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Spectral Criteria for Almost Periodicity of Solutions of Periodic Evolution Equations

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In this work we look for conditions on the spectrum of the monodromy operator $P(t)$ determined from a periodic evolutionary process $(U(t,s))_{t\geq s}$ and that of the given almost periodic function f in order that the following integral equation

$$x(t)=U(t,s)x(s)+\int_{s}^{t}U(t,\xi)f(\xi)d\xi \tag{1}$$

to have a unique almost periodic solution.

Into this model one can include many kinds of evolutionary differential equations such as ordinary, functional and partial differential equations. Here, the process $(U(t,s))_{t\geq s}$ plays the role of evolution operators which arise naturally from well-posed evolution equations.

The problem has been discussed in [Pr] for the autonomous case. In the periodic case with Floquet representation the problem can be solved in the same way as in the previous case. The general case has been conjectured in [V]. Our paper is motivated by this and especially by the fact that Floquet representation does not exist for many infinite dimensional systems which are frequently met in applications, for instance functional differential equations with finite delay and parabolic equations.

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The main results we obtain in this work is Eq.(1) has an almost periodic solution x_f if

$$\sigma(P(t)) \cap \overline{e^{i\mathrm{sp(f)}}} = \oslash, \tag{2}$$

where $P(t)$ is monodromy operators of the underlying evolutionary process. This solution is unique if one requires

$$sp(x_f) \subset \overline{\{\lambda + 2\pi k, \lambda \in sp(f), k \in \mathbf{Z}\}} \tag{3}$$

In particular, when $\sigma(P(t))$ is saparated from the unit circle, i.e. $P(t)$ is hyperbolic , it turns out that the unique sobvability of Eq.(1) is equivalent to the hyperbolicity of $P(t)$. And in turn, this is equivalent to the exponential dichotomy of the process $(U(t,s))_{t\geq s}$. This result generalizes the well-known one of ODE. Partial results have been obtained also in [AMZ], [R].

The method we employ to prove the above mentioned results is the so-called "evolution semigroup" associated with the process $(U(t,s))_{t\geq s}$ by the formula

$$T^h v(t) = U(t, t-h)v(t-h), \forall t \in \mathbf{R}, h \geq 0, \tag{4}$$

where v belongs to a suitable subspace of $AP(\mathbf{X})$ (of all almost periodic $\mathbf{X}$-valued functions). If the process is strongly continuous, the generator of this semigroup is nothing but the integral operator determined by Eq. (1). Invertibility of this operator means the unique solvability of Eq. (1). Using the spectral inclusion of C_0-semigroups one can find a sufficient condition for the invertibility in the following form $1 \in \rho(T^1)$. (In the paper we always assume, for simplicity of notations, that the underlying process is 1-periodic.) The connection between $\rho(T^1)$ and $\rho(P(t))$ is easily established by using the periodicity of the process.

As a particular case which may be of independent interest we consider the case when the forcing term f is also 1-periodic. Is true the following assertion Eq. (1) has a unique 1-periodic solution if and only if $1 \in \rho(P(t))$. In the autonomous case, this assertion implies the well-known Gearhart's spectral mapping theorem for Hilbert spaces.

As a conclusion we emphasize that our treatment is concerned only with periodic evolutionary processes. This gives rise to diverse applications. In this work, we apply the obtained results to parabolic equations of the form

$$\frac{dx}{dt} = (-A + B(t))x + f(t) \tag{5}$$

where $-A$ is a sectorial operator in a Banach space $\mathbf{X}$ and $B(\cdot)$ is a 1-periodic continuous function taking values in $L(\mathbf{X}^\alpha, \mathbf{X})$. All conditions of our theorems are satisfied for this model of applications.

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