# Processes Integration: Multiplant Complex vs Multipurpose Plant Assessment 

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#### Abstract

This paper presents a tool for the generation and evaluation of different alternatives of product production. The importance of this approach lies in the integration of synthesis, design, operation, and scheduling decisions. When simultaneously approached, these decisions allow consideration of the relationships and effects between critical elements of the problem that are usually analyzed sequentially. Two different scenarios are modeled in order to evaluate and select which production process is convenient to develop: multipurpose plant and multiplant complex with single-product batch plants. The methodology is illustrated by an industrially motivated case study involving the production of three products: torula yeast, bakery yeast, and brandy. According to different production conditions, both scenarios are solved, and the analysis and conclusions with regards to the most favorable production scheme are reported.


## 1. Introduction

The constant shift toward the production of higher-valueadded products in chemical processing industries has encouraged modeling and optimization studies of batch processes. Whereas significant development has been made in the design, planning, and scheduling of batch plants with single or multiple production routes, the problem of synthesis, design, and operation in batch multiplant complexes has received much less attention.

Previous works have been focused on specific decision levels. Tools and models that solve separately the different aspects of a process have been developed. However, to obtain a good preliminary production process, it is important to consider design, synthesis, operation, and scheduling simultaneously. In the last years, a few authors have incorporated scheduling constraints into the synthesis problem of multiproduct and multipurpose batch plants. Birewar and Grossmann ${ }^{1,2}$ have addressed the problem of simultaneous sizing and scheduling of multiproduct batch plants that account for the Unlimited Intermediate Storage and Zero Wait (ZW) policies with mixedproduct campaigns. In that work, they developed a nonlinear programming (NLP) model for fixed time and size factors. Their later work incorporates the structural decision of parallel units for some stages resulting in a mixed-integer nonlinear programming (MINLP) model.

Zhang and Sargent ${ }^{3,4}$ developed a general scheduling formulation based on a variable-event time representation. This continuous-time formulation for scheduling can be easily extended to the design and synthesis of batch plants. However, when nonlinear task models (processing time, utility usage, unit availability, etc.) and nonlinear capital cost functions are considered, a nonconvex MINLP problem will arise. Even for the locally linearized models, a large number of auxiliary variables and constraints for linearization of bilinear terms of integer and continuous variables must be typically introduced to reduce the MINLP into a mixed-integer linear programming (MILP) model, which makes the problem very large in scale and difficult to solve, as stated by Xia and Macchietto. ${ }^{5}$ These authors presented a formulation based on the variable-event time scheduling model of Zhang and Sargent. ${ }^{3,4}$ A stochastic method is used to solve the resulting nonconvex MINLP problem directly, instead of introducing a large number of auxiliary

[^0]variables and constraints to reduce the MINLP into a MILP. Lin and Floudas ${ }^{6}$ extended the continuous-time scheduling formulation proposed by Ierapetritou and Floudas ${ }^{7}$ and Ierapetritou et al. ${ }^{8}$ to address the problem of integrated design, synthesis, and scheduling of multipurpose batch plants. They studied both linear and nonlinear cases, which resulted in MILP and MINLP problems, respectively. MILP problems were solved with an LP-based branch-and-bound method. Nonconvex MINLP problems were solved with MINOPT, ${ }^{9}$ and global optimal solutions can be obtained for a class of problems with special structures.

On the other hand, there are few works in the literature dealing with multiplant complex integration. Lee et al. ${ }^{10}$ developed a capacity expansion model of an integrated production and distribution system comprised of multisite batch plants and several warehouses. The proposed MINLP model determines a revised plant configuration, sizes and operation modes of newly added units, shipments from plants to warehouses, and batch processing variables. Kallrath ${ }^{11}$ addresses the issue of process and plant representation by describing a tool for simultaneous strategic and operational planning in a multisite production network. The total net profit of a global network is optimized, where key decisions include the following: operating modes of equipment in each time period, production and supply of products, minor changes to the infrastructure (e.g., the addition and removal of equipment from sites), and raw material purchases and contracts. Jackson and Grossmann ${ }^{12}$ considered a problem that involves the production and distribution of products manufactured at several geographically distributed sites to different global markets. They developed a multiperiod nonlinear programming optimization model for the planning and coordination of production, transportation, and sales of geographically distributed multiplant facilities.

In this work, detailed NLP models are developed in order to obtain the optimal synthesis, design, and operation of two scenarios: multipurpose (MP) plant and multiplant complex (MC). In both cases, the structure of the plant is decided on simultaneously with the design, operating, and scheduling issues. A separate mother plant provides material and energy resources for these processes. Thus, residuals from one plant might eventually become raw materials for others. For both schemes, the objective function employed is the maximization of annualized net profit given by the total expected selling price minus the total investment and operation cost.


Figure 1. Flowsheet for a sequential MP batch plant.

The paper is focused on providing a tool for generating and evaluating different alternatives of product production. The importance of this approach lies in the integration of the synthesis, design, operation, and scheduling decisions. When simultaneously approached, these decision aspects allow consideration of the relationships and effects between critical elements of the problem that are usually analyzed sequentially.

## 2. Problem Definition

2.1. MP Plant. In this work, a sequential MP plant is considered. In a sequential MP plant, it is possible to recognize a specific direction in the plant floor that is followed by the production paths of all products. ${ }^{13}$ However, some processing units are used only by some products. Obviously, the presented model is also valid for the multiproduct batch plant where all products use all stages. The MP plant is also integrated to a mother plant that provides the material and energy resources.

Figure 1 shows a sequential MP plant where two products, A and B, and a byproduct of B, product C, are produced. Product A follows production path $\mathrm{U} 1 \rightarrow \mathrm{U} 4$, and U 1 receives an extra feeding (blend of batches) and a recycled batch from U4. Product B follows path U1 $\rightarrow \mathrm{U} 2 \rightarrow \mathrm{U} 3 \rightarrow \mathrm{U} 4$. At U3, batches are split to produce product C through U5. U3 also has an extra feeding.

The synthesis, design, and operation problem for a MP plant is defined as follows:

Given $N_{\mathrm{p}}$ products to be processed in the sequential MP plant that has batch and semicontinuous units, the production targets, and the selling prices, determine the plant configuration including unit sizes, the mixed-product campaign, the material and energy resources for each production, and the various processing variables considering mixing, splitting, and recycle of batches to attain the best economical and environmental solution. In addition, the model contains material and energy balances for each process production and interconnection constraints between processes.

In many cases, some stream in the production of a product can be recycled to some stage of another product production process. In such a case, the single-product campaign is impracticable because the material to be recycled should be stored. In addition, single-product campaigns require high inventory values, and many products cannot be stored because they are degraded in a short time. From the point of view of the design model, mixed-product campaigns pose greater challenges than those that arise from the formulations for singleproduct campaigns. The campaign configuration and its scheduling must be considered. In this work, a mixed-product campaign model for the MP plant is adopted.
2.2. MC Model. In a MC, there are two or more interconnected plants that are supplied with raw materials and energy by a mother plant. The model contains the material and energy


Figure 2. MC scheme integration.
balances for each plant and the interconnection constraints between the plants. The example in Figure 2 shows that, with only two derivative plants, there are interconnections between the derivative plants given by residue recycles and intermediate productions. Both derivative plants can produce a product and a byproduct. A byproduct is a product that is obtained by splitting a batch or processing it in a different manner. Besides producing material and energy resources for derivative plants, the mother plant produces sale products.

The derivative plants have batch and semicontinuos units, and mixing, splitting, and recycle batches are allowed.

The synthesis, design, and operation problem for multisite production is defined as follows:

Given $N_{\mathrm{p}}$ derivative products to be processed, its production target, the selling price, and the production routes for each product, determine the plant configuration for each product including unit sizes, material and energy streams between the plants, and the various processing variables to obtain the best economical and environmental solution.

