Title	Optimal Timing of Environmental Policy under Asymmetric Information
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Citation	京都大学大学院経済学研究科Working Paper (2005), 80
Issue Date	2005-08
URL	http://hdl.handle.net/2433/26701
Right	
Type	Research Paper
Textversion	author

Optimal Timing of Environmental Policy under Asymmetric Information

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August, 2005

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(First Version: August 10, 2005)

Abstract. This paper examines the optimal timing strategy of environmental policy in the presence of agency conflict due to asymmetric information. When the policy maker delegates the adoption of environmental policy to agents, contracts must be designed to provide incentive for agents to truthfully reveal private information. Using a contingent claims approach, this paper shows that an underlying option value of social welfare can be decomposed into two components: a policy maker's option and an agent's option. The value of social welfare in the asymmetric information setting is strictly lower than that in the full-information setting. In particular, the implied adoption strategy in the asymmetric information setting differs significantly from that in the full-information setting.

Keywords: environmental policy; option value; agency conflict; asymmetric information

JEL classification: Q28; L51; H23

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*I would like to thank Carl Chiarella and Masaaki Kijima for helpful comments and suggestions. This research was partially supported by a Grant-in-Aid for Scientific Research 2B 17710128.

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1 Introduction

The purpose of this paper is to derive the optimal timing strategy of environmental policy in the presence of agency conflict due to asymmetric information. We incorporate asymmetric information into the optimal timing problem in environmental economics.

In the recent research about the optimal timing of environmental policy, real options model has been often used[1]. The reasons are as follows. First, there is always uncertainty over the future costs and benefits of adopting a particular environmental policy. With global warming, for example, we do not know how much average temperatures will raise with or without the economic impact of higher temperatures. Second, there are usually important irreversibilities associated with environmental damage itself, but also with respect to the costs of adopting policies to reduce the damage. Third, policy adoption is rarely a now or never proposition; in most cases it is feasible to delay action and wait for new information. These uncertainties, irreversibilities, and possibilities of delay can significantly affect the optimal adoption timing of environmental policy. Thus a number of recent studies have examined the optimal timing of environmental policy, at times drawing upon the theory of uncertainties, irreversibilities, and possibilities of delay. For example, see Dixit and Pindyck (1994) and Pindyck (2000, 2002).

In practical situation, the policy maker has often delegated the adoption of environmental policies to agents, taking advantage of agents' special skills. In this situation, there is likely to be agency conflict due to asymmetric information. For example, the agent can observe the realized value of the cost expenditure over which there is uncertainty, while the policy maker cannot observe the realized value.

No principal-agent conflicts arise in the standard environmental economics literature, as it is assumed that the policy maker makes the adoption decision. However, when the policy maker delegates the adoption of environmental policy to agents under asymmetric information, it leads to what is called principal-agent conflicts. Then, the policy maker's problem is to design an optimal contract to provide incentives for agents to truthfully reveal their private information. Otherwise, the policy maker suffers some losses due to asymmetric information. What is of great interest is to derive the optimal contract under asymmetric information, and to calculate the value of social welfare using the contingent claims approach under principal-agent conflict.

The principal-agent setting leads to a decomposition of the underlying option into two components: a "policy maker's option" and an "agent's option." Importantly, there is a

[1] An excellent overview of the real options model is found in Dixit and Pindyck (1994). In the real options model, the project value of investment opportunity can be calculated by the option pricing theory in financial engineering. See, e.g., Chiarella (2002) or Kijima (2002) for details of the option pricing theory.

conflict between the interests of the policy maker and those of the agent, i.e., there is a conflict between a policy maker's option and an agent's option value. In such principal-agent conflicts, the agent attempts to increase his option value by using private information. This action of the agent, at the same time, decreases the policy maker's option value due to principal-agent conflicts. The contracts must be designed to provide incentive for agents to truthfully reveal private information and preserve the value of the policy maker's option.

In this paper, under the presence of principal-agent conflicts due to asymmetric information, we derive the optimal contracts and calculate the value of social welfare. This paper shows that the value of social welfare in the asymmetric information setting is significantly different from that implied by the first-best full-information setting. The result comes from the fact that agents display greater inertia in adoption of policy, in that they adopt the environmental policy later than implied by the first-best full-information solutions. Importantly, the value of social welfare in the asymmetric information setting is strictly lower than that in the full-information setting.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model. Section 3 simplifies the optimization problem and solves for the optimal contracts. In Section 4, I analyze the implications of the model in terms of the agency cost and the expected time lag due to principal-agent problem, and the comparative statics of the optimal contracts with respect to the key parameter (volatility). Section 5 concludes. Appendices contain the proofs and the solutions of the optimal contracts.

2 Model

In this section, we begin with a description of the model, we then define the value function. As a useful benchmark, we provide the solution to the first-best full-information problem. Finally, we present the problem in the asymmetric information setting.

2.1 Setup

Throughout our analysis, we suppose that agents are risk neutral. The policy maker (principal) has an option to adopt an environmental policy. We assume that the policy maker delegates the adoption of environmental policy to an agent. In this paper, we incorporate agency conflict into the optimal timing problem of environmental economics developed by Dixit and Pindyck (1994), and Pindyck (2000, 2002). This model is similar to the one developed by Grenadier and Wang (2005)[2]

Let M_t be a stock of environmental pollutants, e.g., the average concentration of CO_2 in the atmosphere or the acidity level of a lake. And let E_t be a rate of emission of the pollutant that controls M_t. We also begin with a restrictive assumption about the evolution of E_t: Until a policy is adopted, E_t stays at the constant level $E \in \mathbb{R}_+$. Once the policy is adopted, E_t falls immediately to zero, where it remains. The evolution of M_t is then given as:

$$\frac{\mathrm{d}M_t}{\mathrm{d}t} = \gamma E - \lambda M_t, \quad M_0 = m, \tag{1}$$

where λ is the natural rate at which the stock of pollutant dissipates over time.

