Toward Optimal Rate-Delay Tradeoff for Computation over Multiple Access Channel

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***Abstract*—Computation over multiple access channel (CoMAC) scheme provides a promising solution to future large-scale wireless networks by utilizing the superposition property of the wireless channel to compute a class of functions with a summation structure (e.g., mean, norm, etc.). However, its implementation usually requires all nodes' channel state information (CSI) and its performance is limited by the channel condition of the worst node. In order to avoid massive CSI aggregation and improve the limited performance, we propose an automatic repeat request (ARQ)-aided CoMAC scheme in this paper. The transmitters and signaling procedures are designed to achieve the tradeoff between the achievable function rate and the transmission delay. The corresponding performance of the proposed ARQ-aided CoMAC scheme and the traditional ARQ-aided communication scheme are compared for both homogeneous networks and heterogeneous networks. By optimizing the ARQ level, we further maximize the achievable function rate of the proposed scheme. Asymptotic closed-form expressions are derived by resorting to the extreme value theory and point mass approximation. Monte Carlo simulations are given to illustrate and verify the performance of the proposed designs.**

***Index Terms*—ARQ, function rate, multiple access channel, signaling procedure, transmission delay, transmitter.**

I. Introduction

5G cellular networks are predicted to provide Internet of Things (IoTs) that interconnect up to 1 trillion devices, and a million connections per square kilometer [1]. Thus, it imposes great challenges to future wireless networks to connect an enormous number of nodes and aggregate distributed information, where massive radio access in multiple access channel (MAC) would result in excessive network latency [2]–[4].

In order to overcome this bottleneck, most existing works mainly focus on reducing the data traffic [5], [6] or improving the communication efficiency [7], [8]. Due to the sparsity property of data traffic, compressive sensing-enabled approaches could be adopted to enhance the spectrum efficiency and energy efficiency of 5G cellular networks and IoT networks [5]. Exploiting spatial and temporal correlation among the generated traffic, the work in [6] provided two efficient data transmission reduction schemes to increase energy and bandwidth efficiencies in IoT networks. In [7], the application of non-orthogonal multiple access (NOMA) techniques was investigated in ultra dense networks to support massive connectivity. A unified framework of NOMA networks was proposed in [8], where the improved performance was derived based on stochastic geometry. These works were designed for data-centric networks, where the challenge incurred by massive radio access was inevitable.

Computation over MAC (CoMAC) scheme provides a promising solution for function-centric networks, which utilizes the superposition property of wireless channel to compute a class of functions with a summation structure (e.g., sum, norm, etc.) directly. Compared to the traditional "communicate-then-compute" scheme, CoMAC scheme avoids individual data aggregation and allows concurrent transmission of all nodes. Thus, it dramatically reduces the air interface latency without orthogonal multiple access. The idea of CoMAC scheme traces back to the pioneering work in [9], where it pointed out the benefit of CoMAC scheme from the information theoretic viewpoint. Various experimental platforms have been built to verify the idea of CoMAC scheme in [10]–[12]. Recently, CoMAC scheme has been applied into the communication-efficient federated learning over wireless networks [13]–[15].

Despite the advantage of CoMAC scheme, its implementation faces a lot of challenges. One is the distributed synchronization of all nodes for coherent combining at the fusion center (FC). In [16], a solution called "AirShare" was developed for synchronizing distributed nodes by broadcasting a reference clock signal over the air. Compared with connecting an external clock to all nodes via wires, this solution realizes distributed synchronization without sacrificing mobility and flexibility. To cope with the synchronization phase offset, a random phase rotation design was proposed in [17], which turned synchronization error of the nodes into random noise at the FC. Further, considering the misaligned initial clock instants between different nodes, an estimation and equalization design for CoMAC scheme was developed in [18]. By estimating the synchronization error, the aliasing error was reconstructed and subtracted from the output signal. Modeling the channel state information (CSI) including synchronization uncertainty by the worst case model, a robust design problem

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for CoMAC scheme was formulated and optimized in [19].

Communication noise is another challenge to achieve reliable CoMAC scheme. Although uncoded analog function computation is the most direct way to realize CoMAC scheme from the implementation's point of view, the computation error caused by noise is inevitable [20]. The work in [21] proposed a novel compute-and-forward scheme to improve the transmission rate. The key idea of compute-and-forward is to compute integer combinations of codewords from multiple source nodes adopting nested lattice coding, rather than to decode individual codewords by treating others as noise. By introducing compression operation at receivers, compute-and-forward was extended to a more general compression framework, termed generalized compute-compress-and-forward in [22]. The application of compute-and-forward has been investigated for NOMA in [23] and visible light communications in [24]. Inspired by the idea of computing the noisy modulo sum, the work in [25] proposed a unified digital scheme to compute the structured functions over MAC, where the achievable function rate was derived assuming the uniform fading MAC.