## 3. Model Formulations

3.1. MP Plant Model. The model proposed by Corsano et al. ${ }^{14}$ is adopted for the optimal synthesis, design, operation, and scheduling of a sequential MP plant with a mixed-product campaign. That work presents a heuristic strategy for determining the possible configurations of the campaign. This methodology consists of first the solution of a relaxed NLP model, with the aim of finding the optimal plant configuration and ratio between the number of batches of each product. Afterward, a NLP model is solved to obtain the optimal feasible design and operation, implementing a novel representation of the scheduling constraints. This representation arises from the adopted ZW transfer policy. The heuristic strategy can be summarized in the following steps:
(i) First, a model whose constraints consider the design and operation of a MP plant without considering the task scheduling constraints is solved. This model is a relaxation of the mixedproduct campaign problem and is solved as a NLP problem. The model has an embedded superstructure that considers different configuration options for the plant synthesis. ${ }^{15}$ The solution of the relaxed model provides the estimated number of batches of each product and the configuration of the plant.
(ii) Relationships between the number of batches of each product, which are obtained from the relaxed model, are established, and the possible sequences of the multiproduct campaigns are constructed for the plant structure obtained in the relaxed model solution.
(iii) For each proposed campaign configuration, specific constraints are added to the relaxed model in order to ensure that the production processes of two different products do not overlap in the same unit. In this way, each mixed-product campaign model is formulated for the optimal plant configuration obtained from the relaxed model solution and the corresponding set of scheduling constraints. At this step, the plant configuration is fixed and the sizing problem is solved. The solution of the relaxed model is used to propose an initial solution of the model with mixed-product campaigns.
(iv) The models are solved, and the solution with a maximum net annual profit is chosen as the optimal solution.

Relaxed Model Formulation. A plant with $N_{j}$ batch units and $N_{k}$ semicontinuous units is considered. $N_{\mathrm{p}}$ products are manufactured in the plant, not necessarily following the same production path.

The design equations corresponding to the batch and semicontinuous units are stated as

$$
\begin{align*}
V_{j} \geq V_{i j} & \forall i=1, \ldots, N_{\mathrm{p}}, \forall j \in \mathrm{~EB}_{i}  \tag{1}\\
R_{k} \geq R_{i k} & \forall i=1, \ldots, N_{\mathrm{p}}, \forall k \in \mathrm{ES}_{i} \tag{2}
\end{align*}
$$

where $V$ and $R$ are the batch and semicontinuous sizes, respectively, and $\mathrm{EB}_{i}$ and $\mathrm{ES}_{i}$ represent the set of batch and semicontinuous units included in the production path of product $i$.

Let $t_{i j}$ be the processing time for product $i$ at stage $j, \theta_{i k}$ the processing time for product $i$ at semicontinuous stage $k, \mathrm{CT}_{i}$ the cycle time for the production of product $i$, and $\mathrm{Nb}_{i}$ the number of batches of product $i$ over the horizon time HT.

$$
\begin{gather*}
T_{i j}=\theta_{i k^{\prime}}+t_{i j}+\theta_{i k^{\prime \prime}} \quad \forall i=1, \ldots, N_{\mathrm{p}}, \forall j \in \mathrm{~EB}_{i}  \tag{3}\\
\mathrm{CT}_{i} \geq T_{i j} \quad \forall i=1, \ldots, N_{\mathrm{p}}, \forall j \in \mathrm{~EB}_{i} \tag{4}
\end{gather*}
$$

Note that eq 3 defines the time that batch unit $j$ will be occupied with product $i$, which contemplates the material loading $\left(\theta_{i k^{\prime}}\right)$ and unloading ( $\theta_{i k^{\prime \prime}}$ ) time if this unit is located between semicontinuous units. It is worth noting that in this approach it is assumed that variables $t_{i j}$ and $\theta_{i k^{\prime}}$ are involved in detailed submodels, some of them written as differential equations and included in the actual model. ${ }^{15}$

For products $i$ that share unit $j\left(i \in I_{j}\right)$, the following constraints are considered:

$$
\begin{equation*}
\sum_{i \in I_{j}} \mathrm{Nb}_{i} T_{i j} \leq \mathrm{HT} \quad \forall j=1, \ldots, N_{j} \tag{5}
\end{equation*}
$$

In the same way, for all products $i$ that share unit $k\left(k \in I_{k}\right)$

$$
\begin{equation*}
\sum_{i \in I_{k}} \mathrm{Nb}_{i} \theta_{i k} \leq \mathrm{HT} \quad \forall k=1, \ldots, N_{k} \tag{6}
\end{equation*}
$$

If all products follow the same production path, then eq 6 becomes redundant because the batch processing time considers the semicontinuous processing times upstream and downstream from the batch unit.

A characteristic of this model is that, for certain batch stages, the number of units is a priori unknown. For these stages, a superstructure that contemplates all possible configurations, or those chosen by the designer as feasible, is modeled and embedded in the global model. This superstructure model is formulated as an NLP, and the details can be obtained in ref 15. To simplify the formulation in this work, only units in series are considered as possible configurations.

The production rate constraints for each product are

$$
\begin{gather*}
\mathrm{Nb}_{i} B_{i} / \mathrm{HT}=Q_{i} \quad \forall i=1, \ldots, N_{\mathrm{p}}  \tag{7}\\
Q_{i}^{\min } \leq Q_{i} \leq Q_{i}^{\max } \quad \forall i=1, \ldots, N_{\mathrm{p}} \tag{8}
\end{gather*}
$$

where $Q_{i}$ is the production rate of product $i$, which is bounded by $Q_{i}^{\min }$ and $Q_{i}^{\max }$, and $B_{i}$ is the batch size of product $i$.

Components and total mass balances at each stage, connection constraints between stages, and design equations for each stage and for each product are considered as a detailed model. If there are recycles or interconnections between the production processes, as really occurs in the study cases, the balances that correspond to these connections are also considered. Mass balances for some units are given by differential equations such as

$$
\begin{equation*}
\mathrm{d} C_{x i j} / \mathrm{d} t=h(t, x) \quad \forall x \tag{9}
\end{equation*}
$$

where $C_{x i j}$ is the concentration of component $x$ (biomass, substrate, product, etc.) at stage $j$ of process production $i$. These dynamic equations are discretized and included in the overall model. Note that the discretized equations involve the processing time of the batch item and the time integration step, all of which are considered variables. The number of grid points is problem data, but because the final processing time is variable, the discretization step length is determined according to the final time for each unit. For these models, the trapezoidal method was adopted. ${ }^{16}$ For example, if the biomass balance is

$$
\begin{equation*}
\mathrm{d} X_{i j} / \mathrm{d} t=\left(\mu_{i j}-v_{i j}\right) X_{i j} \tag{10}
\end{equation*}
$$

where $X$ represents the biomass concentration, $\mu$ the specific growth rate of biomass, and $v$ the biomass death rate, the corresponding set of algebraic equations is

$$
\begin{equation*}
X_{i j}^{k+1}=X_{i j}^{k}+\frac{l_{i j}}{2}\left[\left(\mu_{i j}^{k+1}-v_{i j}\right) X_{i j}^{k+1}+\left(\mu_{i j}^{k}-v_{i j}\right) X_{i j}^{k}\right] \tag{11}
\end{equation*}
$$

where $l$ is the step length and $k=1, \ldots, K$ are the grid points.
Also, mass balances between units in the same production path are considered:

$$
\begin{align*}
\mathrm{VS}_{i j}= & \sum_{r=1}^{N_{r}} f_{i j r}+\mathrm{VS}_{i, j-1} \quad \forall i=1, \ldots, N_{\mathrm{p}}, \forall j \in \mathrm{~EB}_{i}  \tag{12}\\
\mathrm{VS}_{i j} C_{x i j}^{\mathrm{ini}}= & \sum_{r=1}^{N_{r}} C_{x i j}^{r} f_{i j r}+C_{x i, j-1}^{\mathrm{fin}} \mathrm{VS}_{i, j-1} \\
& \forall i=1, \ldots, N_{\mathrm{p}}, \forall j \in \mathrm{~EB}_{i} \tag{13}
\end{align*}
$$

where superscripts ini and fin represent the initial and final concentrations, respectively, and $\mathrm{VS}_{i j}$ represents the batch
volume at stage $j$ in the production of product $i . f_{i j r}$ is the amount of resource $r$ added to unit $j$ in the production of product $i$ (blending of batches). Equations similar to eqs 12 and 13 are stated for semicontinuous units.

The material and energy resources that each process production requires can be obtained from another plant that belongs to the same industrial complex, called "mother plant", or can be imported from another plant. The unused amount of resource $r$, i.e., the amount of $r$ that is not consumed by the MP plant, can be sold to other complexes. The constraints for the consumed resources are

$$
\begin{equation*}
F_{r}+F_{r}^{\mathrm{trans}}=\sum_{i=1}^{N_{\mathrm{p}}}\left(\sum_{j=1}^{N_{j}} \frac{f_{i j r}}{\mathrm{CT}_{i}}+\sum_{k=1}^{N_{k}} \frac{f_{i k r}}{\mathrm{CT}_{i}}\right)+F_{r}^{\mathrm{ex}} \tag{14}
\end{equation*}
$$

where $F_{r}$ is the amount (per hour) of resource $r$ produced by the mother plant, $F_{r}^{\text {trans }}$ represents the amount of resource $r$ that must be transported from another plant, and $F_{r}^{\mathrm{ex}}$ is the unused $r$ sold to other complexes. $f_{i j r}$ and $f_{i k r}$ are the amounts of resource $r$ consumed for units $j$ and $k$, respectively, in the production of product $i$.

