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Price Formation in a Competitive Market When the Payoff of an Asset Depends on the Market Price

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We consider a competitive market where the final payoff of a risky asset depends on the market price of the asset. It is shown that when the final payoff depends on the market price, there are multiple equilibria, and that even a small change of parameter setting may cause big price movements. It is also shown that, in contrast to Easley and O'Hara (2004), the shift of information from private to public may increase the required return of the risky asset.

Keywords: Market microstructure, CARA-Normal, private information, cost of capital, multiple equilibria

JEL Classification Numbers: G3, G12, G82

1. Introduction

In this paper, we consider a competitive market where the final payoff of a risky asset depends on the market price of the asset. In the previous literature of market microstructure like Grossman and Stiglitz (1980), Hellwig (1980), Admati (1985), and Easley and O'Hara (2004), the final payoff of a risky asset follows a normal distribution, and is exogeneously realized after market participants trade the asset. However, it is quite natural to assume that the payoff depends on the market price of the risky asset. For example, the higher is the price of a new issuing stock, the more money the firm can finance, and invest in a project. The firm can purchase a more advanced machine with the additional money, or spend the extra cash for market research or advertizing, which affects the performance of the firm. Another example is due to the signaling effect. If the stock price of a firm is high, market participants think that the firm is in a good financial condition. Then, the firm can make better contracts with customers. Therefore, the business performance of a firm whose stock price is high will be better than the case of a low stock price even when other conditions are the same.

The main purpose of this paper is to analyze how the functional form of the payoff affects the price formation in a rational expectations equilibrium. We do not specify the functional form of the final payoff of a risky asset with respect to its market price. In this paper, we simply assume that the final payoff v is given by

$$v = g(p) + \omega$$

where p is the market price of the asset, $g(\cdot)$ is some function that specifies the relationship between the market price and the final payoff, and ω is a random term which is exogenously given and follows a normal distribution. Thus, by changing the functional form, our model is applicable to a wide variety of relationships between the asset price and the business performance.

If g is assumed to be constant, then the payoff has no relation to the market price. This reduces to the previous literature. For example, when g is constant and the investors' private signals are perfectly correlated, it is the case of Easley and O'Hara (2004). When g is constant and the investors' private signals are independent among each other, the model is the same as Hellwig (1980). Therefore, our model can be regarded as an extention of Hellwig (1980), Easley and O'Hara (2004) and most of other CARA-Normal models.

In a CARA-Normal model, all market participants have an exponential utility function, and all random variables follow normal distributions. Although the assumption of CARA-Normal is restrictive, its tractability is very advantageous. With a CARA-Normal model, we obatain analytic solutions, and so have many economic implications, such as how the functional form of the payoff or the information structure affects the price, and the market equilibrium.

Our key findings are as follows. First, when the payoff of a risky asset depends on the market price, there are multiple equilibria. It is shown that the stability of an equilibrium is determined by the economies of scale of financing, i.e., marginal return of the asset's payoff with respect to the asset's price. When the economies of scale of financing hold, i.e., when $g'(\cdot)$ is greater than one at equilibrium, the equilibrium price is unstable. Therefore, a small change in the parameter setting may cause a big price movement, because of shifting from one equilibrium price to another one. This result explains the phenomenon of the price instability of a venture company. Many venture companies or rapidly growing firms seem to have high marginal return with respect to the financing money, i.e., their business performaqnce is highly dependent on the market price of their stock or bond. In that situation, even market news that seem irrelevant to the companies actually affects the market price and their business results.

Second, although a greater dispersion of private information monotonically increases the market price and decreases the cost of capital, a shift from private to public information may not decrease the cost of capital in our setting. This result is in sharp constrast to Easley and O'Hara (2004). They showed that both the dispersion and shift of private inforamtion to public information reduce the cost of capital We show in this paper that shifting information from private to public has two opposite effects. The first effect is that traders has more accurate public information about the payoff, which makes investors, especially uninformed traders who do not have private information trade the asset more aggressively. The second effect is that informed trader has less accurate priavate information, which leads informed traders to less aggressive trades. In this situation, the price includes less information that informed traders privately observes. The less informative is the market price, the less aggressively both informed and uninformed traders trade. Thus, it is ambiguous whether the total aggressiveness of both traders increase or decrease, and so the informativeness of the price and the cost of capital can increase or decrease, depending on the parameter setting.

The organization of this paper is as follows. In Section 2, we set up the model. In Section 3, we derive the equilibrium price. In Section 4, we analyze some results on how a change in the information structure affects the price, payoff, and cost of capital. In Section 5, we discuss the case when there are multiple equilibria. Section 6 presents our conclusions.

2. The Setup

In this section, we set up a one-shot two-period noisy rational expectations model in which a firm's final payoff depends on the firm's stock price. Our model is an extention of Hellwig (1980), and Easely and O'Hara (2004).

A firm plans to issue new stock to raise funds for investment in a risky project. At $t = 0$, the market opens, stocks are issued and traded, the firm raises funds, and then invests the money in the project. At $t = 1$, the result of the project is realized, the firm pays a dividend to stockholders, and all positions are cleared. The stock price is denoted by p. The final payoff of a stock, denoted by v, is dependent on the asset price as

$$v = g(p) + \omega,$$

where $g(\cdot)$ is some function that characterizes the relationship between the securities price and the payoff. The random term of the payoff ω is normally distributed with mean 0 and precision (inverse of variance) ρ. The functional form of g is assumed to be known to all market participants. This means that all traders know how the market price of the asset affects the payoff of the asset. In Section 2 to Section 4, we assume that $g'(\cdot) < 1$ in order to guarantee the uniqueness of the equilibrium. We generalize g in Section 5 for the case of multiple equilibria. When g is a constant, the liqudation value is exactly the same as in Hellwig (1980) or Easley and O'Hara (2004).

It is quite natural to assume that the final payoff of a project is dependent on the market price. For example, when a firm issues a corporate bond, and the price is high, the firm saves interest payments. This means that the firm can invest the money in additional equipments or more advanced instruments, and the project is likely to earn more. Another example is that high share prices can be regard as a signal of good business performance of the firm, and so the firm can deal with customers or vendors in better terms.

