



# Vacuum fluctuation effects due to an Abelian gauge field in $2 + 1$ dimensions, in the presence of moving mirrors

C.D. Fosco<sup>a,\*</sup>, M.L. Remaggi<sup>b</sup>, M.C. Rodríguez<sup>a</sup>

<sup>a</sup> Centro Atómico Bariloche and Instituto Balseiro, Comisión Nacional de Energía Atómica, R8402AGP San Carlos de Bariloche, Argentina

<sup>b</sup> Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Cuyo, 5500 Mendoza, Argentina

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## ABSTRACT

We study the Dynamical Casimir Effect (DCE) due to an Abelian gauge field in  $2 + 1$  dimensions, in the presence of semitransparent, zero-width mirrors, which may move or deform in a time-dependent way. We obtain general expressions for the probability of motion-induced pair creation, which we render in a more explicit form, for some relevant states of motion.

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## 1. Introduction

In the Dynamical Casimir Effect (DCE), *real* quanta of a field are created, out of the quantum vacuum, due to the presence of external, time-dependent conditions. Such external conditions may adopt different guises, the most prominent among them being perhaps a time-dependence in the geometry of the boundary conditions. In this context, the creation of particles in a one-dimensional cavity containing a moving perfect mirror has been extensively studied, since the pioneering works by Moore [1] and, subsequently, by Davies and Fulling [2]. In more recent works, different yet related aspects of the DCE have been the focus of attention, because of their relevance to phenomena like cavity quantum electrodynamics, superconducting waveguides subjected to time-dependent boundary conditions, quantum friction, etc., (for some reviews, see, for example [3]). Interestingly, systems that exhibit DCE may also shed light on the information loss problem, entanglement entropy, and its relation to the mirror's state of motion [4]. Besides, exact solutions for physical observables in the moving mirror models [5] have been applied to understand the physics of the related evaporating black-hole system. A moving mirror, on the other hand, provides an example of entanglement harvesting from the vacuum [6], and its radiation has also been studied recently from the point of view of the Equivalence Principle [7].

A time-dependence in the boundary conditions may be due to different causes, and manifest itself in a variety of ways, de-

pending on the nature of the quantum field and of the model for the 'mirror' (the boundary conditions it imposes). In principle, one could consider a time-dependence in the properties of the mirror, keeping its geometry constant. Or the contrary situation, namely, changing only the geometry, or even changing both the geometry and the properties. On the other hand, the nature (spin, mass, number of spacetime dimensions) of the quantum field, does play a role in determining which kind of boundary conditions it may be subjected to. In this respect, a higher spin field is expected to allow for a richer spectrum of boundary conditions in comparison with a scalar field, because of its larger number of components.

In this letter, we study dissipative effects in a concrete model where the DCE is realized. It consists of a  $2 + 1$ -dimensional system, with either one or two semitransparent mirrors, in a non-trivial state of motion, coupled to a quantum Abelian gauge field. Abelian gauge field theories in  $2 + 1$  dimensions play a special role in Quantum Field Theory models which are of relevance to Condensed Matter Physics applications [8–10], in effective descriptions. It is our aim to consider the phenomenon of motion induced radiation in that sort of system, because of its potential relevance in models, besides its intrinsic, theoretical interest. We recall that motion induced radiation with non-perfect mirrors has already been considered, as in Ref. [11], for a mirror in nonrelativistic motion in  $1 + 1$  dimensions. Other models and situations have been considered by several authors, like cavities consisting of media without internal dissipation [12], imperfect mirrors filling half-spaces and moving sideways [13], expanding spherical mirrors [14], many interesting configurations in  $1 + 1$  dimensions [15] and zero-width mirrors described by couplings to singular potentials, containing Dirac's  $\delta$  or its derivatives [16]. Also, rather general

\* Corresponding author.

E-mail address: [fosco@cab.cnea.gov.ar](mailto:fosco@cab.cnea.gov.ar) (C.D. Fosco).

formulations have been presented, based on a scattering theory approach [17].

It is our intention to extend here the idea of [18] to the case of an Abelian gauge field, and to more general states of motion. In particular, we aim to allow for time-dependent deformations of the mirrors. We recall that the approach of [18] consisted of considering imperfect semitransparent mirrors undergoing accelerated motion, coupled to a real scalar field. The approximation involved in treating the mirrors perturbatively allowed for disentangling the purely quantum calculation (due to the field) from the treatment of the mirrors' motion, which can be incorporated at the end of the calculation. The fact that the vacuum field is now an Abelian gauge field will, as we shall see, be responsible for the emergence of qualitatively different results. Indeed, gauge invariance greatly restricts the possible couplings between the field and mirrors: now derivatives of the gauge field necessarily appear. This, in turn, means that the mass dimensions of the coupling constant will be different, and, as a consequence, so will be the momentum dependence of the results.

