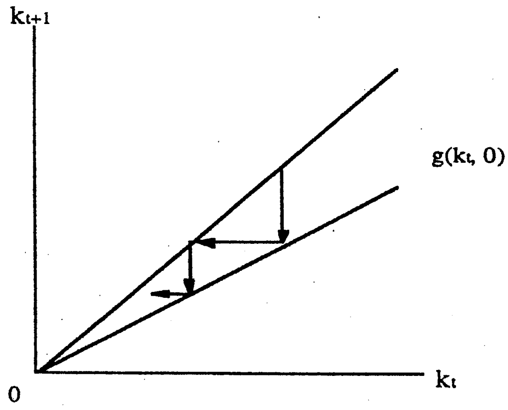


Figure 2.1 Development when the number of cooperators is s. Figure 2.2. Development under no cooperation.

as s_t increases, that is, the more individuals cooperate, the more society develops. This implies that the potential function k(s) is an increasing function of the number s of cooperators. Finally, property (3) implies that if no individuals cooperate, development variable k_t decreases and that the level of development converges to the worst level, zero, as the state of no cooperation is repeated over generations (see Figure 2.2).

The second dynamical system describes population growth. We assume that the population growth has the form

$$n_{t+1} - n_t = \Delta(k_t, n_t) \ , \ t = 1, 2, ... \tag{2.3}$$

where $\Delta(k_t, n_t)$ represents the change of population from generation t to generation t+1.

Assumption 2.3. For any k_t in R_+, there exists a real number $n(k_t)$ such that

(1) $\Delta(n_t, k_t) > 0$ if $n_t < n(k_t)$, $\Delta(n_t, k_t) = 0$ if $n_t = n(k_t)$, and $\Delta(n_t, k_t) < 0$ if $n_t > n(k_t)$,

(2) for every t, $n_{t+1} < n(k_t)$ if and only if $n_t < n(k_t)$.

(3) $\Delta(n_t, k_t)$ is continuous on R^2_+.

(4) $n(k_t)$ is a continuous and increasing function of k_t, and there exists N such that $n(k_t) \leq$ N for all k_t.

This assumption implies the global stability of population change. Given any value of development variable k_t, population monotonically increases (decreases, respectively) and converges to the level $n(k_t)$ when an initial level is below (above, respectively) it. The n(k) is called the *population capacity function*, describing the long-run level of population when development variable is fixed at k over generations.

The following proposition shows the dynamics of the society when the prisoner's dilemma game is played noncooperatively in every generation.

Proposition 2.1. (no cooperation) For any initial point (k_0, n_0), let $\{(k_t, n_t)\}_{t=1}^{\infty}$ be a sequence of development variables and population generated by (2.2) and (2.3) under no cooperation ($s_t = 0$ for all t). Then, $\{(k_t, n_t)\}_{t=1}^{\infty}$ converges to (0, n(0)) as t goes to infinity.

When players of every generation fail to attain cooperation, the development of the society will decline to the worst (zero) level and the population will approach its capacity level n(0). Proposition 2.1 can be regarded as a dynamic version of the "tragedy of commons". To avoid this undesirable equilibrium in the long-run, some suitable mechanism for attaining cooperation by players is needed within the society.

3. The organization of cooperation

As a mechanism attaining cooperation in a society, we consider an organization of players of which primary function is to enforce cooperative actions on its members. We are interested in whether or not such an organization can be voluntarily created by players and, if any, how large an organization is.

An *organization* is formulated by a quintuplet $\varphi = (S, p, i^*, w, TC)$. The first element S, a subset of the player set $N_t = \{1, ..., n_t\}$, represents the set of all members in the organization. Let s denote the number of all players in S. The second element p, a nonnegative real number, is a level of punishment imposed on any member in case of defection. When punished, the payoff of the member is reduced by the amount p. The level p of punishment is exogenously given such that cooperation becomes the dominant action to every member of the organization. The enforcement of cooperation is applied only to members of the organization. Any non-member of the organization can free-ride on cooperative actions by the members. The third element i* is a special agent, called the *enforcement agent*, who does enforcement work within the organization. All members except the enforcement agent are called *regular members*. It is assumed that agent i* sincerely performs the enforcement job, and that he does not play the original prisoner's dilemma game. The fourth element w is the wage for the enforcement agent. The final element TC represents total cost of enforcing cooperation (except the wage for the enforcement agent). It may include bargaining and monitoring costs in the organization. The TC is called the *organization cost,* and assume that it is described as a function TC(s-1) of the number of all regular members in S. Finally, the organization cost plus wage for the enforcement agent is equally allocated to s-1 regular members.

