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# Properties of global monopoles with an event horizon

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We investigate the properties of global monopoles with an event horizon. We find that there is an unstable circular orbit even if a particle does not have an angular momentum when the core mass is negative. We also obtain the asymptotic form of solutions when the event horizon is much larger than the core radius of the monopole, and discuss if they could be a model of galactic halos.

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## I. INTRODUCTION

In unified theories, various kinds of topological defects have been predicted and may appear in cosmological phase transitions in the early Universe. Therefore it is important to investigate such defects both theoretically and observationally. Concerning global monopoles, important facts were found recently: (i) Solutions with an event horizon exist [1–3]; (ii) regular global monopoles coupled nonminimally to gravity have a stable circular orbit and may explain rotation curves in spiral galaxies, as shown by Nucamendi, Salgado, and Sudarsky (NSS) [4].

The result (i) is interesting since the static solutions of regular global monopoles are always repulsive [5]. Moreover, black hole solutions are stable though topological charge is lost in the strict sense [6] and there are solutions with zero mass which are somewhat pathological [2]. Thus, we need to understand their properties. In particular, it would be interesting to investigate a particle motion around the horizon. This is one of our main concerns in this paper.

Possibility (ii) shows that global monopoles can be locally attractive in the nonminimally coupled theory of gravity. Although there have been many attempts to explain rotation curves in spiral galaxies, there is no definite one at present. Among solitonic objects, global monopoles have the remarkable property that energy density decreases with the distance  $r^{-2}$  [7], which may be desirable to explain the flatness of rotation curves. To remove the unpreferable repulsive property of global monopoles, NSS introduced nonminimal coupling and succeeded to obtain locally attractive solutions.

Taking both (i) and (ii) into consideration, we notice a possibility that global monopoles with an event horizon can explain rotation curves since they would be attractive. There are several advantages in this model compared with the NSS model. First, we need not require the nonminimal coupling, which are constrained astrophysically [8]. Second, they would also be model black holes in the central galaxies.

Third, the core mass can be chosen to be astronomically large, contrary to the NSS model, where the core mass is necessarily microscopic. Therefore it is important to study the properties of such global monopoles, and discuss whether or not they can be a realistic candidate as galactic halos, taking astrophysical bounds into account [9].

This paper is written as follows. In Sec. II, we explain our model and basic equations. In Sec. III, we investigate global monopoles with an event horizon in two situations separately. In Sec. III A, we consider the case where the size of event horizon is comparable to the core radius of the monopole to compare with regular monopoles. In Sec. III B, we consider the case where the size of an event horizon is astrophysically large. In Sec. IV, we denote concluding remarks and discuss problems concerning the restriction from observation.

## II. BASIC EQUATIONS FOR NUMERICAL ANALYSIS

We begin with the action

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{(\nabla\Phi^a)^2}{2} - \frac{\lambda}{4} (\Phi^a\Phi^a - v^2)^2 \right], \quad (1)$$

where  $G$  and  $\Phi^a$  are the gravitational constant and the real triplet Higgs field, respectively. The theoretical parameters  $v$  and  $\lambda$  are the vacuum expectation value and the self-coupling constant of the Higgs field, respectively.

We assume that the space time is static and spherically symmetric, in which the metric is written as

$$ds^2 = -f(r)e^{-2\delta(r)}dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2, \quad (2)$$

where  $f(r) := 1 - 2Gm(r)/r$ . We adopt the hedgehog ansatz given by

$$\Phi^a = vr^a h(r), \quad (3)$$

where  $r^a$  is a unit radial vector.

