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Convergence of attractors for simplified magnetic Bénard system

Dedicated to the 60-th birthday of Professor Hideo Kawarada

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Abstract: We consider the relation between the attractors for the simplified magnetic Bénard system(SMBS) and the Lorenz system(L). (SMBP) is a extension of (L), which contains a parameter Q associated with the magnetic field. In the previous work we showed the upper semiconvergence of attractors of (SMBP), $\mathcal{A}_Q$ to that of (L), $\bar{\mathcal{A}}$ in the Hausdorff sense as $Q \to 0$. However the numerical computation of Lyapunov dimensions showed they do not converge. We carried out the computation by the 4th order Runge-Kutta method. In this talk we shall show the computation of dimensions by the spectral collocation methods in the quadruple precision. We obtain the same results, i.e., the fractal dimension does not converge.
Key words: simplified magnetic Bénard system, attractors, fractal dimension, spectral collocation method

0. Introduction

Controlling the chaotic phenomena is the very interesting and important problem both in mathematics and engineering. The various differential systems derived from the phenomena in the engineering have the chaotic attractors, which have finite fractal dimensions [16].

We consider the attractors of the 2-dimensional simplified magnetic Bénard system(SMBS) [8, 9, 4, 5]. That is derived from the 2-dimensional magnetic Bénard system as the extension of the Lorenz system(L) [12, 10, 8, 9]. We studied the relations of the attractors for (SMBS) and (L) [5].

In \$ 1 and \$ 2 we introduce the magnetic Bénard system and (SMBS) [2, 13, 14]. In \$3 we summarizes the attractors for the systems and in \$4 we will prove the theorem about the Hausdorff upper-semi-convergence of the attractors of (SMBS) to that of (L) as Q tends to 0 [5]. In \$5 we will show the numerical computations of Lyapunov dimensions of attractors with various values of Q [11, 5, 15, 17]. We used the spectral collocation method with 11 collocation points in the quadruple precision [1, 3, 5, 6, 7].

1. Magnetic Bénard system

We consider in R^2, an layer of a homogeneous, viscous and electrically conducting fluid filled the region $0 < x_2 < 1$, limited by the surfaces $x_2 = 0$ and $x_2 = 1$. The fluid layer is heated from below in such a way that the lower plane is maintained at a temperature T_0 while the upper one is maintained at $T_1(< T_0)$, where T_0 and T_1 are two constants. Furthermore the layer is permeated by an impressed uniform magnetic field $\boldsymbol{H}_0$ normal to the layer. Then in the Boussinesq approximation the velocity $\boldsymbol{u} = (u_1, u_2)$, the pressure p, the temperature θ of the fluid, and the magnetic field $\boldsymbol{h} = (h_1, h_2)$ are governed in the region $0 < x_2 < 1$ by

$$(1)\quad \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \mathrm{P}_m(\boldsymbol{h}\cdot\nabla)\boldsymbol{h} + \mathrm{P}_m(\frac{1}{2}\nabla|\boldsymbol{h}|^2) = \Delta\boldsymbol{u} - \nabla p + \mathrm{R}\theta\boldsymbol{e} + \mathrm{Q}\{\frac{\partial \boldsymbol{h}}{\partial x_2} + \nabla h_2\},$$

$$(2)\quad \mathrm{P}_m(\frac{\partial \boldsymbol{h}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{h} - (\boldsymbol{h}\cdot\nabla)\boldsymbol{u}) = -\widetilde{\mathrm{rot}}\,\mathrm{rot}\,\boldsymbol{h} + \mathrm{Q}\frac{\partial \boldsymbol{u}}{\partial x_2},$$

$$(3)\quad \mathrm{P}_r(\frac{\partial \theta}{\partial t} + (\boldsymbol{u}\cdot\nabla)\theta) = \Delta\theta + \mathrm{R}u_2,$$