We formulate a model of organization formation in each generation as a three-stage game (see Okada 1993, 1996 for related models).

(1) Participation decision stage:

Given a level of development k_t, every player of generation t (=1, 2, ...) decides independently whether to participate in an organization. Let S be the set of all participants and let s be the number of all participants in S. If $s < 2$, then no organization is possible.

(2) Organization formation stage:

An enforcement agent $i^* \in S$ is randomly selected out of all participants. The wage w for i* is determined by a certain (but unspecified here) process of negotiations among participants. Thereafter, all members in S decide independently whether to approve the organizational decisions: punishment p, wage w for i* and the equal allocation of the total costs w + TC(s -1) for the organization. The organization $\varphi = (S, p, i^*, w, TC)$ is formed if and only if all members in S unanimously approve it.

(3) Action decision stage:

When an organization φ is formed, all players in the society except enforcement agent i* select independently one of two actions, C or D. The payoff to every player is given by

$$
\begin{array}{ll}
f(k_t, a_i, h_{-i}) - \dfrac{w + TC(s-1)}{s-1} & i \in S \ (i \neq i^*),\ a_i = C \\
f(k_t, a_i, h_{-i}) - \dfrac{w + TC(s-1)}{s-1} - p & i \in S \ (i \neq i^*),\ a_i = D \\
f(k_t, a_i, h_{-i}) & i \notin S \\
w & i = i^*,
\end{array}
$$

where a_i is i's action, h_{-i} is the number of all individuals except i selecting C. When no organization is formed, all individuals play the original prisoner's dilemma game described in the last section.

We analyze a subgame perfect equilibrium point for the multi-stage game of organization formation by the following three propositions.

Proposition 3.1. Whenever an organization $\varphi = (S, p, i^*, w, TC)$ is formed, the action decision stage has a unique Nash equilibrium point in which all (regular) members of the organization select cooperation C and all non-members select defection D.

Proposition 3.2. The organization formation stage has a Nash equilibrium point in which an organization $\varphi = (S, p, i^*, w, TC)$ is formed, if and only if

(1) $f(k_t, C, s-2) - \dfrac{w + TC(s-1)}{s-1} \geq f(k_t, D, 0)$

(2) $w \geq f(k_t, D, 0)$

where s (≥ 2) is the number of participants in the organization.

The proposition implies that an organization can be supported by a Nash equilibrium point if all participants in the organization enjoy payoffs greater than, or equal to the noncooperative payoffs of the prisoner's dilemma. In the following, only Nash equilibrium points supporting the formation of organizations are considered.

There exists some wage w satisfying the two conditions in Proposition 3.2 if and only if the *organization surplus*

$$W(k_t, s) \equiv (s-1)\, f(k_t, C, s-2) - TC(s-1) - s\, f(k_t, D, 0) \qquad (3.1)$$

is non-negative.

Assumption 3.1. The organization surplus $W(k_t, s)$ is increasing in the number s of participants.

Given any value of development variable k_t, let $s(k_t)$ be the smallest integer satisfying $W(k_t, s) \geq 0$.

Proposition 3.3. (1) When $n_t \geq s(k_t)$, the participation decision stage has a Nash equilibrium point leading to the formation of an organization if and only if the number of all participants is exactly equal to $s(k_t)$. (2) When $n_t < s(k_t)$, the participation decision stage has no Nash equilibrium point leading to the formation of an organization.

The proposition demonstrates that the smallest feasible organization with exactly $s(k_t)$ participants can be formed in the Nash equilibrium point of the participation decision stage. By this reason, $s(k_t)$ is called the *organization size function.* When the population is less than this level, no organization is formed and thus society fails to develop.

