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Author(s)	Cieliebak, Kai; Sere, Eric
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PSEUDO-HOLOMORPHIC CURVES AND MULTIBUMP HOMOCLINIC ORBITS

Kai Cieliebak
ETH Zürich
Switzerland

Eric Séré
Université de Cergy-Pontoise
France

Abstract

We present a multiplicity result for homoclinics of a nonconvex Hamiltonian system, obtained variationally thanks to a "topological shadowing lemma". As a consequence we exhibit a chaotic behavior of the Hamiltonian system, under assumptions that are weaker than the standard transversality condition. The main tool in our proof is the theory of pseudo-holomorphic curves of Gromov-Floer.

1. Results.

Let M be a compact smooth manifold of dimension n and $T^*M \to^{\tau} M$ its cotangent bundle. T^*M carries a canonical 1-form θ which writes in geodesic normal coordinates (q_i, p_i) as

$$\theta = \sum p_i \, dq_i \quad .$$

$\omega := d\theta$ is then a symplectic form on T^*M.

To a smooth Hamiltonian $H \in C^\infty(S^1 \times T^*M, \mathbb{R})$, 1-periodic in time, we associate the Hamiltonian system

$$\dot{x} = X_H(t,x) \ , \tag{1}$$

where the Hamiltonian vector field X_H is defined by

$$dH_t(x) = \omega(X_H(t,x), \cdot) \ . \tag{2}$$

We make the following assumptions on H :

(H1) (saddle point)

There exists a point $x_0 = (q_0, 0) \in T^*M$ such that

$$\begin{aligned} H(t, q_0, p) &\geq 0 \qquad \text{for all } t, p \\ H(t, q, 0) &< 0 \qquad \text{for all } q \neq q_0 . \end{aligned}$$

Consequently,

$$H(t, x_0) = 0 \quad , \quad H'(t, x_0) = 0 \qquad \text{for all } t .$$

(H2) (growth conditions)

(i) $|dH(t,x)| \leq a\, d(x, x_0)$

(ii) $H(t, q, p) \geq b_1 |p|^2 - b_2$

(iii) There exists a vector field η on TM satisfying

$$\begin{aligned} d(i_\eta \omega) &= \omega \\ |\eta(x)| &\leq c_1\, d(x, x_0) \\ < H'(t,x), \eta(x) > -H(t,x) &\geq c_2 \left(d(x, x_0) \right)^2 \quad . \end{aligned}$$

As a consequence, the energy levels $\{x \in T^*M\,;\, H(t,x) = h\}$ are of contact type, for all $t \in S^1$, $h > 0$.

To formulate $(H2)$, we have chosen a Riemannian metric on M and denoted by $|\cdot|$ and $d(\cdot,\cdot)$ the induced metric and distance on T^*M. Note, however, that $(H2)$ does not depend on the choice of metric.

Example (classical Hamiltonians).

The hypotheses $(H1-2)$ are satisfied by classical Hamiltonians

$$H(t, q, p) = \frac{|p - A(t,q)|^2}{2} + V(t, q) ,$$

with the following properties :

(j) V has a unique nondegenerate absolute maximum q_0, i.e.

$$V(t, q_0) = 0 , \; V_q'(t, q_0) = 0 , \; V_{qq}''(t, q_0) < 0 , .$$

(jj) $A(t, q_0) = 0$, $A'(t, q_0) = 0$,

$$|A"_{qq}(t, q_0) \cdot \xi|^2 + V"_{qq}(t, q_0) \cdot \xi \cdot \xi < 0 , \; \forall \xi \in T_{q_0} M ,$$

and $|A(t,q)|^2 + V(t,q) < 0$ for any $q \neq q_0$.

In this case η is the vector field defined by $\omega(\xi, \cdot) = \theta$.

Remark 1.

The hypotheses $(H1-2)$ are symplectically invariant in the following sense :

Let $\psi : T^*M \to T^*M$ be a symplectomorphism mapping the zero section onto itself and the fibre $T^*_{x_0}M$ onto the fibre $T^*_{\psi(x_0)}M$, and satisfying

$$\frac{1}{a_1} d(x, x_0) \leq d\big(\psi(x), \psi(x_0)\big) \leq a_1 d(x, x_0)$$
$$\frac{1}{a_1}|v| \leq |D\psi(x) \cdot v| \leq a_1 |v| \quad \text{for all} \quad (x,v) \in T(T^*M)\,.$$

Then a Hamiltonian $H(t,x)$ satisfies $(H1-2)$ with fixed point x_0 iff $H\big(t, \psi^{-1}(x)\big)$ satisfies $(H1-2)$ with fixed point $\psi(x_0)$.