By solving the ODE given (1), we can determines M_t as a function of time. Suppose the policy is adopted at time τ, so that $E_t = E$ for $0 \le t < \tau$ and $E_t = 0$ for $t \ge \tau$. Then,

$$M_t = \begin{cases} \left(m - \frac{\gamma E}{\lambda}\right) \mathrm{e}^{-\lambda t} + \frac{\gamma E}{\lambda} & \text{if} \quad 0 \le t < \tau, \\ \frac{\gamma E}{\lambda}\left(\mathrm{e}^{\lambda \tau} - 1\right) \mathrm{e}^{-\lambda t} + m\mathrm{e}^{-\lambda t} & \text{if} \quad t \ge \tau. \end{cases} \tag{2}$$

If the environmental policy is never adopted, the first line of (2) applies for all t, so that M_t asymptotically approaches $\gamma E/\lambda$. If the policy is adopted at time 0, then $M_t = m\mathrm{e}^{-\lambda t}$.

We will assume that the flow of social welfare (negative benefit) associated with the stock variable M_t can be specified by a function $B(M_t, X_t; E_t)$, where X_t shifts stochastically over time. For simplicity we will assume that B is linear in M:

$$B(X_t, M_t; E_t) = -X_t M_t. \tag{3}$$

And we will assume that X_t follows a geometric Brownian motion:

$$\frac{\mathrm{d}X_t}{X_t} = \mu \,\mathrm{d}t + \sigma \,\mathrm{d}z_t, \quad X_0 = x, \tag{4}$$

where μ, σ are constants, $(z_t)_{t \ge 0}$ is a standard Brownian motion.

We will assume that the expenditure in adopting the policy is completely sunk, and its cost expenditure at the time of adoption is I. The cost expenditure, I, may take one of two possible values: I_1 or I_2 with $I_2 > I_1$ where $I_i \in \mathbb{R}_+$ for $i \in \{1, 2\}$. We denote $\Delta I := I_2 - I_1$. We may regard a draw of I_1 as a "lower cost" expenditure and a draw of I_2 as a "higher cost" expenditure. The probability of drawing I_1 equals p, an exogenous variable.

Now we assume the cost expenditure is privately observed by the agent. Immediately after making a contract with the owner at time zero, the agent observes whether the

[2] Grenadier and Wang (2005) examines the investment timing under asymmetric information. We apply the model developed by Grenadier and Wang (2005) into the adoption timing on environmental economics.

cost expenditure is of "lower cost" or "higher cost". On the other hand, although the policy maker cannot observe the true value of I, he does observe the amount transferred to himself at the adoption time of environmental policy to be handed over by the agent. While the agent could attempt to hand over I_2 when the true value is I_1, it will be seen in equilibrium that the amount transferred to the policy maker at the adoption time of policy will always be the true value.

Although the policy maker cannot contract on the cost expenditure privately observed by the agent, he can contract on the observable component of the value, X_t. Contingent on the level of X_t at policy adoption when $I = I_i$, $x(I_i)$, the policy maker designs the optimal compensation $s(I_i)$ paid to the agent.

The assumption that a portion of the value is privately observed only by one (e.g., agent) and not observed by the other (e.g., policy maker) is quite common in the asymmetric information literature. This asymmetric information invites a host of principal-agent issues. An excellent overview of asymmetric information approach is found in Mas-Collel et al. (1995) and Salanié (2005).

In summary, the policy maker faces an optimization problem with asymmetric information. The policy maker needs to provide compensation incentive to induce the agent to reveal private information voluntarily and truthfully, by choosing the equilibrium policy adoption strategy.

2.2 Value Functions

Since the policy maker delegates the adoption of environmental policy to the agent, the underlying option value of social welfare is decomposed into two components: the policy maker's option and the agent's option. Thus the sum of these option values is equal to the option value of social welfare.

Let $V(x, m; I_i)$ denote the option value of the policy maker for $I = I_i$. The value, $V(x, m; I_i)$, can be written as

$$\begin{aligned} V(x, m; I_i) &= \mathsf{E}^x \left[-\int_0^{\tau_i} \mathrm{e}^{-rs} B(X_s, M_s; E) \mathrm{d}s + \int_{\tau_i}^{\infty} \mathrm{e}^{-rs} B(X_s, M_s; 0) \mathrm{d}s \right] \\ &\quad - \mathsf{E}^x \left[\mathrm{e}^{-r\tau_i} (I_i + s_i) \right], \end{aligned}$$

where r is the discount rate, τ_i is the adoption (stopping) time that the policy is implemented when $I = I_i$, s_i is the compensation paid to the agent for $I = I_i$, and $\mathsf{E}^x[\cdot]$ denotes the expectation operator given that $X_0 = x$. Using the fact that the strong Markov property and the time homogeneity of the geometric Brownian motion, X, we can rewrite the value function as follows:

$$
\begin{aligned}
V(x,m;I_i) \;&=\; \mathsf{E}^x[\mathrm{e}^{-r\tau_i}]\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_i - I_i - s_i\right) \\
&\quad -\frac{xm}{(r+\lambda-\mu)} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},
\end{aligned}
$$

Here, we define the Laplace transform of the adoption time by:

$$
\mathsf{E}^x[\mathrm{e}^{-r\tau_i}] = W(x;x_i), \quad x < x_i,
$$

for trigger $x_i := x(I_i)$, $i \in \{1,2\}$. Then it is important to note that $\tau_i := \inf\{t \geq 0 : X_t = x_i\}$. Since the realized value of I can be either I_1 or I_2, we denote the triggers by $x_1 = x(I_1)$ and $x_2 = x(I_2)$, respectively. Then, the Laplace transform of the adoption time satisfies the ordinary differential equation:

$$
\frac{x^2\sigma^2}{2}\,\mathrm{d}^2W(x;x_i) + \mu x\,\mathrm{d}W(x;x_i) - r\,W(x;x_i) = 0, \tag{5}
$$

subject to the boundary condition that $W(x_i;x_i) = 1$ and $W(0;x_i) = 0$. As we show in Appendix, solving the ordinary differential equation for $W(x;x_i)$, one obtains the following results:

