

Efficiency-wage Hypothesis and the Operational Production Pattern

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Abstract - An economy's production set is the collection of all net output vectors that the economy is capable of producing with a given technology and fixed quantities of primary factors of production. The boundary of this set is called the production possibility frontier or PPF. We show that, if the efficiency-wage hypothesis holds, a country's PPF, though conceptually valid, is an operationally irrelevant concept, because the economy never operates on the PPF, which is a view that ought to be appreciated in light of persistent unemployment in the new structure of economies of the post-21st-Century-crisis world.

Keywords: general equilibrium, production set, production possibility frontier, efficiency wage, economic policy, labor market, factors of production

1. Introduction

In mainstream economics, an economy's production set is taken to be the collection of all net output vectors that the economy is capable of producing, by transforming inputs into outputs, with a given technology and with fixed quantities of primary factors of production.¹ The boundary of this set is called the production possibility frontier (PPF). An essential feature of the production set of an economy is that it is invariant to changes in market prices.² In such a case, there is a *unidirectional* relationship insofar as exogenous changes in technology or fixed factor supplies can induce changes in general equilibrium market prices, but changes in market prices do *not* affect the structure of the production set, nor shift its boundary in any manner whatsoever. At a theoretical level, this paper forms part of a program, the purpose of which is to establish that *operationally* the economy's market-invariant PPF is an irrelevant concept insofar as there is a *bidirectional* relationship between an economy's *actual* production possibilities on the one hand, and the general equilibrium market prices, on the other. We accomplish the goal of establishing this claim by utilizing the efficiency-wage hypothesis in the manner of Solow (1979).

Egbert and Naqvi (2011) also achieve this objective. However, they do so by embedding a small open economy in an integrated world capital market, so that under endogenous international capital mobility, a government policy change can alter the quantity of capital that locates in the country, and thereby change the structure of its economy's production set, and *inter alia* shift the boundary of such a set. In this paper, we show that if the efficiency-wage hypothesis holds, the *operational* production pattern in the economy has nothing to do with the concept of a PPF in economics as we know it. Moreover, in contrast with Egbert and Naqvi, we demonstrate that this is so both (a) without international capital mobility in a small open economy, and (b) in a closed economy.

Related work by Albert and Meckl (2001) provides a canonical formulation of the Heckscher-Ohlin (HO) model with the efficiency-wage phenomenon exhibited in both sectors of the economy. This is based on the formulation of Summers (1988). By making the effort function in each sector depend on fixed, though intersectorally differential, mark-ups on the economy-wide, weighted-average wage rate (their reference wage), their model simultaneously exhibits, both involuntary unemployment and stable inter-industry wage differentials. In addition, they demonstrate that (1) all properties, including the Stolper-Samuelson theorem, the Ryczynski theorem, and so on, hold, if factor intensities are interpreted in the cost-share sense, and (2) that the result of immigration can be a lowering of the unemployment rate, whereas Foreign Direct Investment (FDI) can raise unemployment. These latter results are both quite startling – though entirely correct and intuitive – once the dependence of unemployment changes on sectoral distribution of employment is noted.

Our purpose here is different from that of Albert and Meckl (2001). It is to demonstrate the endogenous character of the operational PPF in the face of persistent unemployment, in a Specific Factors model.³ To this end, we adopt the efficiency wage function used by Solow (1979).⁴

Shiller (2010) and Stiglitz (2011), among others, call for endogenizing some variables that were in the pre-21st-Century-crisis world taken to be exogenously specified in economic models. Blinder (2010) goes further and asks for a complete overhaul of the macroeconomics curriculum, and talks about planning to drop the assumption of a single-interest-rate economy from macroeconomic models in the next, 12th Edition, of his joint text with Baumol.⁵ These observations are driven by the failure of macroeconomics to adequately predict or fully comprehend the economic crisis impacting the global economy starting in 2007 to 2008. Our primary purpose in writing the current paper is to heed this call for change in the spirit of Blinder, Shiller and Stiglitz.

Section 2 lays out the Specific Factors model with the efficiency-wage hypothesis, and shows how the pattern of production and the unemployment rate are market dependent for a small open economy without international capital mobility. Section 3 considers the effects of government-policy induced market price changes on the production pattern in the economy. Section 4 extends our result to the case of a closed economy, so as to rule out any presumption that such market-dependence of production pattern arises from the openness of the economy. Section 5 contains concluding remarks.