This paper is organized as follows: in Sect. 2, we introduce the kind of model that we consider in our study, and we also set up our notation and the conventions we have adopted. Then, in Sect. 3, we evaluate the effective action and its imaginary part perturbatively in the coupling of the mirror to the field. We do that for the cases of one and two mirrors, and derive general expressions for the imaginary part of the effective action. In Sect. 4, we evaluate the general expressions derived in the previous section for some particular kinds of motion, therefore obtaining explicit measures for the relation between motion and dissipation. In Sect. 5, we present our conclusions.

## 2. The system

The system that we consider throughout this paper has, as its quantum dynamical variable, an Abelian gauge field  $A_\mu(x)$  in  $2+1$  dimensions.<sup>1</sup> The dynamics of this field, and its coupling to the moving 'mirrors', will be encoded into an Euclidean action  $\mathcal{S}(A)$ , for which we assume the structure:

$$\mathcal{S}(A) = \mathcal{S}_0(A) + \mathcal{S}_I(A), \quad (1)$$

where  $\mathcal{S}_0$  denotes the free gauge-field action:

$$\mathcal{S}_0(A) = \int d^3x \left[ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{\lambda}{2} (\partial \cdot A)^2 \right], \quad (2)$$

which includes a gauge-fixing term (we shall use  $\lambda \equiv 1$ ), while  $\mathcal{S}_I$  deals with the coupling between the field and the mirror(s). In our conventions, the Euclidean space-time metric is tantamount to the identity matrix  $\delta_{\mu\nu}$ . There will be, therefore, no difference between a given expression and another one obtained by raising or lowering one (or more) of its space-time indices.

Let us now construct the explicit form of  $\mathcal{S}_I$ , for just a single mirror; to consider more than one mirror, we just add analogous terms for each one of them. The mirrors are assumed to be localized, i.e., to occupy a spatial curve at any given time, and therefore  $\mathcal{S}_I(A)$  is an integral over the worldsheet(s) swept by the mirror(s) during time evolution. Thus, the worldsheet  $\mathcal{M}$  for that mirror may be parametrized using two coordinates  $\sigma^\alpha$ , as follows<sup>2</sup>:

$$\sigma \equiv (\sigma^0, \sigma^1) \rightarrow y^\mu(\sigma). \quad (3)$$

<sup>1</sup> Indices from the middle of the Greek alphabet ( $\mu, \nu, \lambda, \dots$ ) are assumed to run over the values 0, 1 and 2.

<sup>2</sup> Indices from the beginning of the Greek alphabet ( $\alpha, \beta, \gamma, \dots$ ) run from 0 to 1.

Note that, for indices corresponding to the two-dimensional (generally curved) worldsheet of the mirrors, their raising or lowering may indeed be relevant, since there is an induced non-trivial metric (see Eq. (7) below).

Assuming locality, a simple gauge and reparametrization-invariant form for the interaction term  $\mathcal{S}_I = \mathcal{S}_{\mathcal{M}}(A, y)$  is the following:

$$\mathcal{S}_{\mathcal{M}}(A, y) = \frac{1}{4\xi} \int_{\mathcal{M}} d^2\sigma \sqrt{g(\sigma)} g^{\alpha\alpha'}(\sigma) g^{\beta\beta'}(\sigma) \times \mathcal{F}_{\alpha\beta}(\sigma) \mathcal{F}_{\alpha'\beta'}(\sigma), \quad (4)$$

where we have introduced:

$$\mathcal{F}_{\alpha\beta}(\sigma) \equiv \partial_\alpha \mathcal{A}_\beta(\sigma) - \partial_\beta \mathcal{A}_\alpha(\sigma), \quad (5)$$

with  $\mathcal{A}_\alpha(\sigma)$  denoting the projection of  $A_\mu(x)$  onto the surface  $\mathcal{M}$ :

$$\mathcal{A}_\alpha(\sigma) \equiv A_\mu[y(\sigma)] e_\alpha^\mu(\sigma), \quad (6)$$

$e_\alpha^\mu(\sigma)$  being the tangent vectors  $e_\alpha^\mu(\sigma) = \partial y^\mu(\sigma) / \partial \sigma^\alpha$ . Indices corresponding to objects living on  $\mathcal{M}$  are raised or lowered with the induced metric tensor:

$$g_{\alpha\beta}(\sigma) = e_\alpha^\mu(\sigma) e_\beta^\nu(\sigma) g_{\mu\nu}(\sigma), \quad (7)$$

and  $g(\sigma) \equiv \det[g_{\alpha\beta}(\sigma)]$ .

On the other hand, the constant  $\xi$  (which has the dimensions of a mass) controls the strength of the boundary conditions; namely,  $\xi \rightarrow 0$  corresponds to a perfect conductor, and  $\xi \rightarrow \infty$  to no boundary conditions being imposed on  $\mathcal{M}$ . Imperfect boundary conditions shall mean a non-vanishing, finite value for  $\xi$ . Note that, in the  $\xi \rightarrow 0$  limit, the term  $\mathcal{S}_{\mathcal{M}}$  becomes divergent, unless the tangent components of the electromagnetic field tensor vanish (the integrand in that term) identically, namely, unless perfect-conductor conditions are satisfied. If that sort of term were due to the integration of microscopic, charged degrees of freedom, the constant  $\xi$  could be related to the inverse of the corresponding coupling constant (the would-be fine structure constant).

