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Author(s)	Toyoda, Tadashi
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Nonperturbative Canonical Formulation of Information Theory *

Tadashi Toyoda

Department of Physics, Tokai University,

Kitakaname 1117, Hiratsuka, Kanagawa 259-1292, Japan

e-mail: toyoda@keyaki.cc.u-tokai.ac.jp

In quantum field theory, the most significant role is played by the Green's functions, from which physical quantities can be calculated. The Green's functions can be formally derived from a generating functional, i.e., the Schwinger functional [2]. The physical meaning of the Schwinger functional becomes clear in the Euclidean field theory, particularly in Matsubara's finite temperature Green's function theory [3] [4]. If we consider a many-boson system, we can write the finite temperature Schwinger functional explicitly using the functional integral [5],

$$Z[J,J^{*}]=\int D\phi^{*}D\phi\exp(S[J,J^{*}]),$$

where S is the Matsubara action and ϕ is a classical field variable corresponding to the second quantized boson Schrödinger field. The Matsubara action S contains a term coupled to external c-number field variables J and J^{*} [5] [6]. The one-particle Matsubara temperature Green's function can be calculated as a functional derivative of $\log Z[J,J^{*}]$ [6]

$$G(x,x')=\lim_{J,J^{*}\rightarrow 0}\frac{\delta^{2}\log Z[J,J^{*}]}{\delta J(x)\delta J^{*}(x')},$$

where $x\equiv(\mathbf{x},\tau)$ with Matsubara's imaginary time τ. The Schwinger functional is nothing else but a characteristic functional for the finite temperature many-body correlation func-

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tions. *The one-particle Matsubara temperature Green's functions can be regarded as quantum many-body Fisher's matrices.*

In practical calculations of Matsubara temperature Green's functions for various models such as interacting electron gas, the use of approximations is necessary. In 1957 Ezawa, Tomozawa and Umezawa extended Wick's theorem to Matsubara temperature Green's functions [7]. Since then, the powerful field theoretic perturbative expansion method for Green's functions with the famous Feynman diagrams have been widely applied to Matsubara temperature Green's functions for a number of microscopic models in condensed matter physics. Then it has been realized that it is inevitable to consider an infinite series of perturbative expansion to obtain a physically meaningful result, because the poles of Green's functions contain relevant information such as the energy spectrum or the lifetime of the quasi-particles and the shift of the pole cannot be evaluated without taking account of an infinite series. Consequently, it has become a popular practice to consider a simple infinite series that can be straightforwardly calculated. The geometric series has been the most popular one. However, such an approximation obviously goes beyond the theoretical limitation of the perturbative approximation scheme, since only a particular class of terms are considered arbitrarily neglecting the other.

In quantum field theory it has been known that there are certain exact relations between Green's functions. Well-known examples are Ward-Takahashi relation in quantum electrodynamics (QED) [8] and Slanov-Taylor identities in non-Abelian field gauge theory [9]. The basic idea underlying these relations has been generalized and extended to Matsubara's finite temperature Green's functions [10]. The new approach is based on the canonical formulation of quantum field theory. Starting with the basic equal-time canonical (anti-)commutation relations, the canonical generators for relevant transformations can be defined. By making use of the canonical generators one can derive various exact relations for Matsubara temperature Green's functions, i.e., the finite temperature generalized Ward-Takahashi relations (FTGWTR) [10]. It has been shown that the FTGWTR give a natural extension of the basic Ward identities in Landau's Fermi liquid theory [4] as well as a generalization of

Baym-Kadanoff's conserving approximation [11] [12]. There are a number of useful results derived from the FTGWTR [13] [14]. Recenlty, it has been shown that the linear response formula for the spin response of interacting electron gas can be rigorously generalized to the nonlinear response formula within the FTGWTR formalism [15].

In view of the fact that the Matsubara temperature Green's functions can be formulated within the framework of nonperturbative canonical quantum field theory, it seems natural to expect a similar formulation of Fisher's information matrices, i.e., *the canonical information field theory (CIFT).* The first step to construct a CIFT model is to define the dynamical field variables that describe the information system under consideration. Then, by considering symmetry properties of the system, a model *information action* for the characteristic functional can be constructed in terms of the dynamical field variables. Following the same procedure in the FTGWTR theory, rigorous relations between the Fisher matrices that reflect the symmetry properties of the information system can be derived [16].

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