2. Specific Factors Model with the Efficiency-wage Hypothesis

Consider the standard Specific Factors model with the efficiency-wage hypothesis built into it.⁶ Let all economic activity in an economy be divided into two parts: Manufactured goods, M , and Services, S , produced by the technology embodied in the following production functions.

$$M = F(e(w_L)L^d, K_m) \quad (1)$$

and

$$S = G(\bar{H}, K_s), \quad (2)$$

where L^d is the number of unskilled workers demanded in the manufacturing sector, and K_m the endogenously determined quantity of capital employed in the manufacturing sector of the economy, whereas \bar{H} and K_s are, respectively, the number of skilled workers and the amount of capital employed in service-sector production. In this economy, capital is intersectorally mobile, but unskilled labor is specific to manufacturing whereas skilled labor is specific to services. Moreover, in Eq. (1), $e(w_L)$ is the number of efficiency units of labor delivered by each worker, with the property that it rises as the unskilled wage rate rises, but at a diminishing rate, so that $e'(w_L) > 0$ and $e''(w_L) < 0$, and w_L is the unskilled wage rate.⁷ If unskilled labor

employed in manufacturing is seen in terms of efficiency units, instead of in terms of the number of workers employed, it would be $N = e(w_L)L^d$ effective units.

Each firm in the manufacturing sector is perfectly competitive in the commodity market and the capital market, where it behaves as a price taker. It chooses how much capital to rent and how many unskilled workers to hire. However, it also chooses what wage rate to offer the unskilled workers so as to maximize profit. In the labor market, therefore, the manufacturing firms have a limited ability to determine the wage offer. The unskilled wage rate is thus a decision variable for each firm because a higher wage rate induces each hired worker to deliver greater efficiency. All firms are identical, as are all workers, and capital is homogenous as well.

Here $F(e(w_L)L^d, K_m)$ and $G(\bar{H}, K_s)$ are concave production functions that are characterized by (i) the Inada conditions, including indispensable inputs, (ii) constant returns to scale, and (iii) the law of diminishing returns, which together imply that (iv) inputs are co-operative.⁸

First consider the case in which this is a small open economy. Further, let services be the *numéraire* commodity, so that the price of the service sector output equals one. Then, p_m is the relative price of the manufactures in terms of services. Additional relationships that hold are

$$p_m F_K(e(w_L)L^d, K_m) = G_K(\bar{H}, K_s) = r \quad (3)$$

and

$$K_m + K_s = \bar{K}. \quad (4)$$

Equation (3) asserts that, given that capital is mobile across sectors, the values of marginal product of capital are equal in both sectors, and their common value equals the endogenously determined domestic rental rate of capital, r , measured in terms of services, and Eq. (4) asserts that the demand and hence employment of capital in the two sectors equals its fixed supply \bar{K} , on the assumption that r is perfectly flexible.

Additionally, the unskilled wage rate in the economy is endogenously determined by

$$p_m F_L(e(w_L)L^d, K_m) = w_L/e(w_L) \quad (5)$$

and

$$\frac{e'(\hat{w}_L)\hat{w}_L}{e(\hat{w}_L)} = 1, \quad (6)$$

where Eq. (6) is the well-known Solow elasticity condition.⁹ The unskilled wage rate, while in principle perfectly flexible, is sticky at \hat{w}_L , which is

determined solely by Eq. (6), due to the incentive of the manufacturing firms to offer the wage that minimizes cost with respect to a unit of effective, effort-adjusted unskilled labor, rather than with respect to raw, unskilled labor that is not adjusted for efficiency.¹⁰

Finally, we have

$$G_H(\bar{H}, K_S) = w_H, \quad (7)$$

which determines the skilled wage rate. Notice that in the model, the two wage rates are also expressed in terms of the same *numéraire* commodity, *viz.*, services.

Without loss of generality, assume that manufactures are imported (implying that some services are outsourced to this country by some foreign countries). Then p_m is the *domestic* relative price of the manufactures in terms of services, and $p_m = p_m^* + t$, where p_m^* is the *world* price of manufactures that this country takes as exogenously given, since p_m^* is determined on the world market for manufactured goods. Here, $t \geq 0$ is the import tariff that is exogenously imposed by the country's government. This tariff has a value that is less than t_p , which is the prohibitive tariff that snuffs out all imports.

