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The Joinability and Unification Problems for Confluent Semi-Constructor TRSs

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Abstract

The unification problem for term rewriting systems (TRSs) is the problem of deciding, for a TRS R and two terms s and t , whether s and t are unifiable modulo R . Mitsuhashi et al. have shown that the problem is decidable for confluent simple TRSs. Here, a TRS is simple if the right-hand side of every rewrite rule is a ground term or a variable. In this paper, we extend this result and show that the unification problem for confluent semi-constructor TRSs is decidable. Here, a semi-constructor TRS is such a TRS that every subterm of the right-hand side of each rewrite rule is ground if its root is a defined symbol. We first show the decidability of joinability for confluent semi-constructor TRSs. Then, using the decision algorithm for joinability, we obtain a unification algorithm for confluent semi-constructor TRSs.

1 Introduction

The unification problem for term rewriting systems (TRSs) is the problem of deciding, for a TRS R and two terms s and t , whether s and t are unifiable modulo R . This problem is undecidable in general and even if we restrict to either right-ground TRSs [9] or terminating, confluent, monadic, and linear TRSs [7]. Here, a TRS is monadic if the height of the right-hand side of every rewrite rule is at most one [12]. On the other hand, it is known that unification is decidable for some subclasses of TRSs [2, 4, 5, 8, 11]. Recently, Mitsuhashi et al. have shown that the unification problem is decidable for confluent simple TRSs [7]. Here, a TRS is simple if the right-hand side of every rewrite rule is a ground term or a variable. In this paper, we extend the result of [7] and show that unification for confluent semi-constructor TRSs is decidable. Here, a semi-constructor TRS is such a TRS that every subterm of the right-hand side of each rewrite rule is ground if its root is a defined symbol.

In order to obtain this result, we first show the decidability of joinability for confluent semi-constructor TRSs. Joinability of several subclasses of TRSs has been shown to be decidable so far [13]. Many of these decidability results have been proved by reducing these problems to decidable ones for tree automata, so that these decidable subclasses are restricted to those of right-linear TRSs. In this paper, we provide a decidability result of joinability for possibly non-right-linear TRSs. To our knowledge, such attempts were very few so far.

Moreover, in this paper we show that confluence is necessary to show the decidability of joinability for semi-constructor TRSs, that is, joinability for (non-confluent) linear semi-constructor TRSs is undecidable.

2 Preliminaries

We assume that the reader is familiar with standard definitions of rewrite systems (see [1, 14]) and we just recall here the main notations used in this paper.

Let X be a set of variables, F a finite set of operation symbols graded by an arity function $\text{ar}: F \rightarrow \mathbb{N}$, $F_n = \{f \in F \mid \text{ar}(f) = n\}$, $\text{Leaf} = X \cup F_0$ the set of leaf symbols, and T the set of terms constructed from X and F . We use x, y, z as variables, b, c, d as constants, and r, s, t as terms. A term is *ground* if it has no variable. Let G be the set of ground terms and let $S = T \setminus (G \cup X)$. Let $V(s)$ be the set of variables occurring in s . The *height* of s is defined as follows: $h(a) = 0$ if a is a leaf symbol and

$h(f(t_1, \dots, t_n)) = 1 + \max\{h(t_1), \dots, h(t_n)\}$. The *root symbol* is defined as $\text{root}(a) = a$ if a is a leaf symbol and $\text{root}(f(t_1, \dots, t_n)) = f$.

A position in a term is expressed by a sequence of positive integers, which are partially ordered by the prefix ordering \leq . To denote that positions u and v are disjoint, we use $u|v$. The subset of all minimal positions (w.r.t. \leq) of W is denoted by $\text{Min}(W)$. Let $\mathcal{O}(s)$ be the set of positions of s .

