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Weak topologies, and determining covers

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We assume that spaces are regular T_1, and maps are continuous and onto.

For a cover $\mathcal{P}$ of a space X, X *is determined by* $\mathcal{P}$ [6], if X has the *weak topology* with respect to $\mathcal{P}$ [3]; that is, $G \subset X$ is open in X if $G \cap P$ is open in P for each $P \in \mathcal{P}$. Here, we can replace "open" by "closed". We call such a cover $\mathcal{P}$ a **determining cover** in [20].

We recall that a space X is respectively a *sequential space* [4]; *k-space*; *quasi-k-space* [11] if X has a determining cover by (compact) metric subsets; compact subsets; countably compact subsets. Sequential spaces are k-spaces, and k-spaces are quasi-k-spaces.

As is well-known, every sequential space; k-space; quasi-k-space is respectively characterized as a quotient space of a (locally compact) metric space; locally compact (paracompact) space; M-space.

Let $\mathcal{P}$ be a collection of subsets of a space X. Then, $\mathcal{P}$ is *closure-preserving* (abbreviated by CP), if for any subfamily $\mathcal{P}'$ of $\mathcal{P}$, $cl(\bigcup\{P : P \in \mathcal{P}'\}) = \bigcup\{clP : P \in \mathcal{P}'\}$. Also, $\mathcal{P}$ is *hereditarily closure-preserving* (abbreviated by HCP), if for any subcollection $\mathcal{P}' = \{P_\alpha : \alpha\}$ of $\mathcal{P}$, and any $\{A_\alpha : \alpha\}$ such that $A_\alpha \subset P_\alpha$, the collection $\{A_\alpha : \alpha\}$ is CP.

For a closed cover $\mathcal{F}$ of a space X, X *is dominated by* $\mathcal{F}$ [7] if $\mathcal{F}$ is a CP cover, and any $\mathcal{P} \subset \mathcal{F}$ is a determining cover of the union of $\mathcal{P}$. (Sometimes, we also say that X has the *Whitehead weak* topology; *Morita weak topology* (in the sense of [9]); or *hereditarily weak topology*, with respect to $\mathcal{F}$). We call such a closed cover $\mathcal{F}$ a **dominating cover** in [20].

A space X having an increasing determining cover $\{X_n : n \in N\}$ is called the *inductive limit* of $\{X_n : n \in N\}$. When the X_n are closed in X, $\{X_n : n \in N\}$ is a dominating cover of X. Also, every CW-complex has a dominating cover by compact metric subsets.

Open covers $\Rightarrow$ Determining covers $\Leftarrow$ Dominating covers $\Leftarrow$ HCP closed covers $\Leftarrow$ Locally finite closed covers.

Eery space having a determining cover by sequential spaces (resp. k-spaces; quasi-k-spaces) is a sequential space (resp. k-space; quasi-k-space).

While, every space having a dominating cover by paracompact spaces (resp. normal spaces) is paracompact (resp. normal); see [7] or [10].

Concerning "*preservations*" of weak topologies, we have the following natural questions (Q1), (Q2) and (Q3), and the same questions which are replaced "determining" by "dominating".

(Q1) Let $f : X \to Y$ be a map, and let $\mathcal{P}$ be a determining cover of X (resp. Y). Under what conditions, is $f(\mathcal{P}) = \{f(P) : P \in \mathcal{P}\}$ (resp. $f^{-1}(\mathcal{P}) = \{f^{-1}(P) : P \in \mathcal{P}\}$) a determining cover of Y (resp. X) ?

(Q2) Let $\mathcal{P}$ be a determining cover of a space X. For a (or any) subset $S \subset X$, under what conditions, is $\mathcal{P}|S = \{P \cap S : P \in \mathcal{P}\}$ a determining cover of S ?

(Q3): For each i = 1, 2, let $\mathcal{P}_i$ be a determining cover of a space X_i. Under what conditions, is $\mathcal{P}_1 \times \mathcal{P}_2 = \{P_1 \times P_2 : P_i \in \mathcal{P}_i\}$ a determining cover of $X_1 \times X_2$?

In [20], we gave some related answers to the question (Q3) (containing countable products of weak topologies), and their applications to products of paracompact spaces. For products of weak topologies (determining covers), see [19]. In this paper, let us give some related answers to the questions (Q1) and (Q2) in Section 1 and 2, respectively. Related to (Q3), we also give some results on countable products of spaces having certain determing covers in Section 3, containing additional matters to [20].

We recall some elementary facts which will be used in this paper. For basic matters on weak topologies, see [17] or [18], for example.

