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On the Fate of an Education Obsessed Society

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Abstract

.This paper constructs an economic growth model with overlapping generations. Agents' ability in the model can be high or low. Agents with high ability incur low costs to obtain education. On the other hand, agents with low ability incur high costs to obtain an education. With physical capital accumulation, the wage becomes high enough, and then the low-ability agents want to be thought as a high-ability agent. In order to separate from the low-ability agents, the high ability agents must send a signal to firms by obtaining high level of education. This incurs unnecessarily high costs to the high ability agents and absorbs their saving. This reduces physical capital accumulation and can bring up trade cycles.

JEL Classification Numbers: J24, O41
Keywords: Signal, Human Capital, Cycles

1 Introduction

In almost advanced countries, people eager to have high levels of education. In order to get a good position in a company, people tries to enter highly ranked universities and get degrees. However, abilities are quite different among individuals. Some people have high abilities and can easily get human capital with little effort. On the other hand, another people needs much effort

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to get human capital and further can get only a low level of human capital compared to individuals with high abilities. Spence's seminal paper (1973, 1974) investigates this situation and shows that the high-ability agents have an incentive to send signals to firms in order to discriminate them from the low-ability individuals by getting a higher level of educations than the low-ability individuals.

Now let's consider this issue in a macroeconomic framework. Getting high levels of educations of course needs much cost. Ordinary people sometimes must borrow money from banks to enter universities. In particular, in Japan, primary schoolchildren or junior high school students often attend cram schools (called Jyuku) in order to enter famous private schools. It cost much. This may reduce the saving which was once invested into physical capital. Consequently, this may reduce output level of the education obsessed society..

This paper constructs an economic growth model with overlapping generations in order to examine this issue. Agents with high ability incur low costs to obtain education. On the other hand, agents with low ability incur high costs to obtain an education. With physical capital accumulation, the wage becomes high enough, and then the low-ability agents become to want to be thought as a high-ability agent. In order to separate from the low-ability agents, the high ability agents must send a signal to firms by obtaining a higher level of education. This incurs unnecessarily high costs to the high ability agents and absorbs their saving. This reduces physical capital accumulation and can bring up cycles.

The rest of this paper is structured as follows: Section 2 builds up the model to be considered. Section 3 examines the signaling game among the agents. Section 4 defines equilibrium and dynamics of the model. Section 5 gives some concluding remarks.

2 Model

The model of this paper is an overlapping generations economy of endogenous growth with physical and human capital. The consumption good, Y is produced by using physical, K and human capital, H. The production function takes the following Cobb-Douglas form, $Y = AK^{\alpha}H^{1-\alpha}$, $A > 0$, $0 < \alpha < 1$. Both physical capital and human capital depreciate completely after the production.

Each agent lives three periods (young, adult, old). We assume population is normalized to be one. There are be two types of agents. One is an agent with an ability to accumulate human capital cheaply. The other is an agent without such ability. The population ratio of the agents with the ability is θ, and the ratio of the agents without the ability is $1-\theta$. These ratios remain constant. When young, the agent with the ability (the agent without the ability) receives an education, e_t^a (e_t^n) and get human capital as follows:

$$h_{t+1}^a = e_t^a + (1-\delta)H_t,\ h_t^a \le \gamma H_t,\ \gamma > 1 \tag{1}$$

$$h_{t+1}^n = e_t^n + (1-\delta)H_t,\ h_t^n \le H_t \tag{2}$$

where H_t means aggregate human capital of the economy. Each agent can have a part of human capital of their parents' generation without any cost. The agent with the ability can advance the human capital level than their parents' level, however, the agents without the ability at most get the same level of human capital as their parents' level. The agents incur costs to get human capital, but different level. The high-ability agent can accumulate human capital more cheaply than the low-ability agent. The high-ability agent must pay $\beta^a e^a$, on the other hand, the low-ability agent must pay $\beta^n e^n$. and $0 < \beta^a < \beta^n$. In order to finance this cost, they borrow from adult agents at the asset market.

When adult, they work by selling their human capital to firms and get wage income according to their human capital level which firms believe they have. They save this income all for the old period. There are two saving methods, one is to invest it into physical assets, the other is to lend it to young agents who want to get educations. By arbitrage, the rates of returns of these savings become the same.

When old, they consume all their wealth, both principal and interest. This is the only source of their utility. Consequently, their objective becomes the maximization of their wage income minus their repayment.

At the first period, there are only adult agents and old agents.

3 Job Market Signaling

Because there are two types of agents, we have to consider the signaling game situation at each period. The low-ability agents may have an incentive to mimic the high-ability agents. The timing of the game is the following:

1. Agents choose a level of education with knowing their ability.
2. Firms observe agents' education level, but not knowing their ability (therefore, their borrowing levels), and make wage offers to the agentsagents
3. The agents accept or reject the wage offer.