A relationship that becomes useful in the forthcoming derivations, is that  $\mathcal{S}_{\mathcal{M}}$  may be shown to be equivalent to:

$$\mathcal{S}_{\mathcal{M}}(A, y) = \frac{1}{2\xi} \int_{\mathcal{M}} d^2\sigma \sqrt{g(\sigma)} (\hat{n}_\mu(\sigma) \tilde{F}_\mu[y(\sigma)])^2, \quad (8)$$

where  $\tilde{F}_\mu(x) = \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda(x)$ , and  $\hat{n}_\mu(\sigma)$  is the unit normal to the surface:

$$\hat{n}_\mu(\sigma) = \frac{N_\mu(\sigma)}{\sqrt{N^2(\sigma)}}, \quad N_\mu(\sigma) = \frac{1}{2} \epsilon^{\alpha\beta} \epsilon_{\mu\nu\lambda} e_\alpha^\nu(\sigma) e_\beta^\lambda(\sigma), \quad (9)$$

(it is straightforward to verify that  $\sqrt{N^2(\sigma)} = \sqrt{g(\sigma)}$ ).

In the case of two mirrors, denoted by  $L$  and  $R$ , rather than  $\mathcal{S}_I = \mathcal{S}_{\mathcal{M}}$  as before, we shall have:

$$\mathcal{S}_I(A) = \mathcal{S}_L(A, y_L) + \mathcal{S}_R(A, y_R), \quad (10)$$

where we have introduced two parametrizations, denoted respectively by  $y_L^\mu(\sigma_L)$  and  $y_R^\mu(\sigma_R)$  for the respective mirrors. Besides having not necessarily equal coupling constants  $\xi_L, \xi_R$ , the actions are assumed to have exactly the same structure as  $\mathcal{S}_{\mathcal{M}}(A, y)$ .

The observable we shall be concerned with here is the pair-creation probability  $\mathcal{P}$ , which in turn may be obtained from  $\Gamma$ , the effective action obtained by integrating out the vacuum fluctuations of  $A$  in the presence of the mirror(s):

$$e^{-\Gamma} = \mathcal{Z} = \int \mathcal{D}A e^{-\mathcal{S}(A)}. \quad (11)$$

By its very definition,  $\Gamma$  is a functional of the geometry of the mirror(s), and a function of the constants that control the strength of the coupling between them and the field.

From Eq. (11), we see that the effective action may be written as follows:

$$e^{-\Gamma} = e^{-\Gamma_0} e^{-\Gamma_I}, \quad (12)$$

where

$$e^{-\Gamma_0} \equiv \mathcal{Z}_0 = \int \mathcal{D}A e^{-S_0(A)}, \quad (13)$$

is the effective action in the absence of the mirror(s), and:

$$e^{-\Gamma_I} \equiv \langle e^{-S_I} \rangle, \quad (14)$$

where we have introduced the  $\langle \cdot \rangle$  symbol to denote functional averaging, with a Gaussian weight determined by the free action, namely:

$$\langle \dots \rangle \equiv \frac{\int \mathcal{D}A \dots e^{-S_0(A)}}{\int \mathcal{D}A e^{-S_0(A)}}. \quad (15)$$

Therefore, since only the interaction term may produce a non-vanishing imaginary part, the probability  $\mathcal{P}$  may be written as follows:

$$\mathcal{P} = 2 \text{Im}[\Gamma_I], \quad (16)$$

where  $\Gamma_I$  denotes the continuation to real time of the equally defined functional.

Let us consider, in the next Section, the perturbative calculation of  $\Gamma_I$  and of its imaginary part, without specifying the state of motion of the mirror(s).

### 3. Perturbation theory

When  $\Gamma_I$  is expanded in powers of  $S_I$ ,  $\Gamma_I = \Gamma_I^{(1)} + \Gamma_I^{(2)} + \dots$ , the first and second-order terms are given by:

$$\Gamma_I^{(1)} = \langle S_I \rangle, \quad (17)$$

and

$$\Gamma_I^{(2)} = \frac{1}{2} \langle S_I^2 \rangle - \frac{1}{2} \langle S_I \rangle^2 = -\frac{1}{2} \langle (S_I - \langle S_I \rangle)^2 \rangle. \quad (18)$$

For a single mirror,  $\Gamma_I \rightarrow \Gamma_{\mathcal{M}}$  is obtained by making the substitution:  $S_I \rightarrow S_{\mathcal{M}}$  (with  $S_{\mathcal{M}}$  as defined in Eq. (4)) in the expressions above, while for two mirrors the substitution  $S_I \rightarrow S_L + S_R$  leads to:

$$\Gamma_I^{(1)} \equiv \Gamma_L^{(1)} + \Gamma_R^{(1)}, \quad \Gamma_I^{(2)} \equiv \Gamma_L^{(2)} + \Gamma_R^{(2)} + \Gamma_{LR}^{(2)}, \quad (19)$$

where, in a self-explaining notation:

$$\begin{aligned} \Gamma_R^{(1)} &\equiv \Gamma_{\mathcal{M}}^{(1)} \Big|_{\mathcal{M} \rightarrow L,R}, \quad \Gamma_R^{(2)} \equiv \Gamma_{\mathcal{M}}^{(2)} \Big|_{\mathcal{M} \rightarrow L,R}, \\ \Gamma_{LR}^{(2)} &= -\langle (S_L - \langle S_L \rangle)(S_R - \langle S_R \rangle) \rangle. \end{aligned} \quad (20)$$