In the market, there are three types of traders: informed traders, uninformed traders, and noise traders.

Noise traders trade for exogenous reasons such as liquidity constraints, political circumstances, etc. Due to the noise traders, the supply of an risky asset becomes an random variable. The per capita supply of the asset, denoted by x, is normally distributed with mean $\overline{x}$ and precision η.

We assume that there are J investors who maximizes their expected utility. Some of the investors are informed traders, who privately observe information on the random term of the liquidation value ω. The fraction of the informed traders are denoted by $\mu \in [0, 1]$, i.e. there are μJ informed traders in the market[1]. The informed traders observe public signals and his/her private signals before the market opens. The other investors are uninformed traders. The uninformed traders only oberve public signals. The private signals of Informed trader j are $(s_{j1}, \ldots, s_{j,\alpha I})$. The public signals are $(s_{\alpha I+1}, \ldots, s_I)$. We assume that the signals are of the form

$$s_{ji} = \omega + \epsilon_{ji}, \quad i = 1, \ldots, \alpha I,\ \ j = 1, \ldots, \mu J$$

and

$$s_i = \omega + \epsilon_i, \quad i = \alpha I + 1, \ldots, I,$$

where each ϵ is normally distributed with mean 0 and precision γ. We assume that $\{\epsilon_i\},\ i = \alpha I + 1, \ldots, I$ are independent among all random variables, and that ϵ_{ji} and ϵ_{jk} are independent for $j \neq k$ but ϵ_{ji} and ϵ_{hi} can be correlated. In Easley and O'Hara (2004), ϵ_{ji} is assumed to be the same among all informed traders, i.e. the correlation coefficient of ϵ_{ji} and ϵ_{hi} is unity for $j \neq h$. In this paper, we assume that $(\epsilon_{1i}, \ldots, \epsilon_{\mu Ji})$ follows a multi-dimensional symmetric normal distribution with a correlation coefficient. The correlation coefficients are common for $i = 1, \ldots, \alpha I$. The parameter α represents the proportion of private signals to all available signals[2]. When $\alpha = 1$ and all noise terms $\{\epsilon_{ji}\}$ are independent, the model reduces to Hellwig (1980).

When the distribution are normal and the signals are symmetrically distributed, the mean of the signals are sufficient statistics. Let

$$N_j = \frac{1}{\alpha I} \sum_{i=1}^{\alpha I} s_{ji}$$

and

$$M = \frac{1}{(1-\alpha)I} \sum_{\alpha I+1}^{I} s_i.$$

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[1] We of course assume $\mu \in \left\{\frac{1}{J}, \frac{2}{J}, \ldots, 1\right\}$.

[2] As in the case of μ, $\alpha \in \left\{\frac{1}{I}, \frac{2}{I}, \ldots, 1\right\}$.

Then, from the assumption of symmetric normal distributions, we can regard the private and public signals as

$$N_j = \omega + \epsilon_j, \quad j = 1, \ldots, \mu J \tag{1}$$

and

$$M = \omega + \epsilon_c,$$

where $(\epsilon_1, \ldots, \epsilon_{\mu J}, \epsilon_c)$ is normally distributed with mean 0 and variance

$$\mathrm{Var}[\epsilon_j] = \frac{1}{\alpha I \gamma}, \quad j = 1, \ldots, \mu J$$

and

$$\mathrm{Var}[\epsilon_c] = \frac{1}{(1-\alpha) I \gamma}. \tag{2}$$

See Lemma 1 in Appendix A

Let

$$\mathrm{Cov}[\epsilon_j, \epsilon_k] = \frac{\phi}{\alpha I \gamma} \quad \text{for } j \neq k.$$

The correlation coefficient ϕ is determined by the correlation coefficent of ϵ_{ji} and ϵ_{ki}[3].

Investor j submits a trade order to maximize his/her conditional expected utility

$$\mathrm{E}\left[-\mathrm{e}^{-\delta_j (v-p) z_j} \middle| \mathcal{F}_j\right], \quad j = 1, \ldots, J,$$

where z_j is the position (the number of buy orders) of Trader j, and where δ_j is his/her absolute risk-aversion coefficient. The information $\mathcal{F}_j$ consists of all available information for Trader j. When the signals of the informed traders are imperfectly correlated, the price itself has some information on the liquidatin value for both the informed and the uninformed traders. Hence, $\mathcal{F}_j$ is given by

$$\mathcal{F}_j = \begin{cases} \sigma\{N_j, M, p\} & j = 1, \ldots, \mu J \\ \sigma\{M, p\} & j = \mu J + 1, \ldots, J \end{cases},$$

where $\sigma\{X\}$ is a σ-field generated by the random variable X.

In contrast to Easley and O'Hara (2004), the investors of our model are heterogeneous in the sense that (i) their risk-aversion coefficients are distinct, and (ii)

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[3] It is apparent that when $\phi = 0$, the model is the same as Hellwig (1980), and when $\phi = 1$, the model reduces to Easley and O'Hara (2004).

the private signals are different. Assumption (i) is a natural generalization. We can justify Assumption (ii) because each informed traders can have some bias about a private signal even if they observe the same signal. For example, suppose that all informed traders obtain the information that the firm will deal a big contract. In this situation, one informed trader may think that the firm's performance will critically improve, while another informed trader may not think so. In contrast to Easley and O'Hara (2004), our model captures the effect of the heterogeneity of private information.

Before proceeding, we explain the role of α, the parameter which determines the precision of private and public information. All informed traders observe I signals, while uninformed traders only observe $(1-\alpha)I$ signals. This means that αI signals are private for each informed trader. Therefore, α can be thought of as the fraction of private signals to all signals. By varying α, we keep the total information quality of signals for each informed trader constant while varying the amount of private versus public information. This is the same structure as Easley and O'Hara (2004).

Finally, we define the equilibrium of this market model.

Definition 1. The market equilibrium is defined by the following two conditions:

(i) Each informed or uninformed trader maximizes his/her conditional expected utility given all available information.