Under the above assumptions, the basic equations are

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$$\bar{m}' = \bar{r}^2 \bar{v}^2 \left[ \frac{f}{2} (h')^2 + U \right], \quad (4)$$

$$\delta' = -\bar{v}^2 \bar{r} (h')^2, \quad (5)$$

$$h'' = -\frac{h'}{\bar{r}} + \frac{1}{f} \left[ h(h^2 - 1) + \frac{2h}{\bar{r}^2} + 2\bar{r}\bar{v}^2 h' U - \frac{h'}{\bar{r}} \right], \quad (6)$$

where  $' = d/d\bar{r}$  and

$$U = \frac{(h^2 - 1)^2}{4} + \frac{h^2}{\bar{r}^2}. \quad (7)$$

We have introduced the following dimensionless variables:

$$\bar{r} = v \sqrt{\lambda} r, \quad \bar{m} = G v \sqrt{\lambda} m, \quad \bar{v} = v \sqrt{4\pi G}. \quad (8)$$

We assume the regular event horizon at  $r = r_H$ :

$$\bar{m}_H = \frac{\bar{r}_H}{2}, \quad \delta_H < \infty, \quad (9)$$

$$h'_H = \frac{h_H [2 + \bar{r}_H^2 (h_H^2 - 1)]}{\bar{r}_H (1 - 2\bar{r}_H^2 \bar{v}^2 U_H)}. \quad (10)$$

The variables with subscript  $H$  are evaluated at the horizon.

We introduce the variable

$$\bar{m}_{\text{core}}(\bar{r}) := \bar{m}(\bar{r}) - \bar{v}^2 \bar{r} \quad (11)$$

and assume the boundary conditions at spatial infinity as

$$\bar{m}_{\text{core}}(\infty) =: \bar{M} = \text{const}, \quad \delta(\infty) = 0, \quad h(\infty) = 1, \quad (12)$$

which means that the space time is asymptotically “flat” with deficit angle. Here  $\bar{M}$  corresponds to a core mass of the monopole, which determines a particle motion we will see below. We will obtain the black hole solutions numerically by solving Eqs. (4)–(6) iteratively with the boundary conditions (9), (10), and (12). For our numerical calculation, we use the double-precision Bulirsch-Stoer Method based on Ref. [10].

### III. PROPERTIES

The space-time structure of global monopole black holes depends on the expectation value of the Higgs field [1]. For  $\bar{v}^2 < 1/2$ , there is a solution with asymptotically “flat” space time. We concentrate on this realistic case.

#### A. Small horizon

Typically, it is supposed that a global monopole has a nontrivial structure in the core  $r \lesssim r_{\text{core}} := 2/v \sqrt{\lambda}$  [5], while the field is almost constant,  $h \cong 1$ , outside the core  $r$