Remark 2.

The hypothesis $(H2)$ implies that x_0 is a hyperbolic fixed point of the time-one map of (1), i.e.

$$D\varphi_1(x_0) \; : \; T_{x_0}T^*M \;\to\; T_{x_0}T^*M$$

has no eigenvalue of modulus 1.

Let

$$\mathcal{C} \;=\; \Big\{ x \in C^\infty(\mathrm{I\!R}, TM) \;|\quad \dot{x} = X_H(t,x)\;, \lim_{t \to \pm\infty} x(t) = x_0 \Big\}$$

be the set of all solutions of (1) which are doubly asymptotic to x_0. The elements of $\mathcal{C} \setminus \{x_0\}$ are called orbits homoclinic to x_0.

Homoclinics for the kind of system described in Remark 1 were first studied variationally by Bolotin [Bo], and later by many authors (see e.g. [B-Gi], [R1], [Gi], [Gi-R]). However, our assumptions are more general: we don't assume convexity of H in the fibres. The first result we know for nonconvex Hamiltonians on manifolds is due to Felmer [Fe], in the case of the cotangent bundle of the Torus.

Since H is one-periodic in time, the integers act on $\mathcal{C}$ via

$$\begin{aligned} * : \mathbf{Z} \times \mathcal{C} &\to \mathcal{C} \\ (n, x) &\to n * x(t) = x(t-n) \end{aligned}$$

In [Ci-S-1] the following result is proved :

Theorem 1. *Assume that* $(H1-2)$ *are satisfied. Then there are infinitely many orbits homoclinic to* x_0 *, which are in different classes of* $\mathcal{C}/\mathbf{Z}$.

The method is based on ideas due to Gromov [Gr] and Floer [Fl]. The structure of our proof is inspired of [H-W] and [Ci].

We point out that the hypotheses on H in [Ci-S-1] are slightly stronger than $(H1-2)$. But it is not very difficult to weaken these assumptions, this is explained in [Ci-S-2], Section 5.

Now near a homoclinic orbit one expects, under certain assumptions, to find chaotic behavior. This goes back to Poincaré who observed in 1899 that in the neighborhood of a homoclinic orbit there may exist an infinite number of further homoclinic orbits giving rise to a very complicated orbit structure: "On sera frappé de la complexité de cette figure, que je ne cherche même pas à tracer" ([Po], p. 387). Later this was made precise by Birkhoff, Smale, Silnikov and others in terms of symbolic dynamics. Recall the definition of a Bernoulli shift. Let

$$\Sigma = \{0, 1\}^{\mathbf{Z}}$$

be the set of all doubly infinite sequences endowed with the metric

$$d(a, b) = \sum_{n \in \mathbf{Z}} \frac{|b_n - a_n|}{2^{|n|}} .$$

The Bernoulli shift is given by the homeomorphism

$$\begin{aligned} \sigma : \Sigma &\to \Sigma \\ (a_n)_{n \in \mathbf{N}} &\to (a_{n+1})_{n \in \mathbf{N}} \end{aligned}$$

We say that a homeomorphism ϕ on an invariant subset A is semi-conjugate to a Bernoulli shift if there exists a continuous surjection $\tau : A \to \Sigma$ such that the following diagram commutes:

$$\begin{array}{ccc} A & \xrightarrow{\phi} & A \\ \tau \downarrow & & \downarrow \tau \\ \Sigma & \xrightarrow{\sigma} & \Sigma \end{array}$$

It is conjugate to a Bernoulli shift if τ is a homeomorphism.

It is a classical result that if x_0 is a hyperbolic fixed point (or periodic point) of a diffeomorphism ϕ, and the stable and unstable manifolds of x_0 have a transverse intersection outside x_0, then there exists a set A on which the iterate ϕ^N is conjugate to a Bernoulli shift, for $N \in \mathbf{N}$ sufficiently large (see for instance [Mo]).

However, it is quite unnatural to presuppose tranversality of orbits which are to be found by variational methods. Instead we use a weaker global hypothesis in [Ci-S-2]:

(C): Any connected component of $\mathcal{C}$ for the $H^{1,2}(\mathbb{R}, T^*M)$ topology, is compact for this topology.

Here we use an embedding $T^*M \subset \mathbb{R}^{2a}$ identifying x_0 with 0, to define the Hilbert manifold $H^{1,2}(\mathbb{R}, T^*M)$.