As already noted, from Eq. (6) alone, the value of $w_L = \hat{w}_L$ is uniquely determined. With this, for $t = 0$, which means that in free trade equilibrium, from Eq. (3), Eq. (4) and Eq. (5), $K_m = \hat{K}_m$, $K_S = \hat{K}_S$ and $L^d = \hat{L}^d$ are uniquely determined, as is $r = \hat{r}$, all of which are functions of the exogenous variables \bar{K} , \bar{H} , and p_m . Finally, substituting for \hat{K}_S in Eq. (7) uniquely determines \hat{w}_H as a function of the three exogenous variables. Notice also that unemployment of unskilled workers in the economy is equal to $\hat{u} = \bar{L} - \hat{L}^d > 0$, which is a function also of the fourth exogenous variable \bar{L} , and this unemployment arises as an equilibrium phenomenon, because the manufacturing firms do not reduce the unskilled wage rate for fear of facing reduced efficiency of incumbent workers.

Once the equilibrium values of these endogenous variables are plugged into the production functions Eq. (1) and Eq. (2), we also obtain the pattern of production in the economy, \hat{M} and \hat{S} , which lies strictly inside the PPF because of the unemployment of unskilled labor, despite full employment of both capital and skilled labor in the economy. It can be verified that the restrictions placed on the production functions and on the efficiency function ensure that the general equilibrium supply curves for both final commodities are upward sloping. This is true in spite of the fact that the economy operates strictly inside the PPF.

3. Policy Intervention

To see the effects of policy intervention, in particular the effect of an import tariff, a diagram is helpful.

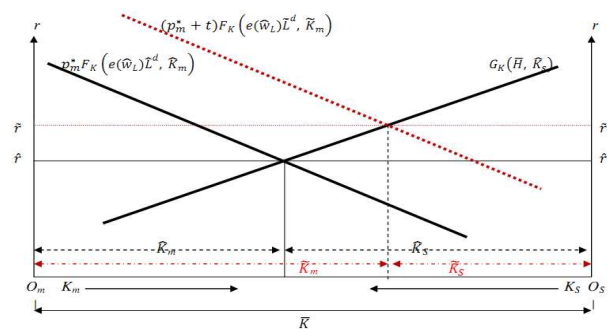


Figure 1
Graph of Equations (3) and (4), with $t > 0$

In Figure 1, the fact that \hat{w}_L is determined from Eq. (6), which also determines $e(\hat{w}_L)$, from the efficiency wage function, is utilized. Substituting for these two values in Eq. (3), Eq. (4) and Eq. (5), we obtain \hat{K}_m , \hat{K}_S and \hat{L}^d under free trade, for $t = 0$, as displayed in Figure 1. Suppose next that the government imposes a positive tariff on the imports of manufactures. That means $t > 0$. Then, the left-hand side (LHS) of Eq. (3) becomes higher, displayed by a rightward shift of the curve representing the value of marginal product of capital in manufacturing, as the (red) dashed curve. To restore equilibrium, given the law of diminishing returns, the employment of capital must rise in manufacturing. In Figure 1, this is given by $\tilde{K}_m > \hat{K}_m$. With a given quantity \bar{K} of capital in the economy, this means that less capital is now employed in the service sector as a consequence, given by $\tilde{K}_S < \hat{K}_S$ in Figure 1.

Notice also that with more capital employed in manufacturing, given that inputs are cooperative, the value of marginal product of unskilled labor rises, so that the LHS of Eq. (5) becomes higher. Since the RHS of Eq. (5) remains fixed due to Eq. (6), given the law of diminishing returns, the employment of unskilled labor must rise in manufacturing for equilibrium to be maintained, so that $\tilde{L}^d > \hat{L}^d$, thereby reducing unskilled unemployment in the economy, which is now $\tilde{u} = \bar{L} - \tilde{L}^d < \hat{u}$.

Due to the positive import tariff on manufactures, a change in income distribution in favor of unskilled workers occurs, simply because the unskilled wage rate remains unchanged but unemployment is lower. Further, since the supply of skilled labor, \bar{H} , is exogenously given, the decline in capital employed in the services sector lowers the productivity of skilled workers, and *inter alia* reduces the skilled wage rate, w_H . This means that $\tilde{w}_H < \hat{w}_H$ and total earnings of skilled workers decline. Also, with a fixed quantity of capital in the country, a higher rental rate of capital implies that the aggregate

earnings of capital owners rise. Notice that the rightward shift of the curve representing the value of marginal product of unskilled labor, the (red) dashed curve, occurs due to two forces: an increase in the domestic relative price of manufactures *and* an increase in the employment of unskilled labor in manufacturing.