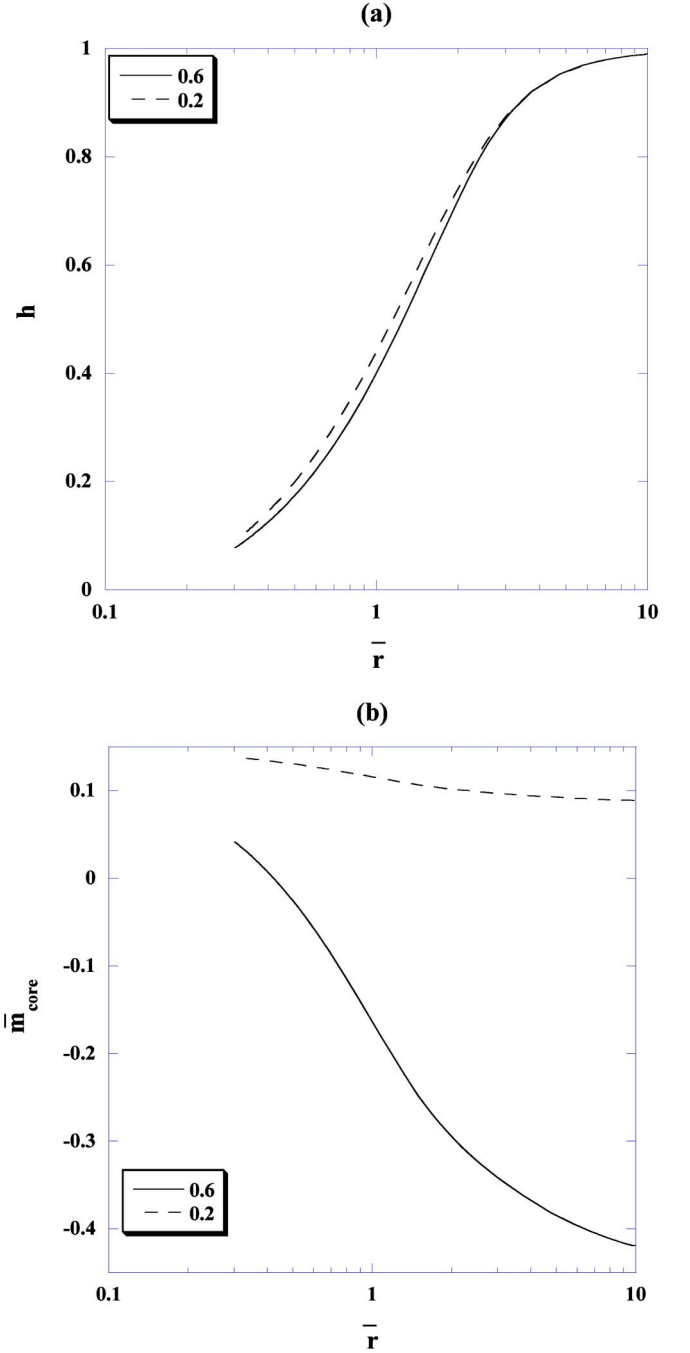


FIG. 1. The behavior of (a)  $h$ , (b)  $\bar{m}_{\text{core}}$  for  $\bar{r}_H = 0.3$  and  $\bar{v} = 0.6, 0.2$ .

$\geq r_{\text{core}}$ . Therefore, we expect that a monopole black hole with a small horizon  $r_H \cong r_{\text{core}}$  (i.e.,  $\bar{r}_H \cong 1$ ) may have new properties.

First, we show field distributions of black hole solutions in Figs. 1(a)  $\bar{r}$ - $h$  and 1(b)  $\bar{r}$ - $\bar{m}_{\text{core}}$ . We choose  $\bar{v} = 0.6, 0.2$  and  $\bar{r}_H = 0.3$ . In Fig. 1(a), we find that the Higgs field has a nontrivial structure extending to  $\sim 10$  and it does not depend on the expectation value of the Higgs field, which is important in the later analysis. In this solution,  $h$  increases mono-

tonically with  $\bar{r}$ . Although solutions where  $h$  is not monotonic exist [3], here we only consider monotonic solutions for simplicity. Figure 1(b) shows that  $\bar{m}_{\text{core}}$  for  $\bar{v}=0.6$  decreases with  $\bar{r}$  much faster than that for  $\bar{v}=0.2$ . This is natural because of the factor  $\bar{v}^2\bar{r}$  in Eq. (11). The important point is that  $\bar{m}_{\text{core}}$  becomes negative for  $\bar{v}=0.6$  and positive for  $\bar{v}=0.2$  in the asymptotic region. As we will discuss below, the sign of  $\bar{m}_{\text{core}}$  at large  $r$  (i.e.,  $\bar{M}$ ) determines the qualitative behavior of particle motions around the monopole.

Let us consider the geodesic equation of a test particle in the equatorial plane. In our coordinate system (2), this is expressed as

$$\frac{E^2}{2} = \frac{\dot{r}^2}{2} e^{-2\delta} + V_{\text{eff}}, \quad (13)$$

where  $\dot{\phantom{r}} = d/dt$  and  $E$  is the energy of the particle per unit rest mass.  $V_{\text{eff}}$  is defined as

$$V_{\text{eff}} := e^{-2\delta} \left( 1 - \frac{2Gm}{r} \right) \left( 1 + \frac{L^2}{r^2} \right), \quad (14)$$

where  $L$  is the angular momentum of the particle per unit rest mass.