Note that independently of [Ci-S-2], an assumption similar to (C) is introduced by Rabinowitz [R2], in the context of singular Lagrangian systems. (C) is a weakening of an assumption introduced in [CZ-E-S], and used in several works, under several forms (see e.g [S1-2], [CZ-R], [Li], [Gi-R]). The variants of this assumption give additional compactness properties to functionals invariant by a discrete group of translations.

Theorem 2 [Ci-S-2].
If H satisfies (H1-2) and (C), then for each sufficiently large $T \in \mathbf{N}$ there exists a compact subset $A_T \subset T^*M$, invariant under the time-T map ϕ_T of (1), such that ϕ_T is semi-conjugate to a Bernoulli shift on A_T.

Such a result has been obtained in [S2] for convex Hamiltonians on $\mathbb{R}^{2n}$, under the hypothesis

($\mathcal{H}$): $\mathcal{C}$ is at most countable.

Obviously, ($\mathcal{H}$) implies (C).

The rate at which a system is chaotic can be measured by the topological entropy as defined by Bowen (see [O], p. 182-183):

$$h_{top}(\phi) = \sup_{R>0} \lim_{\epsilon \to 0} \Big(\limsup_{n \to \infty} \frac{\ln s(n, \epsilon, R)}{n} \Big),$$

where $s(n, \epsilon, R) = \max\{\mathrm{card}(E) \mid E \subset B(0, R), (\forall x \neq y \in E)(\exists k \in [0, n]) : |\phi^k(x) - \phi^k(y)| \geq \epsilon\}$.

Theorem 2, and the fact that the entropy of a Bernoulli shift is $\ln(2)$, immediately imply

Corollary 3 [Ci-S-2].
If (H1-2), (C) are true, then the time-one map ϕ_1 has a positive topological entropy.

Remark. If (C) is not satisfied, then either $W^s_{loc} \cap W^u$ or $W^u_{loc} \cap W^s$ contains a compact connected set Λ with $x_0 \in \Lambda$ and $\Lambda \neq \{x_0\}$. This set will be constructed in the proof of Lemma 3.2.

Example ($n = 1$). In the case of one-dimensional systems, Montechiarri and Nolasco [M-N] have recently proved the following alternative: either $W^u = W^s$ or the system is semi-conjugate to a Bernoulli shift. In some perturbative cases, it is possible to check that $W^u \neq W^s$, see the earlier work of Bessi [B1]. Theorem 1.1 can be considered as a generalization of [B1] and [M-N]. Indeed, let $M = S^1$ and H satisfy $(H1 - 2)$. If (C) is not satisfied, then the connected set Λ of the remark above must be an interval containing x_0. Suppose for instance that $\Lambda \subset W^u_{loc} \cap W^s$. Then Λ *is* a local unstable space, and $W^u = \bigcup_{N \geq 0} \varphi_N(\Lambda) \subset W^s$, hence $W^u = W^s$.

The presence of a Bernoulli shift near a transversal homoclinic orbit is derived from the so-called "Shadowing Lemma", which states that near an approximate solution one finds a real one.

Theorem 2 also follows from a kind of shadowing lemma. However, since the usual proof of the Shadowing Lemma relies heavily on the transversality assumption (which allows to use the contraction mapping principle), we cannot expect the Shadowing Lemma to hold in its classical form. Rather, we obtain a "Topological Shadowing Lemma", as we explain now.

Using an isometric embedding $M \subset \mathbb{R}^a$ we will talk of norms $|x|$ and differences $x - y$ for x, y in $T^*M \subset \mathbb{R}^{2a}$.

For $x : \mathbb{R} \to T^*M$ and $n \in \mathbf{Z}$ define $n * x : \mathbb{R} \to T^*M$ by

$$n * x(t) = x(t - n).$$

Let $x_i : \mathbb{R} \to T^*M$ be given for $1 \leq i \leq p$, and let $R_i > 0$ be such that $|x_i(t)|$ is smaller than the injectivity radius r_0 of M outside the interval

$[-R_i, R_i]$. If $n_i \in \mathbf{Z}$ are such that $n_{i+1} - n_i \geq R_i + R_{i+1} + 1$, then using cut-off functions one can define the "multibump" function $\sum n_i * x_i$ (see [Ci-S-1]). This function coincides with $n_i * x_i$ on $[n_i - R_i, n_i + R_i]$, and it is smaller than r_0 outside these intervals.