Let $s|_u$ be the subterm of s at position u . We use $s[t]_u$ to denote the term obtained from s by replacing the subterm $s|_u$ by t . For a sequence (u_1, \dots, u_n) of pairwise disjoint positions and terms r_1, \dots, r_n , we use $s[r_1, \dots, r_n]_{(u_1, \dots, u_n)}$ to denote the term obtained from s by replacing each subterm $s|_{u_i}$ by r_i ($1 \leq i \leq n$).

A *rewrite rule* is defined as a directed equation $\alpha \rightarrow \beta$ that satisfies $\alpha \notin X$ and $V(\alpha) \supseteq V(\beta)$. Let \leftarrow is the inverse of \rightarrow , $\leftrightarrow = \rightarrow \cup \leftarrow$ and $\downarrow = \rightarrow^* \cdot \leftarrow^*$. Let $\gamma: s_1 \xrightarrow{u_1} s_2 \cdots \xrightarrow{u_{n-1}} s_n$ be a *rewrite sequence*. This sequence is abbreviated to $\gamma: s_1 \leftrightarrow^* s_n$ and $\mathcal{R}(\gamma) = \{u_1, \dots, u_{n-1}\}$ is the set of the redex positions of γ . If the root position ε is not a redex position of γ , then γ is called ε -invariant. For any sequence γ and position set W , $\mathcal{R}(\gamma) \geq W$ if for any $v \in \mathcal{R}(\gamma)$ there exists a $u \in W$ such that $v \geq u$. If $\mathcal{R}(\gamma) \geq W$, we write $\gamma: s_1 \xrightarrow{\geq W} s_n$.

Let $\mathcal{O}_G(s) = \{u \in \mathcal{O}(s) \mid s|_u \in G\}$. For any set $\Delta \subseteq X \cup F$, let $\mathcal{O}_\Delta(s) = \{u \in \mathcal{O}(s) \mid \text{root}(s|_u) \in \Delta\}$. Let $\mathcal{O}_x(s) = \mathcal{O}_{\{x\}}(s)$. The set D of *defined symbols* for a TRS R is defined as $D = \{\text{root}(\alpha) \mid \alpha \rightarrow \beta \in R\}$. A term s is *semi-constructor* if, for every subterm t of s such that $\text{root}(t)$ is a defined symbol, t is ground.

Definition 1 A rule $\alpha \rightarrow \beta$ is *ground* if $\alpha, \beta \in G$, *right-ground* if $\beta \in G$, *semi-constructor* if β is semi-constructor, and *linear* if $|\mathcal{O}_x(\alpha)| \leq 1$ and $|\mathcal{O}_x(\beta)| \leq 1$ for every $x \in X$.

Example 2 Let $R_e = \{\text{nand}(x, x) \rightarrow \text{not}(x), \text{nand}(\text{not}(x), x) \rightarrow t, t \rightarrow \text{nand}(f, f), f \rightarrow \text{nand}(t, t)\}$.

R_e is semi-constructor, non-terminating and confluent [3]. We will use this R_e in examples given in Section 3.

Definition 3 [11] An equation is a pair of terms s and t denoted by $s \approx t$. An equation $s \approx t$ is *unifiable modulo* a TRS R (or simply *R-unifiable*) if there exists a substitution θ and a rewrite sequence γ such that $\gamma: s\theta \leftrightarrow^* t\theta$. Such θ and γ are called an *R-unifier* and a *proof* of $s \approx t$, respectively. This notion is extended to sets of term pairs: for $\Gamma \subseteq T \times T$, θ is an *R-unifier* of Γ if θ is an *R-unifier* of every pair in Γ . In this case, Γ is *R-unifiable*. As a special case of *R-unifiability*, $s \approx t$ is \emptyset -unifiable if there exists a substitution θ such that $s\theta = t\theta$, i.e., \emptyset -unifiability coincides with the usual unifiability. If $s \downarrow t$ then $s \approx t$ is *joinable*. If $s \rightarrow^* t$ then $s \approx t$ is *reachable*.