(ii) The amount of total orders is equal to the supply of assets:

$$Jx = \sum_{j=1}^{J} z_j. \tag{3}$$

Definition 1 is standard in most of noisy rational expectations models. In this setting, there is no market maker who takes his own position to clear the market. Instead, an auctioneer adjusts the price to equate the demand and supply. If the demand is more than (or less than) the supply, the auctioneer raises (or lowers) the price until demand and supply meet. Trades occur only after the price is adjusted so that the market clears all orders.

3. Asset Prices in Equilibrium

In this section, we derive the equilibrium price p as a function of $M, N_1, \ldots, N_{\mu J}$ and x. The procedure is standard in CARA-Normal models.

By the assumption of the relationship between the asset price and the final payoff, the conditional expected utility of Trader j can be rewritten as

$$\mathrm{E}\left[-\mathrm{e}^{-\delta_j (g(p)+\omega-p) z_j} \,\middle|\, \mathcal{F}_j \right].$$

Both $g(p)$ and p are measurable with respect to $\mathcal{F}_j$, and the only unknown variable is ω. Thus the demand function of Trader j can be derived as

$$z_j = \frac{1}{\delta_j \mathrm{Var}[\omega|\mathcal{F}_j]}(g(p) - p + \mathrm{E}[\omega|\mathcal{F}_j]), \quad \text{for } j = 1, \ldots, J. \tag{4}$$

To derive a partially revealing equilibrium price, we first conjecture that the asset price satisfies the following functional form:

$$p = \varphi\left(\sum_{h=1}^{\mu J} b_h N_h + cM - dx + e\overline{x}\right), \tag{5}$$

where φ is some function that has an inverse function, and where $\{b_h\}_{h=1}^{\mu J}$, c, d and e are all constants that are known to both informed and uninformed traders.

We define θ as

$$\theta := \frac{1}{\sum_{h=1}^{\mu J} b_h}\{\varphi^{-1}(p) - cM + (d-e)\overline{x}\} = \omega + \frac{\sum_{h=1}^{\mu J} b_h \epsilon_h}{\sum_{h=1}^{\mu J} b_h} - \frac{d}{\sum_{h=1}^{\mu J} b_h}(x - \overline{x}).$$

It is easily seen that θ is measurable with respect to the information of both informed and uninformed traders, i.e., θ is an observable signal of both informed and uninformed traders. It is also apparent that

$$\sigma\{M, p\} = \sigma\{M, \theta\}.$$

Hence, the demand function of each uninformed trader is given by

$$z_u = \frac{1}{\delta_u \mathrm{Var}[\omega|M, \theta]}\{\mathrm{E}[\omega|M, \theta] - (p - g(p))\}, \quad \text{for } u = \mu J + 1, \ldots, J. \tag{6}$$

Denote the set of informed traders by $\mu\mathcal{J}$, and define

$$\begin{aligned} \theta_{-j} :&= \frac{1}{\sum_{h\in\mu\mathcal{J}\setminus j} b_h}\{\varphi^{-1}(p) - cM + (d-e) - b_j N_j\} \\ &= \omega + \frac{\sum_{h\in\mu\mathcal{J}\setminus j} b_h \epsilon_h}{\sum_{h\in\mu\mathcal{J}\setminus j} b_h} - \frac{d}{\sum_{h\in\mu\mathcal{J}\setminus j} b_h}(x - \overline{x}), \end{aligned} \tag{7}$$

for $j = 1, \ldots, \mu J$. As in the case of uninformed traders, the demand functin of Informed Trader j is given by

$$z_j = \frac{1}{\delta_j \mathrm{Var}[\omega|M, N_j, \theta_{-j}]}\{\mathrm{E}[\omega|M, N_j, \theta_{-j}] - (p - g(p))\}, \quad \text{for } j = 1, \ldots, \mu J. \tag{8}$$

Using the above information structure, we can obtain the following proposition. The proof is given in the Appendix A.

Proposition 1. *There exists a partially revealing rational expectations equilibrium in which the asset price is given by*

$$p = \varphi^{-1}\left(\sum_{j=1}^{\mu J} b_j N_j + cM - dx + e\bar{x}\right),$$

where $\varphi(y) = y - g(y)$, if there exists a solution $(b_1, \dots, b_{\mu J}, c, d, e)$ of the following system of the equations,

$$b_j = \frac{1}{C}\left[\frac{\alpha I\gamma}{\delta_j} + b_j\left(\sum_{h\in\mu\mathcal{J}\setminus j} \frac{(1-\phi)B_h}{\delta_h} + \sum_{h=\mu J+1}^{J} \frac{B}{\delta_h}\right) - \sum_{h\in\mu\mathcal{J}\setminus j} b_h \frac{\phi(1-\phi)B_j}{\delta_j}\right], \quad (9)$$

$$c = \frac{1}{C}\sum_{j=1}^{J} \frac{(1-\alpha)I\gamma M}{\delta_h}, \quad (10)$$

$$d = \frac{1}{C}\left[J + d\left(\sum_{h=1}^{\mu J} \frac{(1-\phi)B_h}{\delta_h} + \sum_{h=\mu J+1}^{J} \frac{B}{\delta_h}\right)\right], \quad (11)$$

$$e = \frac{1}{C}(d - J) \quad (12)$$

and

$$C = \sum_{h=1}^{J} \frac{\rho}{\delta_h} + \sum_{h=1}^{\mu J} \frac{I\gamma}{\delta_h} + \sum_{h=\mu J+1}^{J} \frac{(1-\alpha)I\gamma}{\delta_h} + \sum_{h=1}^{\mu J} \frac{(1-\phi)^2 B_h}{\delta_h} + \sum_{h=\mu J+1}^{J} \frac{B}{\delta_h}, \quad (13)$$

where B and $\{B_j\}_{j=1}^{\mu J}$ is given in Appendix A

We consider the following special cases.

Easley and O'Hara (2004). Suppose that $\delta_j \equiv \delta$ for all $j = 1, \dots, J$, $\phi = 1$ and $g \equiv \bar{v}$. Since $\phi = 1$, the private signals are the same among the informed traders, i.e., $N_1 = \cdots = N_{\mu J} \equiv N$. It is apparent that b_j are identical as well, i.e. $b_1 = \cdots = b_{\mu J} \equiv b$.