Fact A: (1) Let $\mathcal{C}$ be a determining cover of X. Let $\mathcal{P}$ be a cover of X. If each element of $\mathcal{C}$ is contained in some element of $\mathcal{P}$, then $\mathcal{P}$ is a determining cover of X.

(2) Let $\{P_\alpha : \alpha\}$ be a determining cover of X. If each P_α has a determining cover $\mathcal{P}_\alpha$, then $\bigcup\{\mathcal{P}_\alpha : \alpha\}$ is a determining cover of X.

(3) Let $\mathcal{P}$ be a determining cover of X. If S is a closed or open subset of X, then $\mathcal{P}|S$ is a determining cover of S.

(4) For a determining cover $\mathcal{P}$ of a space X^ω, $\mathcal{P}_1 \times \mathcal{P}_2 \times \cdots$ is a determining cover of X^ω, where $\mathcal{P}_i = P_i(\mathcal{P})$ for the projection P_i from X^ω onto the i-th coordinate space X.

A cover $\mathcal{P}$ of X is *point-countable* if every $x \in X$ is in at most countably many $P \in \mathcal{P}$. A decreasing sequence (A_n) of non-empty subsets of X is a q-

sequence (resp. *k-sequence*) [8], if $C = \bigcap\{A_n : n \in N\}$ is countably compact (resp. compact) in X, and each open subset U with $C \subset U$ contains some A_n (equivalently, for any $x_n \in A_n$, $\{x_n : n \in N\}$ has an accumulation point in C).

Fact B: (1) Let $\mathcal{P}$ be a point-countable determining cover of X. Then, for each q-sequence (A_n) in X, some A_n is contained in a finite union of elements of $\mathcal{P}$ ([14, Lemma 6]).

(2) Let $\mathcal{F} = \{X_\alpha : \alpha \leq \gamma\}$ be a dominating cover of X. For each $\alpha \leq \gamma$, let $L_\alpha = X_\alpha - \bigcup\{X_\beta : \beta < \alpha\}$. Then $\{clL_\alpha : \alpha \leq \gamma\}$ is a determining cover of X such that, for each q-sequence (A_n) in X, some A_n meets only finitely many L_α (cf. [16, Lemma 2.5]).

1. Maps

Example 1.1. (1) An open map $f : X \to Y$ with each $f^{-1}(y)$ at most two points, and X has a discrete, closed and open cover $\mathcal{F}$ by compact subsets, but $f(\mathcal{F})$ is not a CP cover (hence, not a dominating cover).

(2) An open map $g : X \to Y$ with each $g^{-1}(y)$ at most two points, and Y has a countable determining cover $\mathcal{F}$ by convergent sequences (or, a dominating cover by compact metric subsets), but $g^{-1}(\mathcal{F})$ is not a determining cover of X.

Theorem 1.2. (1) Let $f : X \to Y$ be a quotient map. If $\mathcal{P}$ is a determining cover of X, as is well-known, $f(\mathcal{P})$ is a determining cover of Y.

(2) Let $f : X \to Y$ be a closed map. Then the following hold.

(a) If $\mathcal{F}$ is a dominating cover of X, $f(\mathcal{F})$ is a dominating cover of Y.

(b) If $\mathcal{P}$ is a determining (resp. dominating) cover of Y, $f^{-1}(\mathcal{P})$ is a determining (resp. dominating) cover of X ([13, Lemma 1.2]).

Corollary 1.3. Let $f : X \to Y$ be a closed map such that each $f^{-1}(y)$ is compact (resp. countably compact; first countable). Then X is a k-space ([1]) (resp. quasi-k-space; sequential space ([13])) if (and only if) Y is so respectively.

Corollary 1.4. Let $f : X \to Y$ be a map. Then the following hold.

(1) Let X be a k-space. If $\mathcal{P}$ is a determining cover of Y, then $f^{-1}(\mathcal{P})$ is a determining cover of X.

(2) Let X be a quasi-k-space. If $\mathcal{F}$ is a dominating (resp. point-countable closed) cover of Y, then $f^{-1}(\mathcal{F})$ is a dominating (resp. point-countable closed) cover of X.

The author doesn't know whether the above (1) remains true under X being a quasi-k-space. This is affiramative if any countably compact subset of Y is closed (as Y is a sequential space, or a space whose points are G_δ-sets, for example).