Definition 4 TRSs R and R' are *equivalent* if $\leftrightarrow_R^* = \leftrightarrow_{R'}^*$.

3 Joinability

First, we show that the joinability and reachability problems for (non-confluent) semi-constructor TRSs are undecidable.

Theorem 5 The joinability and reachability problems for linear semi-constructor term rewriting systems are undecidable. **Proof** [sketch] The proof is by a reduction from the Post's correspondence problem (PCP). Let $P = \{\langle u_i, v_i \rangle \in \Sigma^* \times \Sigma^* \mid 1 \leq i \leq k\}$ be an instance of the PCP. The corresponding TRS R_P is constructed as follows: Let $F_0 = \{c, d, \$\}$, $F_1 = \Sigma \cup \{f, h\}$, $F_2 = \{g\}$; $R_P = \{c \rightarrow h(c), c \rightarrow d, d \rightarrow f(d)\} \cup \{d \rightarrow g(u_i(\$), v_i(\$)), f(g(x, y)) \rightarrow g(u_i(x), v_i(y)) \mid 1 \leq i \leq k\} \cup \{h(g(a(x), a(y))) \rightarrow g(x, y) \mid a \in \Sigma\}$. $u(x)$ is an abbreviation for $a_1(a_2(\dots a_k(x)))$ where $u = a_1 a_2 \dots a_k$ with $a_1, \dots, a_k \in \Sigma$. R_P is linear and semi-constructor. For R_P , the following three propositions (1)–(3) are equivalent: (1) $c \downarrow g(\$)$, (2) $c \rightarrow^* g(\$)$, and (3) PCP P has a solution. \square

3.1 Standard Semi-Constructor TRSs

From now on, we consider only confluent semi-constructor TRSs, for which joinability is shown to be decidable. In order to facilitate the decidability proof, we transform a TRS into a simpler equivalent one.

Definition 6 For TRS R , we use R_{rg} and $\overline{R_{\text{rg}}}$ to denote the sets of right-ground and non-right-ground rewrite rules in R , respectively.

If R is clear from the context, we write \rightarrow_{rg} instead of $\rightarrow_{R_{rg}}$.

Definition 7 A TRS R is *standard* if the following condition holds: for every $\alpha \rightarrow \beta \in R$ either $\alpha \in F_0$ and $h(\beta) \leq 1$ or $\alpha \notin F_0$ and for every $u \in \mathcal{O}(\beta)$ if $\beta|_u \in G$ then $\beta|_u \in F_0$.

Let R_0 be a confluent semi-constructor TRS. The corresponding standard TRS $R^{(1)}$ is constructed as follows. First, we choose $\alpha \rightarrow \beta \in R_k (k \geq 0)$ that does not satisfy the standardness condition. If $\alpha \in F_0$ then let $\{u_1, \dots, u_m\} = \text{Min}(\mathcal{O}_G(\beta) \setminus \mathcal{O}_{F_0}(\beta) \setminus \{\varepsilon\})$, else let $\{u_1, \dots, u_m\} = \text{Min}(\mathcal{O}_G(\beta) \setminus \mathcal{O}_{F_0}(\beta))$. Let $R_{k+1} = R_k \setminus \{\alpha \rightarrow \beta\} \cup \{\alpha \rightarrow \beta[d_1, \dots, d_m]_{(u_1, \dots, u_m)}\} \cup \{d_i \rightarrow \beta|_{u_i} \mid 1 \leq i \leq m\}$ where d_1, \dots, d_m are new pairwise distinct constants which do not appear in R_k . This procedure is applied repeatedly until the TRS satisfies the condition of standardness. The resulting TRS is denoted by $R^{(1)}$. For example, $\{f_1(x) \rightarrow g(x, g(a, b)), f_2(x) \rightarrow f_2(g(c, d))\}$ is transformed to $\{f_1(x) \rightarrow g(x, d_1), d_1 \rightarrow g(a, b), f_2(x) \rightarrow d_2, d_2 \rightarrow f_2(d_3), d_3 \rightarrow g(c, d)\}$. This transformation preserves confluence, joinability and unifiability.