In this parameter setting, (9)–(12) are rewritten as

$$b = \frac{\alpha I\gamma + (1-\mu)Jb\frac{\mu Jb}{\frac{(\mu Jb)^2}{\alpha I\gamma} + \frac{d^2}{\eta}}}{\rho J + \{(1-\alpha) + \mu\alpha\}\gamma IJ + (1-\mu)J\frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma} + \frac{d^2}{\eta}}}, \quad (9')$$

$$c = \frac{(1-\alpha)IJ\gamma}{\rho J + \{(1-\alpha) + \mu\alpha\}\gamma IJ + (1-\mu)J\frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma} + \frac{d^2}{\eta}}}, \quad (10')$$

$$d = \frac{\frac{J}{\delta} + (1-\mu)Jd\frac{\mu Jb}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}{\rho J + \{(1-\alpha)+\mu\alpha\}\gamma IJ + (1-\mu)J\frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}} \tag{11'}$$

and

$$e = \frac{(1-\mu)Jd\frac{\mu Jb}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}{\rho J + \{(1-\alpha)+\mu\alpha\}\gamma IJ + (1-\mu)J\frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}. \tag{12'}$$

From (9′) and (11′), we have

$$\frac{d}{\mu Jb} = \frac{\delta + d(1-\mu)\frac{\mu Jb}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}{\mu\alpha I\gamma + (1-\mu)\frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}. \tag{14}$$

Arranging (14) leads to

$$\frac{d}{\mu Jb} = \frac{\delta}{\mu\alpha I\gamma}.$$

Noting that $\varphi^{-1}(y) = \bar{v} + y$, we have

$$\begin{aligned} p = \bar{v} &+ \frac{\mu\alpha I\gamma + (1-\mu)\rho_\theta}{\rho + \{(1-\alpha)+\mu\alpha\}\gamma I + (1-\mu)\rho_\theta}N \\ &+ \frac{(1-\alpha)I\gamma}{\rho + \{(1-\alpha)+\mu\alpha\}\gamma I + (1-\mu)\rho_\theta}M \\ &- \frac{\delta + \frac{(1-\mu)\rho_\theta\delta}{\alpha I\mu\gamma}}{\rho + \{(1-\alpha)+\mu\alpha\}\gamma I + (1-\mu)\rho_\theta}x \\ &+ \frac{\frac{(1-\mu)\rho_\theta\delta}{\alpha I\mu\gamma}}{\rho + \{(1-\alpha)+\mu\alpha\}\gamma I + (1-\mu)\rho_\theta}\bar{x}, \end{aligned}$$

where $\rho_\theta := \frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}} = \frac{1}{\frac{1}{\eta}\left(\frac{\delta}{\mu\alpha I\gamma}\right)^2 + \frac{1}{\alpha I\gamma}}$. Finally, by setting $N \equiv \frac{\sum_{i=1}^{\alpha I} s_i}{\alpha I} - \bar{v}$ and $M \equiv \frac{\sum_{i=\alpha I+1}^{I} s_i}{(1-\alpha)I} - \bar{v}$, we obtain

$$
\begin{aligned}
p = {} & \frac{\rho}{\rho + \{(1-\alpha) + \mu\alpha\}\gamma I + (1-\mu)\rho_\theta}\overline{v} \\
& + \frac{\mu\alpha I\gamma + (1-\mu)\rho_\theta}{\rho + \{(1-\alpha) + \mu\alpha\}\gamma I + (1-\mu)\rho_\theta}\sum_{i=1}^{\alpha I} s_i \\
& + \frac{(1-\alpha)I\gamma}{\rho + \{(1-\alpha) + \mu\alpha\}\gamma I + (1-\mu)\rho_\theta}\sum_{\alpha I+1}^{I} s_i \\
& - \frac{\delta + \frac{(1-\mu)\rho_\theta\delta}{\alpha I\mu\gamma}}{\rho + \{(1-\alpha) + \mu\alpha\}\gamma I + (1-\mu)\rho_\theta} x \\
& + \frac{\frac{(1-\mu)\rho_\theta\delta}{\alpha I\mu\gamma}}{\rho + \{(1-\alpha) + \mu\alpha\}\gamma I + (1-\mu)\rho_\theta}\overline{x},
\end{aligned}
$$

which is the same equiation as the one derived in Easley and O'Hara (2004).

Hellwig (1980). Consider the case that $\mu \equiv 1$, $\alpha \equiv 1$, $\phi = 0$, $I = 1$ and $g = \overline{v}$. Then, the price function becomes

$$
\begin{aligned}
p = \overline{v} + {} & \frac{\gamma\overline{r} + \sum_{j=1}^{J}\left\{\frac{b_j}{J}\sum_{h\in\mathcal{J}\setminus j}\frac{\frac{\sum_{k=1}^{J} b_k - b_h}{J}}{\delta_h J\left(\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}\right)}\right\}}{(\rho+\gamma)\overline{r} + \sum_{h=1}^{J}\frac{1}{\delta_h J}\left\{\frac{\left(\frac{\sum_{k=1}^{J} b_k - b_h}{J}\right)^2}{\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}}\right\}}(\omega + \epsilon_i) \\
& - \frac{1 + \frac{d}{J}\sum_{h=1}^{J}\frac{\frac{\sum_{k=1}^{J} b_k - b_h}{J}}{\delta_h J\left(\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}\right)}}{(\rho+\gamma)\overline{r} + \sum_{h=1}^{J}\frac{1}{\delta_h J}\left\{\frac{\left(\frac{\sum_{k=1}^{J} b_k - b_h}{J}\right)^2}{\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}}\right\}} x \\
& + \frac{\frac{d}{J}\sum_{h=1}^{J}\frac{\frac{\sum_{k=1}^{J} b_k - b_h}{J}}{\delta_h J\left(\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}\right)}}{(\rho+\gamma)\overline{r} + \sum_{h=1}^{J}\frac{1}{\delta_h J}\left\{\frac{\left(\frac{\sum_{k=1}^{J} b_k - b_h}{J}\right)^2}{\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}}\right\}} x,
\end{aligned}
\tag{15}
$$