$$(4)\quad \mathrm{div}\ \boldsymbol{u} = 0,$$

$$(5)\quad \mathrm{div}\ \boldsymbol{h} = 0.$$

where R(square root of Rayleigh number) $= \sqrt{\dfrac{g\alpha\beta d^4}{\kappa\nu}}$, Q(square root of Chandrasekhar number) $= \sqrt{\dfrac{\mu H^2 d^2}{4\pi\rho\nu\eta}}$, P_r(Prandtl number)$= \dfrac{\nu}{\kappa}$, P_m(magnetic Prandtl number) $= \dfrac{\nu}{\eta}$.
Here g is the gravitational acceleration, α the coefficient of volume expansion, κ the thermal diffusivity coefficient, ν the kinematic viscosity, μ the magnetic permeability, ρ the density, η the resistivity, d the depth of the layer and temperature gradient $\beta = \dfrac{T_0 - T_1}{d}$. $\boldsymbol{e}$ is a unit vector in the x_n direction.
This is the perturbation system to the equilibrium state which is the pure conduction.

We supplement the above system with boundary conditions for the free surface and impose a periodicity condition.

$$(6)\quad u_2(x,t) = \frac{\partial u_1}{\partial x_2} = 0 \quad \text{at} \quad x_2 = 0, 1,$$

$$(7)\quad \boldsymbol{h}(x,t) = 0 \quad \text{at} \quad x_2 = 0, 1,\ t \geq 0,$$

$$(8)\quad \theta(x,t) = 0 \quad \text{at} \quad x_2 = 0, 1, \quad t \geq 0,$$

$$(9)\quad p, \boldsymbol{u}, \boldsymbol{h}, \theta, \frac{\partial \boldsymbol{u}}{\partial x_j}, \frac{\partial \boldsymbol{h}}{\partial x_j}, \frac{\partial \theta}{\partial x_j} \quad \text{are periodic in the } x_1 \text{ direction}$$

REMARK 1.1

$$(10)\quad \mathrm{rot}\,\boldsymbol{u} = \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \quad \text{for every vector function } \boldsymbol{u},$$

$$(11)\quad \widetilde{\mathrm{rot}}\,\phi = (\frac{\partial \phi}{\partial x_2}, -\frac{\partial \phi}{\partial x_1}) \quad \text{for every scalar function } \phi,$$

(12)

2. simplified Bénard system (SMBS) and the Lorenz system (L)

The Lorenz system is derived from the 2-dim Bénard convection and the system (SMBS) is derived from the 2-dim magnetic Bénard convection as an extension of (L).
Lorenz system:

$$(13)\qquad \begin{cases} \dot{x} &= -P_r x + P_r y, \\ \dot{y} &= Rx - y - xz, \\ \dot{z} &= -bz + xy. \end{cases}$$

Simplified magnetic Bénard system:

$$(14)\qquad \begin{cases} \dot{x} &= -P_r x + P_r y - P_r Q w, \\ \dot{y} &= Rx - y - xz, \\ \dot{z} &= -bz + xy, \\ \dot{w} &= P_r P_m^{-1} Q x - P_r P_m^{-1} w, \end{cases}$$

where $\dot{} = \frac{d}{dt}$, and P_r, P_m, R and Q denote Prandtl, magnetic Prandtl, Rayleigh, and Chandrasekhar numbers, respectively. b denotes the aspect ratio of the roll.

Remark; If $Q = 0$, then w of (SMBS) is independent of other components, and three other components (x, y, z) are governed by the Lorenz system.

3. Attractors for (L) and (SMBS)

Both systems (L) and (SMBS) have global attractors $\bar{\mathcal{A}}$ and $\mathcal{A}_Q$ respectively. We studied the relation between the attractors when $Q \to 0$, and obtained the following result.

First we shall show that there exist absorbing sets for both systems.

