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A short-run and long-run analysis of a quasi-stochastic monetary economy

by

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Abstract

To analyze the liquidity trap in a dynamic optimization framework, most studies assume that money has inherent utility. Instead of assuming money-in-utility, this paper considers uninsurable idiosyncratic risks as the source of money demand to investigate the short-run economic fluctuations as well as the long-run economic growth. Individuals face uncertainty over the return to capital, and hence invest both physical capital and money as a risk diversification. To distinguish the effect of intertemporal substitution from that of risk-aversion, we utilize a non-expected utility maximization approach. In the short-run, due to uncertainty, the economy may falls into the liquidity trap in which an increase in money supply does not push down the interest rate because the money demand based on precautionary motives absorb all the money. In the long-run, there exists the optimal growth rate of money supply, which depends not only on the degree of risk-aversion but also crucially on the elasticity of intertemporal substitution.

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1. Introduction

As is well-known, the Japanese economy has been in the serious slump for more than a decade with very-low nominal interest rates and low inflation or even deflation, as Figure-1 shows. [1] This kind of situation is treated as a special case as a "liquidity trap" in the IS-LM models.

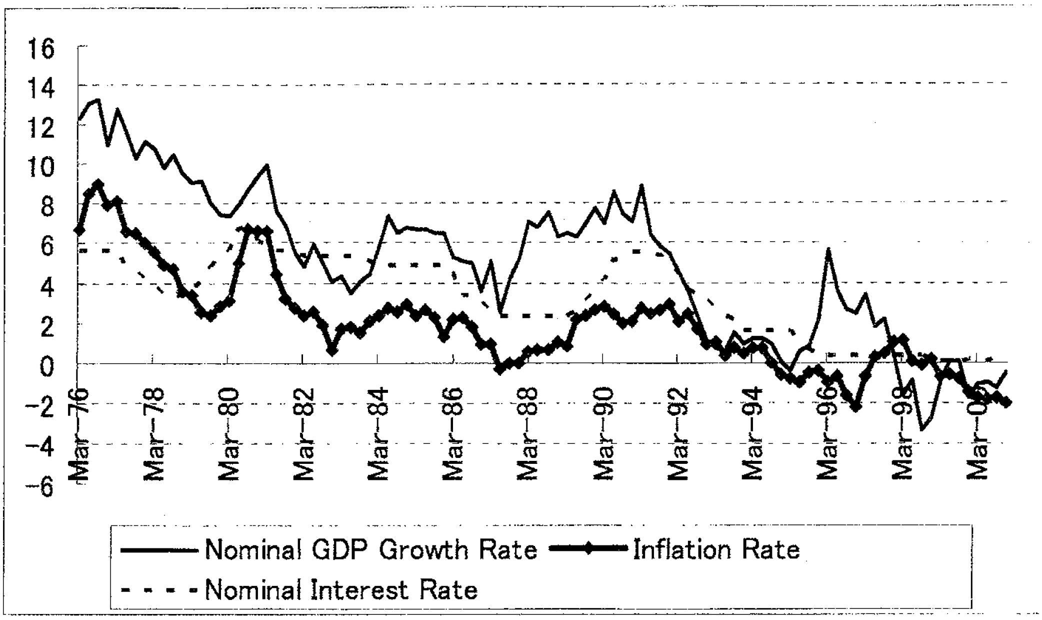

Figure-1 Recent Movements of GDP, Inflation Rates, and Nominal Interest Rates in Japan

There is the growing literature that focuses on this problem based upon rigorous and explicit microfoundations. For example, Benhabib, Schmitt-Grohe and Uribe (2001, 2002), Buiter and Panigirtzoglou (2003), and Ono (1994, 2001), capturing the interactions between forward-looking prices and the agents' intertemporal maximizing behavior, discuss the possibility of liquidity trap and the economic policies to avoid or to escape from the trap. In essence, they are variants of the seminal model of Brock (1975), where money has inherent utility. Hence, similarly to Brock, most studies on the liquidity trap assume an endowment

[1] The inflation rates here are the GDE (Gross Domestic Expenditure) deflators (=GDP deflators) while the nominal interest rates are the yields of the short-term (13 week) government. The data on the GDP and deflator are taken from the Cabinet Office homepage (http://www.cao.go.jp/) while the data on the interest rates are from the Bank of Japan homepage (http://www.boj.or.jp/).

economy, i.e., no capital accumulation.