Lemma 2.1 *The Laplace transform of the adoption time is obtained by:*

$$
\mathsf{E}^x[\mathrm{e}^{-r\tau_i}] = \left(\frac{x}{x_i}\right)^{\beta}, \quad x < x_i, \tag{6}
$$

where β is the positive root of quadratic equation:

$$
Q(y) = y(y-1)\frac{\sigma^2}{2} + y\mu - r = 0. \tag{7}
$$

Moreover, the expected adoption time is obtained by:

$$
\mathsf{E}^x\left[\tau_i\right] = \frac{1}{\mu - \frac{1}{2}\sigma^2}\log\left(\frac{x_i}{x}\right), \quad x < x_i. \tag{8}
$$

We assume that $\mu > \frac{1}{2}\sigma^2$ for this expectation to exist. So, under such a condition, the expected adoption time is decreasing in the mean growth rate μ. In particular, the expected adoption time is increasing in the volatility, σ. This result is exactly the same as the one in the standard option-pricing literature.

Using Lemma 2.1, we can rewrite the option value of the policy maker for $I = I_i$:

$$
\begin{aligned}
V(x,m;I_i) \;&=\; \left(\frac{x}{x_i}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_i - I_i - s_i\right\} \\
&\quad -\frac{xm}{r+\lambda-\mu} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},
\end{aligned} \tag{9}
$$

where $x < x_i$.

We denote the options value of the policy maker by $\pi_p(x, m)$. Then, since the option value of the policy maker is defined by $\pi_p(x, m) = p\,V(x, m; I_1) + (1-p)\,V(x, m; I_2)$, the option value of the policy maker, $\pi_p(x, m; x_1, x_2, s_1, s_2)$, is equal to:

$$\begin{aligned}\pi_p(x, m; x_1, x_2, s_1, s_2) \;=\;& p\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1 - s_1\right) \\ &+(1-p)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2 - s_2\right) \\ &-\frac{xm}{r+\lambda-\mu} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},\end{aligned} \tag{10}$$

where $x < x_1$.

On the other hand, the agent has a payoff function s_1 if $I = I_1$ and s_2 if $I = I_2$. We will assume that the agent incurs a cost ξ if he make employment contract at time zero. Similar to the derivation of the policy maker's value, the options value of the agent, $\pi_a(x; x_1, x_2, s_1, s_2)$, can be written as:

$$\pi_a(x; x_1, x_2, s_1, s_2) = p\left(\frac{x}{x_1}\right)^{\beta} s_1 + (1-p)\left(\frac{x}{x_2}\right)^{\beta} s_2 - \xi, \tag{11}$$

where $x < x_1$.

Thus, since the value of social welfare is defined as $\pi = \pi_p + \pi_a$, we can write it as follows:

$$\begin{aligned}\pi(x, m; x_1, x_2) \;=\;& p\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1\right) \\ &+(1-p)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2\right) - \xi \\ &-\frac{xm}{r+\lambda-\mu} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},\end{aligned} \tag{12}$$

where $x < x_1$.

In this section, the value function has been derived. However, the triggers to which the environment policy is adopted have not been decided. Before analyzing the optimal trigger in the asymmetric information, we first briefly review the full-information solution used as the benchmark.

2.3 A Full-Information Setting

It is useful to begin our analysis by looking at the optimal contract problem when I is publicly observable by both the policy maker and the agent. Deriving the full-information solution, we will show that it turns out to be the first-best solution.

When the policy maker can observe the realized value of I, the policy maker must make the contract to the agent by $s_1 = s_2 = s$. Hence we form the following optimization problem:

$$\begin{aligned}\max_{x_1,x_2,s} \quad & p\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1 - s\right) \\ & +(1-p)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2 - s\right) \\ & -\frac{xm}{r+\lambda-\mu} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},\end{aligned} \tag{13}$$

subject to one constraint:

$$p\left(\frac{x}{x_1}\right)^{\beta} s + (1-p)\left(\frac{x}{x_2}\right)^{\beta} s - \xi \geq 0. \tag{14}$$

Constraint (14) ensures that the agent accept the employment contract. The policy maker's problem can be summarized as the solution to the objective function (13) subject to (14).

Now describe the optimal contract in the full-information setting. The proofs detailing the solutions are provided in Appendix.

Proposition 2.1 *In the full-information setting, the optimal contracts* (x_1, x_2, s) *are as follows:*

$$x_1 = x_1^*, \quad x_2 = x_2^*, \quad s = s^* := \left(p\left(\frac{x}{x_1^*}\right)^{\beta} + (1-p)\left(\frac{x}{x_2^*}\right)^{\beta}\right)^{-1}\xi,$$

where

$$x_i^* = x^*(I_i) := \frac{\beta}{\beta-1}\frac{(r-\mu)(r+\lambda-\mu)}{\gamma E} I_i. \tag{15}$$

Moreover, the optimal triggers in the full-information setting turn out to be those to maximize the value of social welfare. That is,

$$(x_1^*, x_2^*) = \arg\max_{(x_1,x_2)} \pi(x, m; x_1, x_2). \tag{16}$$

From the second statement of Proposition 2.1, triggers (x_1^*, x_2^*) will be called the first-best full-information ones. From the first statement, we state the following remark:

Remark 2.1 *The first-best full-information trigger for the realized state* I_1 *is strictly smaller than that for* I_2*, i.e.,* $x_1^* < x_2^*$

Remark 2.1 implies that the trigger for having the "low cost" expenditure is strictly smaller than for having "high cost." In general, the agent having the "high cost" expenditure will display greater inertia in its behavior than the agent having "low cost."