4. The dynamics of social development

In this section, we investigate how a society can develop through the voluntary creation of an organization for cooperation. In the dynamic process of development, every generation plays the organization formation game introduced in the previous section, and the number of cooperators (i.e. regular members) in the organization determines the level of a "gear" of the development engine.

We first extend the domain of variable s_t, the number of cooperators from natural numbers to real numbers for convenience of analysis. Specifically, we assume:

Assumption 4.1. The domain of the transition function $g(k_t, s_t)$ is R_+^2, and for any k_t, $g(k_t, \cdot)$ is constant on semi-closed interval (m, m+1] for any m = 0, 1, 2, ...

Recall that the potential function $k(s_t)$ of development is defined by a solution of $k = g(k, s_t)$. This assumption, together with Assumption 2.2, implies that the potential function $k(s_t)$ of development is an increasing step-function on R_+ with jumping at integer points.

As a bench mark of analysis, we first consider the development process under the presumption that all individuals cooperate in all generations. The dynamics under full-cooperation is given by

$$\begin{aligned} &(1)\ \ k_{t+1} = g(k_t, n_t) \\ &(2)\ \ n_{t+1} = n_t + \Delta(k_t, n_t). \end{aligned} \tag{4.1}$$

Assumption 4.2. The population capacity function $n = n(k)$ and the potential function $k = k(n)$ of development has a unique intersection (k^+, n^+).

Proposition 4.1. (full cooperation) For any initial point (k_0, n_0) with $k_0 < k(n_0) < k^+$ and $n_0 < n(k_0) < n^+$, let $\{(k_t, n_t)\}_{t=1}^{\infty}$ be a sequence of development variables and population generated by (4.1). Then, k_t and n_t monotonically increase and $\{(k_t, n_t)\}_{t=1}^{\infty}$ converges to (k^+, n^+) as t goes to infinity.

If all players cooperate, then the society develops at the highest speed. However, since the population capacity function $n(k)$ is bounded above (Assumption 2.3), the population has an upper limit during the development process. This constraint of the population also imposes an upper limit of the development variable. Proposition 4.1 shows that when the population and the development are initially at low levels, these variables monotonically increase and converge to the intersection point (k^+, n^+) of the population capacity curve and the potential curve of development (see Figure 4.1).

We now consider the dynamic process of development through the organization of cooperation. In the continuous version of the model, the equilibrium size $s(k_t)$ of the organization is determined by the solution of

$$W(k_t, s) = 0 \tag{4.2}$$

where $W(k_t, s)$ is the organization surplus defined in (3.1).

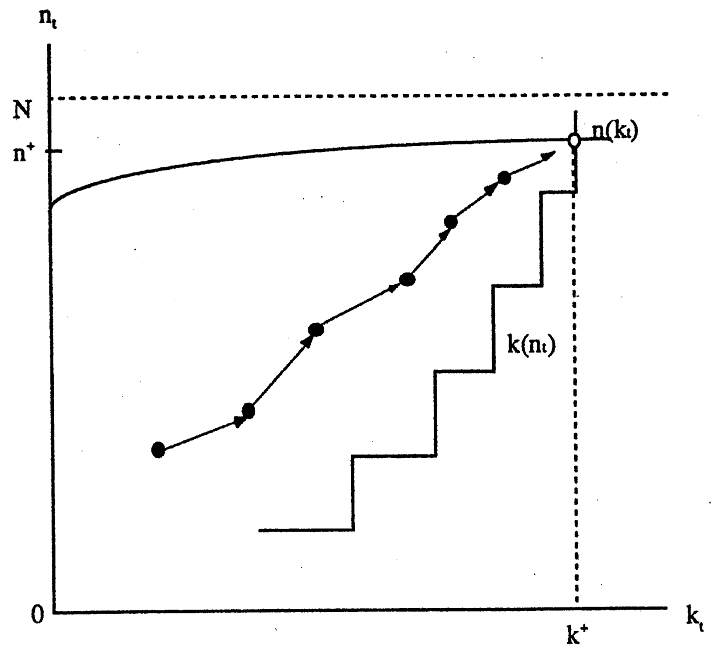


Figure 4.1. Development with full cooperation .