Instead of assuming money-in-utility, this paper focuses on the money demand as a risk diversification measures. Of course, people are happy with money. But, most of their happiness comes from its purchasing power, and hence indirectly from consumption. Cash-in-advance models capture this feature of money. As Keynes (1936) emphasizes, however, people have money as the asset especially in an uncertain world since the value of money is considered more stable than that of other assets. Introducing productivity shocks to physical capital, the model presented in this paper derives the precautional demand for money and discussed the possibility of liquidity trap in the short-run.

Allowing the capital stock to change, the paper also investigates the relationship between growth and inflation in the long-run. Intertemporal substitution plays a key role in the analysis. In order to distinguish the intertemporal substitution from the risk-aversion, we use Kreps-Porteus non-expected utility preferences instead of time- and state-separable isoelastic preferences.

The organization of the rest of this paper is as follows. Section 2 presents the simple stochastic optimization model of an individual with a non-expected utility preference. Section 3 investigates the short-run and long-run properties of a quasi-stochastic macroeconomy. The final section provides some concluding remarks.

2. The Model

Consider an economy that consists of a continuum of identical individuals: each owns a firm and produces a homogenous good according to a stochastic production function:

$$dY(t) = AK(t)[dt + \sigma dz(t)] \quad \text{with } A > 0, \tag{1}$$

where $K(t)$ is the individual's capital stock. In each period, the deterministic flow of each firm's production is $AK(t)dt$. In addition to this deterministic part, there is also a stochastic component of production $AK(t)\sigma dz(t)$ due to idiosyncratic technology shocks, where $dz(t)$ is a Wiener process with mean zero and unit variance, and parameter σ is the instantaneous standard deviation of the technology shock.

Since the rate of return to capital of each firm is equal to the marginal product of capital, it becomes

$$r(t) = A[dt + \sigma dz(t)], \tag{2}$$

Each individual knows that each firm faces its idiosyncratic risks, and hence wants to hedge the risks by holding money as a risk diversification. Although these risks are assumed to be uninsurable at the individual level, there is no aggregate uncertainty assuming that the individuals' risks are cancelled out each other.

The budget constraint of the representative consumer is given by

$$P(t)C(t)dt + dM(t) + P(t)I(t)dt = r(t)P(t)K(t), \tag{3}$$

where $P(t)$ is output price, $dM(t)$ is the nominal money demand, $C(t)$ is consumption and $I(t)$ is the fixed investment. Assuming no depreciation in physical capital for simplicity, the capital per capita evolves according to the following:

$$dK(t) = I(t)dt. \tag{4}$$

The budget constraint in real term is expressed as

$$C(t)dt + \frac{dM(t)}{P(t)} + dK(t) = r(t)K(t). \tag{5}$$

Defining the total nominal asset as $W(t)$, or

$$W(t) = M(t) + P(t)K(t), \tag{6}$$

the real asset becomes

$$w(t) = m(t) + K(t), \tag{7}$$

where $w(t) \equiv W(t)/P(t)$ and $m(t) \equiv M(t)/P(t)$. Hence, the budget constraint in real term can be rewritten as:

$$dw(t) = [A(w(t) - m(t)) - \pi(t)m(t) - C(t)]dt - \sigma A(w(t) - m(t))dz. \tag{8}$$

This can also be expressed as follows:

$$dw(t) = [r_R w(t) - r_N(t)m(t) - C(t)]dt - \sigma A(w(t) - m(t))dz, \tag{9}$$

where $r_R = A$ is the mean of a real interest rate, $r_N(t) = A + \pi(t)$ is the mean of a nominal interest rate.

The utility of the individual depends only on consumption $C(t)$. To distinguish the effect of intertemporal substitution from that of risk-aversion, we employ a non-expected utility maximization setup.[2] We assume that at point in time t the individual maximizes the intertemporal objective $V(t)$ by recursion,

$$f([1-\gamma]V(t)) = \left(\frac{1-\gamma}{1-1/\varepsilon}\right)C(t)^{1-1/\varepsilon}h + e^{-\rho h} f([1-\gamma]E_t V(t+h)), \tag{10}$$

where the function $f(x)$ is given by

$$f(x) = \left(\frac{1-\gamma}{1-1/\varepsilon}\right)x^{(1-1/\varepsilon)/(1-\gamma)}. \tag{11}$$

In (10), h is the economic decision interval, E_t is a mathematical expectation conditional on time-t information, and $\rho > 0$ the subjective discount rate. The parameter $\gamma > 0$ measures the relative risk-aversion while the parameter $\varepsilon > 0$ is the intertemporal substitution elasticity.[3] When $\gamma = 1/\varepsilon$, so that $f(x) = x$, our setup is the standard state- and time-separable expected-utility setup, which does not allow independent variation in risk aversion and intertemporal substitutability over time.[4]

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[2] For detailed treatment of "recursive utility," see Duffie and Esptein (1992).