The objective function of the problem is the maximization of annualized net profits given by the total expected selling price minus the investment and operative cost. The unused resources of the mother plant are considered as a benefit for sale, so the objective function is

$$
\begin{align*}
& \operatorname{Max} \sum_{i=1}^{N_{\mathrm{p}}} p_{i} \mathrm{Nb}_{i} B_{i}+\sum_{r=1}^{N_{r}} q_{r} F_{r}^{\mathrm{ex}}- \\
& \quad\left(\sum_{j} \alpha_{j} V_{j}^{\beta_{j}}+\sum_{k} \alpha_{k} R_{k}^{\beta_{k}}+\mathrm{HT} \sum_{r=1}^{N_{r}} c_{r} F_{r}^{\mathrm{trans}}+\mathrm{Res}\right) \tag{15}
\end{align*}
$$

where $p_{i}$ is the expected net profit of product $i, q_{r}$ is the selling price of resource $r$, and $c_{r}$ is the cost of raw material $r$. $\alpha$ and $\beta$ are the cost coefficients, and Res is the disposal cost, which varies according to the effluent.

Mixed-Product Campaign Model. The relaxed model optimal solution provides the plant configuration (number of units in series) and the number of batches of each product.

Let $i^{\prime}$ be the product with the smallest number of batches in the relaxed model solution. Let us define $r_{i}=\operatorname{round}\left(\mathrm{Nb}_{i} / \mathrm{Nb}_{i^{\prime}}\right)$ as the rounding of the relationship between the number of batches of each product $i$ and product $i^{\prime}$.

Let Nb be an optimization variable defined as the number of times that the mixed-product campaign is repeated. For the mixed-product campaign model, the following constraints are imposed:

$$
\begin{gather*}
\mathrm{Nb}=\mathrm{Nb}_{i^{\prime}}  \tag{16}\\
\mathrm{Nb}=\mathrm{Nb}_{i} r_{i}^{-1} \quad \forall i=1, \ldots, N_{\mathrm{p}} \tag{17}
\end{gather*}
$$

According to the estimated proportion among the number of batches of all products, different campaigns can be proposed by the designer. For each mixed-product campaign, the following constraints are posed.

Let $\mathrm{SL}_{i j}$ be the idle time at unit $j$ after processing a batch of product $i$ and before processing the next batch and $\mathrm{CT}_{j}$ be the cycle time for unit $j$ defined by

$$
\begin{equation*}
\mathrm{CT}_{j}=\sum_{i \in I_{j}}\left(T_{i j}+\mathrm{SL}_{i j}\right) \quad \forall j=1, \ldots, N_{j} \tag{18}
\end{equation*}
$$

Analogously

$$
\begin{equation*}
\mathrm{CT}_{k}=\sum_{i \in I_{k}}\left(\theta_{i k}+\mathrm{SL}_{i k}\right) \quad \forall k=1, \ldots, N_{k} \tag{19}
\end{equation*}
$$

If there is more than one batch of some product in the campaign, the processing and idle times for that product must be added as many times as repetitions occur.

For each unit, the modeler must establish the order in which products will be processed, using the relationship between the previously determined number of batches. Next, the constraints that must be implemented according to the production path that each product follows are settled down. These constraints are established for two consecutive products to ensure that the production processes of two different products do not overlap in the same unit. A detailed description of these scheduling constraints can be found in work by Corsano et al. ${ }^{14}$ Once all of the proposed mixed-product campaign models are solved, the best economical solution is chosen as the optimal plant design, operation, and scheduling solution.
3.2. MC Model. In this case, each product is produced in a different plant and some plants can also produce byproducts. Given $N_{\mathrm{p}}$ different derivative products to be processed in $L$ different plants, the model considers the following:

Objective function: the maximization of annualized net profits given by the total expected selling price minus the investment and operative cost:

$$
\begin{align*}
& \operatorname{Max} \sum_{i=1}^{N_{\mathrm{p}}} p_{i} \mathrm{Nb}_{i} B_{i}+\sum_{r=1}^{N_{r}} q_{r} \mathrm{~F}_{r}^{\mathrm{ex}}- \\
& \quad\left[\sum_{l=1}^{L}\left(\sum_{j \in \mathrm{~EB}_{l}} \alpha_{l j} V_{l j}^{\beta_{l j}}+\sum_{k \in \mathrm{ES}_{l}} \alpha_{l k} R_{l k}^{\beta_{l k}}+\operatorname{Res}_{l}\right)+\mathrm{HT} \sum_{r=1}^{N_{r}} c_{r} F_{r}^{\mathrm{trans}}\right] \tag{20}
\end{align*}
$$

where $V$ and $R$ are the batch $(j)$ and semicontinuous $(k)$ unit sizes, $\mathrm{EB}_{l}$ and $\mathrm{ES}_{l}$ represent the set of batch and semicontinuous units included in the production path in plant $l$, and $\operatorname{Res}_{l}$ represents the disposal cost for each plant $l$.

Mass balances at every unit of each plant: some of them are given by differential equations, which are discretized and included in the global model as algebraic equations, for example

$$
\begin{equation*}
\mathrm{d} C_{x l j} / \mathrm{d} t=h(t, x) \quad \forall x, \forall l=1, \ldots, L \tag{21}
\end{equation*}
$$

where $C_{x l j}$ is the concentration of component $x$ (biomass, substrate, product, etc.) at stage $j$ in plant $l$. These dynamic equations are discretized and included in the overall model as algebraic equations in the same way as was previously posed for MP models.

Mass balances between units of the same plant:

$$
\begin{align*}
& \mathrm{VS}_{l j}= \sum_{r=1}^{N_{r}} f_{l j r}+\mathrm{VS}_{l, j-1} \quad \forall l=1, \ldots, L, \forall j \in \mathrm{~EB}_{l}  \tag{22}\\
& \mathrm{VS}_{l j} C_{x l j}^{\mathrm{ini}}=\sum_{r=1}^{N_{r}} C_{x l j}^{r} f_{l j r}+C_{x l, j-1}^{\mathrm{fin}} \mathrm{VS}_{l, j-1} \\
& \forall l=1, \ldots, L, \forall j \in \mathrm{~EB}_{l} \tag{23}
\end{align*}
$$

where superscripts ini and fin represent the initial and final concentrations, respectively, $\mathrm{VS}_{l j}$ represents the batch volume at stage $j$ in plant $l$, and $f_{l j r}$ is the amount of resource $r$ used in unit $j$ at plant $l$.

Interconnection constraints between the mother plant and derivative plants:

$$
\begin{equation*}
F_{r}+F_{r}^{\mathrm{trans}}=\sum_{l=1}^{L} \sum_{j=1}^{N_{l}} \frac{f_{l j r}}{\mathrm{CT}_{l}}+F_{r}^{\mathrm{ex}} \quad \text { for each resource } r \tag{24}
\end{equation*}
$$

where $f_{r l j}$ is the amount of $r$ consumed at stage $j$ in the derivative plant $l . \mathrm{CT}_{l}$ represents the cycle time of plant $l$. Because each plant is a monoproduct plant, $\mathrm{CT}_{l}$ is similar to $\mathrm{CT}_{i}$ unless the plant produces byproducts. If the plant produces byproducts, then

$$
\begin{equation*}
\mathrm{CT}_{l} \geq \mathrm{CT}_{i} \quad \text { for all } i \text { produced in plant } l \tag{25}
\end{equation*}
$$

All products must be produced within the production horizon. In this work, the time horizon is the same for all plants (HT), so

$$
\begin{equation*}
\mathrm{Nb}_{i} \mathrm{CT}_{l} \leq \mathrm{HT} \quad \text { for each } l=1, \ldots, L \text { and } i \text { produced in } l \tag{26}
\end{equation*}
$$

The transfer policy adopted in this work is the ZW transfer, so

$$
\begin{equation*}
\mathrm{CT}_{l} \geq T_{l j} \quad \text { for } l=1, \ldots, L, j \in \mathrm{~EB}_{l} \tag{27}
\end{equation*}
$$

Interconnection constraints between derivative plants: which are mass and energy equations and the recycle equations between two different plants and/or from one stage to another in the same production process

$$
\begin{equation*}
F_{l j}^{e} \geq \sum_{l \in L_{e} \in \mathrm{~EB}_{e}} \sum_{l j} f_{l j}^{e} \quad \forall e \tag{28}
\end{equation*}
$$

where $F_{l j}^{e}$ is the amount of recycle $e$ produced at stage $j$ in plant $l$ and $f_{l j}^{e}$ is the amount of recycle $e$ consumed at stage $j$ in plant $l . L_{e}$ and $\mathrm{EB}_{e}$ represent the set of plants and units that consume $e$, respectively. For model simplification purposes, the store cost of raw materials and effluents is not considered in this work.