We denote by v and $\bar{v}$, the following norms,

$$(15)\qquad v^2 = x^2 + y^2 + (z - P_r - R)^2 + P_m w^2 \text{ for (SMBS)},$$

$$(16)\qquad \bar{v}^2 = x^2 + y^2 + (z - P_r - R)^2 \text{ for (L)}.$$

Then we can obtain the following inequalities,

$$(17)\qquad \begin{aligned} \frac{d}{dt} v(t)^2 &\leqq -2\min\{1, P_r, P_r P_m^{-1}, \frac{b}{2}\} v(t)^2 + b(P_r + R)^2, \\ \frac{d}{dt} \bar{v}(t)^2 &\leqq -2\min\{1, P_r, \frac{b}{2}\} \bar{v}(t)^2 + b(P_r + R)^2. \end{aligned}$$

Hence if we define as follows

$$(18)\qquad \begin{aligned} \mathcal{U} &= \{(x, y, z, w) \,|\, v^2 < \frac{b}{\min\{1, P_r, P_r P_m^{-1}, b/2\}} (P_r + R)^2\}, \\ \bar{\mathcal{U}} &= \{(x, y, z) \,|\, \bar{v}^2 < \frac{b}{\min\{1, P_r, b/2\}} (P_r + R)^2\}, \end{aligned}$$

then $\mathcal{U}$, and $\bar{\mathcal{U}}$ are the absorbing sets for (SMBS) and (L) respectively.

Let us denote by $\mathcal{A}_Q$, and $\bar{\mathcal{A}}$, the attractors for (SMBS) and (L) respectively, then they are obtained as the ω limit sets of the absorbing sets $\mathcal{U}$, and $\bar{\mathcal{U}}$.

(19)

$$\begin{aligned} \mathcal{A}_Q &= \bigcap_{t \geq o} \text{Cl.} \bigcup_{s \geq t} \{u(s) = (x(s), y(s), z(s), w(s)) | u(s) \text{ is a solution of } (1), u(0) \in \mathcal{U}\}.\} \\ \bar{\mathcal{A}} &= \bigcap_{t \geq o} \text{Cl.} \bigcup_{s \geq t} \{\bar{u}(s) = (x(s), y(s), z(s)) | \bar{u}(s) \text{ is a solution of } (2), \bar{u}(0) \in \bar{\mathcal{U}}\}. \end{aligned}$$

$u = (x, y, z, w) \in \mathcal{A}_Q$ means that there exist the sequence $t_1 < t_2 < \cdots \to \infty$ and the solutions $u_i(t)$ with initial values in $\mathcal{U}$ which satisfy $\lim_{i\to\infty} u_i(t_i) = u$.

4. Convergence of attractors

We studied the relation between the attractors when $Q \to 0$, and obtained the following result.

Define the projection P of $\mathbb{R}^4$ onto $\mathbb{R}^3$ and the immersion P_0^{-1} of $\mathbb{R}^3$ into $\mathbb{R}^4$ as follows,

$$P(x, y, z, w) = (x, y, z),$$
$$P_0^{-1}(x, y, z) = (x, y, z, 0).$$

Here we introduce the distance d between $\bar{\mathcal{A}}$ and $\mathcal{A}_Q$,

$$d(\mathcal{A}_Q, \bar{\mathcal{A}}) = \sup_{u\in\mathcal{A}_Q} \inf_{v\in P_0^{-1}\bar{\mathcal{A}}} |u - v|.$$

REMARK 4.1 Above distance d is a pseudo metric, so $d(A, B) = 0$ means only $B \subset A$.

If we set $\rho(\mathcal{A}_Q, \bar{\mathcal{A}}) = \max\{ \sup_{u\in\mathcal{A}_Q} \inf_{v\in P_0^{-1}\bar{\mathcal{A}}} |u - v|, \sup_{v\in P_0^{-1}\bar{\mathcal{A}}} \inf_{u\in\mathcal{A}_Q} |u - v|\}$,

then ρ becomes a metric between $\mathcal{A}_Q$ and $\bar{\mathcal{A}}$, which is called the Hausdorff metric.

We obtained the following result with respect to d.

Theorem

$\mathcal{A}_Q$ converges to $\bar{\mathcal{A}}$ upper semi-continuously in the Hausdorff sense as $Q \to 0$, i.e.

$$d(\mathcal{A}_Q, \bar{\mathcal{A}}) \to 0 \quad \text{as} \quad Q \to 0.$$

REMARK 4.2 As the above remark, this theorem only means the following fact.