The main theorem in [Ci-S-2] is the following:

Theorem 4. (Topological Shadowing)
Let M, H be as above and (H1-2), (C) be satisfied. Then we find a compact set $\mathcal{C}_0 \subset \mathcal{C}\backslash\{0\}$, and for every $\epsilon > 0$, an integer $N(\epsilon) > 0$ with the following property:

If $(n_i)_{i\in I}$ is a family of integers, the index set I being either $\mathbf{Z}$, $\mathbf{N}$, $-\mathbf{N}$ or $\{1, ..., l\}$, and $n_{i+1} - n_i \geq 2N(\epsilon)$, then there exist $y_i \in \mathcal{C}_0$ and a solution y of $\dot{y} = X_H(t, y)$ such that

$$(i) \qquad |y_i(t) - x_0| \leq \epsilon \quad \textit{for } |t| \geq N(\epsilon)$$

$$(ii) \qquad |y(t) - \sum_{i\in I} n_i * y_i(t)| \leq \epsilon\,, \quad \textit{for all } t \in \mathbb{R}\,.$$

Moreover,

if $I = \pm\mathbf{N}$ then $y(t) \to x_0$ as $t \to \mp\infty$.

If $I = \{1, ..., l\}$ then y is a homoclinic orbit "with l bumps".

If $I = \mathbf{Z}$ and (n_i) is periodic, i.e. $n_{i+p} = n_i + q$ for some $p, q \in \mathbf{N}$ and all $i \in \mathbf{Z}$, then y may be chosen periodic.

In contrast to the classical versions of the shadowing lemma, we cannot prescribe precisely which y_i are to occur. This is due to the fact that in the proof we 'glue together' sets of orbits supporting certain cohomology classes, rather than individual orbits. But the structure we find is rich enough to give Theorem 1.1.

Proof of Theorem 2 as a consequence of Theorem 4.

Since $\mathcal{C}_0$ is compact and does not contain x_0, there is $\epsilon > 0$ such that $|y(0) - x_0| \geq 4\epsilon$ for all $y \in \mathcal{C}_0$. Let $T = 2N(\epsilon)$. For each $n \in \mathbf{Z}$ the map which, to each $x \in TM$, associates $\phi_{nT}(x) \in TM$, is continuous. Hence

$$A_T = \left\{x \in TM \,:\, |\phi_{nT}(x) - x_0| \in [0, 2\epsilon] \cup [3\epsilon, R]\,, \forall n \in \mathbf{Z}\right\}$$

is compact, where $R > \sup\{|y(0) - x_0| \,:\, y \in \mathcal{C}(\alpha)\} + \epsilon$.

Clearly A_T is ϕ_T-invariant. Moreover, the map $\tau : A_T \to \Sigma$ defined by

$$[\tau(x)]_n = 1 \text{ if } |\phi_{nT}(x) - x_0| \geq 3\epsilon$$
$$[\tau(x)]_n = 0 \text{ if } |\phi_{nT}(x) - x_0| \leq 2\epsilon$$

is continuous, and one readily checks that $\tau \circ \phi_T = \sigma \circ \tau$.

For the surjectivity of τ, let $(a_n) \in \Sigma$ be given. Write the set $\{n \in \mathbf{Z} : a_n = 1\}$ in increasing order as $(k_i)_{i\in I}$, where I is chosen as in Theorem 1.3 according to whether $\{n \in \mathbf{Z} : a_n = 1\}$ is unbounded, bounded from below and/or from above (the choice of (k_i) is not unique in the case of a doubly infinite sequence).

By the choice of T, the sequence $(k_iT)_{i\in I}$ satisfies the hypothesis of Theorem 4, with ϵ as above. Hence there exists a solution y and $y_i \in \mathcal{C}_0$ such that

$$|y(t) - \sum_{i\in I}(k_iT) * y_i| \leq \epsilon\,, \quad \forall t \in \mathbb{R}\,.$$

Let $x = y(0)$.

If $n \in \mathbf{Z}$ with $a_n = 1$, we have

$$\begin{aligned} \phi_{nT}(x) &= |y(nT) - x_0| \\ &\geq |\sum_{i\in I}(k_iT) * y_i(nT) - x_0| - \epsilon \\ &= |y_j(0) - x_0| - \epsilon \\ &\geq 3\epsilon\,, \end{aligned}$$

where $k_j = n$. Hence $[\tau(x)]_n = 1 = a_n$. Similarly $[\tau(x)]_n = 0$ if $a_n = 0$. This proves the equality $\tau(x) = (a_n)_{n\in\mathbf{Z}}$. $\square$

There has been a lot of works in the recent years on variational gluing of homoclinic orbits (see e.g. [S1-2], [CZ-R], [Be], [Gi-R], [B-S]). In these works, the Hamiltonian presents some convexity, so that a first homoclinic can be found by mountain-pass or category theory. Then a gluing method introduced in [S1] is used, to get an analogue of Theorem 1.3. The paper [Ci-S-2] is the first one where a nonconvex Hamiltonian is considered.