In other words, to this order, we have terms that involve just one of the mirrors, plus one which mixes both of them. Therefore, we just need the effective action  $\Gamma_{\mathcal{M}}^{(1,2)}$ , corresponding to an interaction term  $S_{\mathcal{M}}$ , plus  $\Gamma_{LR}^{(2)}$ ; the remaining ones may be obtained by performing the appropriate substitutions in a set of 'independent' functionals.

The explicit form of the independent terms we need in order to determine all the rest (up to the second order) is:

$$\Gamma_{\mathcal{M}}^{(1)} = \frac{1}{2\xi} \int_{\mathcal{M}} d^2\sigma \sqrt{g(\sigma)} \hat{n}_\mu(\sigma) \hat{n}_\nu(\sigma) \langle \tilde{F}_\mu[y(\sigma)] \tilde{F}_\nu[y(\sigma)] \rangle, \quad (21)$$

$$\begin{aligned} \Gamma_{\mathcal{M}}^{(2)} &= -\frac{1}{2(2\xi)^2} \int_{\mathcal{M}} d^2\sigma \sqrt{g(\sigma)} \hat{n}_\mu(\sigma) \hat{n}_\nu(\sigma) \\ &\times \int_{\mathcal{M}} d^2\sigma' \sqrt{g(\sigma')} \hat{n}_{\mu'}(\sigma') \hat{n}_{\nu'}(\sigma') \\ &\times \langle : \tilde{F}_\mu[y(\sigma)] \tilde{F}_\nu[y(\sigma)] : : \tilde{F}_{\mu'}[y(\sigma')] \tilde{F}_{\nu'}[y(\sigma')] : \rangle, \end{aligned} \quad (22)$$

and

$$\begin{aligned} \Gamma_{LR}^{(2)} &= -\frac{1}{2\xi_L 2\xi_R} \int_{\mathcal{M}_L} d^2\sigma \sqrt{g_L(\sigma)} \hat{n}_\mu^L(\sigma) \hat{n}_\nu^L(\sigma) \\ &\times \int_{\mathcal{M}_R} d^2\sigma' \sqrt{g_R(\sigma')} \hat{n}_{\mu'}^R(\sigma') \hat{n}_{\nu'}^R(\sigma') \\ &\times \langle : \tilde{F}_\mu[y_L(\sigma)] \tilde{F}_\nu[y_L(\sigma)] : : \tilde{F}_{\mu'}[y_R(\sigma')] \tilde{F}_{\nu'}[y_R(\sigma')] : \rangle, \end{aligned} \quad (23)$$

where we have used the notation  $:G: \equiv G - \langle G \rangle$ .

Let us now evaluate each one of the previous terms in turn. All of them involve the  $\langle \tilde{F}_\mu(x) \tilde{F}_\nu(x') \rangle$  correlator, which may be obtained from the gauge-field propagator. The outcome is

$$\langle \tilde{F}_\mu(x) \tilde{F}_\nu(x') \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot (x-x')} \delta_{\mu\nu}^\perp(k), \quad (24)$$

where we have introduced the object:  $\delta_{\mu\nu}^\perp(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$ .

Therefore, the first-order term becomes

$$\Gamma_I^{(1)} = \frac{1}{2\xi} \int_{\mathcal{M}} d^2\sigma \sqrt{g(\sigma)} \hat{n}_\mu(\sigma) \hat{n}_\nu(\sigma) \int \frac{d^3k}{(2\pi)^3} \delta_{\mu\nu}^\perp(k), \quad (25)$$

which is UV-divergent; indeed, using an Euclidean cutoff  $\Lambda$ , we see that

$$\int_{|k| \leq \Lambda} \frac{d^3k}{(2\pi)^3} \delta_{\mu\nu}^\perp(k) = \frac{\Lambda^3}{9\pi^2} \delta_{\mu\nu}. \quad (26)$$

Finally,

$$\Gamma_{\mathcal{M}}^{(1)} = \frac{\Lambda^3}{9\pi^2 \xi} \int_{\mathcal{M}} d^2\sigma \sqrt{g(\sigma)} = \frac{\Lambda^3}{9\pi^2 \xi} \text{area}(\mathcal{M}). \quad (27)$$

As indicated, it is a divergent term proportional to the area of the worldsheet. This may be absorbed into a renormalization of the tension associated to the curve, and therefore, does not contribute to dissipative effects associated to the motion of the boundary.