Lemma 8

- (1) $R^{(1)}$ is confluent.
- (2) For any terms s, t which do not contain new constants, $s \downarrow_{R_0} t$ iff $s \downarrow_{R^{(1)}} t$.
- (3) For any terms s, t which do not contain new constants, $s \approx t$ is R_0 -unifiable iff $s \approx t$ is $R^{(1)}$ -unifiable.

The proof is straightforward, since R_0 is confluent. By this lemma, we can assume that a given confluent semi-constructor TRS is standardized without loss of generality. By standardization, for any $\alpha \rightarrow \beta \in R_{rg}$, $\alpha \in F_0$ or $\beta \in F_0$ holds and $h(\beta) \leq 1$. However, by the transformation algorithm given in Section 3.2, the heights of the right-hand sides of ground rules (called R_C type rules later) may increase. This is the only exceptional case.

3.2 Adding Ground Rules

The joinability for right-ground TRSs is decidable [10]. In this paper, we show that the joinability for confluent semi-constructor TRSs is decidable, by reducing to the joinability for right-ground TRSs.

Let R_1 be a confluent TRS and R_2 be such a TRS that $\rightarrow_{R_2} \subseteq \downarrow_{R_1}$. Then, obviously $R_1 \cup R_2$ is equivalent to R_1 and confluent. Thus, even if we add pairs of joinable terms of R_1 to R_1 as new rewrite rules (called shortcuts), confluence, joinability and unifiability properties are preserved. Note that reachability is not necessarily preserved. Now, we show that the joinability of confluent semi-constructor TRSs reduces to that of right-ground TRSs by adding new finite ground rules. For this purpose, we need some definitions.

Definition 9 A rule $\alpha \rightarrow \beta$ has *type C* if $\alpha \in F_0, \beta \notin F_0$ and $\mathcal{O}_{D \setminus F_0}(\beta) = \emptyset$, and has *type F₀* if $\alpha, \beta \in F_0$. Let $R_\tau = \{\alpha \rightarrow \beta \in R \mid \alpha \rightarrow \beta \text{ has type } \tau\}$.

That is, R_C is the subset of R_{rg} satisfying that for every rule $\alpha \rightarrow \beta \in R_C$, α is a constant, and β is non-constant and contains no defined non-constant symbol. Henceforth, we assume that $R \setminus R_C$ is standard.

Definition 10

$$h_D(s) = \begin{cases} w + \max\{h_D(s_i) \mid 1 \leq i \leq n\} & (\text{if } s = f(s_1, \dots, s_n), n > 0, f \in D) \\ 1 + \max\{h_D(s_i) \mid 1 \leq i \leq n\} & (\text{if } s = f(s_1, \dots, s_n), n > 0, f \notin D) \\ 0 & (\text{if } s \in \text{Leaf}) \end{cases}$$

where $w = 1 + 2 \max\{h(\beta) \mid \alpha \rightarrow \beta \in R\}$. Note that we give weight w to each defined non-constant symbol and 1 to each other non-constant symbol and define new heights derived from these weights. We define $H_D(s) = \{h_D(s|_u) \mid u \in \mathcal{O}(s)\}_m$, which is a multiset of heights of all subterms of s . Here, we use $\{\dots\}_m$ to denote a multiset and \sqcup to denote multiset union. For TRS R_e of Example 1, $w = 3$ and $H_D(\text{nand}(\text{not}(x), x)) = \{0, 0, 1, 4\}_m$.