We show the effective potential  $V_{\text{eff}}$  for  $\bar{r}_H=0.3$  in Figs. 2(a)  $\bar{v}=0.2$  and 2(b)  $\bar{v}=0.6$ . The angular momentum is chosen as  $(L/r_H)^2=0$  and 5. For  $(L/r_H)^2=5$ , there is no potential minimum for  $\bar{v}=0.6$ , while there is for  $\bar{v}=0.2$ . For  $(L/r_H)^2=0$ , there is a potential *maximum* for  $\bar{v}=0.6$ , while there is not for  $\bar{v}=0.2$ . While the properties of the monopole black hole with  $\bar{v}=0.2$  are essentially the same as those of the Schwarzschild black hole, larger  $\bar{v}$  changes properties qualitatively. These different properties are determined by the sign of  $\bar{M}$ .

To evaluate the potential minimum, we substitute the asymptotic form  $\delta \rightarrow 0$  and  $\bar{m} \rightarrow \bar{v}^2\bar{r} + \bar{M}$  into Eq. (14). Then,  $dV_{\text{eff}}/d\bar{r}=0$  is satisfied at the positions

$$\bar{r}_{\pm} = \frac{\bar{L}^2(1-2\bar{v}^2) \pm \sqrt{\bar{L}^4(1-2\bar{v}^2)^2 - 12\bar{L}^2\bar{M}^2}}{2\bar{M}}, \quad (15)$$

where  $\bar{L}$  is defined as  $\bar{L} := v\sqrt{\lambda}L$ . We find  $\bar{r}_{\pm} < 0$  when  $\bar{L}^2(1-2\bar{v}^2)^2 > 12\bar{M}^2$  and  $\bar{M} < 0$ . Therefore, there is no potential minimum even if a test particle has large angular momentum.

On the other hand, the potential maximum cannot be evaluated from the asymptotic form of the solution because it is near the horizon if it exists, as shown by Fig. 2. However, we can discuss its existence as follows. If  $\bar{M} < 0$ ,  $V_{\text{eff}}$  decreases with  $\bar{r}$  and approaches  $1-2\bar{v}^2$  asymptotically, since  $V_{\text{eff}}=0$  at the horizon, there is at least one local maximum at some  $\bar{r}$ . If  $\bar{M} > 0$ , on the other hand,  $V_{\text{eff}}$  increases with  $\bar{r}$