For any $\varepsilon > 0$, there exists Q_ε such that

$$\mathcal{N}_{\mathbf{R}^4}(P_0^{-1}\bar{\mathcal{A}}, \varepsilon) \supset \mathcal{A}_Q \quad (0 < Q < Q_\varepsilon),$$

where $\mathcal{N}_{\mathbf{R}^4}(\cdot, \varepsilon)$ is a ε neighborhood in $\mathbb{R}^4$.

Sketch of the proof

(Step i) First estimate the fourth component w by Q.

For $u = (x, y, z, w) \in \mathcal{A}_Q$, we obtain

$$|w| \le C_1 Q.$$

In particular for $u(t, u_0), u_0 \in \mathcal{U}$, and $t \geqq t_Q = P_r^{-1} P_m \log(1/Q)$ we have

$$|w(t)| \le C_1 Q \quad \text{where} \quad C_1 = (1 + \frac{1}{\sqrt{P_m}}).$$

From the equation of w, we obtain the following equality,

$$w(t) = w(0)e^{-P_r P_m^{-1} t} + \int_0^t P_r P_m^{-1} Q x(s) e^{-P_r P_m^{-1}(t-s)} ds.$$

Because $\mathcal{U}$ is bounded in R^4, we can get the estimate.

Here we must remark that this estimate means that when Q converges monotonously to 0, then $|w|$ also converges to 0.

So the immersion P_0^{-1} which embeds the attractors $\mathcal{A}$ into the hyperplane $\{w = 0\} \subset R^4$ is well meaning.

(Step ii) Estimate of the orbits.

Define $\delta_Q(t)$ which presents the distance between the orbits as follows.

$$\delta_Q(t) = \sup_{u_0 \in \mathcal{U}} |u(t+t_Q, u_0) - P_0^{-1}\bar{u}(t, Pu(t_Q, u_0))|$$

Then there exist constants C_2,and C_3 which are independent of Q, and satisfy the following inequality,

$$\delta_Q(t) \leqq C_2 \cdot Q e^{C_3 t}$$

where $\bar{u}(t, Pu(t_Q, u_0))$ is a solution of (2) that satisfy $Pu(t_Q, u_0) \in \bar{\mathcal{U}}$ when t is zero.

Let $\delta_Q(t, u_0)$ be defined as follows

$$\begin{aligned} \delta_Q(t, u_0) &= u(t+t_Q) - P_0^{-1}\bar{u}(t, Pu(t_Q, u_0)) \\ &= (\delta x, \delta y, \delta z, \delta w) \end{aligned}$$

then $\delta_Q(t, u_0)$ satisfies the following equation,

$$(20) \qquad \left\{ \begin{aligned} (\dot{\delta x}) &= -P_r(\delta x) + P_r(\delta y) - P_r Q(\delta w) \\ (\dot{\delta y}) &= R(\delta x) - (\delta y) - z(\delta x) - \bar{x}(\delta z) \\ (\dot{\delta z}) &= -b(\delta z) + y(\delta x) + \bar{x}(\delta y) \\ (\dot{\delta w}) &= P_r P_m^{-1} Q x - P_r P_m^{-1}(\delta w) \end{aligned} \right.$$

where $u(t+t_Q) = (x, y, z, w), P_0^{-1}\bar{u}(t, Pu(t_Q, u_0)) = (\bar{x}, \bar{y}, \bar{z}, 0)$.

Hence the estimate can be easily verified from the estimate

$$|\delta_Q(0, u_0)| = |w(t_Q)| \leq C_1 Q.$$

(Step iii) From the definition of the ω limit set $\bar{\mathcal{A}}$, we can find t_ε which satisfies the following relation,

$$(21) \qquad \mathcal{N}_{\mathbf{R}^4}(P_0^{-1}\bar{\mathcal{A}}, \varepsilon) \supset \mathcal{N}_{\mathbf{R}^4}(P_0^{-1}\bar{u}(t, \bar{\mathcal{U}}), \frac{\varepsilon}{2}) \quad \text{for } t > t_\varepsilon,$$

where $\bar{u}(t, \mathcal{U})$ is the solution with initial value in $\mathcal{U}$.

