Title	開ソリッドトーラス上の接触構造(実特異点のトポロジーとその関連話題)
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Citation	数理解析研究所講究録 (1997), 1006: 69-80
Issue Date	1997-08
URL	http://hdl.handle.net/2433/61469
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

数理解析研究所講究録
1006 巻 1997 年 69-80

開ソリッドトーラス上の接触構造

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0 Introduction

To classify contact structures on a manifold is a basic problem in differential topology. It is largely open for open manifolds in particular. It was shown by Ya. Eliashberg in [E2] that there is a continuous family of non-equivalent tight contact structures on an open solid torus $S^1 \times \mathbb{R}^2$. This result indicates that the situation seems to be more complicated on open manifolds than closed manifolds. In this note we study contact structures on the open solid torus, which infinite end has linear characteristic foliation.

First of all, let us recall some notions and results to formulate our results. A *contact structure* on a 3-manifold M is a completely non-integrable tangent plane field ξ. It is defined, at least locally, as a kernel of a differential 1-form α which satisfies the non-integrable condition $\alpha \wedge d\alpha \neq 0$. We consider, in this note, *cooriented* contact structures, which are defined by global 1-forms by definition. Since the sign of $\alpha \wedge d\alpha$ depends only on the contact structure ξ, the manifold M is orientable. When M is oriented, the contact structure ξ is called *positive* or *negative* whether the orientation induced by ξ is coincide with the given orientation or not. In what follows, we consider only positive contact structures.

For an embedded surface F in a contact 3-manifold (M, ξ), ξ traces on F a singular foliation. Such a foliation is called the *characteristic foliation* on F with respect to ξ, and denoted by F_ξ. A contact structure ξ is called *tight* if the characteristic foliation D_ξ has no limit cycle for any embedded disc D. A contact structure which is not tight is called *overtwisted* (see [E3], [E1],). For non-compact manifolds the following notions are introduced in

[E4]. Let M be non-compact. A contact structure on M is said to be *tight at infinity* if it is tight outside a compact subset of M. We say contact structure ξ on a non-compact manifold M is *overtwisted at infinity* if, for any relatively compact open subset U of M, each non-compact component of $(M \setminus U, \xi)$ is overtwisted.

It was shown by Ya. Eliashberg in [E4] that the classification of contact structures overtwisted at infinity on an open manifold, similarly to that of overtwisted contact structures on a closed manifold (see [E1]), coincides with the homotopical classification of tangent plane fields. Therefore, we treat only contact structures tight at infinity in this note.

We introduce the notion of the characteristic foliation on the infinite end of $S^1 \times \mathbb{R}^2$ as follows. Let (θ, r, ϕ) be the cylindrical coordinate of $S^1 \times \mathbb{R}^2$. Let $S(\varepsilon) \subset \mathbb{R}^2$ be a circle of radius ε and $D \subset \mathbb{R}^2$ a unit disc. We denote the interior of D by $\overset{\circ}{D}$. Let $\Psi : S^1 \times \mathbb{R}^2 \to S^1 \times \overset{\circ}{D}$ be a diffeomorphism defined by

$$(\theta, r, \phi) \mapsto \left(\theta, \frac{2}{\pi}\tan^{-1}(r), \phi\right) .$$

A contact open solid torus $(S^1 \times \mathbb{R}^2, \xi = \{\alpha = 0\})$ is contact diffeomorphic to $(S^1 \times \overset{\circ}{D}, \Psi_*(\xi))$. We denote $\Psi_*(\xi)$ by $\bar{\xi}$, and $(\Psi^{-1})^*\alpha$ by $\bar{\alpha}$. Let $T(\varepsilon) := S^1 \times S(\varepsilon)$, $\varepsilon < 1$, be an embedded 2-torus in $S^1 \times \overset{\circ}{D}$. The characteristic foliation $(T(\varepsilon))_{\bar{\xi}}$ is determined by $\bar{\alpha}|_{T(T(\varepsilon))}$. When $\lim_{\varepsilon \to 1} \bar{\alpha}|_{T(T(\varepsilon))}$ converges, this determines a characteristic foliation on a 2-torus $T(1)$. We call it the *infinite characteristic foliation* and denote it by $\mathcal{F}^\infty(\xi)$. We call a characteristic foliation on T^2 is *linear* if there exists a coordinate (θ, ϕ) of T^2 for which the restriction of the contact form α to $T(T^2)$ is denoted by $\alpha|_{T(T^2)} = a \cdot d\theta + b \cdot d\phi$ for some constants a, b. We call the number b/a the *inclination* of this linear characteristic foliation.