Let us now consider the second-order term  $\Gamma_I^{(2)}$ : applying Wick's theorem and taking into account the form of the interaction term,

$$\begin{aligned} \Gamma_{\mathcal{M}}^{(2)} &= -\frac{1}{(2\xi)^2} \int_{\mathcal{M}} d^2\sigma \sqrt{g(\sigma)} \hat{n}_\mu(\sigma) \hat{n}_\nu(\sigma) \\ &\times \int_{\mathcal{M}} d^2\sigma' \sqrt{g(\sigma')} \hat{n}_{\mu'}(\sigma') \hat{n}_{\nu'}(\sigma') \\ &\times \langle \tilde{F}_\mu[y(\sigma)] \tilde{F}_{\mu'}[y(\sigma')] \rangle \langle \tilde{F}_\nu[y(\sigma)] \tilde{F}_{\nu'}[y(\sigma')] \rangle, \end{aligned} \quad (28)$$

which, recalling Eq. (24), can be rendered as follows:

$$\Gamma_{\mathcal{M}}^{(2)} = \frac{1}{2\xi^2} \int \frac{d^3k}{(2\pi)^3} f_{\mu\nu}(-k) \tilde{\Pi}_{\mu\nu;\mu'\nu'}(k) f_{\mu'\nu'}(k) \quad (29)$$

where

$$f_{\mu\nu}(k) \equiv \int d^2\sigma \sqrt{g(\sigma)} \hat{n}_\mu(\sigma) \hat{n}_\nu(\sigma) e^{-ik \cdot y(\sigma)}, \quad (30)$$

and

$$\tilde{\Pi}_{\mu\nu;\mu'\nu'}(k) = -\frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \delta_{\mu\mu'}^\perp(p) \delta_{\nu\nu'}^\perp(k-p). \quad (31)$$

An entirely analogous analysis for  $\Gamma_{LR}^{(2)}$  leads to:

$$\Gamma_{LR}^{(2)} = \frac{1}{\xi_L \xi_R} \int \frac{d^3k}{(2\pi)^3} f_{\mu\nu}^L(-k) \tilde{\Pi}_{\mu\nu;\mu'\nu'}(k) f_{\mu'\nu'}^R(k) \quad (32)$$

where  $f_{\mu\nu}^L$  and  $f_{\mu\nu}^R$  are defined as in Eq. (30), for the respective mirror, and the same kernel  $\tilde{\Pi}_{\mu\nu;\mu'\nu'}$  as in  $\Gamma_{\mathcal{M}}^{(2)}$ . Let us then consider the calculation of this kernel, which determines all the second-order terms. After some algebra, we see that the structure of that object is:

$$\tilde{\Pi}_{\mu\nu;\mu'\nu'}(k) = A \delta_{\mu\mu'} \delta_{\nu\nu'} + B_{\mu\nu;\mu'\nu'}(k), \quad (33)$$

where

$$A \equiv -\frac{1}{6} \int \frac{d^3p}{(2\pi)^3}, \quad (34)$$

and

$$B_{\mu\nu;\mu'\nu'}(k) \equiv -\frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \frac{p_\mu p_{\mu'} (k-p)_\nu (k-p)_{\nu'}}{p^2 (k-p)^2}. \quad (35)$$

The term proportional to  $A$  in the previous expression may also be absorbed into a renormalization of the tension, as it was the case for the first order calculation. Since we are interested in dissipative effects, we neglect this kind of contribution (which dimensional regularization renormalizes away) from now on.

Using dimensional regularization and introducing a Feynman parameter representation, the  $D$  dimensional version of the remaining term is:

$$B_{\mu\nu;\mu'\nu'}(k) = -\frac{1}{2} \int_0^1 d\alpha \int \frac{d^D p}{(2\pi)^D} \frac{p_\mu p_{\mu'} (p+k)_\nu (p+k)_{\nu'}}{[(1-\alpha)p^2 + \alpha(p+k)^2]^2}. \quad (36)$$

For  $D=3$ , the result is:

$$\tilde{\Pi}_{\mu\nu;\mu'\nu'}(k) = -\frac{1}{2048} \left( \frac{2}{5} \delta_{\mu\mu'} \delta_{\nu\nu'} |k|^3 + 3 k_\mu k_{\mu'} k_\nu k_{\nu'} \frac{1}{|k|} \right), \quad (37)$$

which, we recall, allows us to determine all the second-order terms, both for one or two mirrors.

Therefore, performing the rotation back to real time, and taking imaginary parts afterwards, we see that:

$$\begin{aligned} \text{Im}[\Gamma_{\mathcal{M}}^{(2)}] &= \frac{1}{4096 \xi^2} \int \frac{d^3k}{(2\pi)^3} f_{\mu\nu}(-k) f_{\mu'\nu'}(k) \\ &\times \left[ \frac{2}{5} g^{\mu\mu'} g^{\nu\nu'} \text{Im}(|k|^3) + 3 k^\mu k^{\mu'} k^\nu k^{\nu'} \text{Im}(|k|^{-1}) \right], \end{aligned} \quad (38)$$

while for  $\text{Im}[\Gamma_{LR}^{(2)}]$  we see that:

$$\begin{aligned} \text{Im}[\Gamma_{LR}^{(2)}] &= \frac{1}{2048 \xi_L \xi_R} \int \frac{d^3k}{(2\pi)^3} f_{\mu\nu}^L(-k) f_{\mu'\nu'}^R(k) \\ &\times \left[ \frac{2}{5} g^{\mu\mu'} g^{\nu\nu'} \text{Im}(|k|^3) + 3 k^\mu k^{\mu'} k^\nu k^{\nu'} \text{Im}(|k|^{-1}) \right]. \end{aligned} \quad (39)$$