(Step iv) Take $Q_\varepsilon = \frac{\varepsilon}{4} C_2^{-1} e^{-c_3 t_\varepsilon}$, then for any $Q < Q_\varepsilon$, there exists $t_{\varepsilon,Q}$ such that

$$(22) \qquad \mathcal{N}_{\mathbf{R}^4}(u(t, \mathcal{U}), \frac{\varepsilon}{4}) \supset \omega(\mathcal{U}) = \mathcal{A}_Q \quad \text{for } t \geqq t_{\varepsilon,Q}.$$

(Step v) Since $\mathcal{U}$ is positively invariant with respect to $S_Q(t)$, we infer for $t > t_\varepsilon + t_{\varepsilon,Q} + t_Q$,

$$(23) \qquad S_Q(t)\mathcal{U} = S_Q(t_\varepsilon + t_Q + (t - t_\varepsilon - t_Q))\mathcal{U} \subset S_Q(t_\varepsilon + t_Q))\mathcal{U}.$$

So from the estimates of $\delta_Q(t_\varepsilon)$, and the property $PS_Q(t_Q)\mathcal{U} \subset \bar{\mathcal{U}}$, we obtain the following estimate

$$(24) \qquad \mathcal{N}_{\mathbf{R}^4}(u(t_\varepsilon + t_Q, \mathcal{U}), \frac{\varepsilon}{4}) \subset \mathcal{N}_{\mathbf{R}^4}(P_0^{-1}\bar{u}(t_\varepsilon, \bar{\mathcal{U}}), \frac{\varepsilon}{2}).$$

Thus from (22), (23) and (24), we obtain

$$\mathcal{A}_Q \subset \mathcal{N}_{\mathbf{R}^4}(P_0^{-1}\bar{\mathcal{A}}, \varepsilon).$$

5. Numerical Computation of Lyapunov dimensions

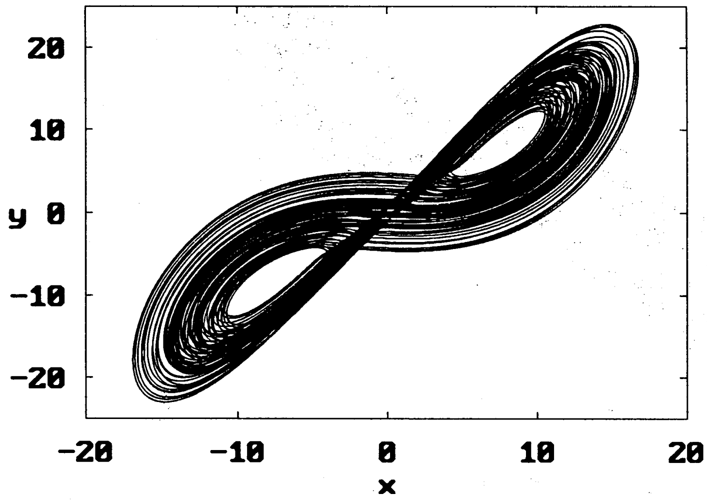

Figure 1. Projection of the attractor $\mathcal{A}_Q$, $Q = 0.5$.

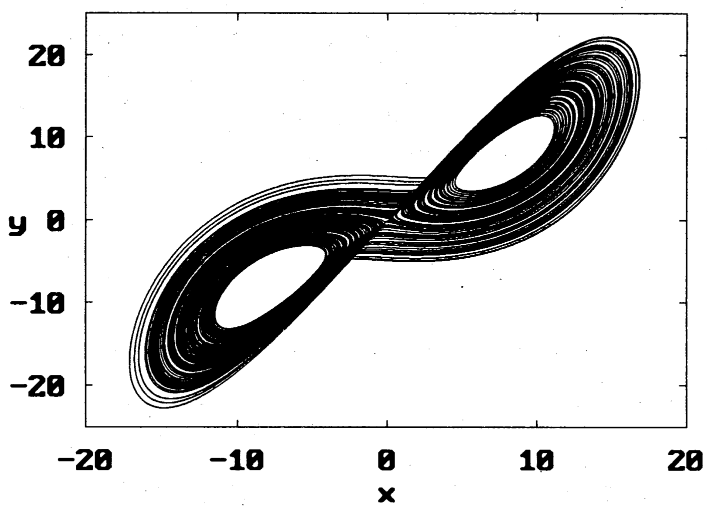

Figure 2. Projection of the attractor $\mathcal{A}_Q$, $Q = 0.1$.