2. Subsets

For an open (resp. HCP closed) cover $\mathcal{P}$ of X, $\mathcal{P}|S$ is a determining (resp. dominating) cover of S for any $S \subset X$. But, we have the following example. Here, the *Arens' space* S_2 is the space obtained from the disjoint union $\Sigma\{L_n : n = 0, 1, \cdots\}$ of the convergent sequence $\{1/n : n \in N\} \cup \{0\}$ by identifying each $1/n \in L_0$ with $0 \in L_n$ $(n \geq 1)$. The quotient space S_2/L_0 is called the *sequential fan* which is denoted by S_ω.

Example 2.1. The Arens' space S_2 has the obvious increasing and dominating countable cover $\mathcal{F}$ by compact metric subsets, but for $S = S_2 - \{1/n \in L_0 : n \in N\}$, $\mathcal{F}|S$ is not a determining cover of S.

A space is *Fréchet*, if whenever $a \in clA$, then there exists a sequence in A converging to the point a. We recall that a space X is a *k'-space* [1], if whenever $a \in clA$, then there exists a compact subset K of X such that $a \in cl(A \cap K)$. Let us recall other related spaces due to [8]. A space X is a *countably bi-quasi-k-space* if, whenever $x \in clA_n$ with $A_{n+1} \subset A_n$ $(n \in N)$, there exists a q-sequence (B_n) such that $x \in cl(A_n \cap B_n)$ for all $n \in N$. If the A_n are all the same set, then such a space is a *singly bi-quasi-k-space.* Féchet spaces and locally compact spaces are k'-spaces. k'-spaces, and M-spaces (generally, countably bi-quasi-k-spaces) are singly bi-quasi-k-spaces. Singly bi-quasi-k-spaces are quasi-k-spaces. For properties related to dominating or point-countable determining covers among singly bi-k-spaces (or, singly bi-quasi-k-spaces), see [15] or [21].

Theorem 2.2. (1) Let $\mathcal{P}$ be a determining cover of X. For $S \subset X$, $\mathcal{P}|S$ is a determining cover of S if S has a determining cover by open or closed sets in X, in particular, S is a k-space. When $\mathcal{P}$ is point-countable and closed, the same result holds if S is a quasi-k-space.

(2) Let $\mathcal{F}$ be a dominating cover of X. For $S \subset X$, $\mathcal{F}|S$ is a dominating cover if S has a determining cover by open or closed subsets in X, or S is a quasi-k-space.

Corollary 2.3. Let $\mathcal{F}$ be a dominating cover of X by Fréchet spaces. For $S \subset X$, the following are equivalent.

(a) S has a dominating cover $\mathcal{F}|S$.

(b) S has a determining cover $\mathcal{F}|S$.

(c) S is a quasi-k-space.

(d) S is a sequential space.

Theorem 2.4. (1) Let $\mathcal{P}$ be a cover of X. Then the following are equivalent.

(a) For any $S \subset X$, $\mathcal{P}|S$ is a determining cover of S.

(b) For any $A \subset X$ and any $a \in clA$, there exists $P \in \mathcal{P}$ such that $a \in cl_P(A \cap P)$.

(2) Let $\mathcal{F}$ be a closed cover of X. Then the following are equivalent.

(a) For any $S \subset X$, $\mathcal{F}|S$ is a dominating cover of S.

(b) For any $S \subset X$, $\mathcal{F}|S$ is CP in X.

Corollary 2.5. Let X be a singly bi-quasi-k-space, and let $\mathcal{F}$ be a dominating (resp. point-countable determining closed) cover of X. Then, for any $S \subset X$, $\mathcal{F}|S$ is a dominating (resp. determining) cover of S.

Corollary 2.6. (1) For a space X, the following are equivalent.

(a) X is Fréchet.

(b) X has a determining cover $\mathcal{P}$ by compact metric subsets such that for any $S \subset X$, $\mathcal{P}|S$ is a determining cover of S.

(c) X is a sequential space, and for any *determining* cover $\mathcal{P}$ of X and any $S \subset X$, $\mathcal{P}|S$ is a determining cover of S.

(2) For a space X, the following are equivalent.

(a) X is a k'-space.

(b) X has a determining cover $\mathcal{P}$ by compact subsets such that for any $S \subset X$, $\mathcal{P}|S$ is a determining cover of S.

Corollary 2.7. Let X be a sequential space. If any subset of X is a quasi-k-space, then X is Fréchet (cf. [5]).

Corollary 2.8. (1) Let X have a determining cover $\mathcal{P}$ by Fréchet spaces. Then the following are equivalent.

(a) X is Fréchet.

(b) For any $S \subset X$, $\mathcal{P}|S$ is a determining cover of S.