[3] For a detailed discussion on the roles of these parameters and more general preference setups, see, for example, Kreps and Porteus (1979, 1979), Epstein and Zin (1989, 1991), Weil (1989), and Obstfeld (1994a, 1994b).

[4] This paper analyzes the individual's behavior in the limit as h becomes infinitesimally small.

Let $J(w(t))$ denote the maximum feasible level of the expected sum of discounted utilities. The value function $J(w(t))$ depends on the contemporaneous variable $w(t)$ only. Applying Ito's lemma to the maximization of $V(t)$ in (10), we get the following stochastic Bellman equation:

$$0 = \max_{C,m}\{[(1-\gamma)/(1-1/\varepsilon)]C^{1-1/\varepsilon} - \rho f([1-\gamma]J(w))$$

$$+(1-\gamma)f'([1-\gamma]J(w))[J'(w)(A(w-m)-\pi m - C)+(1/2)J''(w)(\sigma A(w-m))^2]\}. \quad (12)$$

(For notational convenience, time arguments are suppressed as long as no ambiguity results.) From (12), the first-order conditions with respect to C and m are

$$C^{-1/\varepsilon} - f'([1-\gamma]J(w))J'(w) = 0. \quad (13a)$$

$$(A+\pi)J'(w) + (\sigma A)^2(w-m)J''(w) = 0. \quad (13b)$$

Eq. (10)'s form suggests that $J(w)$ is given by

$$J(w) = (bw)^{1-\gamma}/(1-\gamma), \quad (14)$$

where b is a positive constant to be determined. Eqs. (13a) and (13b) become

$$C = uw. \quad (15a)$$

$$m^d = \left(1 - \frac{A+\pi}{\gamma(\sigma A)^2}\right)w. \quad (15b)$$

where $u \equiv b^{1-\varepsilon}$ and m^d is the money demand. Noticing that $A+\pi$ is a nominal interest rate, (15a) shows that the money demand is a decreasing function of the nominal interest rate, risk-aversion coefficient γ, and the risk σ.

Substituting (14), (15a) and (15b) for (12) gives

When $\gamma = 1/\varepsilon$, (10) implies that as $h \to 0$, $V(t)$ becomes the standard setup of discounted sum of utilities: $V(t) = E_t\left\{(1-\gamma)^{-1}\int_t^{\infty} C(t)^{1-\gamma} e^{-\rho(s-t)} ds\right\}$.

$$u = \varepsilon(\rho+\pi) + \frac{(1-\varepsilon)(A+\pi)^2}{2\gamma(\sigma A)^2} - \pi \ , \tag{16a}$$

and therefore

$$b = \left[\varepsilon(\rho+\pi) + \frac{(1-\varepsilon)(A+\pi)^2}{2\gamma(\sigma A)^2} - \pi \right]^{1/(1-\varepsilon)} . \tag{16b}$$

Evidently from (16a) together with (15a), whether an increase in the nominal interest rate $A+\pi$ raises consumption depends crucially on the intertemporal substitution elasticity ε.

3. The Analysis

For simplicity, the population of individuals is normalized to unity. Since there is no aggregate uncertainty, the aggregate output at each point in time becomes

$$Y = AK \ . \tag{17}$$

Also, the differential equation on the time-path of the aggregate asset is

$$\dot{w} = A(w-m) - \pi m - uw \ . \tag{18}$$

3-1. The Short-Run Analysis

In this subsection, assuming no capital accumulation we will analyze the short-run dynamics of the economy. In what follows, the capital stock is assumed to be fixed at $\overline{K}$, or $K=\overline{K}$. Hence our economy here the stochastic version of endowment economy analyzed in Benhabib, Schmitt-Grohe and Uribe (2001, 2002), Buiter and Panigirtzoglou (2003), Ono (1994, 2001), and others.