Assumption 4.3. For every $k_t \in R_+$, there exists a unique solution $s = s(k_t) \in R_+$ of (4.2). The $s(k_t)$ is called the *organization size function.*

Let F denote the region, $\{(k_t, n_t) \in R_+^2 \mid s(k_t) \leq n_t\}$, of development variables and population, called *the feasible region of cooperation.* When the society has state variables (k_t, n_t) in F, it can develop with the organization of size $s(k_t)$. When not, the society declines with no cooperation.

The dynamics of development through organized cooperation is given by

$$\begin{aligned} (1)\ k_{t+1} &= g(k_t, s(k_t) - 1) \quad \text{if } (k_t, n_t) \in F \\ &= g(k_t, 0) \quad \text{otherwise} \end{aligned} \tag{4.3}$$

(2) $n_{t+1} = n_t + \Delta(k_t, n_t)$

where $s(k_t) - 1$ is the number of regular members selecting cooperative actions in the organization. Unlike the cases of full cooperation, the development by the organization crucially depends upon the graph of the organization size function $s(k_t)$, which shapes the feasible region F of an organization.

Differentiating both sides of (4.2) with respect to development variable k yields

$$\frac{\partial W}{\partial k} + \frac{\partial W}{\partial s}\frac{ds}{dk} = 0.$$

Since $\partial W/\partial s > 0$ from Assumption 3.1, we have $ds/dk > 0$ if and only if $\partial W/\partial k < 0$. Namely, if the organizational surplus decreases as development variable k increases, then the organization must become larger to compensate the surplus decrease.

We are now ready to analyze the development process under the organization of cooperation. In the rest of this section, the organization size s(k) is assumed to be an increasing function of development variable k. The technique of analysis can be applied to other cases.

Assumption 4.4.

(1) The organization size function $s = s(k)$ is an increasing function of development variable k, and has a unique intersection (k^*, n^*) with the population capacity function n(k) satisfying $n^* = s(k^*) = n(k^*)$.

(2) Let m^* be the largest integer which is smaller than n^*. Then,

$$m^* \leq n(k) \text{ for all } k \text{ with } 0 \leq k \leq k^*.$$

The feasible region $F = \{(k_t, n_t) \in R^2_+ \mid s(k_t) \leq n_t\}$ of cooperation is partitioned into sub-regions according to the size of the organization. For every $i = 2, 3, \ldots,$ define the set

$$F(i) = \{ (k_t, n_t) \in F \mid i - 1 < s(k_t) \leq i \}.$$

For every (k_t, n_t) in F(i), an organization with $s(k_t)$ participants is established and the society develops according to the equation $k_{t+1} = g(k_t, i-1)$ by Assumption 4.1. Then, development variable k_t increases if k_t is lower than the potential $k(i-1)$ for $i-1$ cooperators, and decreases, otherwise. When $F(i) \neq \emptyset$, define k^i by

$$s(k^i) = i.$$

k^i indicates the development level at which the number of participants in an organization changes. Since s(k) is an increasing function (Assumption 4.4), we have $k^i < k^{i+1}$ for every i = 2, 3, ... If $k^{i-1} < k_t \leq k^i$ and (k_t, n_t) is in F, then an organization with participants i can be formed.

Given the population capacity function n(k), the development pattern described by dynamic system (4.3) is determined by the two graphs of the organization size function s(k) and of the potential function k(s−1) of development. Theorem 4.1 considers the case that the potential of development is sufficiently high such that the curve of the organization size function is located left of that of the potential function of development (see Figure 4.2).

Theorem 4.1. For any initial point (k_0, n_0) in the feasible region F of cooperation, let $\{(k_t, n_t)\}_{t=1}^{\infty}$ be a sequence of development variables and population generated by (4.3). If

$$k^i < k(i-1) \qquad \text{for every } i = 2, \ldots, m^*,[5] \tag{4.4}$$

then there exists some t such that $m^* \leq n_t$ and $k^{m^*} \leq k_t$.

The theorem shows that when the potential of a society is high, the development level can exceed the largest turning point k^{m^*} below the intersection level k^* of the potential function of development and the organization size function after sufficiently many generations. If we employ a counting rule for the number of players finer than the

[5] We set $k^i = 0$ when $F(i) = \emptyset$.

integer (for example, if we can say meaningfully that 10.322 players cooperate), then the largest switching point k^{m*} can become closer to k^*. Therefore, by employing a sufficiently fine counting rule, if necessary, it is possible to say from the theorem that the state variables (k_t, n_t) of a society can approach the intersection point (k^*, n^*) of the organization size curve and the population capacity curve in the long run.