where $r := \sum_{j=1}^{J}\frac{1}{\delta_j\left(\frac{d}{J}\right)}$, and $\mathcal{J}$ is the set of all traders. By the law of large numbers, the term of ϵ_j disappears when $J \to \infty$. Taking limits, we have

$$\lim_{J\to\infty} \sum_{h=1}^{J} \frac{1}{\delta_h J} \left\{ \frac{\left(\frac{\sum_{k=1}^{J} b_k - b_h}{J}\right)^2}{\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}} \right\} = \frac{\bar{r}\bar{b}^2}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}},$$

$$\lim_{J\to\infty} \frac{d}{J} \sum_{h=1}^{J} \frac{\frac{\sum_{k=1}^{J} b_k - b_h}{J}}{\delta_h J \left(\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}\right)} = \frac{\bar{r}\bar{b}\bar{d}}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}$$

and

$$\lim_{J\to\infty} \sum_{j=1}^{J} \left\{ \frac{b_j}{J} \sum_{h\in\mathcal{J}\setminus j} \frac{\frac{\sum_{k=1}^{J} b_k - b_h}{J}}{\delta_h J \left(\frac{\sum_{k=1}^{J} b_k^2 - b_h^2}{J^2\gamma} + \frac{\left(\frac{d}{J}\right)^2}{\eta}\right)} \right\} = \frac{\bar{r}\bar{b}^2}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}},$$

where $\bar{b} = \frac{1}{J} \lim_{J\to\infty} \sum_{j=1}^{J} b_j$, and where $\bar{d} = \lim_{J\to\infty} \frac{d}{J}$. Therefore, (15) can be rewritten as

$$p = \bar{v} + \frac{\gamma + \frac{\bar{b}^2}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}}{\rho + \gamma + \frac{\bar{b}^2}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}} \omega + \frac{\frac{\bar{b}\bar{d}}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}}{\rho + \gamma + \frac{\bar{b}^2}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}} x - \frac{\frac{1}{\bar{r}} + \frac{\bar{b}\bar{d}}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}}{\rho + \gamma + \frac{\bar{b}^2}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}} \bar{x}.$$

Noting that

$$\frac{\bar{d}}{\bar{b}} = \frac{\frac{\bar{b}\bar{d}}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}}{\gamma + \frac{\bar{b}^2}{\frac{\bar{b}^2}{\gamma} + \frac{\bar{d}^2}{\eta}}},$$

and setting $\omega \equiv (v - \bar{v})$, we finally have

$$p = \frac{\rho}{\rho + \gamma + (\bar{r}\gamma)^2\eta} \bar{v} + \frac{\gamma + (\bar{r}\gamma)^2\eta}{\rho + \gamma + (\bar{r}\gamma)^2\eta} v - \frac{\frac{1}{\bar{r}} + \bar{r}^2\gamma\eta}{\rho + \gamma + (\bar{r}\gamma)^2\eta} x + \frac{\bar{r}^2\gamma\eta}{\rho + \gamma + (\bar{r}\gamma)^2\eta} \bar{x}. \tag{16}$$

Equation (16) was derived in Hellwing (1980).

We can see as above that our model is a generalization of most of the CARA-Normal model.

4. Informational Effect on Asset Returns

Having established the equilibrium, we next analyze how the functional form of the payoff with respect to the asset return as in Easley and O’Hara (2004).

Let

$$\kappa := \sum_{j=1}^{\mu J} b_j N_j + cM - dx + e\overline{x}.$$

Even when the final payoff depends on the stock price, the random asset return can be calculated easily as follows:

$$v - p = g(p) + \omega - p = \omega - \varphi(p) = \omega - \varphi \circ \varphi^{-1}(\kappa) = \omega - \kappa. \tag{17}$$

Therefore, the random asset return is independent of the functional form of g. This is because in CARA-Normal models, the demand function of the asset is given by

$$z_j = \frac{1}{\delta_j \mathrm{Var}[v|\mathcal{F}_j]}(\mathrm{E}[v|\mathcal{F}_j] - p) = \frac{1}{\delta_j \mathrm{Var}[\omega|\mathcal{F}_j]}(\mathrm{E}[\omega - \kappa|\mathcal{F}_j]). \tag{18}$$

Each trader submits his/her demand schedule according to his/her conditional asset return divided by the conditonal variance of the random term. Easley and O'Hara defined the firm's cost of capital as the ex-ante expected return, i.e. $\mathrm{E}[v-p]$. Hence, we can calculate the cost of capital as

$$\mathrm{E}[v - p] = \mathrm{E}[\omega - \kappa] = (e - d)\overline{x} = \frac{J}{C}\overline{x}, \tag{19}$$

where C is given in (13).

Some properties on the cost of capital obtained by Easley and O'Hara (2004) still hold even in our model. For example, if a strictly positive mass of agents is risk-neutral, δ_j becomes 0 and so the risk premium becomes 0. The expected supply of shares affects the risk premium as well.

What interests us is how parameters α and μ affect the risk premium. Easley and O'Hara (2004) showed that the cost of capital is decreasing in μ and increasing in α. Therefore, even if the final payoff depends on the market price, these properties still hold when traders are homogeneous, i.e., δ_j is common among all traders and $\phi = 1$. The monotonicity of the cost of capital with respect to μ implies that a greater dispersion of private information lowers the cost of capital. The monotonicity with respect to α indicates that shifting information from private to public also lowers the cost of capital. Then, do these properties still hold when traders are heterogeneous, i.e., δ_j is different among traders and $\phi \neq 1$? The answer is that the cost of capital is decreasing in μ, but is not necessarily decreasing in α.

We can easily see from (19) that how the increase of μ or α affects the cost of capital depends on the coefficent of C in (19). If the incease in μ lowers the value of C, it implies that a greater dispersion of private information lowers the cost of capital. If the increase in α raises the value of C, it indicates that shifting information from private to public also lowers the cost of capital.

Let us first examine the effect of μ on the cost of capital. We have the following proposition.