The objective of the agents is given by:

$$\max_{e_t^i} \ w_{t+1}h_{t+1}^i - r_{t+1}\beta^i e_t^i, \ i = a, \ n.$$

Taking account of (1) and (2), we must distinguish the following five cases:

(I) Both of the agents do not have an education.

If $r_{t+1}\beta^n/w_{t+1} > r_{t+1}\beta^a/w_{t+1} > 1$, then both types choose not be educated (see Figure 1).

$$e_t^a = e_t^n = 0.$$

(II) The high-ability agents begin to have an education.

When $r_{t+1}\beta^n/w_{t+1} > r_{t+1}\beta^a/w_{t+1} = 1$, then the high-ability agents are indifferent between getting an education and not getting an education. On the other hand, the low-ability agents have no incentive to have an education (see Figure 2).

$$e_t^a \in [0, \ (\gamma + \delta - 1)H_t]$$

$$e_t^n = 0$$

(III) Only the high-ability agents have an education.

When $r_{t+1}\beta^n/w_{t+1} > 1 > r_{t+1}\beta^a/w_{t+1}$, then the high-ability agents have an incentive to be educated up to the maximum level. However, the low-ability agents still have no incentive to have an education (see Figure 3).

$$e_t^a = (\gamma + \delta - 1)H_t$$

$$e_t^n = 0$$

(IV) The low-ability agents begin to have an education.

When $1 = r_{t+1}\beta^n/w_{t+1} > r_{t+1}\beta^a/w_{t+1}$, then the low-ability agents are indifferent between getting an education and not getting an education. The high-ability agents have an incentive to have an education up to their maximum level. If the low ability agents invest up to the maximum level of the high-ability agents, then firms want to distinguish the low-ability agent who mimic the high-ability agents. But, firms cannot separate them from the high-ability agents because they cannot observe the actual ability of the agents. The firms can observe only the education level. Therefore, the high-ability agents have an incentive to invest human capital over their maximum level in order to separate them from the low-ability agents (see Figure 4).[1] Consequently, we obtain the following:

$$e_t^a = (\gamma + \delta - 1)H_t + \varepsilon H_t,\ \varepsilon > 0$$

$$e_t^n \in [0,\ \delta H_t]$$

We assume that the high-ability agents need to overinvest εH_t in order to discriminate them from the low-ability agents.

(V) Both of the agents invest up to their maximum levels.

When $1 > r_{t+1}\beta^n/w_{t+1} > r_{t+1}\beta^a/w_{t+1}$, then both of the agents have an incentive to have an education up to their maximum levels. The same situation as case (IV) occurs. Therefore, the high-ability agents must send a signal to firms to separate them from the low-ability agents. Accordingly there is an unnecessary overinvestment in human capital (see Figure 5).

$$e_t^a = \left[\frac{w_{t+1}}{r_{t+1}\beta^n}(\gamma - 1) + \delta\right] H_t + \varepsilon H_t,\ \varepsilon > 0$$

$$e_t^n = \delta H_t$$

Summarizing the preceding arguments and noting (1) and (2), we can obtain the following human capital accumulation expressions:

[1]There can be pooling equilibria other than the separating equilibrium. However, by resorting to the *Intuitive Criteria* of Cho and Kreps (1987), we can refine the perfect Bayesian equilibrium. We can show that all pooling equilibria cannot survive through the Intuitive Criteria (see Gibbons (1992).

$$
\begin{aligned}
H_{t+1} &= \theta h^a_{t+1} + (1-\theta)h^n_{t+1} \\
&= \begin{cases}
(1-\delta)H_t & when\ \frac{r_{t+1}\beta^a}{w_{t+1}}>1, \\
[0,\ \{\theta\gamma+(1-\theta)(1-\delta)\}H_t] & when\ \frac{r_{t+1}\beta^a}{w_{t+1}}=1, \\
\{\theta\gamma+(1-\theta)(1-\delta)\}H_t & when\ \frac{r_{t+1}\beta^n}{w_{t+1}}>1>\frac{r_{t+1}\beta^a}{w_{t+1}}, \\
[\{\theta\gamma+(1-\theta)(1-\delta)\}H_t,\ \{\theta(\gamma-1)+1\}H_t] & when\ \frac{r_{t+1}\beta^n}{w_{t+1}}=1>\frac{r_{t+1}\beta^a}{w_{t+1}}, \\
\{\theta(\gamma-1)+1\}H_t & when\ 1>\frac{r_{t+1}\beta^n}{w_{t+1}}.
\end{cases}
\end{aligned}
$$

4 Market Equilibrium

Bertrand competition among the firms drives the profit of the firms down to zero. Hence, the following conditions must hold:

$$w_t = (1-\alpha)Ak_t^\alpha, \tag{3}$$

$$r_t = \alpha A k_t^{\alpha-1}, \tag{4}$$

where w_t and r_t stands for the wage rate and the (gross) interest rate respectively, and $k_t \equiv K_t/H_t$.