The design equation for each batch and semicontinuous unit that depends on process variables is

$$
\begin{align*}
& V_{l j}=S_{i j}^{l} B_{i} \quad \forall i \text { produced in } l, \forall j \in \mathrm{~EB}_{l}, \forall l=1, \ldots, L  \tag{29}\\
& R_{l k}=S_{i k}^{l} \frac{B_{i}}{\theta_{i k}} \quad \forall i \text { produced in } l, \forall k \in \mathrm{ES}_{l}, \forall l=1, \ldots, L \tag{30}
\end{align*}
$$

where $S$ represents the size or duty factor of batch and semicontinuous units, respectively, and $\theta_{i k}$ is the processing time of unit $k$ in the production of $i$. These factors are computed as a function of the process variables. Note that, in the last two equations, product $i$ and plant $l$ appear simultaneously. This is so because some plants can produce a subproduct, and thus they manufacture more than one product.

Constraints of each plant's production rate:

$$
\begin{gather*}
\mathrm{Nb}_{i} B_{i} / \mathrm{HT}=Q_{i} \quad \text { for all } i \text { produced in plant } l  \tag{31}\\
 \tag{32}\\
Q_{i}^{\min } \leq Q_{i} \leq Q_{i}^{\max }
\end{gather*}
$$

where $Q_{i}$ is the production rate of product $i$, which is bounded by $Q_{i}^{\min }$ and $Q_{i}^{\max }$.

Just like in the MP model, a superstructure model that considers a set of different alternative configurations is embedded in the overall model so that the model keeps the NLP nature.

It is worth noting that neither campaigns nor scheduling constraints are applied because the MC model considers singleproduct plants.

The MC model considers constraints (20) - (32), which are simultaneously optimized and no heuristic or decomposition strategy is applied. The results are shown in section 5.2.

## 4. Study Case

In this section, the production process of two products and one byproduct integrated to a mother plant that provides material and energy resources is presented for both scenarios: MP and MC.
4.1. Sequential MP Plant: Torula Yeast, Brandy, and Bakery Yeast Production Integrated to a Sugar Plant. The integration of several processes into a sugar cane complex is considered. The sugar plant produces sugar and bagasse for sale and molasses, mill and filter juices, vapor, and electricity that are used in the MP plant. The production of sugar, molasses, vapor, and electricity depends on the amount of mill and filter juice extraction. If more mill and/or filter juice is extracted, molasses and sugar productions are diminished, so the consumption of vapor and electricity in the sugar production process is also diminished, and therefore the amount of electricity and vapor available for derivatives is increased. For the sugar plant, the model optimizes the amount of extracted mill and filter juices. The sugar plant is considered as an existing mother plant.

The derivative plant will be a sequential MP plant with batch and semicontinuous units to produce torula yeast (torula utiliz), brandy, and bakery yeast. The torula yeast is used for cattle feeding. The brandy plant not only produces this alcohol but also generates a nondistilled remainder called vinasses or distillery broth that represents another contribution of sugaring substrate for fermentations of both derivatives. The previous stage to the brandy distillation is a centrifuge that separates the biomass from the alcohol. The centrifuge remainder is a cream with moderate biomass concentration. For this reason, an option is to evaporate and dry this cream to produce bakery yeast.

Molasses and filter juices produced in the sugar plant serve as sugaring substrates for the biomass and alcohol fermentations. In addition, water and vinasses are added to the fermentation feed. Electricity generated in the sugar plant is used in the centrifuge of the derivatives plant, whereas the fermentors, the evaporator, the spray dryer, and the distillation column consume low-pressure steam from the mother plant. In addition, if necessary, steam can be imported from other power stations with the operative cost allocated on the total annual cost. Vapor and electricity that are not consumed by the derivatives plant can be sold.

Four stages for brandy production are considered: biomass fermentation, alcohol fermentation, centrifugation, and distillation. For bakery yeast, semicontinuous evaporation and semicontinuous spray drying are added. The main objective of the first stage is biomass production. This stage operates in batch form, and it is fed with molasses and filter juices from a sugar plant, vinasses, and water. The first biomass fermentor is fed with a broth containing biomass prepared in the laboratory: the inoculums. At this stage, large amounts of air are supplied. The alcohol fermentor is also a batch item, and it is fed with the product of biomass fermentors, molasses, filter juices, vinasses, and water. The brandy production occurs in this stage without air supply. The fermented broth is centrifuged in a disk stack centrifuge that operates in a semicontinuous mode. The objective of this stage is to separate the biomass from the liquid that contains the brandy. The rich solids stream can be evaporated

Table 1. Simultaneous Decisions Made for Each Model

| model | synthesis | design | operation | scheduling (ZW transfer policy) |
| :---: | :---: | :---: | :---: | :---: |
| MP | plant configuration | unit sizes | batch blending, batch splitting, and batch recycle flow rates within the same production process | cycle time of each production process |
|  | number of units in series of some batch stages in the plant; same configuration for all of the production processes blend and recycle allocation | heating and cooling areas | flow rate recycles from one process to another | unit cycle times |
|  |  | power consumption (vapor and electricity) | material and energy resource allocation from the mother plant to the different process production plants | unit idle times |
|  |  | stage number of distillation columns | component concentrations unit processing times for each product | number of batches mixed-product campaign configuration |
| MC | plant configuration | unit sizes | batch blending, batch splitting, and batch recycle flow rates within the same production process | unit idle times in each plant |
|  | number of units in series for some batch stages in each plant; each plant with an independent configuration blend and recycle allocation | heating and cooling areas | flow rate recycles from one process to another | plant cycle time |
|  |  | power consumption (vapor and electricity) | material and energy resource allocation from the mother plant to the different process production plants | number of batches |
|  |  | stage number of distillation columns | component concentrations |  |
|  |  |  | processing times of each unit in |  |

and dried to produce bakery yeast. The last stage of the process is the batch distillation. The batch distiller model is a combination of two batch items-the distiller feed vessel and the distillate tank-and three semicontinuous items-the heating surface to evaporate, the cooling area to condense the vapor, and the column itself. An analytical model presented by Zamar et al. ${ }^{17}$ for batch distillation is adopted. This model relates both the minimum and operational reflux values as well as the minimum and operational numbers of stages.

Four stages for torula yeast production are considered: biomass fermentation, centrifugation, evaporation, and dryer. The batch biomass fermentors are fed with molasses, filter juices, vinasses, and water. The first biomass fermentor in the series is fed with inoculums. The last three units constitute a semicontinuous subtrain.

For the fermentation stages of both production processes, the superstructure optimization model proposed by Corsano et al. ${ }^{15}$ is adopted and integrated to the overall model. So, a synthesis, design, operation, and scheduling problem is solved for the sequential MP plant integrated to the sugar cane plant as an NLP model. The various decisions that are simultaneously made for this scenario are shown in Table 1. Figure 3 shows the integration scheme.

To exemplify how the previously presented equations (eqs $1-19)$ are expressed for the MP model for this study case, we state some of them:

For the fermentors of each alternative in the superstructure, eq 13 is formulated for $j=$ biomass and alcohol fermentors, $i$ $=$ brandy and torula yeast, $x=$ biomass, substrate, and nonactive biomass concentrations for both the production process and product concentration for brandy production in the alcohol fermentors, and $r=$ molasses, filter juice, and distillery vinasses. So, for example, the substrate balance for each fermentor for
each torula production is

$$
\begin{aligned}
\mathrm{VS}_{\text {torula }, j} C_{S, \text { torula }, j}^{\text {ini }}= & C_{S, \text { torula }, j}^{\text {molass }} f_{\text {torula }, j, \text { molass }}+ \\
C_{S, \text { torula }, j \text { torula }, j, \text { fjuice }}^{\text {fjuice }} & +C_{S, \text { torula }, j}^{\text {vinasses }} f_{\text {torula, } j, \text {,vinasses }}+ \\
& C_{S, \text { torula }, j-1}^{\text {fin }} \mathrm{VS}_{\text {torula }, j-1}
\end{aligned}
$$

In the same way, eq 14 is stated for $r=$ molasses, filter juices, vapor, and electricity and $i=$ brandy and torula yeast. For $r=$ molasses and filter juices, $j$ represents the fermentors in the plants. For $r=$ vapor, $j=$ fermentors and distillation column, while $k$ represents the evaporator and the spray dryer. For $r=$ electricity, $k=$ centrifuge. For vapor consumption, for instance, the equation is

$$
\begin{aligned}
F_{\text {vapor }}+F_{\text {vapor }}^{\text {trans }}= & \sum_{i=\text { brandy,torula }}\left(\sum_{j=\text { fermentors }} \frac{f_{i j \text {,vapor }}}{\mathrm{CT}_{i}}+\right. \\
& \left.\sum_{k=\text { evap,dryer }} \frac{f_{i k \text {,vapor }}}{\mathrm{CT}_{i}}\right)+\frac{f_{\text {brandy,dist,vapor }}}{\mathrm{CT}_{\text {brandy }}}+F_{\text {vapor }}^{\mathrm{ex}}
\end{aligned}
$$

4.2. MC To Produce Derivatives from Sugar Cane. The sugar cane complex consists of three plants interconnected by material and energy flow currents. The plants are the sugar plant (mother plant) and two derivative plants: torula yeast and brandy/bakery yeast.