Let superscript "$*$" on the value be the first-best full-information one, for example, $\pi^*(x,m) = \pi(x,m;x_1^*,x_2^*)$. Then substituting the solutions $x_1 = x_1^*, x_2 = x_2^*, s = s^*$ into the agent's value function (11), we can obtain the following result:

Corollary 2.1 *In the first-best full-information setting, the optimal contracts keep the value of the agent zero, i.e.,* $\pi_a^*(x) = 0$.

Since the value of social welfare is defined as $\pi(x,m) = \pi_p(x,m) + \pi_a(x)$, Corollary 2.1 leads to the following result.

Lemma 2.2 *In the first-best full-information setting, the value of social welfare,* $\pi^*(x,m)$*, is equal to:*

$$\begin{aligned} \pi^*(x,m) &= p\left(\frac{x}{x_1^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1^* - I_1\right\} \\ &\quad +(1-p)\left(\frac{x}{x_2^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2^* - I_2\right\} - \xi \\ &\quad -\frac{xm}{r+\lambda-\mu} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)}, \end{aligned} \tag{17}$$

for $x < x_1^*$.

The function in equation (17) has simple intuitive interpretations; the first two terms represent an options value with each probability p and $1-p$, respectively. The last two terms correspond to the discounted value of evolving social welfare.

Finally, we examine the comparative statics with respect to the key parameter (volatility), σ. As we show in Appendix, one can obtain the following result.

Lemma 2.3 *In the first-best full-information setting, the trigger* x_i^* *is increasing in the volatility for* $i \in \{1,2\}$*. Moreover, the value of social welfare is increasing in the volatility.*

Lemma 2.3 implies that uncertainty delays the adoption of environmental policy. This result is exactly the same as the one in the standard option-pricing literature.

2.4 An Asymmetric Information Setting

In an asymmetric information setting, the policy maker will make the employment contract in order to induce the agent to do the truth-telling action at adoption. Otherwise, the principal will suffer some loss due to asymmetric information. Thus the policy maker must attempt to design the contract to reveal truthfully private information.

Since there are only two possible value of I_i, for any s_i scheduled, there can be at most two exercise/subsidy trigger pairs that will be chosen by the agent. Thus, the contract is modeled as a mechanism, $\mathcal{M} = \{x(\tilde{I}), s(\tilde{I}); \tilde{I} \in \{I_1, I_2\}\}$, which may be contingent on a reported $\tilde{I}$. Since the revelation principle ensures that the agent truthfully reveals a true I as private information[3], we will make no distinction between a reported $\tilde{I}$ and a true I. Thus we will drop the suffix "tilde" on the reported $\tilde{I}$ and simply write the reported type as I.

In the asymmetric information setting, the policy maker sets the contract pairs in order to induce the agent to engage in truth-telling action at adoption. In order to accomplish these objectives, the principal must attempt to design two types of constraints: the incentive compatibility and participation constraints.

The incentive compatibility constraint ensures that the agent will adopt the environmental policy in accordance with the policy maker's expectations. Specifically, the agent having a I_1 type privately observed cost will adopt the policy at x_1, and the agent having I_2 will adopt the policy at x_2. To provide such a timing incentive, the agent must not have any incentive to divert value. These conditions ensure that this value diversion does not occur. The incentive compatibility constraints in this model are as follows:

$$\left(\frac{x}{x_1}\right)^{\beta} s_1 \geq \left(\frac{x}{x_2}\right)^{\beta} (s_2 + \Delta I), \tag{18}$$

$$\left(\frac{x}{x_1}\right)^{\beta} (s_1 - \Delta I) \leq \left(\frac{x}{x_2}\right)^{\beta} s_2. \tag{19}$$

Constraints (18) and (19) are the incentive compatibility constraints for the agent in state I_1 and I_2, respectively. Consider, for example, constraint (18). The agent's payoff in state I_1 is $(x/x_1)^{\beta} s_1$ if he tell the truth, but it is $(x/x_1)^{\beta}(s_2 + \Delta I)$ if he instead claims that it is state I_2. Thus, he will tell the truth if (18) is satisfied. Constraint (19) follows similarly. Constraint (19) will be shown not to be binding, so only constraint (19) is relevant to our discussion.

On the other hand, the participation constraints in this model are as follows:

$$s_1 \geq 0, \tag{20}$$

$$s_2 \geq 0. \tag{21}$$

Note that non-negative s_1 and s_2 insures that the agent makes an agreement about employment. For example, if $s_2 < 0$, then the agent would refuse the contract on learning

[3] See, e.g., Mas-Colell et al. (1995), and Salanié (2005) for details of the revelation principle.

that $I = I_2$. Thus, we assume a non-negative compensation. Moreover, we will assume that the policy maker pay the cost incured to the agent at time zero when the policy maker makes employment contract.

In sum, in the asymmetric information setting, the policy maker's problem can be summarized as the maximization of its objective function, subject to the four inequality constraints (18) to (21). Fortunately, we will find in the next section that the problem can be simplified in that we can reduce the number of constraints to only one.

3 Model Solution

In this section, we provide the solution to the optimal contract problem described in the previous section: maximizing the policy maker's value function subject to the four inequality constraints (18) to (21).

3.1 A Simplified Statement of the Principal-Agent Problem

We can form the optimization problem as:

$$\begin{aligned}\max_{x_1,x_2,s_1,s_2} \quad & p\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1 - s_1\right) \\ & +(1-p)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2 - s_2\right) \\ & -\frac{xm}{(r+\lambda-\mu)} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},\end{aligned} \tag{22}$$

subject to four constraints:(18),(19),(20),(21). We now proceed to characterize the solution to problem (22) through a series of steps. Proofs of these procedures are shown in Appendix.