Proposition 2. *The expected asset return* $\mathrm{E}[v - p]$ *is monotonically decreasing in* μ*, i.e., a greater dispersion of private information* (*decreasing* μ) *lowers the cost of capital.*

Proof. The demand function of Trader j is given by (18). Hence, The market clearing condition (3), we have

$$Jx = \sum_{j=1}^{J} \frac{1}{\delta_j \mathrm{Var}[\omega|\mathcal{F}_j]} (\mathrm{E}[v|\mathcal{F}_j] - p). \tag{20}$$

Note that the conditional variance $\mathrm{Var}[\omega|\mathcal{F}_j]$ is deterministic coefficient, not dependent on the realization of the random variables. Taking expectations both sides of (20), we obtain from the tower property of conditional expectations that

$$J\overline{x} = \sum_{j=1}^{J} \frac{1}{\delta_j \mathrm{Var}[\omega|\mathcal{F}_j]} (\mathrm{E}[v - p]).$$

Thus, we have $C = \sum_{j=1}^{J} \frac{1}{\delta_j \mathrm{Var}[\omega|\mathcal{F}_j]}$. Now consider that the trader $\mu J + 1$ changes from the uninformed to the informed. This means that Trader $\mu J + 1$ observes a private signal $N_{\mu J+1}$. Then, $\mathrm{Var}[\omega|\mathcal{F}_{\mu J+1}]$, the conditional variance of trader $\mu J + 1$, decreases. This means that Trader $\mu J + 1$ trades more aggressively and the price becomes more informative. Now, it immediately follows that C increases. □

The above proposition is the same as the result obtained by Easley and O'Hara (2004). The more informed traders are there in the market, the less is the required risk premium. This is because as the conditional precision of newly informed traders increases, they trade more aggressively, and as a result the price reflects the private information owned by informed traders.

Next we investigate the effect of α on the cost of capital. To show that the cost of capital is not monotonic with repsect to α, we only need to give a counterexample. Consider the case that $\phi = 0$ and J goes to infinity. Then, κ is given by

$$\begin{aligned}
\kappa = & \frac{\frac{\mu\alpha}{\delta_i}\gamma + \frac{\eta}{\delta}\left(\frac{\mu\alpha\gamma}{\delta_i}\right)^2}{\frac{\rho}{\delta_i} + \left(\frac{\mu}{\delta_i} + \frac{(1-\mu)(1-\alpha)}{\delta_u}\right)\gamma + \frac{\eta}{\delta}\left(\frac{\mu\alpha\gamma}{\delta_i}\right)^2}\omega \\
& - \frac{1 + \frac{\mu\alpha\gamma\eta}{\delta\delta_i}}{\frac{\rho}{\delta_i} + \left(\frac{\mu}{\delta_i} + \frac{(1-\mu)(1-\alpha)}{\delta_u}\right)\gamma + \frac{\eta}{\delta}\left(\frac{\mu\alpha\gamma}{\delta_i}\right)^2}x \\
& + \frac{\frac{\mu\alpha\gamma\eta}{\delta\delta_i}}{\frac{\rho}{\delta_i} + \left(\frac{\mu}{\delta_i} + \frac{(1-\mu)(1-\alpha)}{\delta_u}\right)\gamma + \frac{\eta}{\delta}\left(\frac{\mu\alpha\gamma}{\delta_i}\right)^2}\overline{x}.
\end{aligned}$$

Here,

$$\frac{1}{\bar{\delta}_i} := \lim_{J\to\infty} \frac{1}{\mu J} \sum_{j=1}^{\mu J} \frac{1}{\delta_j},$$
$$\frac{1}{\bar{\delta}_u} := \lim_{J\to\infty} \frac{1}{(1-\mu)J} \sum_{j=\mu J+1}^{J} \frac{1}{\delta_j}$$

and

$$\frac{1}{\bar{\delta}} := \lim_{J\to\infty} \frac{1}{J} \sum_{j=1}^{J} \frac{1}{\delta_j} = \frac{\mu}{\bar{\delta}_i} + \frac{1-\mu}{\bar{\delta}_u}.$$

Hence, the cost of capital in this case is written as

$$\mathrm{E}[v-p] = \mathrm{E}[\omega-\kappa] = -\frac{1}{\frac{\rho}{\bar{\delta}_i} + \left(\frac{\mu}{\bar{\delta}_i} + \frac{(1-\mu)(1-\alpha)}{\bar{\delta}_u}\right)\gamma + \frac{\eta}{\bar{\delta}}\left(\frac{\mu\alpha\gamma}{\bar{\delta}_i}\right)^2}\bar{x}$$

Now we have the following proposition.

Proposition 3. *It is indeterminant whether shifting information from public to private (decreasing α) lowers the cost of capital or not.*

Proof. Differentiate (19) with respect to α. Thus we obtain

$$\frac{\partial}{\partial\alpha}\Big(\mathrm{E}[v-p]\Big) = \frac{\frac{(1-\mu)\gamma}{\bar{\delta}_u} - \frac{2\left(\frac{\mu\gamma}{\bar{\delta}_i}\right)^2\alpha\eta}{\bar{\delta}}}{\left[\frac{\rho}{\bar{\delta}} + \left(\frac{\mu}{\bar{\delta}_i} + \frac{(1-\mu)(1-\alpha)}{\bar{\delta}_u}\right)\gamma + \frac{\eta}{\bar{\delta}}\left(\frac{\mu\alpha\eta}{\bar{\delta}_i}\right)^2\right]^2} S. \tag{21}$$

Since the sign of the denominator is positive, the numerator determines the sign of (21). However, the sign of the numerator is not indeterminant. For example, when the $\bar{\delta}_u$ is large and $\bar{\delta}_i$ is small, the sign is positive, while it is negative when $\bar{\delta}_u$ is small and $\bar{\delta}_i$ is large. This completes the proof. □

We can explain Proposition 3 intuitively. Changing α has two effects. Suppose α increases, i.e., public information shifts to private information. Then the conditional precision of uninformed traders decreases. This makes uninfomred traders trade less aggressively. As a result, the risk premium increases. This effect appears in the first term of the numerator in (21) $\frac{(1-\mu)\gamma}{\bar{\delta}_u}$. On the other hand, informed traders trade more aggressively because their conditional precision has improved. The market price reflects the private information owned by informed traders through their trades, so that the conditional precision of public information θ increases, and the risk premium decreases. This effect appears in the second term of the numerator in (21).