The production processes of each product were described in the previous section, with the only difference being the fact that in this scenario each product is manufactured in a different plant. Bakery yeast is a byproduct of the brandy plant.

For both derivative plants, the configuration of the fermentation stage is obtained using the superstructure model presented


Figure 3. Flowsheet for sugar cane complex integration with a MP derivative plant.
in Corsano et al. ${ }^{15}$ embedded in the global integration model, considering only in series unit duplication.

The derivative plants are connected by the vinasse recycles, and the connections between sugar and derivative plants are given by material (molasses and filter juices) and energy (vapor and electricity) resources. For the former integration, eq 28 is rewritten as

$$
F_{\text {brandy,dist }}^{\text {vinasses }} \geq \sum_{j \in\{\text { fermentors }\}} f_{\text {torula }, j}^{\text {vinases }}+\sum_{j \in\{\text { fermentors }\}} f_{\text {brandy }, j}^{\text {vinasses }}
$$

For the electricity integration between the mother plant and derivative plants, for instance, eq 24 is rewritten as

$$
F_{\text {elec }}+F_{\text {elec }}^{\text {trans }}=f_{\text {torula,cent,elec }}+f_{\text {brandy,centr,elec }}+F_{\text {elec }}^{\mathrm{ex}}
$$

where cent means centrifuge and the electricity unit is kWh .
In Table 1, the different simultaneous decisions made in this model are described.

Figure 4 shows the sugar cane complex integration. This figure shows that the model considers blending, recyce, and splitting batches. There are few works in the literature that consider this characteristic and that address the MC scenario with the degree of detail provided by this paper.

## 5. Results and Analysis

The presented models and strategies for optimal synthesis, design, operation, and scheduling of MP and MC models provide a tool for the alternative analysis of different configurations and for the evaluation of several solutions. In addition, the economical tradeoffs between each production scheme can be established and assessed. In this section, the results of the model optimizations under different operative conditions and their analysis are shown for the torula yeast, brandy, and bakery yeast process production integrated to a sugar plant, as was presented in the previous section. This study is based on specific data of product profit, equipment and resource costs, etc. These
values can be modified by the manager or designer, taking into account different contexts or scenarios. Precisely, this work tries to show the advantages of considering all of these elements simultaneously to assess the effects or relationships between them. Regardless of the attained results, which depend on several factors, the emphasis is on the analytical capability of this approach.

All models were implemented and solved in $G A M S^{18}$ on a Pentium IV, 1.60 GHz. The code CONOPT2 was employed for solving the NLP problems. The MP plant relaxed model has about 1200 equations and 1300 variables, and the CPU time for resolution is 180 s . The MP plant model has about 550 equations and 620 variables, and the CPU time is approximately 23 s . Finally, the MC model has about 1100 equations and 1200 variables, and the CPU time is about 160 s .

In this section, we show two examples in which different production conditions are considered for both MP and MC scenarios.
5.1. Example 1. Following the strategy for the previously described MP model resolution, the relaxed model is first solved. Table 2 shows the description and optimal values for some optimization variables. The minimum and maximum production rates for each product were fixed at 1 and 5 tons $\mathrm{h}^{-1}$, respectively. The fermentor sizes are upper bounded by 750 $\mathrm{m}^{3}$. Figure 5 shows the Gantt chart for the relaxed model solution. The fermentation stage configuration consists of one biomass fermentor and two alcohol fermentors in series.

As can be observed in Table 2, the number of torula batches is 281 , while for brandy and bakery yeast, the number of batches is 305 . So, for the mixed-product campaign model, the following campaigns are proposed: (a) brandy-torula for all units ( $\mathrm{B}-$ T); (b) brandy-torula for fermentation stages and torula-brandy for semicontinuous units ( $B-T / T-B$ ); (c) brandy-brandytorula for all units $(B-B-T)$.

Campaigns a and b seem to be more suitable, taking into account the ratio between the number of product batches obtained from the relaxed model. Anyway, campaign c is also


Figure 4. Flowsheet for sugar cane complex integration.

Table 2. Optimal Variables for a Sequential MP Plant Relaxed Model

| variable | description | optimal value |
| :--- | :--- | :---: |
| $Q_{\mathrm{T}}$ | torula production [ton $\left.\mathrm{h}^{-1}\right]$ | 1.87 |
| $Q_{\mathrm{BY}}$ | bakery yeast production [ton $\left.\mathrm{h}^{-1}\right]$ | 1.36 |
| $Q_{\mathrm{B}}$ | brandy production [ton $\left.\mathrm{h}^{-1}\right]$ | 5.00 |
| $\mathrm{Nb}_{\mathrm{T}}$ | number of torula batches | 281 |
| $\mathrm{Nb}_{\mathrm{B}}$ | number of brandy and bakery yeast batches | 305 |
| $\mathrm{CT}_{\mathrm{T}}$ | cycle time for torula production $[\mathrm{h}]$ | 16.0 |
| $\mathrm{CT}_{\mathrm{B}}$ | cycle time for brandy production $[\mathrm{h}]$ | 24.5 |
| $\mathrm{CT}_{\mathrm{BY}}$ | cycle time for bakery yeast production $[\mathrm{h}]$ | 11.9 |
| NAP | net annual profit $\left[\$ \mathrm{~h}^{-1}\right]$ | 6903 |

assessed. It should be noted that, because the campaigns are cyclic, $B-B-T$ is similar to $B-T-B$ and $T-B-B$. Because brandy production requires alcohol fermentors that are not used for torula production, the sequence campaign at the semicontinuous subtrain (centrifuge, evaporator, and dryer) is changed in case b.

In addition, according to the proposed methodology, the mixed-product campaign model adopts the plant configuration obtained in the relaxed model optimal solution. So, when that plant configuration is used and the corresponding constraints for each campaign are added, the mixed-product campaign model is solved. The sizing problem is also solved at this stage.

Table 3 shows the objective value for each proposed campaign. The campaign configuration $\mathrm{B}-\mathrm{T} / \mathrm{T}-\mathrm{B}$ attains the best economical results. It is a reasonable campaign in the sense that the vinasses produced upon brandy production would be used in torula fermentation. Another result is that the idle time for the $\mathrm{B}-\mathrm{T}$ campaign is $30 \%$ higher than that for the $\mathrm{B}-\mathrm{T} /$ $\mathrm{T}-\mathrm{B}$ campaign and that the idle time for the $\mathrm{B}-\mathrm{B}-\mathrm{T}$ campaign is increased by $26 \%$.

The results for some optimal processing variables are presented in Table 4. Optimal design variables are shown in Table 5. The production schedule is displayed in a Gantt chart (Figure 6).

The brandy production rate is equal to its upper bound in the optimal solution, and this occurs because this production is more profitable. Then, according to the available material and energy resources, the production of torula is carried out. It is worth mentioning that some units are suboccupied. For example, the biomass fermentor for brandy production is $80 \%$ occupied because different production rates are reached and different units are used for both productions.

When Figures 5 and 6 are compared, it can be observed that, for the single-product campaign, there is a production of 305 batches of brandy/bakery yeast and then 281 batches of torula are produced. Therefore, for these kinds of problems where recycles from one production to another might exist, the singleproduct campaign is not reasonable. On the other hand, in the mixed-product campaign, the campaign $B-T / T-B$ is repeated 320 times, and thus the vinasses do not need large storage tanks.

A novel result of this sequential MP plant model with the mixed-product campaign is the fact that, because some units are not used by all products, the operating times of such stages are larger, thus decreasing the operating costs. This means that a better use of equipment is achieved because it occurs at the distillation stage. This would not happen if the process adopted a single-product campaign, which is usually the case.