Asset market equilibrium condition becomes

$$K_{t+1} + \theta\beta^a e_t^a + (1-\theta)\beta^n e_t^n = w_t H_t - r_t\left[\theta\beta^a e_{t-1}^a + (1-\theta)\beta^n e_{t-1}^n\right]. \tag{5}$$

The left hand side means the demand for funds for physical and human capital investment. On the contrary, the right hand side means supply for the funds. Dividing the both side of (5) by H_t and taking account of (3) and (4), we get the following:

$$
\begin{aligned}
& k_{t+1}\frac{H_{t+1}}{H_t} + \theta\beta^a\frac{e_t^a}{H_t} + (1-\theta)\beta^n\frac{e_t^n}{H_t} \\
= \ & A(1-\alpha)k_t^\alpha - A\alpha k_t^{\alpha-1}\left[\theta\beta^a\frac{e_{t-1}^a}{H_{t-1}} + (1-\theta)\beta^n\frac{e_{t-1}^n}{H_{t-1}}\right]\frac{H_{t-1}}{H_t}
\end{aligned}
\tag{6}
$$

We first examine the demand for the funds. Let's denoting the demand for the funds as D_{t+1}. There are five cases which is described above. The five cases are: (I) $k_{t+1} < \frac{\alpha\beta^a}{1-\alpha}$, (II) $k_{t+1} = \frac{\alpha\beta^a}{1-\alpha}$, (III) $\frac{\alpha\beta^a}{1-\alpha} < k_{t+1} < \frac{\alpha\beta^n}{1-\alpha}$, (IV) $k_{t+1} = \frac{\alpha\beta^n}{1-\alpha}$, (V)$\frac{\alpha\beta^n}{1-\alpha} < k_{t+1}$. By using the factor market equilibrium conditions, (3) and (4), the five cases become as follows:

$$D_{t+1} = \begin{cases} k_{t+1}(1-\delta) & \text{(I)} \\ \left[(1-\delta)\frac{\alpha\beta^a}{1-\alpha},\ \theta(\gamma+\delta-1)\left(\beta^a + \frac{\alpha\beta^a}{1-\alpha}\right) + \frac{\alpha\beta^a}{1-\alpha}(1-\theta)\right] & \text{(II)} \\ k_{t+1}\{\theta\gamma + (1-\theta)(1-\delta)\} + \theta\beta^a(\gamma+\delta-1) & \text{(III)} \\ \left[D_{\text{IV}},\ \bar{D}_{\text{IV}}\right] & \text{(IV)} \\ k_{t+1}\{\theta(\gamma-1)+1\} + \theta\beta^a\left[\frac{1-\alpha}{\alpha\beta^n}(\gamma-1)k_{t+1} + \delta + \varepsilon\right] + (1-\theta)\beta^n\delta & \text{(V)} \end{cases}$$

where $D_{\text{IV}} = \theta(\gamma+\delta-1)\left(\beta^a + \frac{\alpha\beta^n}{1-\alpha}\right) + \frac{\alpha\beta^n}{1-\alpha}(1-\theta)$ and $\bar{D}_{\text{IV}} = D_{\text{IV}} + \theta(1-\delta)\frac{\alpha\beta^n}{1-\alpha} + (1-\theta)\beta^n\delta + \theta\beta^a\varepsilon$. D_{t+1} is a correspondence which assigns a nonempty compact subset to every k_{t+1}. We denote this correspondence as $D(k_{t+1})$.

Next, let's examine the supply side. By denoting the supply for the funds as S_t, we similarly get the following for $t \geq 2$:

$$S_t = \begin{cases} A(1-\alpha)k_t^\alpha & \text{(I)} \\ \left[S_{\text{II}},\ \bar{S}_{\text{II}}\right] & \text{(II)} \\ A(1-\alpha)k_t^\alpha - A\alpha k_t^{\alpha-1}\theta\beta^a\frac{\gamma+\delta-1}{\theta\gamma+(1-\theta)(1-\delta)} & \text{(III)} \\ \left[S_{\text{IV}},\ \bar{S}_{\text{IV}}\right] & \text{(IV)} \\ A(1-\alpha)\left[1 - \frac{\beta^a}{\beta^n}\frac{\theta(\gamma-1)}{\theta(\gamma-1)+1}\right]k_t^\alpha - A\alpha k_t^{\alpha-1}\frac{(\delta+\varepsilon)\theta\beta^a + \delta(1-\theta)\beta^n}{\theta(\gamma-1)+1} & \text{(V)} \end{cases}$$