The relevant imaginary parts are:

$$\text{Im}(|k|^3) = \theta(|k_0| - |\mathbf{k}|) (k_0^2 - |\mathbf{k}|^2)^{3/2}, \quad (40)$$

and

$$\text{Im}(|k|^{-1}) = \theta(|k_0| - |\mathbf{k}|) (k_0^2 - |\mathbf{k}|^2)^{-1/2}, \quad (41)$$

where  $\theta$  denotes Heaviside's step function and  $\mathbf{k} \equiv (k_1, k_2)$ .

Expressions (38) and (39) may be regarded as general results, where the motion is encoded in  $f_{\mu\nu}$  and quantum effect belong to the kernel, depending on  $k$ .

It is worth mentioning here that there is a qualitative difference in the momentum dependence of the integrals in the results above, in comparison with the scalar field model [18]. Indeed, because of the different nature of the field, one now has constraints of the allowed coupling to the mirrors, and that determines the possible form of the momentum dependence, because the effective action should be dimensionless (in natural units).

## 4. Examples

Let us consider here the form adopted by the second-order contributions to the imaginary part, in situations where one can obtain more explicit expressions.

### 4.1. Small departures with respect to a planar world-sheet, single mirror

As a first concrete example, we consider small, time-dependent departures about a static spatial straight line. In other words, small deformations of a planar world-sheet, which we assume to coincide with the  $y^2=0$  plane:

$$y^0 = x^0, \quad y^1 = x^1, \quad y^2 = q(x_{i1}) \quad (42)$$

where  $x_{i1} = (x^0, x^1)$ , and  $q(x_{i1})$  represents small departures from the static straight line configuration. We thus expand  $f_{\mu\nu}(k)$  in powers of  $q(x_{i1})=0$ , obtaining:

$$f_{\mu\nu}(k) = f_{\mu\nu}^{(0)}(k) + f_{\mu\nu}^{(1)}(k) + \dots, \quad (43)$$

where

$$f_{\mu\nu}^{(0)}(k) = \delta_\mu^2 \delta_\nu^2 \delta(k^0) \delta(k^1), \quad (44)$$

and

$$f_{\mu\nu}^{(1)}(k) = -i (\delta_\mu^2 \delta_\nu^2 k^2 + \delta_\mu^2 \delta_\nu^\alpha k_\alpha + \delta_\mu^\alpha \delta_\nu^2 k^\alpha) \tilde{q}(k_{i1}), \quad (45)$$

where we have introduced the Fourier transform of the departure:

$$\tilde{q}(k_{i1}) = \int d^2x_{i1} q(x_{i1}) e^{-ik_{i1} \cdot x_{i1}}. \quad (46)$$

It may be verified that, up to the second order in the departure, the only non-vanishing contribution to the imaginary part comes from using (twice) the first-order term for  $f_{\mu\nu}$  in the general expression. We then find the pair-creation probability  $\mathcal{P} = 2 \text{Im}[\Gamma_{\mathcal{M}}^{(2)}]$  to be:

$$\mathcal{P} = \frac{1}{2^{11} \xi^2} \int \frac{d^3 k}{(2\pi)^3} \theta(|k_0| - |\mathbf{k}|) \times \left[ \frac{2}{5} ((k^2)^2 + 2k_{\parallel}^2) ((k^0)^2 - |\mathbf{k}|^2)^{3/2} + 3(k^2)^2 ((k^2)^2 + 2k_{\parallel}^2) ((k^0)^2 - |\mathbf{k}|^2)^{-1/2} \right] |\tilde{q}(k_{\parallel})|^2. \quad (47)$$

Performing the  $k^2$  integral, we obtain a more compact expression, depending only on the momenta which are parallel to the space-time plane:

$$\mathcal{P} = \frac{941}{2^{16} 5 \xi^2} \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \theta(|k^0| - |k^1|) |k_{\parallel}|^6 |\tilde{q}(k_{\parallel})|^2, \quad (48)$$

which exhibits a power-like spectrum.