The money market is assumed to be in equilibrium at each point in time,

$$m^d = \left(1 - \frac{A+\pi}{\gamma(\sigma A)^2}\right)\left(m^d + \overline{K}\right) = m^s = m \ \text{ or } \ \left(\frac{A+\pi}{\gamma(\sigma A)^2}\right)\left(m + \overline{K}\right) = \overline{K} \ . \tag{19}$$

where m^s is the money supply. The above equilibrium condition of course determines the nominal interest rate and hence the rate of inflation because the real interest rate is fixed at A:

$$\pi = \left(\frac{\overline{K}}{m+\overline{K}}\right)\gamma(\sigma A)^2 - A. \tag{20}$$

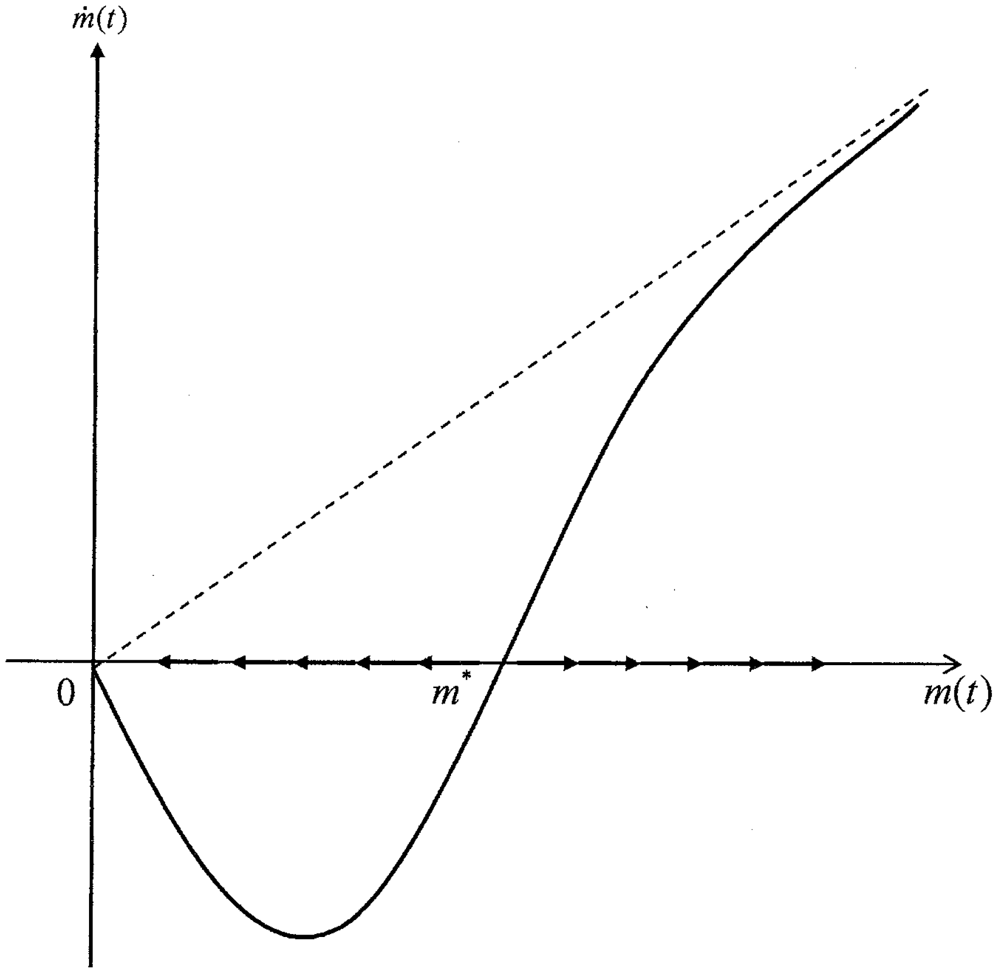

Figure-2 Dynamics of Real Money Balance in the Short-Run

Assuming that there is no growth of nominal money supply, the differential equation on real money balance becomes

$$\frac{\dot{m}}{m} = -\pi = A - \left(\frac{\overline{K}}{m+\overline{K}}\right)\gamma(\sigma A)^2 . \tag{21}$$

The dynamics of the real money balance can be depicted in Figure-1. m^* in Figure-2 is a neoclassical equilibrium in the economy, where there is no inflation, or $\pi = 0$.