where five cases are as follows: (I) $k_t < \frac{\alpha\beta^a}{1-\alpha}$, (II) $k_t = \frac{\alpha\beta^a}{1-\alpha}$, (III) $\frac{\alpha\beta^a}{1-\alpha} < k_t < \frac{\alpha\beta^n}{1-\alpha}$, (IV) $k_t = \frac{\alpha\beta^n}{1-\alpha}$, (V)$\frac{\alpha\beta^n}{1-\alpha} < k_t$. $S_{\text{II}} = A(1-\alpha)\left[\frac{\alpha\beta^a}{1-\alpha}\right]^\alpha$, $\bar{S}_{\text{II}} = A(1-\alpha)\left[\frac{\alpha\beta^a}{1-\alpha}\right]^\alpha \frac{1}{\theta(\gamma+\delta-1)+1-\theta}$, $S_{\text{IV}} = A(1-\alpha)\left[\frac{\alpha\beta^n}{1-\alpha}\right]^\alpha \frac{(\beta^n-\beta^a)\theta(\gamma+\delta-1)+\beta^n(1-\theta)}{\theta(\gamma+\delta-1)+1-\theta}$, and $\bar{S}_{\text{IV}} = A(1-\alpha)\left[\frac{\alpha\beta^n}{1-\alpha}\right]^\alpha \frac{(\beta^n-\beta^a)\theta(\gamma+\delta-1)+\beta^n(1-\delta)-\theta\beta^a\varepsilon}{\theta(\gamma+\delta-1)+1-\theta}$. As for the first period, we get $S_1 = w_1H_1 = A(1-\alpha)k_1^\alpha$. Similar to the demand correspondence, we can define the supply correspondence as $S(k_t)$.

Consequently, we get the following dynamics from the asset market equilibrium condition:

$$k_{t+1} \in \Phi(k_t) \equiv \{k_{t+1} \mid D(k_{t+1}) \cap S(k_t) \neq \emptyset \ for\ k_t\} \quad (7)$$

This define the dynamic path of k_t.Steady State:

Definition 1 *Steady State*

Steady states of the (7) is defined by $k^* \in \Phi(k^*)$.

There can be many patterns of the dynamics. So, let's pick up an interesting case. Figure 6 depicts the demand and supply correspondences.. There is a steady state between point A and point B. As can be seen from this figure, this steady state happens to be case (IV). If k_t enters into this region, then k_{t+1} must be in this region because of (7). In this case, we have to examine the following dynamics of $\frac{e_t^n}{H_t}$. Denoting $\frac{e_t^n}{H_t}$ by x_t^n, taking account of $e_t^a = \gamma + \delta - 1$, $k^* = \frac{\alpha\beta^n}{1-\alpha}$, we can express (6) as follows:

$$\begin{aligned} &\frac{\alpha\beta^n}{1-\alpha}[\{\theta\gamma+(1-\theta)(1-\delta)\}+(1-\theta)x_t]+\theta\beta^a(\gamma+\delta-1+\varepsilon)+(1-\theta)\beta^n x_t^n \quad (8) \\ = \; &(1-\alpha)A\left[\frac{\alpha\beta^n}{1-\alpha}\right]^\alpha \frac{(\beta^n-\beta^a)\theta(\gamma+\delta-1+\varepsilon)+\beta^n(1-\theta)}{\beta^n[\{\theta\gamma+(1-\theta)(1-\delta)\}+(1-\theta)x_{t-1}^n]} \end{aligned}$$

This defines the dynamics of case (IV). Consequently, the steady state of this dynamics is defined by:

$$\begin{aligned} &\frac{\alpha\beta^n}{1-\alpha}\{\theta\gamma+(1-\theta)(1-\delta)\}+\theta\beta^a(\gamma+\delta-1+\varepsilon)+\frac{(1-\theta)\beta^n}{1-\alpha}x^{n*} \quad (9) \\ = \; &(1-\alpha)A\left[\frac{\alpha\beta^n}{1-\alpha}\right]^\alpha \frac{(\beta^n-\beta^a)\theta(\gamma+\delta-1)+\beta^n(1-\theta)}{\beta^n[\{\theta\gamma+(1-\theta)(1-\delta)\}+(1-\theta)x^{n*}]} \end{aligned}$$

We first examine the stability of the steady state of the dynamics of x_t^n. By differentiating the right hand side of (8) with respect to x_{t-1}^n and dividing this by $\frac{(1-\theta)\beta^n}{1-\alpha}$, we get

$$-(1-\alpha)^2 A\left[\frac{\alpha\beta^n}{1-\alpha}\right]^\alpha \frac{(\beta^n-\beta^a)\theta(\gamma+\delta-1+\varepsilon)+\beta^n(1-\theta)}{(\beta^n)^2[\{\theta\gamma+(1-\theta)(1-\delta)\}+(1-\theta)x^{n*}]^2}$$