Let \ll be the multiset extension of the usual relation $<$ on \mathbb{N} and let \leq be $\ll \cup =$. Let $\#(s) = (H_D(s), g(s))$. Here, function $g(s)$ returns a natural number corresponding to s uniquely, and we assume that the ordering derived by this function is closed under context, i.e., for any terms r, s, t and any position $u \in \mathcal{O}(r)$, if $g(s) < g(t)$ then $g(r[s]_u) < g(r[t]_u)$. Such a function g is effectively computable. In order to compare $\#(s)$ and $\#(t)$, we use lexicographic order $<_{\text{lex}}$. Note that $<_{\text{lex}}$ is a total order. A term s_0 is minimum in set Δ iff $s_0 \in \Delta$ and $\#(s_0) = \text{Min}(\{\#(s') \mid s' \in \Delta\})$.

Definition 11

- (1) Function $\text{linearize}(s)$ linearizes non-linear term s in the following manner. For each variable occurring more than once in s , the first occurrence is not renamed, and the other ones are replaced by new pairwise distinct variables. For example, $\text{linearize}(\text{nand}(x, x)) = \text{nand}(x, x_1)$. If function linearize replaces x by x_1 , then we use $x \equiv x_1$ to denote the replacement relation.
- (2) For set $\Delta \subseteq T$, $\text{Psub}(\Delta) = \{s|_u \mid s \in \Delta, u \in \mathcal{O}(s) \setminus \{\varepsilon\}\}$.
- (3) For set $\Delta \subseteq T$, $\text{Bud}(\Delta, R_C) = F_0 \cup \text{Psub}(\Delta \cup \{\beta \mid \alpha \rightarrow \beta \in R_C\})$. Note that if $\Delta \subseteq F_0$ then $\text{Bud}(\Delta, R_C) = \text{Bud}(\emptyset, R_C)$.
- (4) Substitution σ is *joinability preserving under relation* \equiv for TRS R_{rg} if $x\sigma \downarrow_{R_{\text{rg}}} x'\sigma$ whenever $x \equiv x'$. In this case, we write $\sigma \in \downarrow (\equiv, R_{\text{rg}})$.
- (5) For TRS R and term α , $R(\alpha) = \{\beta \mid \alpha \rightarrow \beta \in R\}$.
- (6) Let $\{s_1, \dots, s_m\} = R_C(d)$ and $\{u_1, \dots, u_n\} = \text{Min}(\cup_{1 \leq i \leq m} \mathcal{O}_{F_0}(s_i))$. Let d_j be the minimum term in $\{s_i|_{u_j} \in F_0 \mid 1 \leq i \leq m\}$, $1 \leq j \leq n$. Then we define $\text{Normalize}(d, R_C) = \{d \rightarrow s_1[d_1, \dots, d_n]_{(u_1, \dots, u_n)}\} \cup \{d_j \rightarrow s_i|_{u_j} \mid 1 \leq i \leq m, 1 \leq j \leq n, d_j \neq s_i|_{u_j}\}$. For example, $\text{Normalize}(t, \{t \rightarrow \text{not}(\text{not}(t)), t \rightarrow \text{not}(f)\}) = \{t \rightarrow \text{not}(f), f \rightarrow \text{not}(t)\}$.

Lemma 12 Let $R \setminus R_C$ be standard. Let $\alpha \rightarrow \beta \in \overline{R_{\text{rg}}}$, $\theta : X \rightarrow T$ and $s \rightarrow_{R_{\text{rg}}}^* \alpha\theta$. Let $\alpha' = \text{linearize}(\alpha)$. Then, there exists a substitution $\sigma : V(\alpha') \rightarrow \text{Bud}(\{s\}, R_C)$ such that $s \rightarrow_{R_{\text{rg}}}^* \alpha'\sigma \rightarrow_{R_{\text{rg}}}^* \alpha\theta$, $\beta\sigma \rightarrow_{R_{\text{rg}}}^* \beta\theta$ and $\sigma \in \downarrow (\equiv, R_{\text{rg}})$.