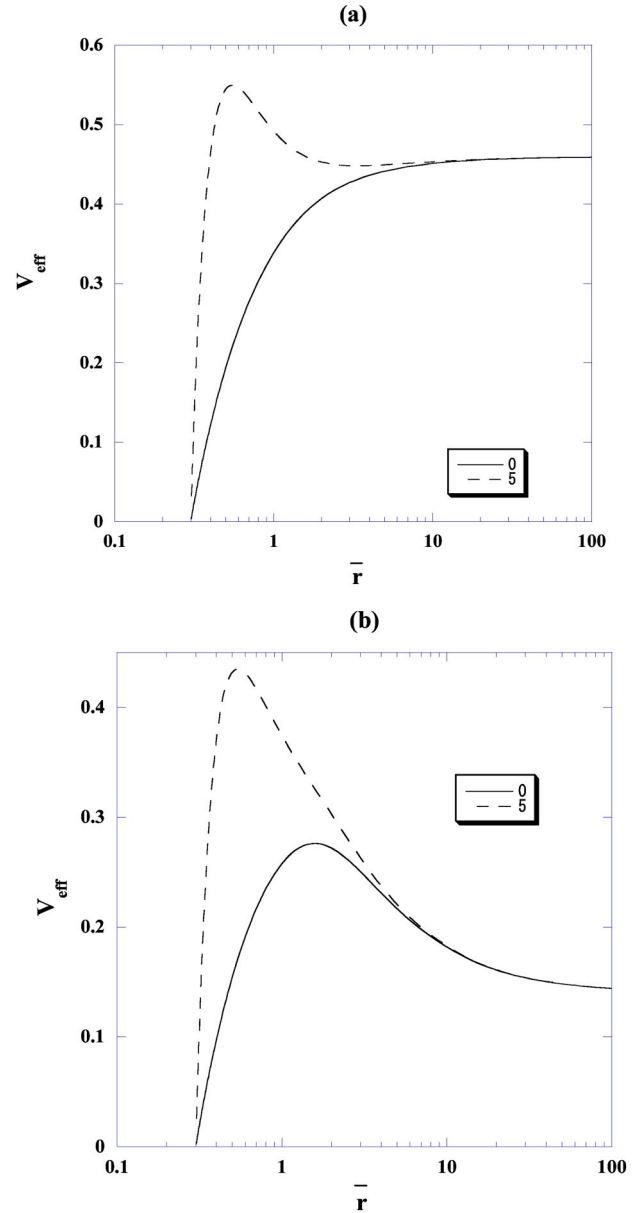


FIG. 2. The behavior of  $V_{\text{eff}}$  for  $\bar{r}_H=0.3$ , and  $(L/r_H)^2=0, 5$ . (a)  $\bar{v}=0.2$  and (b)  $\bar{v}=0.6$ .

and approaches  $1-2\bar{v}^2$  asymptotically; whether or not the potential maximum appear depends on  $\bar{L}$  as in the Schwarzschild black hole. Thus, the sign of  $\bar{M}$  is important to determine the particle motion around the black hole.

Such a convex form of the potential for a  $L=0$  particle motion is characteristic of a global monopole black hole, and does not appear neither in a Schwarzschild black hole nor in a regular global monopole. In the case of a regular global monopole, the whole space time is repulsive [5], which means that  $V_{\text{eff}}$  decreases monotonically.

Figure 3 shows the relation between  $\bar{r}_H$  and  $\bar{M}$  for various  $\bar{v}$ .  $\bar{M}$  is negative in the limit  $\bar{r}_H \rightarrow 0$  as it is expected from the regular solutions. As  $\bar{v}$  increases, the region of  $\bar{M} < 0$  extends to larger  $\bar{r}_H$ . As it was pointed out in Ref. [2], there are

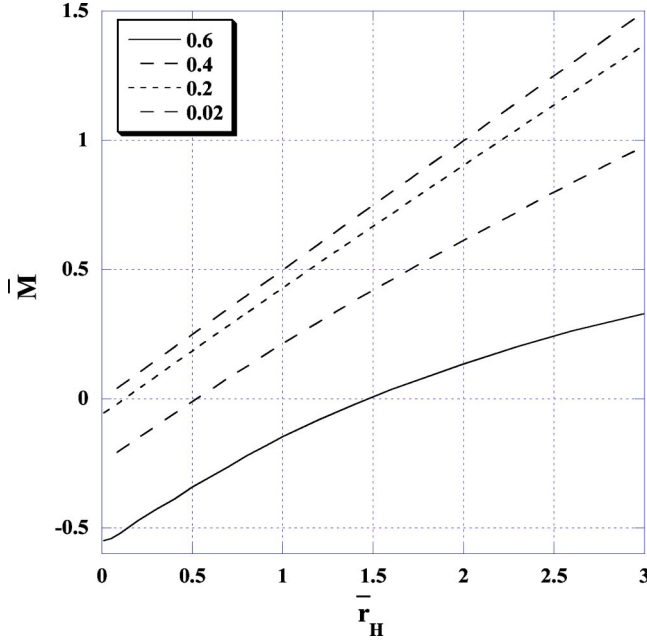


FIG. 3. The relation between  $\bar{r}_H$  and  $\bar{M}$  for various  $\bar{v}$ .

solutions with  $\bar{M}=0$ . In this case, a potential minimum  $\bar{r}_-$  goes to infinity and a test particle cannot feel a black hole.