This is the slope of the graph of the dynamics of x_t^n. Therefore, when this is smaller than -1, then the steady state is unstable (see Figure 7). By making use of (9), we can rearrange this as follows:

$$-\left[1+(1-\alpha)\frac{\beta^a\theta(\gamma+\delta-1+\varepsilon)-\beta^n\{\theta\gamma+(1-\theta)(1-\delta)]}{\beta^n[\{\theta\gamma+(1-\theta)(1-\delta)\}+(1-\theta)x^{n*}]}\right]$$

Hence, the numerator takes a positive value, then the steady state becomes unstable. We get the following condition under which the steady state is unstable:

$$\varepsilon > \left(1 - \frac{\beta^a}{\beta^n}\right)(\gamma + \delta - 1) + \frac{1-\delta}{\theta}$$

This inequality can be consistent with Figure 6 because this inequality does not contain the productivity parameter A.

When this condition holds, even if k_t enters into case (IV), k_t leave case (IV). Then, k_t enters case (III) or case (IV).

5 Concluding Remarks

We have shown that overinvestment to human capital may absorb funds for investment for physical capital. As mentioned in the introduction, the saving is absorbed by expense to getting educations. This reduces income in the adult period and thus saving volume of the adult individuals. Consequently, this leads to a decrease in physical capital and can produce permanent cycles.

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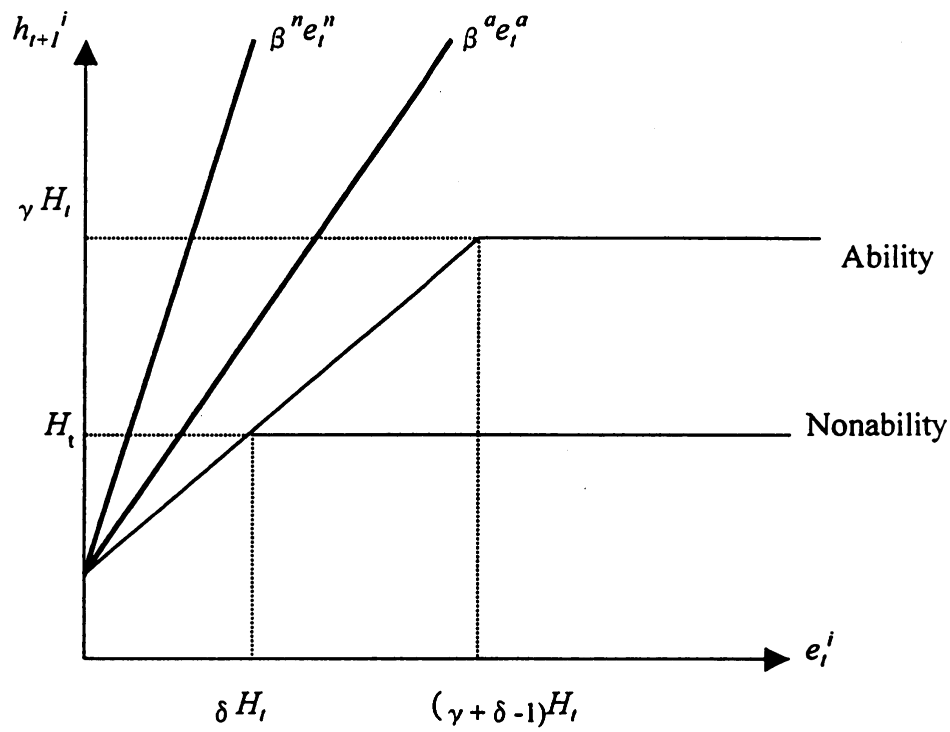