Objective functions of relaxed and mixed-product campaign models cannot be compared to each other because, in the relaxed model, the total production of torula yeast, brandy, and bakery yeast could be held in a larger period of time than the horizon time. In this case, for the relaxed model solution

$$
\begin{aligned}
& \mathrm{CT}_{\mathrm{B}} \mathrm{Nb}_{\mathrm{B}}+\mathrm{CT}_{\mathrm{T}} \mathrm{Nb}_{\mathrm{T}}=24.5 \times 305+ 16 \times 281= \\
& 11968.5 \geq 7500=\mathrm{HT}
\end{aligned}
$$

However, the total production of torula yeast, brandy, and bakery yeast is completed in the mixed-product campaign model in $7500 \mathrm{~h}(\mathrm{HT})$ because for all stages the campaign cycle time is 23.3 h and the campaign is repeated 320 times. Table 6 shows


Figure 5. Gantt chart for a torula yeast, bakery yeast, and brandy production relaxed model.

Table 3. Objective Value of Different Mixed-Product Campaigns for Example 1

|  | $\mathrm{B}-\mathrm{T}$ <br> campaign | $\mathrm{B}-\mathrm{T} / \mathrm{T}-\mathrm{B}$ <br> campaign | $\mathrm{B}-\mathrm{B}-\mathrm{T}$ <br> campaign |
| :---: | :---: | :---: | :---: |
| profit $\left(\$ \mathrm{~h}^{-1}\right)$ | 6839 | 6881 | 6745 |

Table 4. Optimal Variables for the $\mathbf{B}$-T/T-B Model

| variable | description | optimal value |
| :--- | :--- | :---: |
| $Q_{\mathrm{T}}$ | torula production [ton $\left.\mathrm{h}^{-1}\right]$ | 1.98 |
| $Q_{\mathrm{BY}}$ | bakery yeast production [ton $\left.\mathrm{h}^{-1}\right]$ | 1.2 |
| $Q_{\mathrm{B}}$ | brandy production [ton $\left.\mathrm{h}^{-1}\right]$ | 5.0 |
| Nb | number of times the mixed-product | 320 |
|  | $\quad$ campaign is repeated |  |
| $\mathrm{CT}_{\mathrm{T}}$ | cycle time for torula production $[\mathrm{h}]$ | 13.4 |
| $\mathrm{CT}_{\mathrm{B}}$ | cycle time for brandy production $[\mathrm{h}]$ | 23.3 |
| $\mathrm{CT}_{\mathrm{BY}}$ | cycle time for bakery yeast production $[\mathrm{h}]$ | 11.1 |
| NAP | net annual profit $\left[\$ \mathrm{~h}^{-1}\right]$ | 6881 |

Table 5. Optimal Design Variables of the MP and MC Plant Solutions

| equipment | $\begin{aligned} & \text { MP } \\ & \text { plant } \end{aligned}$ | MC |  |
| :---: | :---: | :---: | :---: |
|  |  | torula yeast plant | brandy/bakery yeast plant |
| biomass fermentor $1\left(\mathrm{~m}^{3}\right)$ | 667.2 | 9.2 | 140 |
| biomass fermentor $2\left(\mathrm{~m}^{3}\right)$ |  | 107.2 |  |
| biomass fermentor $3\left(\mathrm{~m}^{3}\right)$ |  | 295.5 |  |
| alcohol fermentor $1\left(\mathrm{~m}^{3}\right)$ | 605.2 | - | 518.3 |
| alcohol fermentor $2\left(\mathrm{~m}^{3}\right)$ | 691.7 | - | 593.1 |
| centrifuge (KWh) | 97.8 | 37.1 | 55.1 |
| evaporator ( $\mathrm{m}^{2}$ ) | 306.3 | 124.2 | 152.6 |
| dryer (diameter [m]) | 7.6 | 2.5 | 1.8 |
| distillation |  |  |  |
| stage number | 9 |  | 10 |
| reflux ratio | 4.2 |  | 5.1 |
| distillate tank ( $\mathrm{m}^{3}$ ) | 147.9 |  | 94.6 |
| still vessel ( $\mathrm{m}^{3}$ ) | 513.8 |  | 442.4 |
| condenser area ( $\mathrm{m}^{2}$ ) | 240.8 |  | 373.5 |
| evaporator area ( $\mathrm{m}^{2}$ ) | 142.5 |  | 221 |
| column area ( $\mathrm{m}^{2}$ ) | 5.9 |  | 9.2 |

processing and idle times. The campaign cycle time is calculated as the sum of processing times and idle times for each stage. For example, for the biomass fermentation stage, the cycle time is equal to the processing time for brandy production ( 9.9 h ) plus the processing time for torula production ( 9.4 h ) plus the unloaded time from fermentation to semicontinuous units for torula production ( 4 h ), with 23.3 h being the total. Observe
that the ZW nature of this model brings about equal cycle times for all stages, so the stage cycle time is equal to the campaign cycle time.

To evaluate different production alternatives, the MC model is solved for the same operative conditions of the previous case, that is, production rates lower and upper bounded by 1 and 5 tons $h^{-1}$, respectively. For bakery yeast production, no bounds are imposed because this production depends on brandy production. Table 5 shows the optimal design variables. The optimal fermentation configuration for the torula plant is three units in series, and it is one biomass fermentor and two alcohol fermentors in series for the brandy/bakery yeast plant. The processing times are displayed in Table 7, and because each product is manufactured in a different plant, no scheduling constraints are applied. The ZW transfer policy avoids idle times, as shown in Figure 7 for both productions. Anyway, it is worth noting that the second alcohol fermentor in the brandy plant has idle time, and this is due to the tradeoff that exists between the processing time of this unit and the substrate concentration of vinasses. Longer processing times imply smaller substrate concentrations because the substrate is consumed at this stage.

The optimal production rates for this scenario are 1.38 tons $\mathrm{h}^{-1}$ for torula, 5 tons $\mathrm{h}^{-1}$ for brandy, and 0.6 ton $\mathrm{h}^{-1}$ for bakery yeast production. Because brandy is the most profitable production, it is produced up to this upper bound, and torula and bakery yeast are produced according to the available raw material: molasses, vapor, and electricity.

The net annual profit is $6883 \$ \mathrm{~h}^{-1}$, which is similar to the MP plant net annual profits. However, because the incomes for sugar, bagasse, and brandy sales are the same in both cases, only the benefits for torula, bakery yeast, and electricity sales and the total annualized cost for both scenarios are compared and displayed in Table 8 for a detailed analysis. The "partial benefits" mean the benefits for torula, bakery yeast, and unused electricity sales minus the investment and operative costs. The difference between both total partial benefits is not significant, but the investment cost for the MP plant is $19 \%$ smaller than that for MC. If the operative costs such as inoculum costs are not considered, the optimal solution may change. In this case, the operative costs are higher for the MP plant because productions are mutually dependent since all products are produced in the same plant, while in the MC plant scenario,


Figure 6. Gantt chart for mixed-product campaign $B-T / T-B$.

Table 6. Processing and Idle Times for the $B-T / T-B$ Campaign

|  | brandy |  |  | torula |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | processing <br> time $(\mathrm{h})$ | idle <br> time $(\mathrm{h})$ | processing <br> time $(\mathrm{h})$ | idle <br> time $(\mathrm{h})$ |  |
| biomass fermentor | 9.9 | 0 |  | 9.4 | 0 |
| alcohol fermentor 1 | 6.5 | 16.8 |  |  |  |
| alcohol fermentor 2 | 6.9 | 12.2 |  | 0 |  |
| semicont. subtrain | 4.2 | 15.1 |  | 4 | 0 |
| distillation | 19.14 | 0 |  |  |  |
|  |  |  |  |  |  |

Table 7. Processing Time of the MC Model Solution

|  | processing time (h) |  |
| :--- | :---: | :---: |
|  | torula plant | brandy/bakery <br> yeast plant |
| biomass fermentor 1 | 7.5 | 15.0 |
| biomass fermentor 2 | 7.5 |  |
| biomass fermentor 3 | 2.8 | 15.0 |
| alcohol fermentor 1 |  | 5.3 |
| alcohol fermentor 2 | 4.7 | 5.8 |
| semicont. subtrain |  | 9.2 |
| distillation | 7.5 | 15.0 |

process performances are good and there is a good use of units because each product is produced in a separate plant. Therefore, it is very important to simultaneously consider all of these characteristics in order to perform a real comparison.

In addition, human resource demand is higher for the MC scenario than for the MP plant scenario. If we assume that one worker is needed for the fermentation stages, one for the semicontinuous units, and one for the distillation stage and considering three work shifts a day, 9 workers will be needed for the MP plant versus 15 for the MC plant. Human resources demand is not considered in the model.