### B. Large horizon

NSS argued that global monopoles with the nonminimally coupled gravity may explain rotation curves in spiral galaxies [4]. However, the nonminimally coupled gravity has been constrained astrophysically. Moreover, in the NSS model the bound orbits exist only in the microscopic region  $r \lesssim r_{\text{core}}$ . Thus, it is desirable to seek for other possibilities.

On the other hand, Wetterich discussed the possibility that a massless scalar field may explain rotation curves of galactic halos [11]. In his solutions, however, physical boundary conditions were not taken into account. If we assume regularity at the center or an event horizon of a black hole, only a trivial solution remains. Nonexistence of nontrivial black hole solutions are guaranteed by no hair theorem [12]. In this sense, his model is also unrealistic.

Then, we turn to a global monopole with an event horizon. This model is free from the above difficulties existing in the NSS model. Furthermore, the existence of an event horizon is realistic because massive black holes are observed in the central regions of galaxies.

Let us consider astrophysical bounds from the mass density of monopoles in the universe at first. If we demand that mass density of monopoles should be less than 10 times of critical density [9], we have

$$n < 10^{-3} \left( \frac{10^{16} \text{ GeV}}{v} \right)^3 \text{ Mpc}^{-3}, \quad (16)$$

where  $n$  is the number density of monopoles. Thus,  $v \ll 10^{16} \text{ GeV}$  is required to explain rotational curves of galactic halos. For definiteness, we choose  $\bar{v} = 0.2 \times 10^{-4}$  and discuss the case where the event horizon is cosmological size.

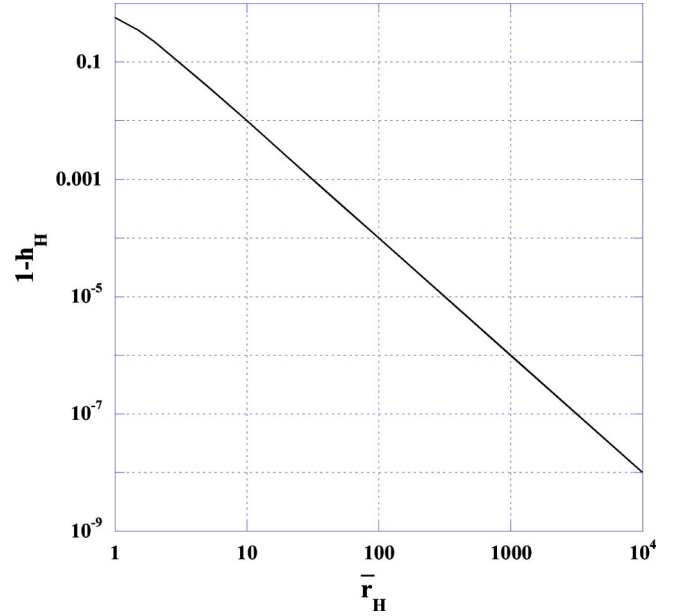


FIG. 4. The relation between  $\bar{r}_H$  and  $(1-h_H)$  for  $\bar{v} = 0.2 \times 10^{-4}$ .

This corresponds to the case  $\bar{r}_H \gg 1$  unless the self-coupling constant of the Higgs field  $\lambda$  takes extremely smaller value than 1.

To see the structure of the Higgs field for  $\bar{r}_H \gg 1$ , we plot  $r_H(1-h_H)$  in Fig. 4. To maintain numerical accuracy, we change the variable from  $h(r)$  to  $s(r) := 1-h(r)$ . In this diagram, the error is below a percent. We find  $h_H \cong 1$  and  $(1-h_H) \cong 1/r_H^2$ . We can check this relation analytically as follows. In the asymptotic region, supposing the asymptotic form

$$h = 1 + \sum_{n=1}^{\infty} C_n r^{-n}, \quad (17)$$

we have  $C_1 = 0$  and  $C_2 = -1$ , which are consistent with our numerical results above.