We introduce a model of contact structures on S^3. Let $(z_1 = r_1 \cdot e^{i\theta_1}, z_2 = r_2 \cdot e^{i\theta_2})$ be a coordinate of $\mathbb{C}^2$. By considering S^3 as a unit sphere in $\mathbb{C}^2$, $\alpha_0 := r_1^2 \cdot d\theta_1 + r_2^2 \cdot d\theta_2$ defines the *standard* contact structures $\xi_{st} := \{\alpha_0 = 0\}$ on S^3. We set $\Gamma_i := \{r_1 = i\} \subset S^3$, $(i = 0, 1)$. Let $K_k \subset S^3$ be a knot transversal to ξ_{st}, which satisfies $lk(K_k, \Gamma_0) = k$ and $lk(K_k, \Gamma_1) = 0$. Applying the Lutz modification of degree 1 which has a center at K_k (see [L] for definition), we obtain a contact structure ξ_k on S^3. Ya. Eliashberg showed in [E3] that any contact structure on S^3 is isotopic to one of ξ_{st}, ξ_k $(k \in \mathbb{Z})$ and they are pairwise non isotopic. k is the Hopf invariants as plane fields.

Ya. Eliashberg gave in [E4] the discrete classification of contact structures on $\mathbb{R}^3$. Let (z, r, ϕ) be the cylindrical coordinate of $\mathbb{R}^3$. The standard contact structure on $\mathbb{R}^3$ is $\zeta_0 := \{dz + r^2 \cdot d\phi = 0\}$. It is known that ζ_0 is tight. The contact structure $\zeta_1 := \{\cos r \cdot dz + r \cdot \sin r \cdot d\phi = 0\}$ is an example of contact structures overtwisted at infinity. Let σ_k be contact structures which are the restriction of ξ_k to $\mathbb{R}^3 = S^3 \setminus \{p\}$, $p \in S^3$, for $k \in \mathbb{Z}$. As ξ_k, defined above, are overtwisted, σ_k are overtwisted contact structure on $\mathbb{R}^3$ but tight at infinity. Ya. Eliashberg showed the following.

Theorem 0.1 (Ya. Eliashberg) *Any contact structure on $\mathbb{R}^3$ is isotopic to one of the above contact structures $\zeta_0, \zeta_1, \sigma_k (k \in \mathbb{Z})$. These contact structures are pairwise non-isotopic.*

It is interesting to consider differences between $S^1 \times \mathbb{R}^2$ and $\mathbb{R}^3$. On account of Darboux's theorem, contact structures on $\mathbb{R}^3$ tight at infinity admit the one point compactification. Ya. Eliashberg used this property. The basic idea of this note is to compactify with solid torus. That is to say, a contact open solid torus corresponds to a contact 3-sphere by pasting some contact solid torus. Thus we obtain the following informations from a contact open solid torus; the contact diffeomorphism class of contact structures on S^3, the transversal knot type of the souls of the compactifying solid torus, and the thickness of the compactifying solid torus. In the process of proving the extension of contact structures to S^3, we obtain a slight extension of a theorem of Ya. Eliashberg in [E2] about open solid tori (see Proposition 1.1).

1 Family of non-equivalent contact structures

1.1 The inclinations of infinite characteristic foliations.

Ya. Eliashberg showed in [E2] that there is a continuous family of non-equivalent contact structures on an open solid torus. It is clear from the construction that they have linear infinite characteristic foliation. However they can take not all the inclination of linear foliations on a 2-torus. We obtain a slight extension of a theorem in [E2] to take all the inclination.