Figure 1

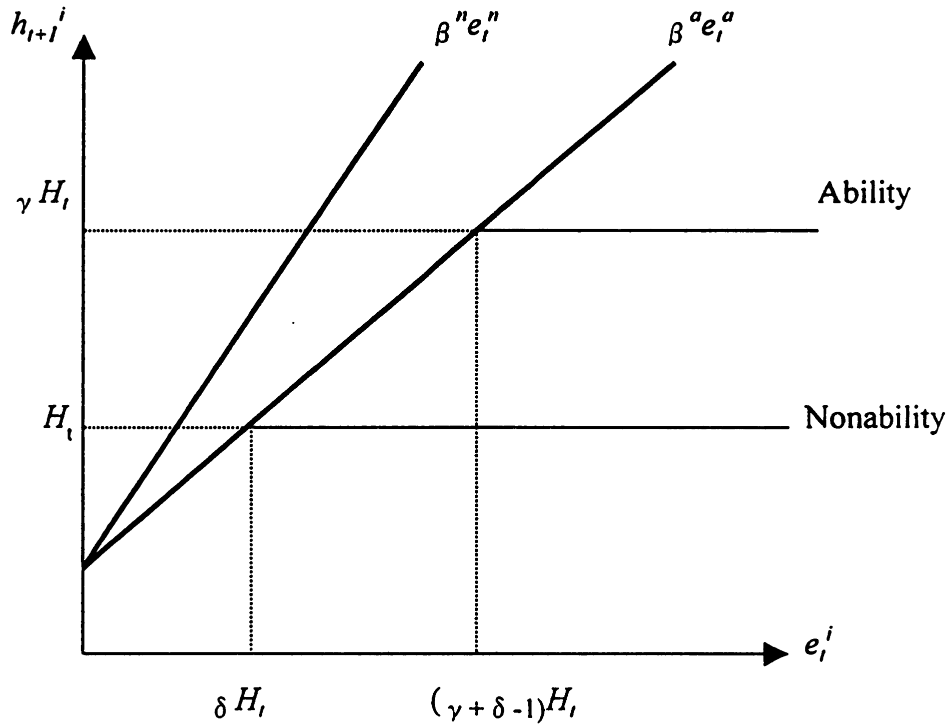

Figure 2

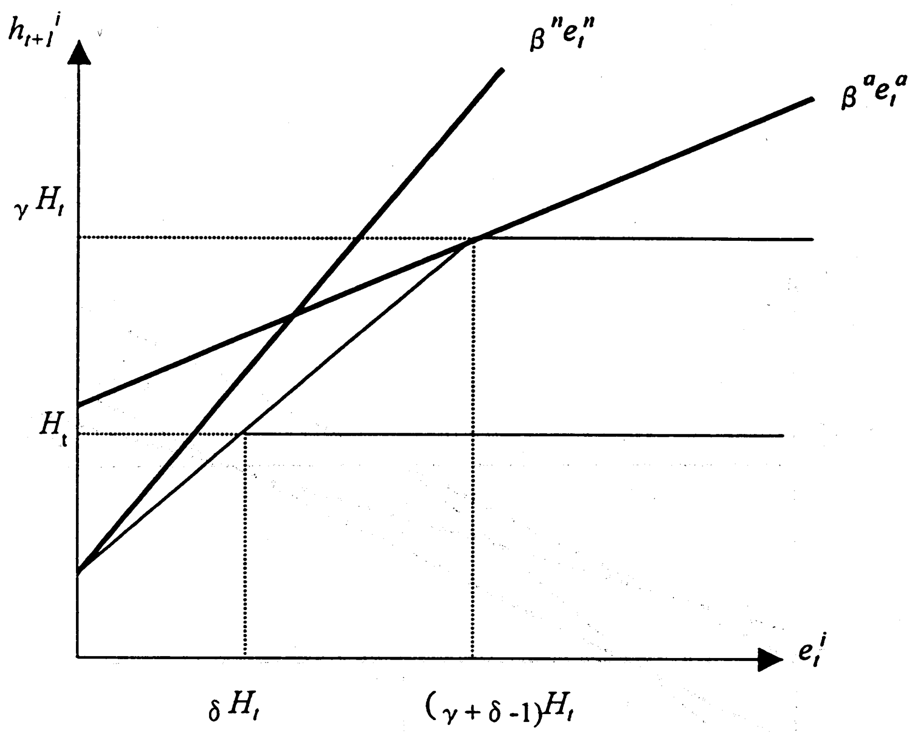

Figure 3

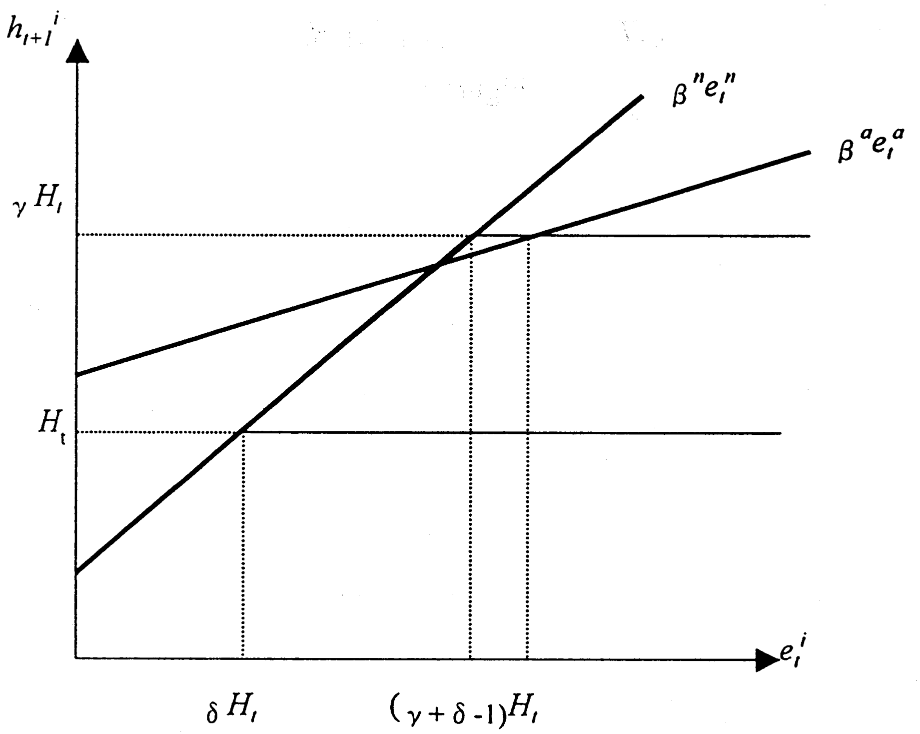