Following the steps given in Appendix, we can simplify the optimization problem noted above as (22) and show that we can determine the optimal contract by solving the following optimization problem:

$$\begin{aligned}\max_{x_1,x_2,s_1} \quad & p\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1 - s_1\right) \\ & +(1-p)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2\right) \\ & -\frac{xm}{(r+\lambda-\mu)} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},\end{aligned} \tag{23}$$

subject to only one constraint:

$$\left(\frac{x}{x_1}\right)^{\beta} s_1 \geq \left(\frac{x}{x_2}\right)^{\beta} \Delta I, \tag{24}$$

where $x < x_1^*$. It is important to note that we can simply substitute $s_2 = 0$ into the problem.

We now simplified optimization problem for the policy maker. In the next subsection, we provide the solution to the optimal contract problem.

3.2 Optimal Contract

Now describe the optimal contracts. The proofs detailing the solutions are provided in Appendix.

Proposition 3.1 *In the asymmetric information setting, the optimal contracts* (x_1, x_2, s_1, s_2) *are as follows:*

$$x_1 = x_1^*, \quad x_2 = x_3^*, \quad s_1 = s_1^* := \left(\frac{x_1^*}{x_3^*}\right)^\beta \Delta I, \quad s_2 = s_2^* = 0. \tag{25}$$

where x_i^* *is defined by (15) and* I_3 *is defined by*

$$I_3 := I_2 + \frac{p}{1-p}\Delta I. \tag{26}$$

Proposition 3.1 implies that the agent adopt the environmental policy at the first time that x hits x_1^* if $I = I_1$, and that x hits if $I = I_2$. Moreover, the policy maker gives s_1 to the agent if the policy is adopted at x_1^*, gives nothing if adopted at x_3^*.

The first property of the solution is that the agent having state I_1 will always adopt the environmental policy at the trigger, x_1^*.

Remark 3.1 *In the asymmetric information setting, the optimal contract has* $x_1 = x_1^*$.

Remark 3.1 implies that the trigger in the asymmetric information setting is exactly the same as that in the first-best full-information setting. As we will show see, it is less costly for the policy maker to distort x_2 from the first-best level than to distort x_1 away from the first-best level in order to provide the appropriate incentive to the agent.

The second property of the solution is that the agent having state I_2 will not adopt the environmental policy at the full-information trigger, x_2^*. Intuitively, the necessity of ensuring that the agent having state I_2 does not imitate the one having state I_1 leads the agent having state I_2 to display a greater option to wait than the first-best full-information solution. In order to dissuade the agent having state I_1 from adopting the policy at x_2^*, the distance between triggers in the asymmetric information setting is sufficiently larger than that in the first-best full-information setting.

Lemma 3.1 *In the asymmetric information setting, the adoption trigger for an agent having high cost, I_2, is strictly bigger than that in the first-best full-information setting, i.e., $x_3^* > x_2^*$.*

The third property of the solution is that the policy maker sets the optimal compensation s_1 according to the level of triggers. The compensation s_1 is the present value of information rent paid to the agent in order to provide the incentive to truthfully reveal private information. Also, as ΔI increases, the agent has a greater incentive to divert this difference in value. Thus, an increase in ΔI increases the optimal compensation s_1.

The last property of the solution is that the policy maker keeps the optimal compensation s_2 zero. The intuition is straightforward. Giving the agent having high cost I_2 positive rent implies higher rent for the agent of state I_1 in order to induce the agent to engage in truth-telling at adoption. In order to minimize these rents, it is optimal for the principal to keep s_2 zero.

Let superscript "$**$" on the value be the asymmetric information one, for example, $\pi^{**}(x,m) = \pi(x,m;x_1^*,x_3^*,s_1^*,s_2^*)$. Substituting the solutions into each value, we can obtain the following results.

Proposition 3.2 *In the asymmetric information setting, the policy maker and the agent options value, π_p^{**} and π_a^{**}, respectively, can be written as:*

$$
\begin{aligned}
\pi_p^{**}(x,m) \;=\; & p\left(\frac{x}{x_1^*}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1^* - I_1\right) \\
& +(1-p)\left(\frac{x}{x_3^*}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_3^* - I_2\right) - p\left(\frac{x}{x_3^*}\right)^{\beta}\Delta I - \xi \\
& -\frac{xm}{(r+\lambda-\mu)} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},
\end{aligned}
\tag{27}
$$

and

$$
\pi_a^{**}(x) \;=\; p\left(\frac{x}{x_3^*}\right)^{\beta}\Delta I, \tag{28}
$$

where $x < x_1^*$. The value of social welfare, π^{**}, can be written as:

$$
\begin{aligned}
\pi^{**}(x,m) \;=\; & p\left(\frac{x}{x_1^*}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1^* - I_1\right) \\
& +(1-p)\left(\frac{x}{x_3^*}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_3^* - I_2\right) - \xi \\
& -\frac{xm}{(r+\lambda-\mu)} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)},
\end{aligned}
\tag{29}
$$

where $x < x_1^*$.

It is interesting to note that the options value of social welfare is equivalent to the first-best full-information value in which x_3^* is replaced by x_2^*. However, that difference follows the next result.

Corollary 3.1 *The value of social welfare in the asymmetric information setting, $\pi^{**}(x, m)$, is strictly lower than that in the first-best full-information setting, $\pi^*(x, m)$, i.e., $\pi^{**}(x, m) < \pi^*(x, m)$.*

Importantly, the agency conflict due to asymmetric information problem leads to a decrease in the value of social welfare.

4 Model Implications

In this section, we analyze several of the more important implications of the model. First, Subsection 4.1 examines the agency cost due to asymmetric information. Second, Subsection 4.2 demonstrates the asset substitution by examining the comparative statics with respect to the key parameter of the model.

4.1 Agency Cost

Although the policy maker chooses the value-maximizing contract to provide an incentive for the agent to truthfully reveal private information, the principal-agent problem ultimately proves costly. So, in this situation, there will be an agency cost due to the suboptimal strategy.