(2) Let X have a *dominating* (or point-countable determining closed) cover $\mathcal{F}$ by k'-spaces. Then the following are equivalent.

(a) X is a k'-space.

(b) For any $S \subset X$, $\mathcal{F}|S$ is a determing cover of S.

Remark 2.9. Not every compact sequential space is Fréchet (the space Ψ^* in [5, Example 7.1], for example). Thus, in (c) $\Rightarrow$ (a) of Corollary 2.6(1), we can't replace "*determining*" by "dominating". While, under X being a k-space, (c) implies X is a k'-space, but the converse need not hold even if

X is compact sequential. Also, in (a) $\Rightarrow$ (b) of Corollary 2.8(2), we can't replace "*dominating*" by "determining".

Question 2.10. (1) Let $\mathcal{P}$ be a determining cover of X. Let $S \subset X$, and S be a quasi-k-space. Is $\mathcal{P}|S$ a determining cover of S ?

(2) Let $\mathcal{F}$ be a dominating cover of X. For any $S \subset X$, let $\mathcal{F}|S$ be a determining cover of S. Is $\mathcal{F}|S$ a dominating cover of S ?

(3) Let X be a k-space. For any determining cover $\mathcal{P}$ of X, and any $S \subset X$, let $\mathcal{P}|S$ be a determining cover of S. Is X Fréchet ?

3. Countable products

In this section, we consider countable products of weak topologies, as additional matters to Section 4 in [20]. For finite products of weak topologies in terms of Question 3, see [19] or [20]. First, let us give the following notations.

For a cover $\mathcal{P}$ of a space, let $[\mathcal{P}] = \{A : A \text{ is a finite union of elements of } \mathcal{P}\}$, $\mathcal{P}^* = \{P \cup F : P \in \mathcal{P}, F \text{ is finite}\}$, and let $\mathcal{P}^\circ = \{intP : P \in \mathcal{P}\}$.

Remark 3.1. (1) For a space $X = F_1 + F_2$, $\mathcal{F} = \{F_1,\ F_2\}$ is a determining cover of X, but $\mathcal{F}^\omega (= \mathcal{F} \times \mathcal{F} \times \cdots)$ is not a determining cover of X^ω.

(2) Let X be the sequential fan S_ω (or the Arens' space S_2). Then, for any (countable) determining closed cover $\mathcal{F}$ by (compact) metric subsets in X, $[\mathcal{F}]^\omega$ is not a determining cover of X^ω by means of Theorem 3.2(2) below.

As a generalization of sequential spaces, we recall that a space X has *countable tightness*, $t(X) \leq \omega$, if whenever $a \in clA$, $a \in clC$ for some countable $C \subset A$ (equivalently, X has a determining cover by countable subsets); see [8]. While, as a generalization of countably bi-quasi-k-spaces, let us consider the following property (P), refering to [6, (3.1)].

(P): For each decreasing sequence (A_n) in X with $\bigcap\{clA_n : n \in N\} \neq \emptyset$, there exists a countably compact set K of X with $K \cap A_n \neq \emptyset$ for all $n \in N$.

Theorem 3.2. (1) Let X^ω be a sequential space. Let $\mathcal{P}$ be a determining cover of X. Then $\mathcal{P}^{*\omega}$ (hence, $[\mathcal{P}]^\omega$) is a determining cover of X^ω ([13]).

(2) Let X^ω be a quasi-k-space. Let $\mathcal{P}$ be a dominating or point-countable determining cover of X. Then the following hold.

(a) $[\mathcal{P}]^\omega$ is a determining cover of X^ω.

(b) If $t(X) \leq \omega$, then X has property (P), hence $[\mathcal{P}]^{\circ\omega}$ is a determining cover of X^ω.

A space X is a *bi-k-space* [8] if, whenever $\mathcal{A}$ is a filterbase with $x \in cl A$ for every $A \in \mathcal{A}$, there exists a k-sequence (B_n) in X such that $x \in cl(A \cap B_n)$ for all $A \in \mathcal{A}$ and $n \in N$. Locally compact spaces, first countable spaces, or paracompact M-spaces are bi-k-spaces. Bi-k-spaces are k-spaces which are countably bi-quasi-k. Every countable product of bi-k-spaces is a bi-k-space ([8]), hence a k-space.

Corollary 3.3. Let X be a bi-k-space, and let $\mathcal{P}$ be a determing cover of X. Then the following hold.