Since $w = m + \overline{K}$, $\dot{w} = \dot{m}$. Substituting these two equations into (18) gives

$$\dot{m} = A\overline{K} - \pi m - u(m + \overline{K}) \quad \text{or} \quad \frac{\dot{m}}{m} = \frac{ES}{m} - \pi, \tag{22}$$

where $ES \equiv Y - C = A\overline{K} - u(m + \overline{K})$ is the excess supply in the goods market. Since $\dot{m}/m = -\pi$ from (21), the goods market is always in equilibrium, or $ES \equiv Y - C = 0$. However, only at the neoclassical equilibrium m^*, there is no inflation nor deflation, or $\pi = 0$.

If the initial money balance m is larger than m^* or $m > m^*$, then m continues to grow, in other words, the deflation continues, or $\pi < 0$. This trajectory also satisfies all the optimality conditions including the following transversality condition:

$$\lim_{t \to \infty} \left[e^{-\rho t} E_0 J(w(t)) \right] = 0 \ . \tag{23}$$

As is shown in Figure-2, on this transitional path the rate of deflation $\dot{m}/m = -\pi$ approaches to A, and hence the nominal interest rate $r_N \equiv A + \pi$ approaches to zero. In other words, the economy falls into a liquidity trap.

Proposition 1

In a stochastic monetary economy there exist deflationary equilibria as well as a unique neoclassical equilibrium without inflation or deflation. Hence, the economy may fall into liquidity trap, which is characterized by the deflationary equilibria.

3-2. The Long-Run Analysis

Since m/w is constant over time, $\dot{m}/m = \dot{w}/w = \dot{K}/K$ on the balanced growth path. Substitution of (15a) and (15b) into (18) gives

$$g_w \equiv \frac{\dot{w}}{w} = \frac{(1+\varepsilon)(A+\pi)^2}{2\gamma(\sigma A)^2} - \varepsilon(\rho+\pi) \ . \tag{24}$$

There are two polar cases for g_w. If the nominal interest rate becomes non-positive, or $\pi \leq -A$, then the people has no incentive to hold the physical capital. The individual hold all the assets in the form of money, or $m/w = 1$. In this case, $g_w = \varepsilon(A-\rho)$, which is the same as the growth rate in the typical AK model.

The other polar case arises when π becomes equal to or larger than $\gamma(\sigma A)^2 - A$. In this case, evidently from (15b), the individual has no incentive to hold money, and hence $m/w = 0$. In other words, the economy behaves just like the stochastic non-monetary economy analyzed by Smith (1996). In this case,

$$g_w \equiv \varepsilon(A-\rho) + \frac{(1-\varepsilon)\gamma(\sigma A)^2}{2},$$

which is of course exactly the same as the growth rate in Smith's model.

Suppose that the grow rate of nominal money supply is constant at μ, or $\dot{M}/M = \mu$. Then,

$$g_m \equiv \frac{\dot{m}}{m} = \mu - \pi \ . \tag{25}$$

Hence, the long-run growth rate g^* and inflation rate π^* are determined by (24) and (25).

The determination of g^* and π^* when the intetemporal substitution is small, or $\varepsilon < 1$, is shown in Figure-3, where $\bar{\pi} = \gamma(\sigma A)^2 - A$. Since an increase in the growth rate of money supply μ shifts g_m-line to the left, it increases the grow rate g^* but decreases the inflation rate π^*. An increase in g_m must be accompanied by the same amount of increase in g_w on the balanced growth path. When the intetemporal substitution ε is small, this is possible only when the nominal interest rate falls. If the nominal interest rate increases when ε is small, both current and future consumptions increase, and hence the capital accumulation fall. With a small

ε, therefore, the inflation rate π^* must decrease because the decreased nominal interest rate fasters the capital accumulation g_w.