Once again, under tariff protection, the new equilibrium values of these endogenous variables can be substituted in Eq. (1) and Eq. (2) to obtain the new pattern of production in the economy, $\tilde{M} > \hat{M}$ and $\tilde{S} < \hat{S}$, which also lies strictly inside the PPF because of the unemployment of unskilled labor despite full employment of both capital and skilled labor in the economy. The manufacturing sector expands, because more unskilled labor and more capital are employed under tariff protection, but the service sector contracts since some capital gets extracted from this sector.

4. Closed Economy

The economy considered so far is a small open economy for which the domestic relative price of manufactures p_m is exogenously fixed, either because of equality with the parametric world price in free trade, or due to that reason and a government specified fixed import tariff. However, if the economy is closed, p_m has to be endogenously determined, which happens due to the fact that domestic demand for manufactures must be met solely by domestic output supply, and by Walras law, the service market will also clear. So, for a closed economy, we have,

$$C^m(p_m, S + p_m M - \tau) + \gamma^m = M(p_m, \bar{K}, \bar{H}) \quad (8)$$

Where, in Eq. (8), C^m is domestic demand for manufactured goods, τ is the *exogenous* lump-sum tax revenue collected by the government, rendering $S + p_m M - \tau$ as the disposable personal income of the country, and γ^m is the *exogenous* government demand for manufactures. With this specification, Eq. (1) – Eq. (8) constitutes the model of a closed economy, and Eq. (8) serves to endogenously determine the Walrasian general equilibrium domestic relative price of manufactures, at a value of $p_m = \tilde{p}_m$.¹¹ In a closed economy, the relative price is an endogenous variable, but it is replaced here by two exogenous variables, γ^m and τ .

Straightforward reasoning will show that an increase in government demand for manufactures will result in a rightward shift of the curve representing the value of marginal product of capital in manufacturing, not unlike that represented in Figure 1 as the (red) dashed curve. This is analogous to an increase in the import tariff on manufactures, because it will also raise the domestic relative price of manufactures, although this will be an *induced*

change rather than one that is exogenous in the tariff case.

To see this, consider an increase in γ^m . As is evident from Eq. (8), this increases the domestic aggregate demand for manufactures, which, by itself, leads to an increase in p_m to, say, \hat{p}_m . As a consequence, given upward sloping general equilibrium supply curves, the output of manufactures rises to \hat{M} , and this is accomplished, one, by increased employment of capital in manufacturing to \hat{K}_m . Moreover, due to the cooperative nature of inputs, greater employment of capital induces increased productivity of unskilled workers, so that the LHS of Eq. (5) rises. Since the RHS of Eq. (5) remains unchanged due to Eq. (6), to restore the equality in Eq. (5), given the law of diminishing returns, manufacturing firms must hire more unskilled workers, say \hat{L}^d , which constitutes the second cause of an expansion of the manufacturing sector output. Naturally, unemployment falls to \hat{u} . Clearly, consequent upon an increase in γ^m , there is a rightward shift of the curve representing the value of marginal product of capital in manufacturing in Figure 1, due to both an increase in K_m and an increase in L^d .

Since capital supply is fixed, more capital employed in manufacturing implies less of it will be employed in the service sector, causing the service sector output to contract to \hat{S} , and the skilled wage rate also to fall to \hat{w}_S because of the reduced productivity of skilled workers, since they have less capital to work with, as seen from Eq. (7).

As with an import tariff on manufactures, there is a similar change in factoral income distribution, with skilled workers earning less, $\hat{w}_S \bar{H}$, while unskilled workers as a group becoming better off due to lower unemployment at the same wage rate, $\hat{w}_L \hat{L}^d$. Also, from (3) it is evident that with lower employment of capital in the service sector, the RHS becomes higher due to the law of diminishing returns, so the rental rate of capital rises to \hat{r} , leading to an increase in the income of capital owners to $\hat{r} \bar{K}$.