In Easley and O'Hara (2004), information observed by informed traders is common. This means that the equilibrium price has no information for informed traders, and so the conditional precision of informed traders does not change when the shift of information occurs. In our model, however, private signals are different among informed traders, and so the equilibrium price p has some useful information about the final payoff v for both informed and uninformed traders. On the other hand, how the equilibrium price reflects information on the payoff v is dependent on the aggressiveness of informed and uninformed traders. This is why shifting information from private to public does not necessarily lower the risk premium, which is the cost of capital for the issuing firm. By Equation (21), we see that μ, the proportion of informed traders, and $\overline{\delta_i}$ and $\overline{\delta_u}$, the average of risk torelance coefficiets, are of critical importance to determine the sign of (21). Thus when we examine the effect of a change in information strategies, we should consider how many informed and uninformed traders participate in the market, and how risk-averse they are.

5. Multiple Equilibria

So far, we have assumed that φ has an inverse function, and so the equilibrium price is unique if there exists a unique solution of (9) – (13). However, there may be multiple equilibria, depending on the functional form of g. In this section, we discuss the case of multiple equilibria.

Equation (A.17) in the Appendix and the definition of κ in Section 4 shows that the following identity still holds in an equilibrium:

$$g(p) = p - \kappa.$$

Therefore, an equilibrium price is a point at which $g(x)$ crosses $x-\kappa$. Figure 1 shows an example of how an equilibrium price is determined. The slope of each straight line is 45°, and the vertical level is dependent on the value of κ. The functional form of g in Figure 1 can be seen as a typical case. In this case, when insufficient funds are raised, the addition of marginal funds has a small influence on the project completion, and so the price change has little effect on the final payoff. When the price is at a moderate level, the project has economies of scale, and so the slope of g is very steep. Finally, when the amount of funds raised exceeds a certain level, economies of scale are disappearing and the slope of g decreases.

Consider the case when the current situation is (i) in Figure 1 and the current equilibrium price is p. The equilibrium price is determined according to the level of κ. As we showed in the previous section, how κ moves with a change in α is indeterminant. Suppose that information shifts from private to public, but κ decreases because uninformed traders are much more risk-averse than informed traders on average. $p - \kappa$ is supposed to move to the level (ii) in Figure 1. Then $g(p) - p + \kappa$ becomes negative. This means that the total supply of stocks exceeds the total demand of all traders, so the stock price decreases. The equilibrium price after information shifts from private to public becomes p', which is much lower than p, although p'' can also be an equilibrium price.

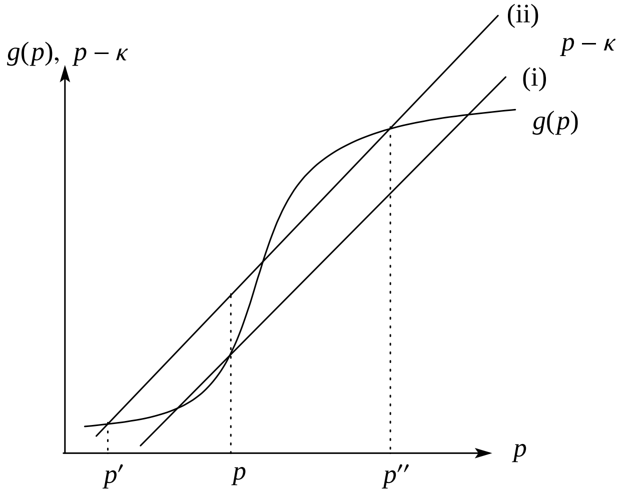

Figure 1 The system of equilibrium solution.

This example demonstrates that when there are multiple equilibria, a small change in α can cause a shift from one crossing point to another, resulting in a big price movement. This result has an important economic implication. Suppose that there are many uninformed traders compared to informed traders, and uninformed traders are not risk-averse in average, i.e., μ is high and $\overline{\delta}_u$ is low. Then the price may move sharply due to a change in the firm's information strategy, which decides the value of α.

This result explains why the stock price of a venture company fluctuates very sharply. A venture firm is typically in a rapid growth, and their business performance is largely dependent on the stock price or bond price. This means that their payoff has the economies of scale with respect to the money they finance, i.e., $g' > 1$. Therefore, the stock price or bond price is an unstable equilibrium price, and so the price moves sharply with a tiny market news or so.

6. Conclusion

We have discussed a competitive market where the final payoff of a risky asset is dependent on the market price of the asset. We show that the cost of capital is independent of the functional form of the payoff. However, in contrast to Easley and O'Hara (2004), when agents are heterogeneous in the sense that their signals and risk-aversion coefficient are different, the shift in information has two kinds of effects, so that the influence is not necessarily monotonic. The aggressiveness of informed traders and uninformed traders, and the proportion of informed traders in the market have a critical effect on the cost of capital. When there are multiple equilibria, a small change in the information structure may cause a big price change.

Our findings explain the unstability of the stock price of a venture company. It is often seen that the business performance of such companies is highly dependent on the market price of their stock price or bond price. One reason of such a phenomenon is that the results of a project by a venture company has economies of scale with respect to the money they finance. In this case, there may be multiple equilibria, and a big price change is caused by the shift from one equilibrium to another.

Our findings suggests many issues for further research. One is a multi-period model as He and Wang (1995), and another is a continuous-time model as Wang (1993). These remain for future research.

A. Proof of Proposition 1

First, we have the following lemma.