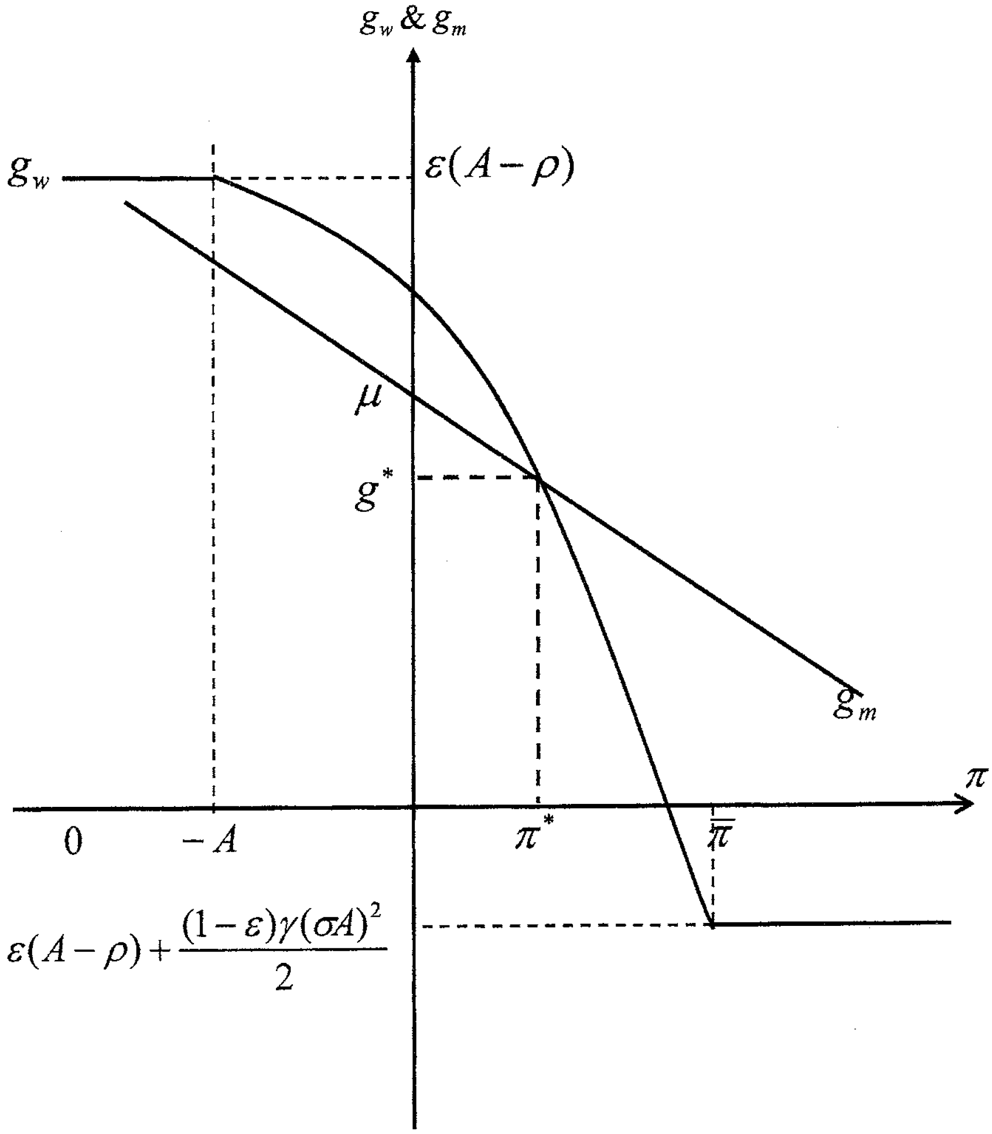

Figure-3 Balanced Growth When $\varepsilon < 1$

The determination of g^* and π^* when the intetemporal substitution is large, or $\varepsilon > 1$, is shown in Figure-4. In this case, an increase in the growth rate of money supply μ increases both the grow rate g^* and the inflation rate π^*. In order for g_w to equate to g_w when μ

increases, g_w should increase. Since the intetemporal substitution ε is large, this is possible only when the nominal interest rate increases. If the nominal interest rate increases when ε is large, then current consumption falls while future consumption increases, and hence the increased interest rate fasters the capital accumulation g_w.