To get an even more concrete expression, we consider a case in which the deformation of the linear boundary amounts to a standing wave. Therefore, we chose  $q(k_{\parallel})$  as follows:

$$q(k_{\parallel}) = \epsilon \cos(\Omega x^0) \cos(p x^1), \quad (49)$$

where  $\epsilon$ ,  $\Omega$  and  $p$  are positive constants. Thus,

$$\tilde{q}(k_{\parallel}) = 4\pi^2 \epsilon [\delta(k^0 - \Omega) \delta(k^1 - p) + \delta(k^0 - \Omega) \delta(k^1 + p) + \delta(k^0 + \Omega) \delta(k^1 - p) + \delta(k^0 + \Omega) \delta(k^1 + p)]. \quad (50)$$

Replacing the previous expression into Eq. 48 and integrating out  $k_{\parallel}$ , we see that the time and space periodicities imply a result proportional to the total time  $T$  and length  $L$  of the mirror, such that the probability per unit length and time becomes:

$$\frac{\mathcal{P}}{LT} = \frac{941 \epsilon^2}{2^{16} 5 \xi^2} \theta(\Omega - p) (\Omega^2 - p^2)^3. \quad (51)$$

Thus, this exhibits a threshold for the frequency of the standing wave, related to its wave number. Since the maximum velocity  $v$  of each point in the mirror is  $v \sim \Omega \epsilon$ , this threshold implies that  $v$  should be larger (in units where the speed of light  $c = 1$ ) than the ratio  $\epsilon/\lambda$ , where  $\lambda$  is the wavelength. Therefore, to overcome the threshold with non-relativistic speeds, the amplitude of the wave needs to be smaller than its wavelength. Namely,  $\epsilon p < v < 1$ .

We may pinpoint the reason for the emergence of a non-trivial threshold for this kind of motion: Indeed, a standing wave provides us with both momentum and frequency scales, which, when combined with the (always present) threshold for the existence of an imaginary part (namely, a time-like momentum), leads to phenomena like the one we have just described.

It is perhaps worth noting the possible values of the momentum and frequency of the standing waves will depend on mechanical properties of the mirror, which do not appear explicitly in the model we are considering.

#### 4.2. Standing waves with small amplitude, two mirrors

This example corresponds to two mirrors, and the contribution we consider is  $\Gamma_{LR}^{(2)}$ . We assume for the  $L$  mirror the parametrization:

$$y_L^0 = x^0, \quad y_L^1 = x^1, \quad y_L^2 = q_L(x_{\parallel}) \quad (52)$$

while for the  $R$  one we include an average distance  $a$ :

$$y_R^0 = x^0, \quad y_R^1 = x^1, \quad y_R^2 = a + q_R(x_{\parallel}). \quad (53)$$

To the second order (first order in each of the departures), we get:

$$\mathcal{P} = \frac{1}{2^{11} \xi_L \xi_R} \int \frac{d^3 k}{(2\pi)^3} \theta(|k^0| - |\mathbf{k}|) \cos(k^2 a) \tilde{q}_L(-k_{\parallel}) \tilde{q}_R(k_{\parallel}) \times \left[ \frac{2}{5} ((k^2)^2 + 2k_{\parallel}^2) ((k^0)^2 - |\mathbf{k}|^2)^{3/2} + 3(k^2)^2 ((k^2)^2 + 2k_{\parallel}^2) ((k^0)^2 - |\mathbf{k}|^2)^{-1/2} \right]. \quad (54)$$

Note the presence of the average distance  $a$ , inside the integrand. We also see that, due to the presence of the Fourier transforms of both departures, in order to have a non-vanishing result we need them to have a non-vanishing overlap between those Fourier transforms. In the special case of motions which involve a single mode this term will only be non-vanishing only if their frequency and wave-number coincide.

For the special case of two standing waves in counterphase:

$$q_L(k_{\parallel}) = 4\epsilon_L \cos(\Omega x^0) \cos(p x^1) \\ q_R(k_{\parallel}) = -4\epsilon_R \cos(\Omega x^0) \cos(p x^1). \quad (55)$$

The probability in this case may be written as follows:

$$\frac{\mathcal{P}}{LT} = \frac{\epsilon_L \epsilon_R}{2^{10} \pi \xi_L \xi_R} \theta(\Omega - p) (\Omega^2 - p^2)^3 \varphi(\sqrt{\Omega^2 - p^2} a), \quad (56)$$

with

$$\varphi(x) = \int_{-1}^1 ds \cos(sx) \times \left[ \frac{2}{5} (s^2 + 2)(1 - s^2)^{3/2} + 3s^2 (s^2 + 2)^2 (1 - s^2)^{-1/2} \right]. \quad (57)$$

Performing the  $s$  integration, we finally obtain

$$\varphi(x) = \frac{3\pi}{5x^5} [x(340 - 111x^2 + 45x^4) J_0(x) + (-680 + 307x^2 - 75x^4) J_1(x)]. \quad (58)$$

It is worth commenting on some qualitative features of this result, corresponding to two mirrors: firstly, the fact that there is non-trivial constraint between the respective motions, in order to have a non-vanishing imaginary part, is valid only to the order  $\frac{1}{\xi_L \xi_R}$ , since, as seen from Eq. (32), the functions corresponding to the motions of each mirror appear evaluated at  $k$  and  $-k$ . One should expect different constraints at higher orders, since more momenta will appear. Regarding the form of the result, we note that the  $p$  dependence is analogous to the one for a single mirror, but the result is modulated by a smooth dimensionless function of  $\Omega a$ . The imaginary part tends to zero when the mirrors are at an infinite distance, as it should be. That decrease is not monotonous, however, but slightly modulated (via the Bessel functions), because of interference phenomena when two mirrors are present.