Because each product is produced in a separate plant in the MC scenario, each process production is performed optimally under the integration conditions because the units in each plant are exclusively used for only one product. The biomass fermentation stage performance is better in MC because a good use of units is achieved since there are no idle times and no suboccupied units. The investment cost for this stage is about $30 \%$ higher for the MC because the total number of units used in this case is greater than that in the MP scenario as a result of production in separate plants. The same occurs for semicon-
tinuous units. It can be noted that the inoculum cost is lower for the MC plant because of the good use of fermentative units. The investment cost of alcohol fermentation is lower in the MC plant because unit sizes of this stage are smaller than those in the MP plant solution as a result of a better yield of biomass fermentation.

The increment of distillation investment cost is a consequence of the processing time of this stage. A reduction of the processing time implies bigger sizes of transference areas and column diameters of this stage.
The increment of vinasse disposal cost is due to a higher volume and substrate concentration of discarded vinasses. If more vinasses were used in the fermentation stages, the unit sizes would be increased and therefore the investment cost of the fermentation stages would also be increased. So, there is a tradeoff between vinasse use and fermentation investment cost.

Therefore, by means of a detailed analysis of the economic results (Table 8), it can be concluded that both scenario solutions are similar. If the synthesis, design, and operation items are not taken into account simultaneously, as is usually the case in the literature, solutions and conclusions might be erroneous.
5.2. Example 2. Now, let us consider smaller production rates for the same productions. The production rate in a small- or medium-sized brandy plant is about 0.8 ton $\mathrm{h}^{-1}$, while for a torula plant, it is 1 ton $\mathrm{h}^{-1}$. Let these values be the upper bound for these productions. Because bakery yeast production depends on brandy production, we leave this production free. In addition, the bakery yeast selling price is doubled.

The optimal solution for the MP plant relaxed model consists of a plant configuration of two biomass fermentors and one alcohol fermentor in series. The number of torula batches is 399 while the number of batches for brandy is 270 , and thus the following campaign configurations are modeled for the optimal plant synthesis of the relaxed model: (a) brandy-torula for all units $(\mathrm{B}-\mathrm{T})$; (b) brandy-torula for fermentation and torula-brandy for semicontinuous units ( $\mathrm{B}-\mathrm{T} / \mathrm{T}-\mathrm{B}$ ); (c) torula-brandy-torula for fermentation and torula-torula-brandy for semicontinuous units ( $\mathrm{T}-\mathrm{B}-\mathrm{T} / \mathrm{T}-\mathrm{T}-\mathrm{B}$ ).

Table 9 shows the objective function for each mixed-product model campaign solution. The best economical solution is $\mathrm{B}-\mathrm{T} /$ $\mathrm{T}-\mathrm{B}$. The total profit is equal to $3587.2 \$ \mathrm{~h}^{-1}$, and the production rates are equal to 0.8 ton $\mathrm{h}^{-1}$ for brandy, 1 ton $\mathrm{h}^{-1}$


Material loading and unloading of semicontinuous units
Brandy - Bakery Yeast
Torula Yeast
Loading and unloading times
Idle times

Figure 7. Gantt chart for the MC scenario.

Table 8. Economic Comparison between MC and MP Plant Model Solutions

|  | MC plant model | MP plant model |
| :--- | :---: | :---: |
| profit for torula yeast sale $\left(\$ \mathrm{~h}^{-1}\right)$ | 828.8 | 687.0 |
| profit for bakery yeast sale $\left(\$ \mathrm{~h}^{-1}\right)$ | 119.8 | 240.7 |
| profit for electricity sale $\left(\$ \mathrm{~h}^{-1}\right)$ | 14.6 | 10.9 |
| total profit for sales $\left(\$ \mathrm{~h}^{-1}\right)$ | 963.2 | 938.6 |
| investment cost $\left(\$ \mathrm{~h}^{-1}\right)$ |  |  |
| biomass fermentors | 90.1 | 63.6 |
| alcohol fermentors | 26.7 | 28.6 |
| centrifuge | 21.6 | 18.1 |
| evaporator | 26.5 | 20.2 |
| dryer | 63.0 | 55.8 |
| distillation | 120.2 | 105.5 |
| total investment cost | 348.1 | 291.8 |
| operative cost $\left(\$ \mathrm{~h}^{-1}\right)$ | 4.7 |  |
| inoculums | 31.2 | 58.5 |
| water (cooling and diluted) | 0 | 27.1 |
| vinasse disposal | 92 | 3.8 |
| imported vapor | 127.9 | 72.6 |
| total operative cost | 487.2 | 162 |
| total partial benefits $\left(\$ \mathrm{~h}^{-1}\right)$ |  | 484.8 |

Table 9. Objective Value of Different Mixed-Product Campaigns for Example 2

|  | B-T <br> campaign | B-T/T-B <br> campaign | $\mathrm{T}-\mathrm{B}-\mathrm{T} / \mathrm{T}-\mathrm{T}-\mathrm{B}$ <br> campaign |
| :---: | :---: | :---: | :---: |
| profit $\left(\$ \mathrm{~h}^{-1}\right)$ | 3292.2 | 3587.2 | 3023.2 |

for torula yeast (upper bounds), and 2.67 tons $^{h^{-1}}$ for bakery yeast production. The optimal processing and idle times are displayed in Table 10, and Figure 8 shows the Gantt chart for this solution. Design variables are shown in Table 11. The fermentation configuration is the one obtained in the relaxed model: two biomass fermentors in series and one alcohol fermentor.

Table 10. Processing and Idle Times for the $B-T / T-B$ Campaign of Example 2

|  | brandy |  |  | torula |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | processing <br> time $(\mathrm{h})$ | idle <br> time $(\mathrm{h})$ | processing <br> time $(\mathrm{h})$ | idle <br> time $(\mathrm{h})$ |  |
| biomass fermentor 1 | 6.1 | 0 |  | 6.1 | 0 |
| biomass fermentor 2 | 6.1 | 0 |  | 2.8 | 1.1 |
| alcohol fermentor | 5.1 | 1.3 |  |  |  |
| semicont. subtrain <br> distillation 5.8 | 4.2 |  | 2.2 | 0 |  |
|  | 6.3 | 0 |  |  |  |

Table 10 shows that the campaign cycle time is equal to 12.2 h. Again, by the ZW nature, all of the stages have equal cycle times. They are calculated as the sum of processing times, load and unload times, and idle times. For example, for the second biomass fermentation stage, it is the brandy processing time, plus the torula processing time, plus the unloaded time from this stage to semicontinuous units for torula production, plus the idle time after processing of the torula batch, that is, $6.1+$ $2.8+2.2+1.1=12.2$. As can be observed, the idle times in this schedule have been reduced.

Under the same operative conditions, the MC model is solved and the optimal plant configurations for fermentation stages are three fermentors in series for torula production and three biomass fermentors and three alcohol fermentors in series for brandy/ bakery yeast production. The total profit is equal to $3057.2 \$$ $\mathrm{h}^{-1}$, which is $15 \%$ lower than that of the MP plant optimal solution. Optimal design variables are reported in Table 11. The processing times are shown in Table 12.
The economic comparison between MP and MC scenarios is presented in Table 13. The benefits for sugar, brandy, and torula sales are not reported because the same values are


Figure 8. Gantt chart for mixed-product campaign $B-T / T-B$ for example 2.