Let η_{st} be a contact structure on $S^1 \times \overset{\circ}{D}$ defined by the contact form

$$\beta_1 := \sin(r\pi)d\phi + \cos(r\pi)d\theta \ .$$

Proposition 1.1 *Two contact open solid tori* $(S^1 \times \overset{\circ}{D}(\varepsilon), \eta_{st})$ *and* $(S^1 \times \overset{\circ}{D}(\varepsilon'), \eta_{st})$, $0 < \varepsilon, \varepsilon' \leq 1$, *are contact diffeomorphic if and only if the difference* $1/\tan(\varepsilon\pi) - 1/\tan(\varepsilon'\pi)$ *is an integer.*

We consider, in this note, not only standard open solid tori as the above Theorem 1.1 but also what is modified by the Lutz modifications. But we treat open solid tori with infinite linear characteristic foliation. As it is shown in Section 2 that it has the standard form, then we can apply the argument used in the following proof of the necessary condition of Proposition 1.1. So we obtain the following Corollary.

Let $(S^1 \times \mathbb{R}^2, \xi)$, $(S^1 \times \mathbb{R}^2, \xi')$ be contact open solid tori with linear characteristic foliation of inclination $\tan(\varepsilon\pi)$, $\tan(\varepsilon'\pi)$.

Corollary 1.2 *If* $(S^1 \times \mathbb{R}^2, \xi)$ *and* $(S^1 \times \mathbb{R}^2, \xi')$ *are contact diffeomorphic, then* $1/\tan(\varepsilon\pi) - 1/\tan(\varepsilon'\pi)$ *is an integer.*

Let us choose a cylindrical coordinate of $(S^1 \times \mathbb{R}^2, \xi)$ as an extension of the standard form of the infinite end. Then the diffeomorphism Φ_k, defined by equation (1.1) in Section 1.3, changes its inclination $\tan(\varepsilon\pi)$ to $\tan(\varepsilon'\pi)$ in such a way that they suffice $1/\tan(\varepsilon\pi) - 1/\tan(\varepsilon'\pi) = k \in \mathbb{Z}$.

According to this fact and Corollary 1.2, we have only to consider open solid tori whose inclination is $\tan \varepsilon\pi$ for $\varepsilon \in (1/4, 1/2]$.

Corollary 1.3 *Any open solid torus with linear infinite characteristic foliation is uniquely contact diffeomorphic to one of that whose inclination is* $\tan(\varepsilon\pi)$ *for* $\varepsilon \in (1/4, 1/2]$.

The proof Proposition 1.1 is an analogue of that of Eliashberg's Theorem in [E2]. In order to make this note reasonably self-contained, we give a brief outline of the proof in the rest of this section.

1.2 Shape-invariant and its properties.

A *symplectic structure* ω on a $2n$-dimensional manifold W is a closed non-degenerate 2-form on it. Let N be a n-dimensional manifold. An embedding $f : N \to (W, \omega)$ is called *Legendrian* if $f^*\omega = 0$. Let $(M, \xi = \ker \lambda)$ be a $(2n-1)$-dimensional contact manifold. Let $\mathcal{S}(M, \xi)$ be the symplectification of (M, ξ) (see [Ar]). That is, $\mathcal{S}(M, \xi) := (M \times \mathbb{R}_+, d(t \cdot \lambda))$, where t is a coordinate of $\mathbb{R}_+$. It is a $2n$-dimensional exact symplectic manifold.

Let N be a n-dimensional connected closed manifold. Let us fix a homomorphism

$$h : H^1(M; \mathbb{R}) \cong H^1(M \times \mathbb{R}_+; \mathbb{R}) \to H^1(N; \mathbb{R}) \ .$$

Let us denote by $I(\mathcal{S}M \mid N, h)$ the subset of $H^1(N; \mathbb{R})$ that consists of elements $z \in H^1(N; \mathbb{R})$ for which there exists a Lagrangian embedding $f : N \to M \times \mathbb{R}_+$ with $f^* = h$ and $[f^*(t\lambda)] = z$. We call the projectivization of $I(\mathcal{S}M \mid N, h)$ the *contact* (N, h)*-shape* of (M, ξ), and denote it by $I_c(M \mid N, h)$. We note that this "projectivization" means the identification of vectors which is different by a positive factor (see [E2]);

$$I_c(M \mid N, h) = I(\mathcal{S}M \mid N, h)/\mathbb{R}_+ \subset PH^1(N; \mathbb{R}) \ .$$

The contact shape is an invariant in the following sense.