It can be verified that the consequences are exactly the opposite if the government increases its demand for services, instead of increasing its demand for manufactures. Are the consequences of a reduction of government taxation on *all* the endogenous variables exactly the same as the effects of an increase in government demand for manufactures? The answer is “no.” This is due to the fact that a reduction in lump-sum taxes leads to an increase in disposable personal income, and given positive marginal propensities to consume *both* manufactures and services (adding up to 1), the consumer demand for services also rises. This leads to an increase in the aggregate demand for services

also, which, by creating an excess demand for services at the pre-tax-reduction relative price of manufactures, generates a force to reduce this relative price. The final outcome, therefore, depends on the *relative magnitude* of the marginal propensity to consume manufactures versus that to consume services.

This much is clear: the increase in the relative price of manufactures, in magnitude, will necessarily be less under a tax reduction scheme than under an increase in government demand for manufactures. It could well be the case that the relative price of manufactures falls, if the marginal propensity to consume services is sufficiently greater than that to consume manufactures. The output supply response of manufactures and services will correspondingly be dependent on the direction and magnitude of the change in the relative price of manufactures. Thus the consequences for the economy are much more complicated due to a tax reduction than due to an increase in government demand for manufactures.

To obtain more precise inferences from the effects of a tax-reduction policy, more restrictions would have to be placed on consumers' preferences, for example, that personal preferences are both identical and homothetic. Such restrictions would be helpful for positive-theoretic purposes of description and prediction, though not for the normative purpose of policy prescription.¹² If all consumers do have preferences that are identical and homothetic, the behavior of all consumers can be portrayed by a single consumption function for descriptive and predictive purposes. Still, more information is needed regarding both marginal propensities to consume the two commodities and regarding the price elasticities of demand for the two commodities, to reach more refined conclusions. This, incidentally, also exposes the weakness of macroeconomic thinking relative to general equilibrium analysis, since in macroeconomic analysis such issues are assumed away.

It is noteworthy that all of the phenomena described thus far, whether for a small open economy or for a closed economy, arise well inside the PPF as we know it. Hence the redundancy of the concept of the PPF, as immune from the influences of market forces, is evident in our examination of the economic issues involved.

5. Concluding Remarks

If stylized facts dictate, the efficiency-wage hypothesis can be built into the service sector and would apply to skilled workers. The exercise would progress along similar lines, except that in this case the skilled wage rate would become sticky, leading to the unemployment of skilled workers, instead of the unemployment of unskilled workers, as in the model

of the current paper. The analysis, however, would trivially be along the same lines, with the only qualification that "unskilled labor" would be replaced by "skilled labor" in terms of the conclusions that we have reached. The actual economy under examination would determine which version of the model is more applicable. Of course, there is nothing wrong in assuming that the efficiency-wage phenomenon arises in both sectors, and with regard to both unskilled workers and skilled workers: for unskilled due to the fact that a higher wage rate provides more nutrition and hence more efficiency, and for the skilled workers because a higher wage rate serves as a disincentive to shirk by raising the opportunity cost of getting fired from the job. Both wage rates would, in such a case, become sticky.

Another extension would be to include an internationally non-traded commodity, in the manner of Batra and Naqvi (1989). This has the advantage of permitting an analysis of real exchange rate variations, since this rate is the ratio of the price of the non-traded commodity to the index of prices of the internationally traded commodities.

Returning to the fundamental reason for our writing this paper, we wish to emphasize, as Egbert and Naqvi (2011) have done in the context of international capital mobility, that a re-evaluation of the concepts and analyses we as economists are prone to undertaking is warranted. In this instance, we suggest a reconsideration with respect to the concept of the PPF, on the so-called supply side of the economy. We are, in fact, attempting to answer the Blinder-Shiller-Stiglitz call for thinking anew the conceptual basis of economic analysis in the 21st Century, post-economic-crisis world.

The fundamental point we wish to emphasize is that extremely valuable conceptual structures have been built by economists in the past 200 years or so, and we are in the enviable position to utilize them, so that only relatively minor, though considerably judicious, decisions need to be made to direct our attention to the economic reality that now faces us, as opposed to the one that faced us in the past. We argue that it is not a case of agency failures. Rather, it is a case of a change in the structure of economies, especially with respect to persistent unemployment, so that a slight change in the *approach to observation* will actually give us a way to comprehend the structure of this new economic reality that has emerged in the post-economic-crisis world.