2. Sketch of the proof of Theorem 4.

From now on, we fix a Riemannian metric on M. It induces an isomorphism $TM \simeq T^*M$ which allows us to transfer all structures such as ω, H, θ to TM. Without changing notation we shall henceforth work with TM instead of T^*M. We call M_0 the zero section of TM.

Theorem 4 is proved by a refinement of the method used in [Ci-S-1]. It is based on the variational principle for the action functional

$$I \,:\, H^{1,2}(\mathbb{R}, TM) \to \mathbb{R}$$
$$I(x) := \int_{\mathbb{R}} x^*(i_\eta \omega) - \int_{\mathbb{R}} H(t, x(t))dt\,.$$

Here we have used an isometric embedding $TM \hookrightarrow \mathbb{R}^{2a}$ mapping x_0 to 0 to define Sobolev classes $H^{m,p}$ for mappings into TM. So $x \in H^{1,2}(\mathbb{R}, TM)$ implies in particular that $x(t) \to x_0$ as $t \to \pm\infty$.

The critical points of I are exactly the elements of $\mathcal{C}$. Let $I'(x)$ be the L^2-gradient of I defined by

$$dI(x)\,\xi \;=\; \langle I'(x), \xi\rangle_{L^2} \quad for\ all\ \xi \in H^{1,2}(\mathbb{R}, x^*TM)\,.$$

With the help of the almost complex structure J on TM defined by

$$\omega(J\cdot,\cdot) = <\cdot,\cdot>\,,$$

we can write $I'(x)$ explicitly as

$$I'(x) = -J(x)\dot{x} - H'(t,x)\,.$$

Hence the equation of gradient lines $u : \mathbb{R}^2 \to TM$, $u_s = I'(u(s))$, becomes the inhomogeneous nonlinear Cauchy-Riemann equation

$$\bar{\partial}u + H'(t,u) \equiv u_s(s,t) + J(u(s,t))u_t(s,t) + H'(t, u(s,t)) = 0\,.$$

Now fix a $T > 1$ and define

$$\Omega_T^\infty := \{\bar{q} \in C^\infty(\mathbb{R}, M) \mid \bar{q}(t) \equiv q_0 \ for\ |t| \geq T\}\,,$$

equipped with the C^∞-topology. For $n = (n_1, \ldots, n_p) \in \mathbf{Z}^p$, $n_{i+1} - n_i \geq 2T$, let

$$n * \Omega_T^\infty := \{\sum_{i=1}^{p} n_i * q_i \mid q_i \in \Omega_T\}\,.$$

For $R > 1$ and $\bar{q} \in n * \Omega_T^\infty$ we study the boundary value problem

$$X_{\bar{q},R} := \left\{ u \in H^{2,2}([-R,R] \times \mathbb{R}, TM) \;\middle|\; u(-R,t) \in M_0 \,, \right.$$
$$\left. u(R,t) \in T_{\bar{q}(t)}M \text{ and } \bar{\partial} u + H'(t,u) = 0 \right\} .$$

The elements of $X_{\bar{q},R}$ should be viewed as finite gradient lines connecting the space of curves in the zero section M_0 to the space of curves over $\bar{q}$. Note that the zero section and the fibres are transversal Lagrangian submanifolds of (TM, ω).

In [Ci-S-1] it has been shown that $X_{\bar{q},R}$ is nonempty for every $\bar{q}$ and R. The crucial point now is to find elements $u \in X_{\bar{q},R}$ which have the same 'multibump shape' as $\bar{q}$ in the sense that

$$|u(s, \frac{n_i + n_{i+1}}{2}) - x_0| \leq \epsilon_0 \quad \text{for all } s \in [-R, R] \text{ and } 1 \leq i < p.$$

To have this estimate independent of R, we use assumption (C) . Then we can let $R \to \infty$ to obtain a space X_∞ of infinite gradient lines of 'multibump shape'. Their asymptotics for $s \to \infty$ are multibump homoclinic orbits of x_0. Finally, from the fact that X_∞ carries a nontrivial product cohomology, we conclude that there must be at least one homoclinic orbit with all bumps nontrivial.

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