Proposition 1.4 ([E2]) *Let* (M_1, ξ_1), (M_2, ξ_2) *be* $(2n-1)$*-dimensional contact manifolds,* N *a connected closed manifold, and* $h : H^1(M; \mathbb{R}) \to H^1(N; \mathbb{R})$ *a homomorphism. If* $\varphi : (M_1, \xi_1) \to (M_2, \xi_2)$ *is a contact embedding, then* $I_c(M_1 \mid N, h) \subset I_c(M_2 \mid N, h \circ \varphi^*)$.

We introduce a result of J. -C. Sikarov ([S1], [S2]) based on [Gro] improved by Ya. Eliashberg ([E2]). Let T^n be a n-torus, and ST^*T^n ($\cong T^n \times S^{n-1}$) an its unit cotangent bundle. ST^*T^n has the canonical contact structure $\xi = \{p \cdot dq = 0\}$ for $q = (q_1, q_2, \dots q_n) \in T^n = \mathbb{R}^n/\mathbb{Z}$, $p = (p_1, p_2, \dots p_n) \in T_qT^n \cong \mathbb{R}^n$. Choosing cohomology classes $[dq_1], \dots [dq_n]$ as a basis of $H^1(T^n;\mathbb{R})$, we identify $PH^1(T^n;\mathbb{R})$ with the fiber S^{n-1} of $ST^*T^n \to T^n$.

Theorem 1.5 ([E2]) *Let $A \subset S^{n-1} = PH^1(T^n;\mathbb{R})$ be an open connected subset, and $i_a : T^n \hookrightarrow T^n \times A$ an inclusion map $x \mapsto (x, a)$ for $a \in A$. Then*

$$I_c(T^n \times A \mid T^n, i^*) = A \, .$$

We use this for contact 3-manifolds, that is $n = 2$. In this case A is an arc.

1.3 Proof of Proposition 1.1.

First, if $\tan\{(1/2 - \varepsilon')\pi\} - \tan\{(1/2 - \varepsilon)\pi\} =: k \in \mathbb{Z}$, $k > 0$, then the map $\Phi_k : S^1 \times \mathring{D}(\varepsilon) \to S^1 \times \mathring{D}(\varepsilon')$ defined by

$$(1.1) \qquad (\theta, r, \phi) \mapsto \left(\theta, \frac{1}{\pi}\tan^{-1}\left(\frac{\tan(r\pi)}{1 + k\tan(r\pi)}\right), \phi - k\theta\right)$$

is contact diffeomorphic.

Next, suppose that there exists a contact diffeomorphism $\Phi : (S^1 \times D(\varepsilon), \eta_{st}) \to (S^1 \times D(\varepsilon'), \eta_{st})$. Let ζ be a contact structure $\{\cos\theta \cdot d\phi_1 + \sin\theta \cdot d\phi_2 = 0\}$ on $ST^*T^2 \cong T^2 \times S^1$, where $(\phi_1, \phi_2), \theta$ are coordinates of T^2 and S^1. We set $V_{\alpha\beta} := \{\alpha < r_1 = |z_1| < \beta\} \subset S^3 \setminus \{z_1 \cdot z_2 = 0\}$ for $0 < \alpha < \beta < 1$, and $\Gamma_{xy} := \{\arctan x < \theta < \arctan y\} \subset S^1$ an open arc. Then the map $g : S^3 \setminus \{z_1 \cdot z_2 = 0\} \to ST^*T^2 \cong T^2 \times S^1$ defined by $(r_1, \theta_1, \theta_2) \mapsto (\theta_1, \theta_2, (1/2 - r_1)\pi)$ is contact diffeomorphic for η_0 and ζ. We note that

$$g(V_{\alpha\beta}) = T^2 \times \Gamma_{h(\beta)h(\alpha)}$$

where $h(t) := 1/\tan(r\pi)$. Let δ, δ' be real numbers which satisfy $0 < \delta < \varepsilon$, $0 < \delta' < \varepsilon'$, and $\Phi(V_{\delta\varepsilon}) \subset V_{\delta'\varepsilon'}$. Let us define the map $i : T^2 \to V_{\delta\varepsilon}$ by $x \mapsto (x, a)$ for a fixed $a \in (\delta, \varepsilon)$. According to Proposition 1.5, we have

$$(1.2) \qquad I_c(V_{\delta\varepsilon} \mid T^2, i^*) \subset I_c(V_{\delta'\varepsilon'} \mid T^2, i^* \circ \Phi^*) .$$