Figure 4

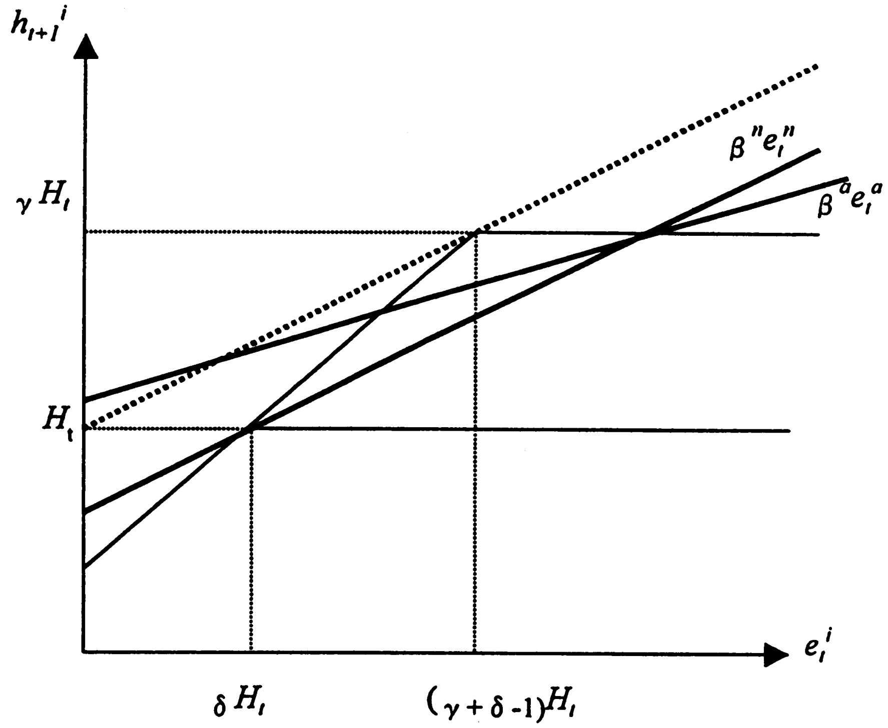
$h_{t+1}{}^{i}$
γH_t
H_t
$\beta^{n}e_t^{n}$
$\beta^{a}e_t^{a}$
e_t^{i}
δH_t
$(\gamma + \delta - 1)H_t$