Lemma 1. *Let ω be normally distributed with mean $\bar{\omega}$ and variance $\frac{1}{\rho}$. Suppose $\{s_i\}_{i=1}^{I}$ are unbiased signals that are conditionally independent, i.e.*

$$s_i = \omega + \epsilon_i, \tag{A.1}$$

where ϵ_i is normally distributed with mean 0 and variance $\frac{1}{\gamma_i}$, and is independent from other random variables. Then,

$$\mathrm{E}[\omega | s_1, \ldots, s_I] = \frac{1}{\rho + \sum_{i=1}^{I} \gamma_i} \left(\rho\bar{\omega} + \sum_{i=1}^{I} \gamma_i s_i \right), \tag{A.2}$$

and

$$\mathrm{Var}[\omega | s_1, \ldots, s_I] = \frac{1}{\rho + \sum_{i=1}^{I} \gamma_i}. \tag{A.3}$$

Proof. This is an application of the projection theorem of normal distributions. See Greene (1995). □

When the variance of ϵ_i in (A.1) is common, i.e. $\gamma_i = \gamma$ for all $i = 1, \ldots, I$, then (A.2) and become (A.3)

$$\mathrm{E}[\omega | \{s_i\}_{i=1}^{I}] = \frac{1}{\rho + I\gamma} \left(\rho\bar{\omega} + \gamma \sum_{i=1}^{I} s_i \right) = \frac{1}{\rho + I\gamma} (\rho\bar{\omega} + I\gamma N) \tag{A.4}$$

and

$$\mathrm{Var}[\omega | \{s_i\}_{i=1}^{I}] = \frac{1}{\rho + I\gamma}. \tag{A.5}$$

Hence, the average of the signals are sufficient statics, and we can justify (1) to (2).

First, we derive the demand function of uninformed traders. Note that

$$\mathrm{Var}[\theta|\omega] = \frac{\sum_{h=1}^{\mu J} b_h^2 + \phi \sum_{h\neq k\in\mu\mathcal{J}} b_h b_k + \frac{d^2}{\eta}}{\left(\sum_{h=1}^{\mu J} b_h\right)^2} \tag{A.6}$$

From Lemma 1 and (4), we easily obtain

$$z_u = \frac{1}{\delta_u}\left[(1-\alpha)I\gamma M + B\theta - (\rho + (1-\alpha)I\gamma + B)\{p - g(p)\}\right], \quad u = \mu J + 1, \ldots, J. \tag{A.7}$$

where

$$B := \frac{\left(\sum_{j=1}^{\mu J} b_j\right)^2}{\frac{\sum_{j=1}^{\mu J} b_j^2 + \phi \sum_{j\neq k\in\mu\mathcal{J}} b_j b_k}{\alpha I\gamma} + \frac{d^2}{\eta}}. \tag{A.8}$$

Now we derive the demand function of informed traders. From some algebra, we obtain

$$\mathrm{E}[\omega|N_j, M] = \frac{\alpha I\gamma N_j + (1-\alpha)I\gamma M}{\rho + I\gamma}, \tag{A.9}$$

$$\mathrm{E}[\theta_{-j}|N_j, M] = (1-\phi)\frac{\alpha I\gamma N_j + (1-\alpha)I\gamma M}{\rho + I\gamma} + \phi N_j, \tag{A.10}$$

$$\mathrm{Var}[\omega|N_j, M] = \frac{1}{\rho + I\gamma}, \tag{A.11}$$

$$\mathrm{Var}[\theta_{-j}|N_j, M] = \frac{(1-\phi)^2}{\rho + I\gamma} + \frac{1}{B_j} \tag{A.12}$$

and

$$\mathrm{Cov}[\omega, \theta_{-j}|N_j, M] = \frac{1-\phi}{\rho + I\gamma}, \tag{A.13}$$

where

$$B_j := \frac{\left(\sum_{h\in\mu\mathcal{J}\setminus j} b_h\right)^2}{\frac{(1-\phi^2)\sum_{h\in\mu\mathcal{J}\setminus j} b_h^2 + \phi(1-\phi)\sum_{h\neq k\in\mu\mathcal{J}\setminus j} b_h b_k}{\alpha I\gamma} + \frac{d^2}{\eta}}. \tag{A.14}$$

Then, from (A.9) – (A.13), the demand function of informed traders can be obtained as

$$z_j = \frac{1}{\delta_j \mathrm{Var}[\omega|N_j, M, \theta_j]}\left[\mathrm{E}[\omega|N_j, M, \theta_j] - \{p - g(p)\}\right] \tag{A.15}$$

$$\begin{aligned} &= \frac{1}{\delta_j}\Big[\{\alpha I\gamma - \phi(1-\phi)B_j\}N_j + (1-\phi)B_j\theta_{-j} + (1-\alpha)I\gamma M \\ &\quad - \{\rho + I\gamma + (1-\phi)^2 B_j\}\{p - g(p)\}\Big], \end{aligned} \tag{A.16}$$

for $j = 1, \ldots, \mu J$.

Plugging (A.7) and (A.16) into (3), solving with respect to $p - g(p)$ leads to

$$p - g(p) = \frac{1}{C}\left[\left\{\frac{\alpha I\gamma}{\delta_j} + b_j\left(\sum_{h\in\mu\mathcal{J}\setminus j}\frac{(1-\phi)B_h}{\delta_h} + \sum_{h=\mu J+1}^{J}\frac{B}{\delta_h}\right) - \sum_{h\in\mu\mathcal{J}\setminus j} b_h\frac{\phi(1-\phi)B_j}{\delta_j}\right\}N_j\right.$$
$$+ \sum_{h=1}^{J}\frac{(1-\alpha)I\gamma}{\delta_h}M - \left\{\frac{\frac{J}{\delta} + (1-\mu)Jd\frac{\mu Jb}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}{\rho J + \{(1-\alpha)+\mu\alpha\}\gamma IJ + (1-\mu)J\frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}\right\}x$$
$$\left. + \left\{\frac{(1-\mu)Jd\frac{\mu Jb}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}{\rho J + \{(1-\alpha)+\mu\alpha\}\gamma IJ + (1-\mu)J\frac{(\mu Jb)^2}{\frac{(\mu Jb)^2}{\alpha I\gamma}+\frac{d^2}{\eta}}}\right\}\bar{x}\right], \tag{A.17}$$

where C is defined in (13). Finally, because $g' < 1$, $y - g(y)$ has an inverse function. Now, the proposition immediately follows. □

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