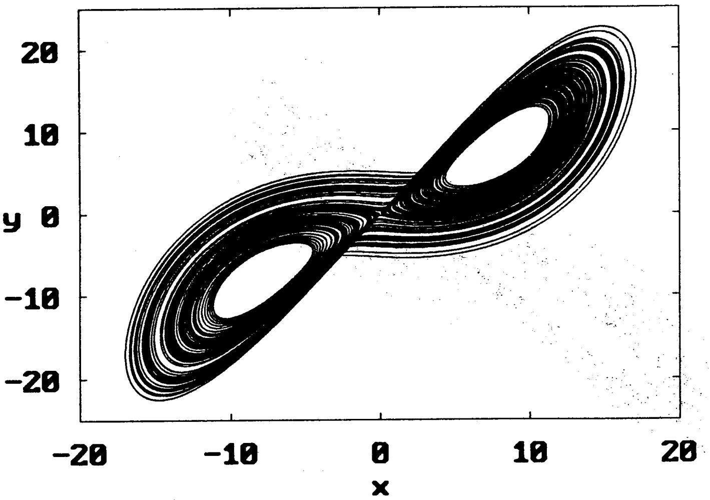

Figure 3. Projection of the attractor $\bar{\mathcal{A}}$.

We showed the theoretical result in the previous section, however it is only the partial result. To know the difference between the attractors of the both systems, we shall study the numerical convergence of the fractal dimensions[1,16,17]. We made the numerical computation of fractal dimensions of both attractors as $Q \to 0$. There are three kind of fractal dimensions of the attractors used as the characteristics of the chaotic structures[4]. Among them we use the Lyapunov dimension as the fractal dimension of them. because both attractors are derived from the ordinary differential systems.

The chaotic structure is very subtle, so the calculation of the dimensions demands both the delicate computations and the higher accuracy.

Table 1. Lyapunov dimension(double precision).

Q	1.0	0.5	0.1	10^{-2}	10^{-3}	0
L_d	2.21	2.38	2.41	2.41	2.41	2.06

Figure 1, 2 and 3 are the projections of the attractors computed by the 4-th order Runge-Kutta method. It seems that $\mathcal{A}_Q$ converge $\bar{\mathcal{A}}$ continuously. However by our former computations using the fourth order explicit Runge-Kutta method with time discrement $\Delta t = 10^{-3}$ in the double precision, dimensions appeared discontinuous as $\mathrm{Q} \to 0$. So we must calculate in the higher order accuracy than the former one.

Because of the reasons above, we adopt the spectral collocation method with quadruple precision using the Shimada-Nagashima's method to calculate the Lyapunov dimensions.

For our computations we shall use the following fixed value as the non-dimensional numbers, $(\mathrm{P_r}, \mathrm{P_m}, \mathrm{R}, \mathrm{b}) = (10, 5, 25, 8/3)$. We made the computations using the system (L) for the value $\mathrm{Q} = 0$ and the system (SMBS) for the values, $\mathrm{Q} > 0$ respectively.

By the above consideration, we should adopt the spectral collocation method(SCM). In this method we can easily change the order of precisions by changing the numbers of collocation points.

Table 2. Lyapunov dimension by SCM with 11 pts(quadruple precision).

Q	1.0	0.5	0.1	10^{-2}	10^{-3}	0
$\mathcal{L}_d$	2.21	2.38	2.41	2.41	2.41	2.06

6. Conclusion

We investigate the convergence of attractors for simplified magnetic Bénard system(SMBS) as Q tends to 0. Analytically we proved that attractors for (SMBS) converge to that of Lorenz system in the sense of Hausdorff upper-semiconvergence.

To study the full convergence of the attractors, we carried out the numerical computations of Lyapunov dimensions of them in both double and quadruple precision arithmetic.

The numerical results show the discontinuity of the dimensions as Q tends to 0.

It is important to study the convergence of attractors both by the analytical investigation and by the numerical computation simultaneously.

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