We refer to the difference between the first-best option value and the suboptimal option value. Thus, we can define the agency cost due to principal-agent issues as C, where $C := \pi^*(x, m) - \pi^{**}(x, m)$ for $x < x_1^*$. The agency cost turns out to be:

$$\begin{aligned} C \quad = \quad & (1-p)\left(\frac{x}{x_2^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2^* - I_2\right\} \\ & -(1-p)\left(\frac{x}{x_3^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_3^* - I_2\right\}. \end{aligned} \tag{30}$$

It is important to note that the agency cost is strictly positive, because of Corollary 3.1. Furthermore, we conclude that the agency cost is driven by the distance of the trigger x_3^* form x_2^*.

Now we consider the expected time lag caused by the distance of the triggers. We define the expected adoption times in the full and asymmetric information setting by:

$$p\,\mathsf{E}^x[\tau_1^*] + (1-p)\,\mathsf{E}^x[\tau_2^*], \quad p\,\mathsf{E}^x[\tau_1^*] + (1-p)\,\mathsf{E}^x[\tau_3^*],$$

respectively. Then the expected time lag can be defined by the difference between the above expected adoption times. So, the expected time lag, $\Psi := (1-p)\mathsf{E}^x\left[\tau_3^* - \tau_2^*\right]$, is given by:

$$\Psi = \frac{(1-p)}{\mu - \frac{1}{2}\sigma^2} \log \left(\frac{I_2 + \frac{p}{1-p}\Delta I}{I_2} \right). \tag{31}$$

The proof is straightforward from Lemma 2.1.

It is important to note that the expected time lag depends upon only five parameters, μ, σ^2, p, I_1, and I_2. An increase in ΔI increases the expected time lag. Thus the greater the distance between triggers, the larger in the expected time lag the agent adopts. In particular, the expected time lag is increasing in the volatility σ.

4.2 Asset Substitution

To get to a deeper understanding of the insights of the model, we now perturb some of the key parameters of the model and analyze their impacts on the optimal contract pairs and the value. In this subsection, we examine the sensitivity of the optimal contract and value with respect to the volatility σ. It may be recalled that the contracts and values, (x_1, x_2, s_1) and (π_p, π_a, π), respectively, are obtained in Proposition 3.1 to 3.2. We begin with the impact to the solutions. The proof is given by Appendix.

Lemma 4.1 *In the asymmetric information setting, an increase in the volatility σ increases the contract pairs, (x_1, x_2, s_1).*

On the other hand, we examine the comparative statics of the policy maker's and agent's options values with respect to the volatility.

Lemma 4.2 *In the asymmetric information setting, an increase in the volatility increases the policy maker's value, has an ambiguous effect on the agent's value.*

Importantly, an increase in the volatility may possibly give rise to what is called "asset substitution." If the underlying state is relatively high, in that,

$$\left| \log \left(\frac{x}{x_3^*} \right) \right| < \left| \frac{1}{\beta - 1} \right|, \tag{32}$$

then an increase in the volatility increases the agent's option value. Therefore, if (32) is satisfied, an increase in the volatility increases the policy maker's value, while it increases the agent's value. These results imply that an increase in the volatility shifts wealth from the agent to the policy maker. This possibility to transfer wealth is known as "asset substitution." Naturally, since the sum of these two values is the value of social welfare, whether this sum is increasing or decreasing in the volatility is an interesting question. The result is as follows:

Corollary 4.1 *In the asymmetric information setting, an increase in the volatility has an ambiguous effect on the value of social welfare.*

Hence in the asymmetric information setting, the impact with respect to the volatility may be different from the one in the first-best full-information setting.

5 Concluding Remark

This paper extends the optimal timing in environmental policy model to account for asymmetric information between the policy maker and the agent. Asymmetric Information leads to principal-agent conflict. When the adoption of policy is delegated to agents under asymmetric information, employment contracts must be designed to provide incentives for agents to truthfully reveal their private information. This paper presents a model of optimal contracting in a continuous-time principal-agent setting in which there is asymmetric information. The implied behavior at adoption differs significantly from that of the first-best full-information solution. In particular, there will be greater inertia in adoption, as the model predicts that the agent will have a more valuable option to wait than the policy maker. The value of social welfare in the asymmetric information setting is strictly lower than that in the first-best full-information setting.

Appendix (Proof of Lemmas and Propositions)

Proof of Lemma 2.1 The first statement is shown by using the standard arguments (e.g., Dixit and Pindyck, 1994). Here, we show the second statement in Lemma 2.1. Differentiating (6) with respect to r yields

$$\mathsf{E}^x[\tau_i \mathrm{e}^{-r\tau_i}] = \left(\frac{x}{x_i}\right)^{\beta} \log\left(\frac{x_i}{x}\right) \left(\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2r\sigma^2\right)^{-1/2}.$$

Taking r into zero gives the result. ¤

Proof of Proposition 2.1 Now we form the Lagrangian as follows:

$$\max_{x_1,x_2,s,\phi} \quad \mathcal{L} \quad = \quad p\left(\frac{x}{x_1}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1 - s\right\}$$

$$
\begin{aligned}
&+(1-p)\left(\frac{x}{x_2}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2 - s\right\} \\
&-\frac{xm}{r+\lambda-\mu} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)} \\
&+\phi\left[p\left(\frac{x}{x_1}\right)^{\beta}s + (1-p)\left(\frac{x}{x_2}\right)^{\beta}s - \xi\right],
\end{aligned}
$$

where ϕ denote the multiplier on the constraint. The first-order conditions with respect to x_i yield:

$$
\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_i - I_i\right\}\frac{-\beta}{x_i} + \frac{\gamma E}{(r-\mu)(r+\lambda-\mu)} = 0, \quad i \in \{1,2\}.
$$

Here we can obtain the solution. Moreover, now we define the first-best solution by x_i^{FB} which maximize the value of social welfare, i.e.,

$$
x_i^{\mathsf{FB}} := \underset{\hat{x}_i}{\operatorname{argmax}} \left\{\left(\frac{x}{\hat{x}_i}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}\hat{x}_i - I_i\right)\right\}, \quad i \in \{1,2\}.
$$