There exists an isomorphism $\varphi : H^1(T^2;\mathbb{R}) \to H^1(T^2;\mathbb{R})$ which satisfies $\varphi \circ i^* = i^* \circ \Phi^*$. Let $P\varphi : PH^1(T^2;\mathbb{R}) \to PH^1(T^2;\mathbb{R})$ be its projectivization. Then we have

$$(1.3) \qquad I_c(V_{\delta'\varepsilon'} \mid T^2, i^* \circ \Phi^*) = P\varphi(I_c(V_{\delta'\varepsilon'} \mid T^2, i^*)) .$$

We note that $P\varphi(\Gamma_{xy}) = \Gamma_{x+k\ y+k}$ for some $k \in \mathbb{Z}$. According to Theorem 1.5, we have

$$(1.4) \qquad I_c(V_{\alpha\beta} \mid T^2, i^*) = I_c(T^2 \times \Gamma_{h(\beta)h(\alpha)} \mid T^2, i^*) = \Gamma_{h(\beta)h(\alpha)} .$$

On account of the equations (1.2), (1.3), and (1.4), we have

$$\Gamma_{h(\varepsilon)h(\delta)} \subset P\varphi(\Gamma_{h(\varepsilon')h(\delta')}) = \Gamma_{h(\varepsilon')+k\ h(\delta')+k} \subset S^1 .$$

Let δ, δ' approach to ε, ε', then we have $h(\varepsilon) = h(\varepsilon') + k$. That is to say, $1/\tan(\varepsilon\pi) - 1/\tan(\varepsilon'\pi) = k \in \mathbb{Z}$.

This completes the proof of Theorem 1.1.

2 Standard form of infinite ends

In this section, we give the standard form of infinite ends on which contact structures determine linear infinite characteristic foliations. It is an analogue of a theorem of E. Giroux in [Gi] about germs of contact structures along embedded surfaces. Our proof is based on the homotopic method due to J. Moser. (see [Mo])

For $0 < r_0 < r_1$ let us denote by $V(r_0, r_1)$ the domain $\{(\theta, r, \phi) \in S^1 \times \mathbb{R}^2 \mid r_0 < r < r_1\}$.

Proposition 2.1 *Let $(S^1 \times \mathbb{R}^2, \xi)$ be a contact open solid torus with a linear infinite characteristic foliation. Then there exist a compact subset*

$N \subset S^1 \times \mathbb{R}^2$ and two numbers $r_0, r_1 \in (0,1]$, we suppose $r_0 < r_1$, which induce the following contact diffeomorphism.

$$\Phi : \left((S^1 \times \mathbb{R}^2) \setminus N, \xi\right) \overset{\cong}{\to} \left(V(r_0, r_1), \eta_{st}\right)$$

This proposition is an easy corollary of the following lemma. Because, we can identify $S^1 \times \mathbb{R}^2$ and $S^1 \times \overset{\circ}{D}$ by the diffeomorphism Ψ in Section 0 and, by the construction of η_{st}, any linear foliation on 2-torus can be represented as an infinite characteristic foliation of $(S^1 \times \overset{\circ}{D}(r), \eta_{st})$ for a number $r \in (0,1]$.

Lemma 2.2 *Let ξ_0, ξ_1 be contact structures on an open solid torus $S^1 \times \overset{\circ}{D}$. We suppose that they have the same infinite characteristic foliation : $\mathcal{F}^\infty(\xi_0) = \mathcal{F}^\infty(\xi_1)$. Then there exists an isotopy of diffeomorphism φ_t, $t \in [0,1]$, defined in a neighborhood of $S^1 \times \partial D$, which satisfies the following conditions.*

(1) *φ_0 is an identity map.*

(2) $(\varphi_1)_* \xi_0 = \xi_1$

(3) *It preserves the infinite characteristic foliation,*
that is to say $\mathcal{F}^\infty((\varphi_t)_*\xi_0) = \mathcal{F}^\infty(\xi_0)$ for all $t \in [0,1]$.