Let us heed the call for a change in perspective, while retaining and preserving the enormous intellectual legacy we have been left by our economist forefathers and foremothers. Amendments to theory that Blinder, Shiller and Stiglitz have called for, i.e., to endogenize some variables and relationships that have hitherto been treated as exogenous in the post-21st-Century-crisis economies,

are quite sufficient for explaining phenomena of the structurally-different economies of the post-crisis world.

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Appendix

This appendix contains notes that are material to the issue, but could detract from the coherence of the narrative in the body of the paper.

- Typically, among other restrictions imposed on this set, it is assumed that the set is (a) non-empty, (b) compact (closed and bounded) and (c) convex.
- For a classic treatment, see Koopmans (1957) and Debreu (1959).
- Unlike our objective, the Albert-Meckl (2001, p. 287) stated goal is to demonstrate that "all HO results have close ... analogues" in the HO model with the efficiency wage function in each sector dependent on a fixed sector-specific mark-up on their reference wage. The more general formulation in Albert and Meckl (1997) has the additional advantage that it unifies and synthesizes a large body of diverse efficiency wage models, primarily because they show that their model entails a very useful property, which also obtains in their 2001 work, that the general equilibrium outcome maximizes the gross domestic product (GDP) of the country, although under an *additional* linear resource constraint, so that the envelope theorem properties of the constrained-GDP function are preserved.
- In our formulation, as in Solow (1979), the efficiency of unskilled workers (only in one sector), depends on the *real* wage rate. While it is measured in terms of services in our formulation, nothing would be lost if the real wage rate were measured in units of manufactures.
- See Baumol and Blinder (2009).
- See Jones (1971) for the original formulation.
- These properties have to hold locally for the existence of equilibrium, though not for uniqueness.

Uniqueness requires additional properties including that these conditions hold $\forall w_L \in (0, \infty)$, with $e'(0^+) = \infty$ and $e'(\infty^-) = 0$.

8. The cross-partial derivatives of the two production functions are both positive. Intuitively this means that more capital increases the marginal productivity of unskilled labor in manufacturing, and conversely. Also, more capital employed in the service sector raises the marginal productivity of skilled labor, and conversely.

9. By substituting the right-hand side (RHS) of Eq. (5) for the value of marginal product of labor, this follows from the first order condition of profit maximization with respect to the unskilled wage rate.

10. Solow (1979) writes, "The upshot is: if the wage enters the short-run production function, a cost minimizing firm will leave its wage offer unchanged no matter how its output varies if and only if the wage enters the production function in a labor augmenting way." (p. 81). This stickiness of the wage rate arising from the behavior of firms, rather than its rigidity, is precisely the cause of unemployment emerging as an equilibrium phenomenon despite the willingness of unemployed workers to offer to work for less. Moreover, this *entailed* constancy, instead of *imposed* constancy of the wage rate, can permit the use of the fixed-factor-price constrained GDP function that Neary (1985) proposes for a fruitful unification and synthesis of the literature on international capital mobility and minimum wage rates in general equilibrium.

11. Other endogenous variable values, \tilde{z} , generically speaking, are to be similarly denoted in general equilibrium for the closed economy.

12. For the soundness (acceptability) of normative policy prescription, it would also have to be the case that all persons in the country actually have exactly the same income, to which equal weights are attached in social *evaluation*; unless, of course, the value judgment is also advanced that equal weights *should* be attached to unequal personal incomes in social evaluation. Indeed, Sen (1979) writes, "given homothetic preferences identical for all, ... the market behavior of a *group* of consumers can be treated as if it were that of one consumer ... While this renders distribution of income irrelevant for explaining or predicting market *behavior*, it does not, of course, make distribution irrelevant for *social welfare*! ... While for the purpose of studying market behavior that assumption [of relevance of distribution] can be dropped still retaining the aggregation over the consumers, the same clearly does not hold for making social welfare judgments." (pp. 27-28)

Thus, if it is the case that homothetic preferences are identical for all persons in society, then this, by itself, fails to constitute a justification as to why equal weights *ought* to be assigned to persons with different incomes, which is the implicit value judgment inherent in the use of a "representative consumer's" *personal* utility function as a *social* welfare evaluation function, simply because "ought" cannot be deduced from "is." On the other hand, in addition to homothetic preferences identical for all persons, if it is also assumed that all persons in society have exactly the same income, in which case equal weighting may be justifiable as a value judgment for social evaluation, then the country becomes indistinguishable from a person, as, for instance, Robinson Crusoe without Man Friday constituting a one-person society. We make no such claim.