This shows that $x_i^* = x_i^{\mathsf{FB}}$ for $i \in \{1,2\}$. $\square$

Proof of Lemma 2.3

For the first statement, differentiating the trigger with respect to σ yields

$$
\frac{\mathrm{d}x_i^*}{\mathrm{d}\sigma} = \frac{\partial x_i^*}{\partial \beta}\frac{\partial \beta}{\partial \sigma} = \left(\frac{-1}{(\beta-1)^2}\frac{(r-\mu)(r+\lambda-\mu)}{\gamma E}I_i\right)\frac{\partial \beta}{\partial \sigma}, \quad i \in \{1,2\}.
$$

Here, differentiating $Q(y)$ defined by (7) with respect to σ yields:

$$
\frac{\partial Q}{\partial \beta}\frac{\partial \beta}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0.
$$

Since $(\frac{\partial Q}{\partial \beta}) > 0$ and $(\frac{\partial Q}{\partial \sigma}) > 0$, we obtain $(\frac{\partial \beta}{\partial \sigma}) < 0$. Thus the triggers are increasing in the volatility, i.e., $(\mathrm{d}x_i^*/\mathrm{d}\sigma) > 0$ for $i \in \{1,2\}$.

For the second statement, differentiating the policy maker's value with respect to σ yields:

$$
\begin{aligned}
\frac{\mathrm{d}\pi_p^*(x,m)}{\mathrm{d}\sigma} &= \left(\frac{\partial \pi_p^*}{\partial \beta} + \frac{\partial \pi_p^*}{\partial x_1^*}\frac{\partial x_1^*}{\partial \beta} + \frac{\partial \pi_p^*}{\partial x_2^*}\frac{\partial x_2^*}{\partial \beta}\right) \\
&= p\left(\frac{x}{x_1^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1^* - I_1\right\}\log\left(\frac{x}{x_1^*}\right) \\
&\quad +(1-p)\left(\frac{x}{x_2^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2^* - I_2\right\}\log\left(\frac{x}{x_2^*}\right),
\end{aligned}
$$

where we have used the envelope theorem, $(\partial \pi_p^*/\partial x_i^*) = 0$ for $i \in \{1,2\}$. Thus we can show $(\mathrm{d}\pi_p^*/\mathrm{d}\sigma) > 0$. On the other hand, since $\pi_a^*(x) = 0$, an increase in the volatility increases the sum of these values, the value of social welfare, $\pi^*(x,m)$. $\square$

Proof of Simplified Statements In order to simplify the optimization problem, we show three lemmas, Lemma A.1 to A.3.

Lemma A.1 *(20) is not binding, i.e.,* $s_1 > 0$.

(proof)

$$s_1 \geq \left(\frac{x_1}{x_2}\right)^{\beta} (s_2 + \Delta I) \geq \left(\frac{x_1}{x_2}\right)^{\beta} \Delta I > 0.$$

The first and second equalities follow from (18) and (21), respectively. $\square$

Lemma A.2 *(21) is binding, i.e.,* $s_2 = 0$.

(proof) Now we can form the Lagrangian as follows:

$$\begin{aligned}
\max_{x_1,x_2,s_1,s_2,\phi_1,\phi_2,\phi_3} \mathcal{L} \quad = \quad & p \left(\frac{x}{x_1}\right)^{\beta} \left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)} x_1 - I_1 - s_1\right) \\
& +(1-p) \left(\frac{x}{x_2}\right)^{\beta} \left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)} x_2 - I_2 - s_2\right) \\
& - \frac{xm}{(r+\lambda-\mu)} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)} \\
& +\phi_1 \left[\left(\frac{x}{x_1}\right)^{\beta} s_1 - \left(\frac{x}{x_2}\right)^{\beta} (s_2 + \Delta I)\right] \\
& +\phi_2 \left[\left(\frac{x}{x_2}\right)^{\beta} s_2 - \left(\frac{x}{x_1}\right)^{\beta} (s_1 - \Delta I)\right] + \phi_3 s_2,
\end{aligned}$$

where ϕ_i denote the multiplier on these constaints ($i = \{1, 2, 3\}$). The first-order conditions with respect to s_1 and s_2 gives:

$$\begin{aligned}
& \left(\frac{x}{x_1}\right)^{\beta} (-p + \phi_1 - \phi_2) = 0, \\
& \left(\frac{x}{x_2}\right)^{\beta} \{-(1-p) - \phi_1 + \phi_2)\} + \phi_3 = 0.
\end{aligned}$$

These two equations show that $\phi_3 > 0$. $\square$

Lemma A.3 *(19) is not binding.*

(proof) Substituting $s_2 = 0$ into (19) gives

$$\left(\frac{x}{x_1}\right)^{\beta} (s_1 - \Delta I) \leq 0.$$

Then, $s_1 \leq \Delta I$ is satisfied from the above equation. There we obtain $s_1 < \Delta I$ readily by the policy maker's payoff maximization problem. $\square$

Proof of Proposition 3.1 We form the Lagrangian as follows:

$$
\begin{aligned}
\max_{x_1,x_2,s_1,\phi} \mathcal{L} \;=\;& p\left(\frac{x}{x_1}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1 - s_1\right)\\
&+(1-p)\left(\frac{x}{x_2}\right)^{\beta}\left(\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2\right)\\
&-\frac{xm}{(r+\lambda-\mu)} - \frac{\gamma E x}{(r-\mu)(r+\lambda-\mu)} + \phi\left[\left(\frac{x}{x_1}\right)^{\beta} s_1 - \left(\frac{x}{x_2}\right)^{\beta}\Delta I\right],
\end{aligned}
$$

where ϕ denote the multiplier on these constraints. The first-order conditions with respect to x_1, x_2, and s_1 gives:

$$
\begin{aligned}
& p\left(1+\frac{-\beta}{x_1}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1 - I_1 - s_1\right\}\right) + \phi\frac{-\beta}{x_1}s_1 = 0,\\
& (1-p)\left(1+\frac{-\beta}{x_2}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_2 - I_2\right\}\right) - \phi\frac{-\beta}{x_2}\Delta I = 0,\\
& -p+\phi = 0.
\end{aligned}
$$