Table 11. Optimal Design Variables of the MP and MC Plant Solution for Example 2

| equipment | $\begin{aligned} & \text { MP } \\ & \text { plant } \end{aligned}$ | MC |  |
| :---: | :---: | :---: | :---: |
|  |  | torula yeast plant | brandy/bakery yeast plant |
| biomass fermentor $1\left(\mathrm{~m}^{3}\right)$ | 258 | 11.34 | 0.14 |
| biomass fermentor $2\left(\mathrm{~m}^{3}\right)$ | 710.2 | 129.9 | 2.1 |
| biomass fermentor $3\left(\mathrm{~m}^{3}\right)$ |  | 200.8 | 31.64 |
| alcohol fermentor $1\left(\mathrm{~m}^{3}\right)$ | 750 |  | 75.7 |
| alcohol fermentor $2\left(\mathrm{~m}^{3}\right)$ |  |  | 85.6 |
| alcohol fermentor $3\left(\mathrm{~m}^{3}\right)$ |  |  | 98 |
| centrifuge (KWh) | 75.1 | 19.3 | 11.5 |
| evaporator ( $\mathrm{m}^{2}$ ) | 224.3 | 62.7 | 39.4 |
| dryer (diameter [m]) | 6.4 | 1.4 | 0.75 |
| distillation |  |  |  |
| stage number | 12 |  | 7 |
| reflux ratio | 24.7 |  | 4.2 |
| distillate tank ( $\mathrm{m}^{3}$ ) | 12.3 |  | 8.3 |
| still vessel ( $\mathrm{m}^{3}$ ) | 556.7 |  | 72.8 |
| condenser area ( $\mathrm{m}^{2}$ ) | 300.6 |  | 80.8 |
| evaporator area ( $\mathrm{m}^{2}$ ) | 177.8 |  | 47.8 |
| column area ( $\mathrm{m}^{2}$ ) | 7.4 |  | 2.0 |

Table 12. Processing Time of of the MC Model Solution of Example 2

|  | processing time (h) |  |
| :--- | :---: | :---: |
|  | torula plant | brandy/bakery <br> yeast plant |
| biomass fermentor 1 | 7.5 | 8.2 |
| biomass fermentor 2 | 7.5 | 8.2 |
| biomass fermentor 3 | 1.4 | 8.2 |
| alcohol fermentor 1 |  | 8.2 |
| alcohol fermentor 2 |  | 8.2 |
| alcohol fermentor 3 | 2 | 3.2 |
| semicont. subtrain |  | 5.0 |
| distillation | 7.5 | 3.2 |
| plant cycle time |  | 8.2 |

obtained for all cases. As can be observed, the MP plant produces a bigger amount of bakery yeast. This occurs because the evaporator and dryer in an MP plant are allocated for torula production and they are available and free during brandy production. However, in the MC plant scenario, the brandy/ bakery yeast plant must use these units only for bakery production, and they are expensive. So, to maintain smaller unit sizes, the bakery yeast is produced in a low quantity.

Table 13. Economic Comparison between MP and MC Plant Solutions for Example 2

|  | MC plant model | MP plant model |
| :--- | :---: | :---: |
| profit for bakery yeast sale $\left(\$ \mathrm{~h}^{-1}\right)$ | 160.0 | 1069.7 |
| profit for electricity sale $\left(\$ \mathrm{~h}^{-1}\right)$ | 16.9 | 12.52 |
| profit for molasses sale $\left(\$ \mathrm{~h}^{-1}\right)$ | 143.7 | 90.32 |
| total profit for sales $\left(\$ \mathrm{~h}^{-1}\right)$ | 320.6 | 1172.52 |
| investment cost $\left(\$ \mathrm{~h}^{-1}\right)$ |  |  |
| biomass fermentors | 72.9 | 102.0 |
| alcohol fermentors | 17.3 | 15.3 |
| centrifuge | 10.2 | 15.1 |
| evaporator | 15.6 | 17.1 |
| dryer | 104.9 | 153.8 |
| distillation | 36.1 | 111.3 |
| total investment cost $\left(\$ \mathrm{~h}^{-1}\right)$ | 257.0 | 414.6 |
| operative cost $\left(\$ \mathrm{~h}^{-1}\right)$ | 3.7 | 66.0 |
| inoculums | 4.2 | 23.0 |
| water (cooling and diluted) | 0 | 3.4 |
| vinasse disposal | 9.6 | 88.8 |
| imported vapor | 17.5 | 181.2 |
| total operative costs $\left(\$ \mathrm{~h}^{-1}\right)$ |  |  |
| total partial benefits $\left(\$ \mathrm{~h}^{-1}\right)$ | 46.1 | 576.74 |

As can be noted in Table 13, the investment cost for MP is $61 \%$ higher than that for MC, but the profit for sale of bakery yeast production is 6.7 times higher in MP than in MC.

In the MP scenario, unit sizes are larger because the bakery yeast production is higher. However, because more alcohol fermentors are used in the MC scenario, the investment cost for this stage is greater in this scenario than in the scenario for the MP plant. For the remaining stages, the investment cost is higher in MP scenarios. Note that the distillation stage in the MP plant is also more expensive because the processed volume is larger (because the fermented broth volume is bigger) and therefore all of the distillation item sizes are larger. Also, the processing time of this stage is not as long as that of example 1 because the processing time of the semicontinuous train is longer in order to have a good performance at the separation, evaporation, and drying stages (for bakery yeast production).

Operative costs are obviously more significant for the MP plant because vapor and cooling water consumption are larger as a result of the semicontinuous unit sizes. The inoculum cost is also higher because of the fermentation stages. A larger amount of bakery yeast is produced, and so more inoculums are needed. On the other hand, the units in series configuration
of the MC scenario reduces the inoculum consumption. This configuration cannot be adopted for MP scenarios because sizes are larger, and thus the investment cost would be incremented. For this reason, larger amounts of inoculums are consumed.

It is worth noting the importance of handling a complete model that simultaneously optimizes synthesis, design, operation, and scheduling of the plant or of the integration of plants. These results cannot be obtained if optimization is carried out in separated formulations, for example, fixing the production rates and minimizing the total annualized cost or giving the plant structure and optimizing the production rates.

## 6. Conclusion

Detailed models for optimal process synthesis, design, and operation were proposed in this work in order to evaluate different production alternatives. Two scenarios were presented: a sequential MP noncontinuous plant and a MC model of single-product batch plants.

For a rigorous analysis, several decisions must be considered simultaneously. The proposed models integrate synthesis, design, operating, and scheduling decisions.

One of these models' characteristics is the high level of detail reached in the processing unit description, with some of them being attained by means of ordinary differential equations. Batch blending, batch splitting, and recycles are allowed as novel components for these types of models, and these decisions are considered as optimization variables in this work.

The models were implemented for a torula yeast, brandy, and bakery yeast production. With the aim of evaluating different alternatives, both scenarios were modeled and solved, varying the upper bound of the production rates. In this case, it was shown that even the most profitable scenario solution might differ according to the value taken for this bound.

The idea presented in this work can be applied to other process productions, and the analysis can be performed according to different variations on the operative conditions.

For the study case, several results were analyzed; some tradeoffs between process and design variables were presented together with the economic impact in each case; and the conclusions for each comparison were reported.

Further on these specific results, this work emphasizes the significant value of formulations that take into account all involved decisions simultaneously to reach effective solutions.

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## Nomenclature

## Indices

B = brandy
$\mathrm{BY}=$ bakery yeast
$e=$ recycle
fin $=$ final
$i=$ product
ini $=$ initial
$j=$ batch unit
$k=$ semicontinuous unit
$l=$ plant
$\max =$ maximum
$\min =$ minimum
$r=$ resource
$\mathrm{T}=$ torula

## Variables and Parameters

$B=$ batch product size (kg)
$\mathrm{CT}=$ cycle time (h)
$c_{r}=$ cost of resource $r\left(\$ \mathrm{~h}^{-1}\right)$
$C_{x}=$ component concentration $\left(\mathrm{k} \mathrm{m}^{-3}\right)$
$f^{e}=$ amount of consumed recycle $e\left(\mathrm{~m}^{3}\right)$
$f_{i j r}=$ amount of resource $r$ consumed in process production of
$i$ in unit $j\left(\mathrm{~m}^{3}\right)$
$F_{r}=$ amount of produced resource $r\left(\mathrm{~m}^{3} \mathrm{~h}^{-1}\right)$
$F_{r}^{\mathrm{ex}}=$ amount of exported resource $r\left(\mathrm{~m}^{3} \mathrm{~h}^{-1}\right)$
$F_{r}^{\text {trans }}=$ amount of transported resource $r\left(\mathrm{~m}^{3} \mathrm{~h}^{-1}\right)$
$\mathrm{HT}=$ horizon time (h)
$l_{\mathrm{s}}=$ step length in discretized equations
$L=$ number of derivative plants
NAP $=$ net annualized profit $(\$)$
$\mathrm{Nb}=$ number of batches
$N_{j}=$ number of batch units
$N_{k}=$ number of semicontinuous units
$N_{p}=$ number of products to be processed
$Q=$ production rate ( $\mathrm{k} \mathrm{h}^{-1}$ )
$R=$ semicontinuous unit size
Res $=$ disposal cost (\$)
$S=$ size factor
$t=$ processing time of the batch unit
$T=$ time that the batch unit is occupied
$V=$ batch unit size $\left(\mathrm{m}^{3}\right)$
$\mathrm{VS}=$ batch volume $\left(\mathrm{m}^{3}\right)$
$\mu=$ specific growth rate of biomass
$\theta=$ processing time of the semicontinuous unit
Sets
$\mathrm{EB}=$ set of batch units
$\mathrm{ES}=$ set of semicontinuous units
$I_{j}=$ set of products that share unit $i$

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