We finally examine the dynamic pattern of development in Theorem 4.1. When an organization with i participants is formed at (k_t, n_t) with $k^{i-1} < k_t \leq k^i$ and its i−1 regular members cooperate, Condition (4.4) implies that development variable k_t increases because it is less than the potential k(i−1). Therefore, development variables increase whenever an organization is successful. If the organization size is unchanged at level i during the process, k_t converges to the potential k(i−1). Therefore, in some generation t development variable k_t goes beyond the switching point k^i, i.e. $k^i < k_t < k(i-1)$. Once this happens, the number of cooperators increases and thus the transition function shifts upward. Thereafter, the same mechanism of development starts again under the new transition function. This process continues as long as an organization is successfully formed. Since the feasible region F of cooperation is bounded by the curve of the organization size function, the state variables (k_t, n_t) of the society may move outside of it. If this happens, no organization is successful, and development variable k_t starts to decrease with the failure of cooperation. Population, however, keeps growing when its initial level is low. Owing to the increased population, the state variable (k_t, n_t) can move back towards the feasible region and can re-enter it after several generations. Again, the process of development starts. In this way, the society develops with the creation of new organizations, repeating growth and decline.

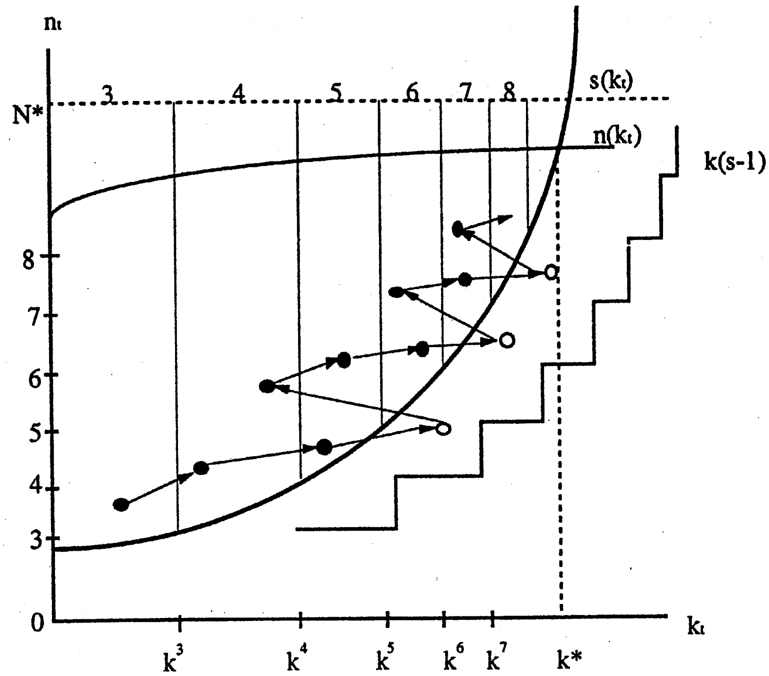


Figure 4.2. Development with organized cooperation when the potential of development is high.

5. Conclusion

A game theoretic model of social development has been presented in this paper by the idea that a society develops with cooperation of individuals. Under the optimistic view that all individuals cooperate in all generations, the society can develop with the highest speed, and the development level monotonically increases and converges to a certain level. The long run development under full cooperation is determined only by the population capacity function and the potential function development. In contrary to this optimistic view of the efficient development, one can not expect that all individuals of the society cooperate in an n-person prisoner's dilemma game. The society needs some suitable mechanism for individuals' cooperation, and we have considered the voluntary creation of an organization to enforce cooperative actions on its participants.

The main result of the paper shows that long-run development with organized cooperation is affected by the institutional conditions on costs of enforcing cooperation.

When the potential of development is high enough so that the society develops whenever cooperation is successfully organized, the development of the society approaches the intersection point of the population capacity curve and the organization size curve (not the potential curve in the optimistic pattern of development), repeating growth and decline.

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