Rearranging these constraints gives the solutions. ¤

Proof of Lemma 4.1 For the first statement, the proof is exactly the same as that in the proof of Lemma 2.3. For the second statement, differentiating the compensation s_1 with respect to σ yields:

$$
\begin{aligned}
\frac{\mathrm{d}s_1}{\mathrm{d}\sigma} &= \left(\frac{\partial s_1}{\partial\beta} + \frac{\partial s_1}{\partial x_1^*}\frac{\partial x_1^*}{\partial\beta} + \frac{\partial s_1}{\partial x_3^*}\frac{\partial x_3^*}{\partial\beta}\right)\frac{\partial\beta}{\partial\sigma}\\
&= \left(\frac{x_1^*}{x_3^*}\right)^{\beta}\Delta I\left\{\log\left(\frac{x_1^*}{x_3^*}\right) + (-\beta)\left((x_1^*)^{-1}\frac{\partial x_1^*}{\partial\beta} - (x_3^*)^{-1}\frac{\partial x_3^*}{\partial\beta}\right)\right\}\frac{\partial\beta}{\partial\sigma}\\
&= \left(\frac{x_1^*}{x_3^*}\right)^{\beta}\Delta I\left\{\log\left(\frac{x_1^*}{x_3^*}\right)\right\}\frac{\partial\beta}{\partial\sigma} > 0,
\end{aligned}
$$

where the step from second to third equation follows from the fact that $\frac{\partial x_i^*}{\partial\beta} = \frac{-x_i^*}{\beta(\beta-1)}$. Since $\log(x_1^*/x_3^*) < 0$ because of $x_1^* < x_3^*$, we obtain the positive sign. Thus the compensation is increasing in the volatility. ¤

Proof of Lemma 4.2 Differentiating the policy maker's value with respect to σ gives:

$$
\begin{aligned}
\frac{\mathrm{d}\pi_p^{**}(x,m)}{\mathrm{d}\sigma} &= \left(\frac{\partial\pi_p^{**}}{\partial\beta} + \frac{\partial\pi_p^{**}}{\partial x_1^*}\frac{\partial x_1^*}{\partial\beta} + \frac{\partial\pi_p^{**}}{\partial x_3^*}\frac{\partial x_3^*}{\partial\beta}\right)\frac{\partial\beta}{\partial\sigma}\\
&= p\left(\frac{x}{x_1^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_1^* - I_1\right\}\log\left(\frac{x}{x_1^*}\right)\frac{\partial\beta}{\partial\sigma}\\
&\quad+(1-p)\left(\frac{x}{x_3^*}\right)^{\beta}\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)}x_3^* - I_3\right\}\log\left(\frac{x}{x_3^*}\right)\frac{\partial\beta}{\partial\sigma} > 0,
\end{aligned}
$$

where the deviation from first to second equation follows from the envelope theorem $\frac{\partial \pi_p^{**}(x,m)}{\partial x_i^*} = 0$. See, e.g., Mas-Collel et al. (1995) for details about the envelope theorem. The positive sign of the last equation follows from the fact that $\left\{\frac{\gamma E}{(r-\mu)(r+\lambda-\mu)} x_i^* - I_i\right\} > 0$, $\log\left(\frac{x}{x_i^*}\right) < 0$, and $\left(\frac{\partial \beta}{\partial \sigma}\right) < 0$ for $i \in \{1, 3\}$.

On the other hand, differentiating the agent's value with respect to σ yields:

$$
\begin{aligned}
\frac{\mathrm{d}\pi_a^{**}(x,m)}{\mathrm{d}\sigma} &= \left(\frac{\partial \pi_a^{**}}{\partial \beta} + \frac{\partial \pi_a^{**}}{\partial x_3^*}\frac{\partial x_3^*}{\partial \beta}\right)\frac{\partial \beta}{\partial \sigma} \\
&= p\left(\frac{x}{x_3^*}\right)^{\beta} \Delta I \left\{\log\left(\frac{x}{x_3^*}\right) + \frac{1}{\beta - 1}\right\}\left(\frac{\partial \beta}{\partial \sigma}\right).
\end{aligned}
$$

Since $\log(x/x_3^*) < 0$ and $(1/(\beta - 1)) > 0$, the sign in parentheses is ambiguous. □

Proof of Corollary 4.1 Since the sum of the policy maker's and agent's values is the value of social welfare, it is straightforward to check the result by using the fact obtained in Lemma 4.2. □

References

[1] Chiarella, C., 2002, An Introduction to Derivative Security Pricing: Techniques, Methods and Applications (Lecture Note).

[2] Dixit, A.K., and Pindyck, R.S. 1994, Investment Under Uncertainty, Princeton University Press.

[3] Grenadier, S.R., and Wang, N., 2005, Investment Timing, Agency and Information, Journal of Financial Economics, 75:493-533.

[4] Kijima, M., 2002, Stochastic Processes with Applications to Finance. Chapman & Hall, London.

[5] Mas-Collel, A., Whinston, M., and Green, J.R., 1995, Microeconomic Theory, Oxford Student Press.

[6] Pindyck, R.S., 2000, Irreversibilities and the timing of environmental policy, Resources and Energy Economics, 22:233-259.

[7] Pindyck, R.S., 2002, Optimal Timing Problems in Environmental Economics, Journal of Economic Dynamics & Control, 26:1677-1697.

[8] Salanié, B., 2005, The Economics of Contracts, Second Edition, MIT Press.