Figure 5

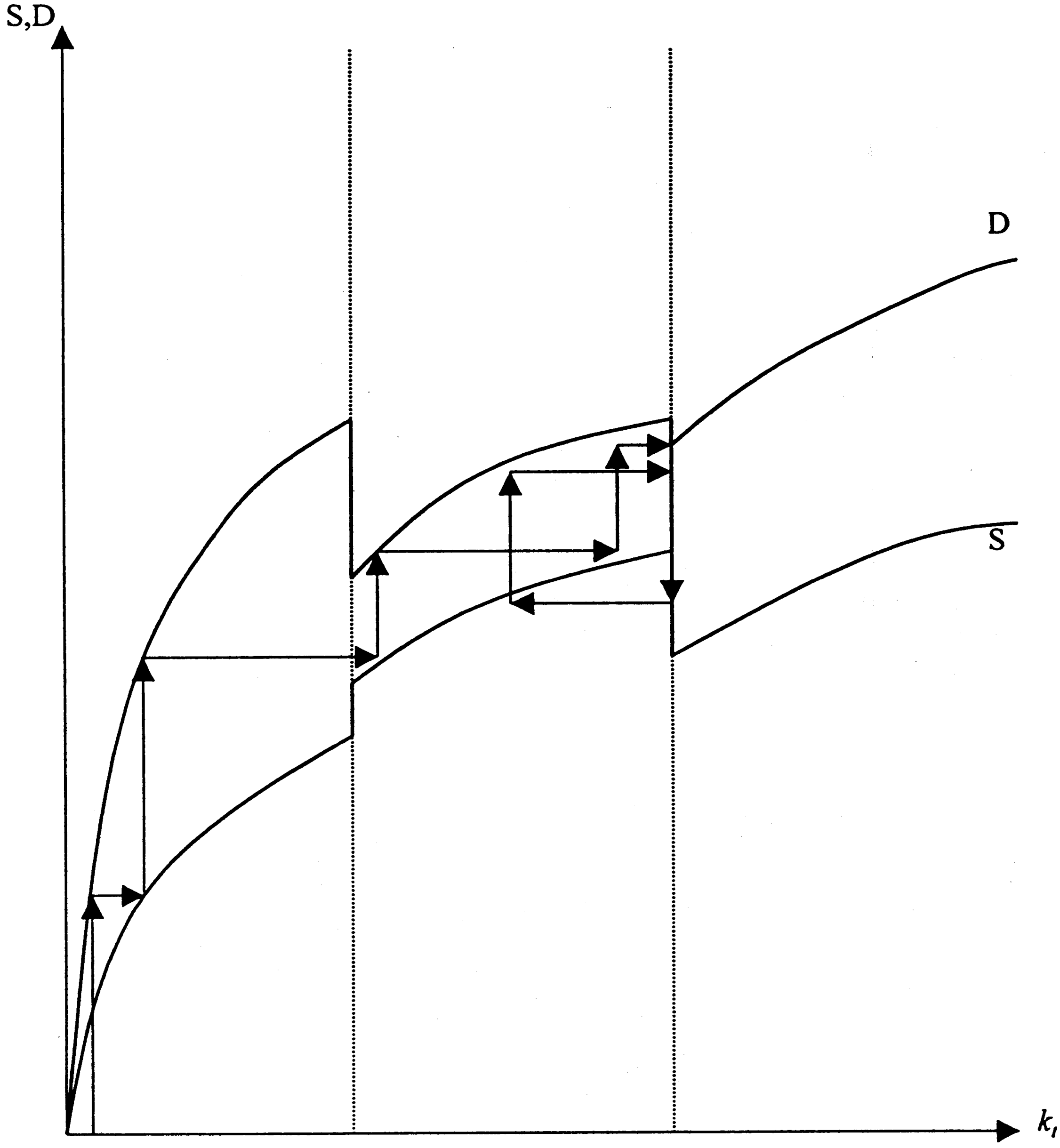

Figure 6

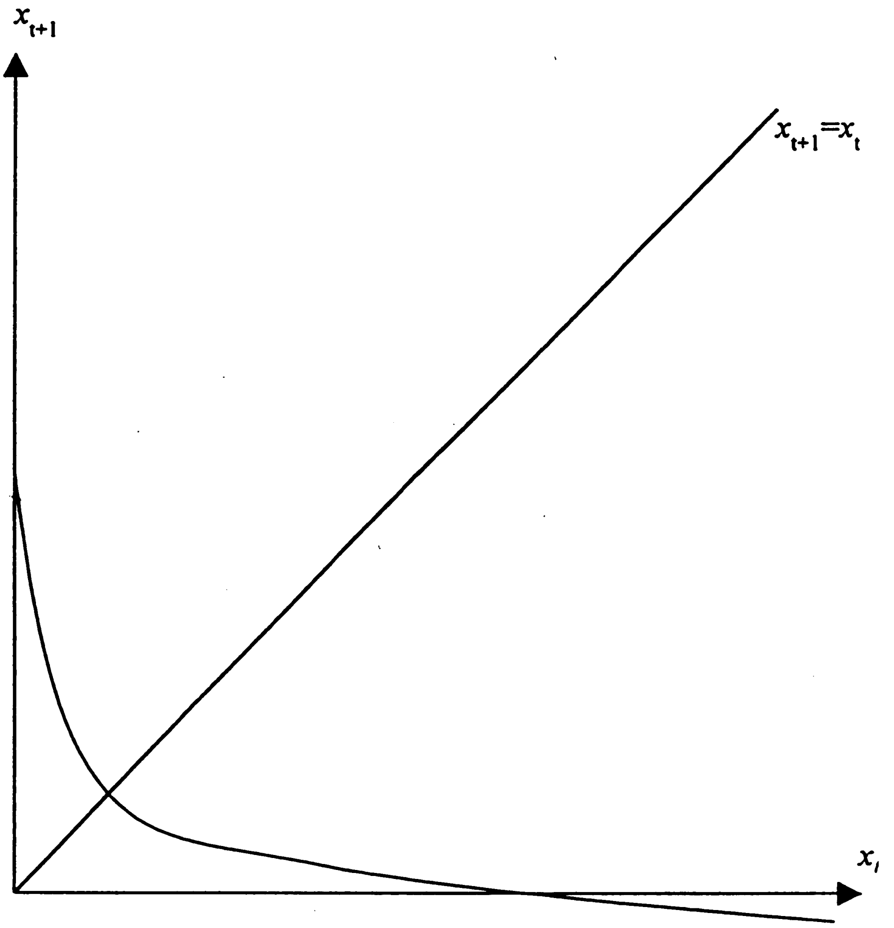

Figure 7