#### 4.3. Stationary waves with arbitrary amplitude

Let us consider here a qualitative difference that appears when one considers stationary waves, or, more generally, a  $q(x_{\parallel})$  function which is periodic in time and space. We assume those periods to be  $\tau = \frac{2\pi}{\Omega}$  and  $\lambda = \frac{2\pi}{p}$ , respectively. This implies that  $C_{\mu\nu}(k^2; x_{\parallel})$ , an object which appears in the integrand which defines the function  $f_{\mu\nu}$ , is also periodic:



$$\begin{aligned} C^{\mu\nu}(k^2; x^0, x^1) &\equiv \sqrt{g(x_{||})} \hat{n}^\mu(x_{||}) \hat{n}^\nu(x_{||}) e^{-ik^2 q(x_{||})} \\ &= C^{\mu\nu}(k^2; x^0 + \tau, x^1 + \lambda), \end{aligned} \quad (59)$$

with the same periodicity as  $q$ .

Then  $C^{\mu\nu}(k^2; x_{||})$  can be expanded in a double Fourier series:

$$C^{\mu\nu}(k^2; x_{||}) = \sum_{l_{||}} \tilde{C}^{\mu\nu}(k^2; l_{||}) e^{-il_{||} \cdot x_{||}}, \quad (60)$$

where  $l_{||} = 2\pi(\frac{n^0}{\tau}, \frac{n^1}{\lambda})$ , with  $n^0$  and  $n^1$  integer numbers. Thus,

$$\tilde{C}^{\mu\nu}(k^2; l_{||}) = \frac{1}{\tau\lambda} \int_0^\tau dx^0 \int_0^\lambda dx^1 C^{\mu\nu}(k^2; x_{||}) e^{il_{||} \cdot x_{||}}. \quad (61)$$

Hence,

$$f^{\mu\nu}(k) = (2\pi)^2 \sum_{n^0, n^1} \tilde{C}^{\mu\nu}(k^2; l_{||}) \delta(k_{||} - l_{||}). \quad (62)$$

Even without knowing the exact form of the  $\tilde{C}^{\mu\nu}$  functions, we see that imaginary part of the effective action will be proportional to the total time and the length of the mirror. Besides, another qualitatively different feature has to do with the threshold for the existence of an imaginary part. Indeed, the existence of a series in  $f^{\mu\nu}$  implies that, in order to have an imaginary part, we need to have:

$$|l^0| > |l^1|, \quad (63)$$

or:

$$\left| \frac{n^0}{n^1} \right| > \frac{\tau}{\lambda} = \frac{p}{\Omega}. \quad (64)$$

In other words, there always be non-vanishing contribution to the imaginary part, regardless of the ratio between the wave frequency and wavelength.

Another analysis that can be done to study a configuration of standing waves result of defining two different waves, with opposite direction. Each wave has the form

$$q(x_{||}) = A \cos(p_{||} x_{||}), \quad (65)$$

and  $p_{||} = (p_0, p_1)$ . In this case, if we assume that the derivative of  $q$  is small, it can be shown that the only contribution comes from  $f_{22}$ . In order to evaluate this function, we use the Jacobi-Anger expansion and we obtain that the imaginary part of the effective action is given by

$$\text{Im}[\Gamma_I^{(2)}] = \frac{1}{2^{12} \xi^2} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk^2}{2\pi} \Theta[n^2((p^0)^2 - (p^1)^2) - (k^2)^2] \quad (66)$$

$$\times \left[ \frac{2}{5} J_n^2(k^2 A) [n^2((p^0)^2 - (p^1)^2) - (k^2)^2]^{3/2} \quad (67)$$

$$+ 3 \frac{(k^2)^4}{\sqrt{n^2((p^0)^2 - (p^1)^2) - (k^2)^2}} \right]. \quad (68)$$

A similar phenomenon appears, of course, in the two-mirror case.

## 5. Conclusions

We have evaluated the probability of vacuum decay, via the imaginary part of the effective action  $\Gamma$ , for semitransparent mirrors in 2 + 1 dimensions, coupled to an Abelian gauge field. We have therefore extended previous analysis to a different kind of field, and to non-rigid motions of the mirror(s). The fact that the vacuum field is not a scalar does lead, as we have shown, to qualitatively different results. The origin of that difference can be traced back to the fact that gauge invariance changes the nature of the allowed coupling between field and mirror, and this affects the results. In particular, that coupling now contains derivatives; that is reflected in the momenta appearing in the numerator of Eq. (35), absent in the scalar field case.

We obtained general expressions for the leading contribution to the imaginary part of  $\Gamma$ , and more explicit ones for the case of small amplitudes, and for standing waves. We believe that standing waves are a natural configuration to consider, since they appear, for example, when one deals with a string-like mirror with fixed ends.

We have shown that, when the motion is periodic both in time and space, the imaginary part is always non-vanishing, with the threshold arising for small amplitudes corresponding to just one of the possible processes leading to pair creation.

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