Proof. Let α_0, α_1 be contact forms which defines contact structures ξ_0, ξ_1 respectively. We may assume that

$$\lim_{t\to 1} \alpha_0|_{T(T(t))} = \lim_{t\to 1} \alpha_1|_{T(T(t))} =: \alpha_\infty$$

as they have the same infinite characteristic foliation. Let $\alpha_s := (1-s)\cdot \alpha_0 + s \cdot \alpha_1$, $s \in [0,1]$, be a homotopy of 1-forms connecting α_0 and α_1. As α_0 and α_1 satisfy the non-degenerate condition, α_s also satisfy it on $T^2 \times (R, 1)$ for some number R sufficiently near 1.

Let us construct an isotopy φ_s of diffeomorphism on $T^2 \times (R,1)$, which satisfies $\varphi_s^* \alpha_s = \lambda_s \cdot \alpha_0$ for non-vanishing functions λ_s. Then we have

$$\frac{d}{ds}(\varphi_s^*\alpha_s) = \frac{d}{ds}(\lambda_s \cdot \alpha_0) = \varphi_s^*\left[\left\{\frac{d}{ds}(\log \lambda_s) \circ \varphi_s^{-1}\right\} \cdot \alpha_s\right] .$$

Let X_s be a time dependent vector field which generates φ_s. Then the following equation holds.

$$\frac{d}{ds}(\varphi_s^*\alpha_s) = \varphi_s^*\left\{\frac{d\alpha_s}{ds} + X_s \lrcorner d\alpha_s + d(X_s \lrcorner \alpha_s)\right\}$$

Thus the vector field X_s have to satisfy

$$\frac{d\alpha_s}{ds} + X_s \lrcorner d\alpha_s + d(X_s \lrcorner \alpha_s) = \mu_s \cdot \alpha_s \tag{2.1}$$

for some non-vanishing function μ_s.

Let X_{α_s} be the Reeb vector field for α_s, which satisfies by definition that $X_{\alpha_s} \lrcorner \alpha_s = 1$ and $X_{\alpha_s} \lrcorner d\alpha_s = 0$. We set $\mu_s := \frac{d\alpha_s}{ds}(X_{\alpha_s})$. Then we take a vector field X_s defined by

$$\begin{cases} X_s \lrcorner \alpha_s = 0 \\ X_s \lrcorner d\alpha_s = -\dfrac{d\alpha_s}{ds} + \mu_s \cdot \alpha_s \end{cases}$$

This clearly satisfy the equation (2.1).

For any vector $Y \in \{\alpha_\infty = 0\} \subset T(T(1))$ which is tangent to the infinite characteristic foliation $\mathcal{F}^\infty(\xi_0) = \mathcal{F}^\infty(\xi_1)$, we have

$$\left(\lim_{r\to 1} \frac{d\alpha_s}{ds}|_{T(T(r))}\right)(Y) = \alpha_\infty(Y) - \alpha_\infty(Y) = 0 \quad .$$

Then the following equation holds.

$$\begin{aligned} \lim_{r\to 1} d\alpha_s|_{T(r)}(X_s, Y) &= \lim_{r\to 1}\left(-\frac{d\alpha_s}{ds}|_{T(r)} + \mu_s \cdot \alpha_s|_{T(r)}\right)(Y) \\ &= \lim_{r\to 1}\left(\mu_s \cdot \alpha_s|_{T(r)}\right)(Y) = 0 \end{aligned}$$

Consequently, X_s is tangent to the infinite characteristic foliation. □

3 Extension of contact structures to S^3

Let $(S^1 \times \mathbb{R}^2, \xi)$ be a contact open solid torus with a linear infinite characteristic foliation. On account of Proposition 2.1, there exist a compact subset $N \subset S^1 \times \mathbb{R}^2$ and two numbers $r_0, r_1 \in (0,1]$, for which we have the following contact diffeomorphism.

$$((S^1 \times \mathbb{R}^2) \setminus N, \xi) \cong (V(r_0, r_1), \eta_{st})$$

According to Proposition 1.1, for this contact toric annulus $(V(r_0, r_1), \eta_{st})$, there exist two numbers $\bar{r}_0, \bar{r}_1 \in (0, 1/2]$, especially $\bar{r}_1 \in (1/4, 1/2]$, for which the following contact diffeomorphism holds.