By Lemma 12, for a rewrite sequence $d \rightarrow_{R_{\text{rg}}}^* \alpha\theta \rightarrow \beta\theta$, there exists $\alpha'\sigma$ such that $d \rightarrow_{R_{\text{rg}}}^* \alpha'\sigma \rightarrow_{R_{\text{rg}}}^* \alpha\theta$ and $\beta\sigma \rightarrow_{R_{\text{rg}}}^* \beta\theta$. So, if we add a new ground rule $d \rightarrow \beta\sigma$ to R , then we have $d \rightarrow_{R'}^* \beta\theta$ for $R' = R_{\text{rg}} \cup \{d \rightarrow \beta\sigma\}$. Thus, by adding shortcut rules such as $d \rightarrow \beta\sigma$, we can omit applications of $\alpha \rightarrow \beta$ which is a non-right-ground rule. Using this technique, the following algorithm takes as input a standard semi-constructor TRS $R^{(i)}$ and produces as output an equivalent semi-constructor TRS $R^{(f)}$ satisfying that if $d \rightarrow_{R^{(i)}}^* s$ then $d \rightarrow_{R^{(f)}}^* s$. We call $R^{(f)}$ a *quasi-right-ground TRS*, hereafter.

function MakeQuasiRightGround(R)

```

 $R := \text{Determinize}(R);$ 
repeat
   $R' := R;$ 
   $R := \text{Determinize}(\text{AddShortcuts}(R'))$ 
until  $R = R'$ ;
return  $R$ 

```

function AddShortcuts(R)

```

 $R' := R;$ 
for each  $\alpha \rightarrow \beta \in \overline{R_{\text{rg}}}$  do
   $\alpha' := \text{linearize}(\alpha);$ 
  for each  $d \in F_0, \sigma : V(\alpha') \rightarrow \text{Bud}(\emptyset, R_C)$  such that  $\sigma \in \downarrow (\equiv, R_{\text{rg}})$  do
    if  $d \rightarrow_{R_{\text{rg}}}^* \alpha'\sigma$  then  $R' := R' \cup \{d \rightarrow \beta\sigma\}$ 
return  $R'$ 

```

function Determinize(R)

```

while there exists  $d$  such that  $|R_C(d)| > 1$  do
   $R := R \cup \text{Normalize}(d, R_C) \setminus \{d \rightarrow s \mid d \rightarrow s \in R_C\}$ 
return  $R$ 

```

Example 13 For TRS R_e of Example 1, $\text{MakeQuasiRightGround}(R_e)$ first computes $\text{Determinize}(R_e)$. It returns the same R_e as output. Next, $\text{AddShortcuts}(R_e)$ is called. Since $t \rightarrow \text{nand}(f, f), \text{nand}(x, x) \rightarrow \text{not}(x) \in R_e$, a new shortcut rule $t \rightarrow \text{not}(f)$ is added to R_e . Similarly, $f \rightarrow \text{not}(t)$ is added. Thus, $\text{AddShortcuts}(R_e) = R'$ where $R' = R_e \cup \{t \rightarrow \text{not}(f), f \rightarrow \text{not}(t)\}$. Next, $\text{Determinize}(R')$ is called and

returns the same R' as output. Then, $\text{AddShortcuts}(R')$ is called. Note that $R'_C = \{t \rightarrow \text{not}(f), f \rightarrow \text{not}(t)\}$. $\text{AddShortcuts}(R')$ returns the same R' and also calls $\text{Determinize}(R')$. Then, the algorithm halts. Let $R_e^{(f)}$ be this result: $R_e^{(f)} = R_e \cup \{t \rightarrow \text{not}(f), f \rightarrow \text{not}(t)\}$, which will be used in later examples.

We apply this algorithm to standard TRS. But by an application of this algorithm, the heights of some right-hand side terms of type C rules may become greater than 1. This algorithm satisfies the following lemmata.

Lemma 14 $\text{MakeQuasiRightGround}$ is terminating.

Lemma 15 Let $R^{(f)} = \text{MakeQuasiRightGround}(R^{(i)})$.