(a) $[\mathcal{P}]^\omega$ is a determing cover of X^ω if X is sequential, or $\mathcal{P}$ is a point-countable cover.

(b) $[\mathcal{P}]^{\circ\omega}$ is a determining cover of X^ω if $\mathcal{P}$ is a dominating cover, a point-countable closed cover, or a point-countable cover with $t(X) \leq \omega$.

Corollary 3.4. Let X have a dominating or point-countable determining closed cover $\mathcal{F}$ by first countable spaces. Then the following properties are equivalent ([20]).

(a) X^ω is a quasi-k-space.

(b) X^ω is a sequential space.

(c) $\mathcal{F}^{*\omega}$ is a determining cover of X^ω.

(d) $[\mathcal{F}]^\omega$ (actually, $[\mathcal{F}]^{\circ\omega}$) is a determining cover of X^ω.

(e) $[\mathcal{F}]^\circ$ is an open cover of X.

(f) X is first countable.

Corollary 3.5. Let X satisfy (a), (b), or (c) below. If X^ω is a quasi-k-space, then X is metric.

(a) X has a dominating cover by metric spaces.

(b) X is a paracompact space having a point-countable determining closed cover by metric spaces.

(c) X has a point-countable determining cover by locally separable, metric spaces.

Corollary 3.6. Let X have a dominating or point-countable determining closed cover $\mathcal{F}$ by locally compact spaces (resp. bi-k-spaces). Then the implications (a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (c), and (d) $\Leftrightarrow$ (e) $\Rightarrow$ (b) hold. When $t(X) \leq \omega$, (a) $\sim$ (e) are equivalent.

(a) X^ω is a quasi-k-space.

(b) X^ω is a k-space.

(c) $[\mathcal{F}]^\omega$ is a determining cover of X^ω.

(d) $[\mathcal{F}]^\circ$ is an open cover of X.

(e) X is a locally compact space (resp. bi-k-space).

Remark 3.7. (CH). "$t(X) \leq \omega$" is essential in Corollary 3.6 (the implication (b) $\Rightarrow$ (d) or (e)), and so is in Theorem 3.2(2). Actually, under (CH), there exists a space X having a countable dominating cover $\mathcal{F}$ by compact subsets, and X^ω is a k-space, but X is not locally compact ([2]) (hence, $[\mathcal{F}]^\omega$ is a determing cover of X^ω, but $[\mathcal{F}]^\circ$ is not an open cover of X, and X doesn't have property (P)).

Finally, let us give questions on products of weak topologies. First, let us review some related mattes.

Remark 3.8. (1) Let X be a sequential space (resp. paracompact space). Then X^ω is a sequential space (resp. k-space) iff X is a quasi-k-space (see [12] for the finite products).

(2) Let $\mathcal{P}$ be a determining cover of X. Then (a) and (b) below hold.

(a) Let X^2 be a k-space. Then $\mathcal{P}^2$ is a determining cover of X^2 if X is a sequential space, or each element of $\mathcal{P}$ is a k-space (see [19] or [20]), in particular, $\mathcal{P}$ is a closed cover.

(b) Let X^ω be a k-space. Then $[\mathcal{P}]^\omega$ is a determining cover of X^ω if $\mathcal{P}$ is a dominating or point-countable cover, or X is sequential (for example, the elements of $\mathcal{P}$ are sequential).

In view of Remark 3.8, the author has Question 3.9 below, in particular. For (1), $X^2 \in [\mathcal{P}]^2$. Also, the compactness of X is essential even if $\mathcal{P}$ is a countable HCP closed cover by separable metric subsets. If (1) is affirmative, then so is the question for X being a bi-k-space (generally, space with X^ω a k-space). For (2), any $\mathcal{F}^n$ ($n \in N$) is a determing cover of X^n. (3) is affirmative if X is sequential, or $\mathcal{P}$ is dominating or point-countable. If X is paracompact, then any $\mathcal{F}^n$ ($n \in N$) is a determing cover of X^n.

Question 3.9. (1) Let X be a compact space, and let $\mathcal{P}$ be a countable determining cover of X. Is $\mathcal{P}^2$ a determining cover of X^2 ?

(2) Let X be a compact space (or space with X^ω a k-space), and let $\mathcal{F}$ be a determining closed cover of X. Is $[\mathcal{F}]^\omega$ a determing cover of X^ω ?

(3) Let $\mathcal{F}$ be a determining cover of X by compact subsets. Let X^ω be a quasi-k-space (in particular, let X be a countably compact space). Is $[\mathcal{F}]^\omega$ a determining cover of X^ω ?

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