$$(V(r_0, r_1), \eta_{st}) \cong (V(\bar{r}_0, \bar{r}_1), \eta_{st})$$

Moreover, since $\bar{r}_1 \in (0, 1/2]$, we have the following contact diffeomorphism.

$$(V(\bar{r}_0, \bar{r}_1), \eta_{st}) \cong (V_{\psi(\bar{r}_0)\psi(\bar{r}_1)}, \xi_{st})$$

where $\psi(r) = 1 / \sqrt{\tan(1/2 - r)\pi + 1}$. As a consequence we have the following contact diffeomorphism.

$$\phi : \left((S^1 \times \mathbb{R}^2) \setminus N, \xi\right) \overset{\cong}{\to} (V_{\psi(\bar{r}_0)\psi(\bar{r}_1)}, \xi_{st})$$

We denote by $\bar{U}_\delta$ an open solid torus $\{(z_1, z_2) \in \mathbb{C}^2 \mid |z_1| > \delta\} \cap S^3$. Then we have

$$(\bar{U}_{\psi(\bar{r}_0)} \setminus \{\bar{U}_{\psi(\bar{r}_1)}\}^{cl}, \xi_{st}) = (V_{\psi(\bar{r}_0)\psi(\bar{r}_1)}, \xi_{st}) \quad .$$

Pasting together by contact diffeomorphism ϕ, we obtain a contact 3-sphere. We denote by $\tilde{\xi}$ the obtained contact structure.

$$(S^3, \tilde{\xi}) = (S^1 \times \mathbb{R}^2, \xi) \cup_\phi (\bar{U}_{\psi(\bar{r}_0)}, \xi_{st})$$

Remark 1 *Contact structures are extended to the* standard contact tube $(\bar{U}_r, \xi_{st})$ *in the above. Then tight (resp., overtwisted) contact structures are extended to tight (resp., overtwisted) contact structures.*

Thus we obtain a contact diffeomorphism class $[\tilde{\xi}]$ of contact structure on S^3. The soul of the compactifying solid torus $(\bar{U}_{\psi(\bar{r}_0)}, \xi_{st})$ is clearly transversal to ξ_{st}. Then there exists a transversal knot K corresponding to the above soul. Thus we get transversal knot class $[K]$ in $(S^3, \tilde{\xi})$. According to Proposition 1.1, $\bar{r}_1 \in (1/4, 1/2]$ is uniquely determined by $(S^1 \times \mathbb{R}^2, \xi)$. It corresponds to the thickness of the compactifying solid torus.

References

[Ar] V. I. Arnol'd, *Mathematical method in classical mechanics*, Grad. Texts in Math. **60**, Springer, New York, 1978.

[E1] Ya. Eliashberg, *Classification of overtwisted contact structures on 3-manifolds*, Invent. Math. **98** (1989), 623-637.

[E2] Ya. Eliashberg, *New invariants of open symplectic and contact manifolds*, J. Amer. Math. Soc. **4** (1991), 513-520.

[E3] Ya. Eliashberg, *Contact* 3*-manifolds twenty years since J. Martinet's work*, Ann. Inst. Fourier (Grenoble) **42** (1992), 165-192.

[E4] Ya. Eliashberg, *Classification of contact structures on* $\mathbb{R}^3$, Internat. Math. Res. Notices **3** (1993), 87-91.

[Gi] E. Giroux, *Convexité en topologie de contact*, Comment. Math. Helv. **66** (1991), 637-677.

[Gro] M. Gromov, *Pseudoholomorphic curves in symplectic manifolds*, Invent. Math. **82** (1985), 307-347.

[L] R. Lutz, *Structures de contact sur les fibres principaux en cercles de dimension trois*, Ann. Inst. Fourier (Grenoble) **27** (1977), 1-15.

[Ma] J. Martinet, *Formes de contact sur les variété de dimension 3*, Lecture Notes in Math. **209** (1971), Springer, 142-163.

[Mo] J. Moser, *On the volume elements of a manifold*, Trans. Amer. Math. Soc. **120** (1965), 286-294.

[S1] J. -C. Sikarov, *Rigidité symplectique dans le cotangent de* T^n, Duke Math. J. **59** (1989), 227-231.

[S2] J. -C. Sikarov, *Quelque propriétés des plongements Lagrangiens*, Mém. Soc. Math. France **46** (1991), 151-167.

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