(1) If $d \rightarrow_{R^{(i)}}^* s$ then $d \rightarrow_{R_{\text{rg}}^{(f)}}^* s$.

(2) $\rightarrow_{R^{(f)}} \subseteq \downarrow_{R^{(i)}}$.

Corollary 16

(1) $R^{(f)}$ is confluent (since $R^{(i)}$ is confluent).

(2) $c \downarrow_{R^{(i)}} d$ iff $c \downarrow_{R_{\text{rg}}^{(f)}} d$.

(3) $s \approx t$ is $R^{(i)}$ -unifiable iff $s \approx t$ is $R^{(f)}$ -unifiable.

3.3 Auxiliary Terms

We have shown that all rewrite sequences from every constant in $R^{(i)}$ (i.e., $d \rightarrow_{R^{(i)}}^* s$) can be obtained by using only right-ground rules (i.e., $d \rightarrow_{R_{\text{rg}}^{(f)}}^* s$). Now, we want to extend this result to that for rewrite sequences from any term. For this purpose, we need the notion of auxiliary terms. For $\Delta \subseteq G$

function $\text{Aux}(\Delta)$

repeat

$\Delta' := \Delta;$

$\Delta := \text{AddTerms}(\Delta')$

until $\Delta = \Delta';$

return Δ

function $\text{AddTerms}(\Delta)$

$\Delta' := \Delta;$

for each $\alpha \rightarrow \beta \in \overline{R_{\text{rg}}^{(f)}}$ **do**

$\alpha' := \text{linearize}(\alpha);$

for each $s \in \Delta, p \in \mathcal{O}_{D \setminus F_0}(s),$

$\sigma : V(\alpha') \rightarrow \text{Bud}(\{s|_p\}, R_C^{(f)})$ such that $\sigma \in \downarrow (\equiv, R_{\text{rg}}^{(f)})$ **do**

if $s|_p \rightarrow_{R_{\text{rg}}^{(f)}}^* \alpha'\sigma$ **then** $\Delta' := \Delta' \cup \{s|\beta\sigma|_p\}$

return Δ'

Example 17 In TRS $R_e^{(f)}$ of Example 2,

$\text{Aux}(\{\text{not}(\text{nand}(t, t))\}) = \text{AddTerms}(\{\text{not}(\text{nand}(t, t))\}) = \{\text{not}(\text{nand}(t, t)), \text{not}(\text{not}(t))\}$.

Lemma 18 For any ground term s ,

(1) For any $s' \in \text{Aux}(\{s\})$, $\text{Aux}(\{s'\}) \subseteq \text{Aux}(\{s\})$.

(2) $\text{Aux}(\{s\})$ is finite and computable.

(3) For any $s' \in \text{Aux}(\{s\})$, $s' \downarrow_{R^{(f)}} s$.

(4) If $s \rightarrow_{R^{(i)}}^* t$ then there exists $s' \in \text{Aux}(\{s\})$ such that $s' \rightarrow_{R_{\text{rg}}^{(f)}}^* t$.

We call s' in Lemma 18(4) an *auxiliary term* of (s, t) . This will be used to transform non-right-ground rewrite sequence to right-ground rewrite sequence.

Example 19 For rewrite sequence $\text{not}(\text{nand}(t, t)) \rightarrow_{\text{rg}}^* \text{not}(\text{nand}(\text{not}(f), \text{not}(f))) \rightarrow \text{not}(\text{not}(\text{not}(f)))$, we can choose $\text{not}(\text{not}(t)) \in \text{Aux}(\{\text{not}(\text{nand}(t, t))\})$ and $\text{not}(\text{not}(t)) \rightarrow_{\text{rg}} \text{not}(\text{not}(\text{not}(f)))$.

3.4 Joinability for Confluent Semi-Constructor TRSs

Lemma 20 For any ground terms s and t , $s \downarrow_{R^{(i)}} t$ iff there exists $s' \in \text{Aux}(\{s\}), t' \in \text{Aux}(\{t\})$ such that $s' \downarrow_{R_g^{(i)}} t'$.