Because of this asymptotic behavior,  $\bar{M}$  can be estimated by substituting  $h=1$  into Eq. (4). Then, we have the asymptotic relation

$$\bar{M} = \frac{\bar{r}_H}{2} (1 - 2\bar{v}^2). \quad (18)$$

We show the relation  $\bar{r}_H - \bar{M}$  in Fig. 5 for  $\bar{v} = 0.2 \times 10^{-4}$  and  $\bar{v} = 0.2$ , which confirms the relation (18). Actually, Fig. 3 shows that this approximation is fairly good even for  $\bar{r}_H \sim 1$  when  $\bar{v}$  is small.

Let us consider particle motions. Setting  $h=1$ , we have

$$V_{\text{eff}} \sim \left( 1 - 2\bar{v}^2 + \frac{2\bar{M}}{r} \right) \left( 1 + \frac{\bar{L}^2}{r^2} \right). \quad (19)$$

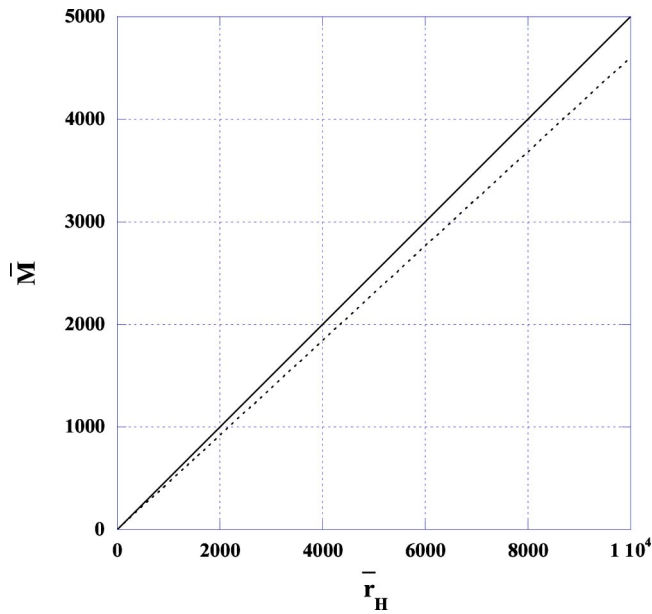


FIG. 5. The relation between  $\bar{r}_H$  and  $\bar{M}$  for  $\bar{v}=0.2\times 10^{-4}$  and  $\bar{v}=0.2$  by a solid line and by a dotted line, respectively.

Since  $\bar{v}$  is small as constrained by Eq. (16), we find that the particle motion is practically the same as that in Schwarzschild black hole. We can regard this kind of behavior as natural since the massive field has its structure comparable to the Compton wavelength. Therefore, as long as we consider an astronomical-sized event horizon, the effect of scalar fields on particle motions is negligible.

#### IV. CONCLUSION AND DISCUSSION

We investigated properties of global monopoles with an event horizon and revealed interesting features which had

not been known so far. The main features of test particle motions are determined by the sign of the core mass; if it is negative and if the event horizon is as small as the core radius, there is an unstable circular orbit even for a particle with zero angular momentum. We also found the asymptotic form of the solutions when the event horizon is much larger than the core radius; the qualitative features of the monopole black hole is the same as that of the Schwarzschild black hole.

Although our model does not explain observed rotation curves very well, we obtain some lessons here. Our results indicates that massive scalar fields would encounter with the same difficulty as in our model. A typical mass scale of particle physics is so large that it generally contributes only to microscopic structure, whose size is of order of the inverse of the mass. In this sense, a massless scalar field considered by Wetterich [11] might be useful. Although his model itself does not satisfy physical boundary conditions neither of a black hole nor of a regular solution, interaction with matter may be a key ingredient to solve this problem. Including this possibility, we also want to consider other solitonlike models such as boson-fermion stars in the future [13].

#### ACKNOWLEDGMENTS

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