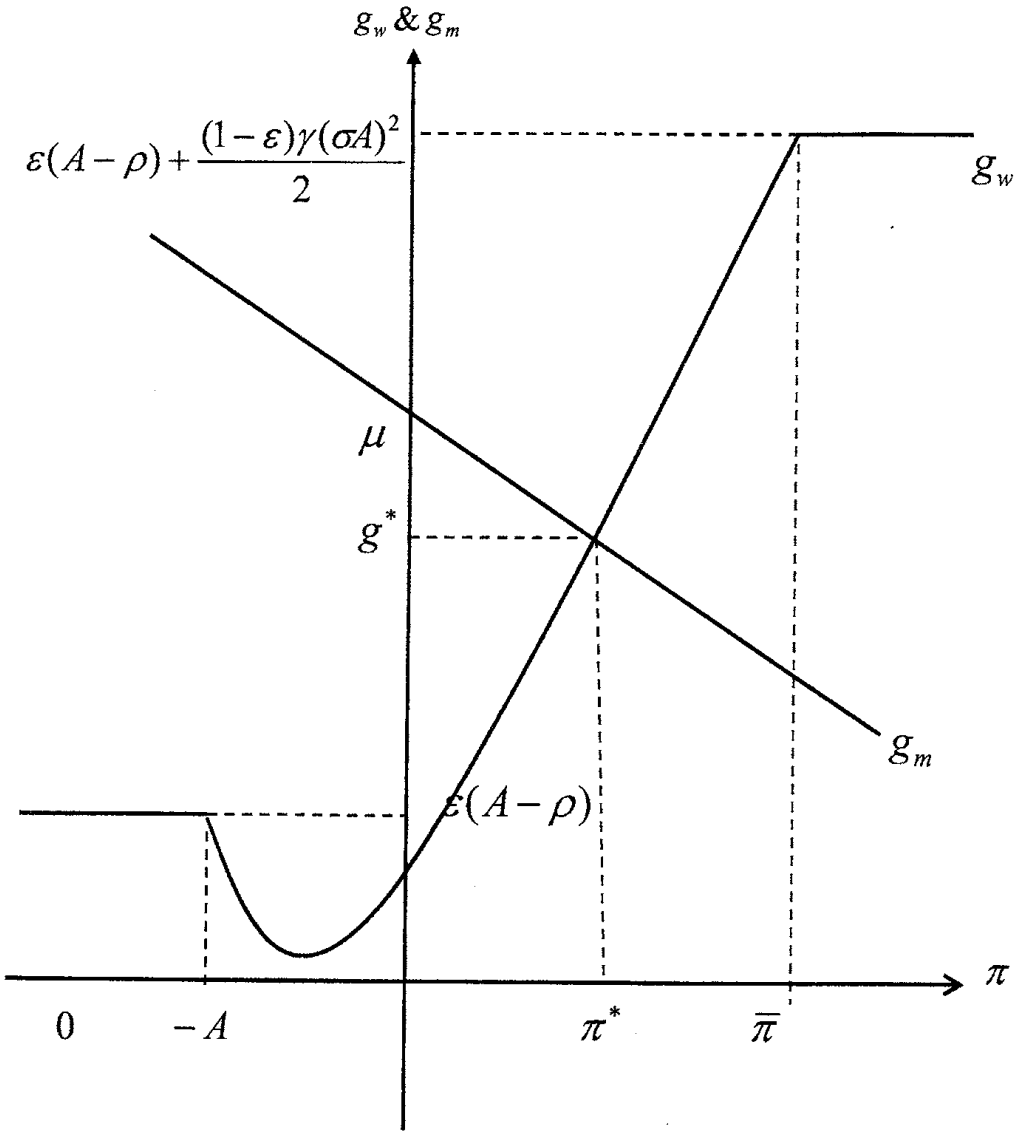

Figure-4 Balanced Growth When $\varepsilon > 1$

Proposition 2

An increase in the growth rate of money supply increases the long-run growth rate. However, it increases the inflation rate when the intertemporal elasticity of substitution is large, but increases the inflation rate when the intertemporal elasticity of substitution is small.

4. Concluding Remarks

As Keynes (1936) emphasizes, the precautional demand for money does matter in an uncertain world. In this paper, instead of assuming money-in-utility, we consider uninsurable idiosyncratic risks as the source of money demand. As a result, it is shown that the individual's risk-averse behavior play a key role in deriving the precautional demand for money, which is in turn the driving force for putting the economy into a liquidity trap in the short-run.

Intetemporal substitution is another important factor characterizing the individual's dynamic preferences. On the balanced growth path, higher growth is accompanied by higher inflation only when the intetemporal substitution is large. When it is small, on the contrary, see higher growth with lower inflation. Regardless of the substitution elasticity, however, the increased money supply growth enhances the growth rate of the economy.

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