By Lemma 18(2) and decidability of $s' \downarrow_{R_g^{(i)}} t'$ [10], $s \downarrow_{R^{(i)}} t$ is decidable for ground terms s and t . If s or t is non-ground, $s \downarrow_{R^{(i)}} t$ is equivalent to $s\sigma \downarrow_{R^{(i)}} t\sigma$ where $\sigma : V(s) \cup V(t) \rightarrow F'_0$ is a bijection and F'_0 is a set of new pairwise distinct constants which do not appear in $R^{(i)}$. Thus, we have the following theorem.

Theorem 21 The joinability for confluent semi-constructor term rewriting systems is decidable.

By confluence, we have the following corollary too.

Corollary 22 The word problem for confluent semi-constructor term rewriting systems is decidable.

4 R-Unification

By using Theorem 21, we have the following theorem.

Theorem 23 The unification problem for confluent semi-constructor term rewriting systems is decidable.

5 Conclusion

In this paper, we have shown that the joinability and unification problems for confluent semi-constructor TRSs are decidable. But, reachability remains open. Obviously, the class of semi-constructor TRSs is a subclass of strongly weight-preserving TRSs, for which several sufficient conditions to ensure confluence are given in [3].

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References

- [1] F. Baader and T. Nipkow. *Term Rewriting and All That*. Cambridge University Press, 1998.
- [2] H. Comon, M. Haberstrau, and J.-P. Jouannaud. Syntacticness, cycle-syntacticness and shallow theories. *Information and Computation*, 111(1):154–191, 1994.
- [3] H. Gomi, M. Oyamaguchi, and Y. Ohta. On the Church-Rosser property of root-E-overlapping and strongly depth-preserving term rewriting systems. *Transactions of Information Processing Society of Japan*, 39(4):992–1005, 1998.
- [4] J.-M. Hullot. Canonical forms and unification. In *Proc. 5th Conf. on Automated Deduction*, pages 318–334. LNCS 87, 1980.
- [5] F. Jacquemard, C. Meyer, and C. Weidenbach. Unification in extensions of shallow equational theories. In *Proc. 9th Rewriting Techniques and Applications*, pages 76–90. LNCS 1379, 1998.
- [6] A. Martelli and G. Rossi. Efficient unification with infinite terms in logic programming. In *Proc. 5th Generation Computer Systems*, pages 202–209, 1984.
- [7] I. Mitsuhashi, M. Oyamaguchi, Y. Ohta, and T. Yamada. On the unification problem for confluent monadic term rewriting systems. *IPSJ Transactions on Programming*, 44(SIG 4(PRO 17)):54–66, 2003.

- [8] R. Nieuwenhuis. Basic paramodulation and decidable theories. In *Proc. 11th IEEE Symp. Logic in Computer Science*, pages 473–482, 1996.
- [9] M. Oyamaguchi. On the word problem for right-ground term-rewriting systems. *Trans. IEICE*, E73(5):718–723, 1990.
- [10] M. Oyamaguchi. The reachability and joinability problems for right-ground term-rewriting systems. *Journal of Information Processing*, 13(3):347–354, 1990.
- [11] M. Oyamaguchi and Y. Ohta. The unification problem for confluent right-ground term rewriting systems. *Information and Computation*, 183(2):187–211, 2003.
- [12] K. Salomaa. Deterministic tree pushdown automata and monadic tree rewriting systems. *Journal of Computer and System Sciences*, 37:367–394, 1988.
- [13] T. Takai, Y. Kaji, and H. Seki. Right-linear finite path overlapping term rewriting systems effectively preserve recognizability. In *Proc. 11th Rewriting Techniques and Applications*, pages 246–260.
- [14] Terese. *Term Rewriting Systems*. Cambridge University Press, 2003.