One more challenge of CoMAC scheme is the fading in wireless channel. Most of the above mentioned works assumed that the fading of the wireless MAC was the same and the uniform summation could always be achieved. Considering the non-uniform fading of practical MAC, the work in [26] first pointed out that the achievable function would decrease as the number of nodes increases, which eventually went to zero. With practical non-uniform fading MAC considered, the performance of digital CoMAC scheme was discussed in [27], and its performance could be worse than that of "communicate-then-compute" scheme with a large number of nodes. In [28], an opportunistic node scheduling was proposed in order to achieve a non-vanishing achievable function rate. Further, a joint power and node scheduling was discussed in [29] for a wide-band MAC, which divided the desired function into sub-functions, allocated these sub-functions into different subcarriers, and reconstructed the desired function at the FC. A uniform-forcing transceiver design was developed in [30] to compensate the non-uniform fading of different nodes into the uniform level. More complicated multi-antenna transceiver designs were provided in [31] and [32] for CoMAC scheme to improve the corresponding performance.

However, the afore-mentioned works for CoMAC scheme, e.g. [27]–[30], require all nodes' CSI and their performance is limited by the channel condition of the worst node. When the number of the nodes is large, the overhead to obtain global CSI would be prohibitive, and the worst node's channel condition would lead to a poor achievable function rate. This motivates our work to propose and analyze an automatic repeat request (ARQ)-aided CoMAC scheme. ARQ protocol has been extensively studied for various communication networks, which provides an efficient solution to avoid massive CSI aggregation and improve the throughput. The use of a thresholding scheme to distributively select users with preferable channel conditions could go back to the works in [33]. In [34], the authors proposed a channel state feedback algorithm with multiple feedback thresholds to reduce the number of users transmitting feedback to a minimum. A joint opportunistic scheduling and limited feedback was proposed in [35], which only collected CSI from the users with sufficiently good channel quality. The discussion was further extended to the multi-antenna scenario in [36] and the multi-channel scenario in [37]. All these works improved the communication performance by opportunistically selecting the best user or channel from multiple ones. In contrast, the performance of CoMAC scheme is determined by the node with the worst channel condition. And unlike traditional networks that focus on communications, the objective of CoMAC scheme is the computation of the target function. As a consequence, the existing designs are inapplicable for the CoMAC scheme. To the best of the authors' knowledge, how to design an ARQ-aided CoMAC scheme and improve the computation performance has never been discussed before.

In this work, we study a CoMAC scheme with the aid of ARQ protocol. The transmitters and signaling procedures are designed to avoid massive CSI aggregation and improve the limited performance. We derive the performance of achievable function rate and transmission delay for the proposed ARQ-aided scheme and the traditional ARQ-aided communication scheme in both homogeneous networks and heterogeneous networks. Then, the tradeoff between rate and delay is optimized to maximize the achievable function rate, where asymptotic closed-form expressions are provided by adopting the extreme value theory and point mass approximation. The main contributions of this work are summarized as follows.

- **Design of an ARQ-aided CoMAC scheme.** We design an ARQ protocol for CoMAC scheme to avoid massive CSI aggregation and overcome the limitation imposed by the worst node's channel condition. The transmitter design and the signaling procedure are provided.
- **Performance analysis of the proposed design.** Based on the proposed scheme, the performance in terms of achievable function rate and transmission delay is derived. Both homogeneous networks with independent and identically distributed (i.i.d.) fading and heterogeneous networks considering path-loss are discussed.
- **The optimal tradeoff between rate and delay.** We further optimize the tradeoff between rate and delay to maximize the achievable function rate. Asymptotic closed-form expressions of the optimal ARQ level are provided using the extreme value theory and point mass approximation.

The remainder of the paper is organized as follows. Section II presents the model of the wireless function-centric networks. Section III provides the design of ARQ-aided CoMAC scheme. The performance analysis and optimization are studied in Section IV for homogeneous networks and in Section V for heterogeneous networks. Simulation results are provided in Section VI, followed by concluding remarks in Section VII.

Notation: We use boldface lowercase letter to denote column vectors. Superscripts $(\cdot)^*$ stands for Hermitian transpose. $\mathcal{C}^{m\times n}$ is the set of complex-valued $m\times n$ matrices. $x\sim\mathcal{CN}(a,b)$ means that x obeys a complex Gaussian distribution with mean a and covariance b. $\mathrm{E}(\cdot)$ denotes the statistical

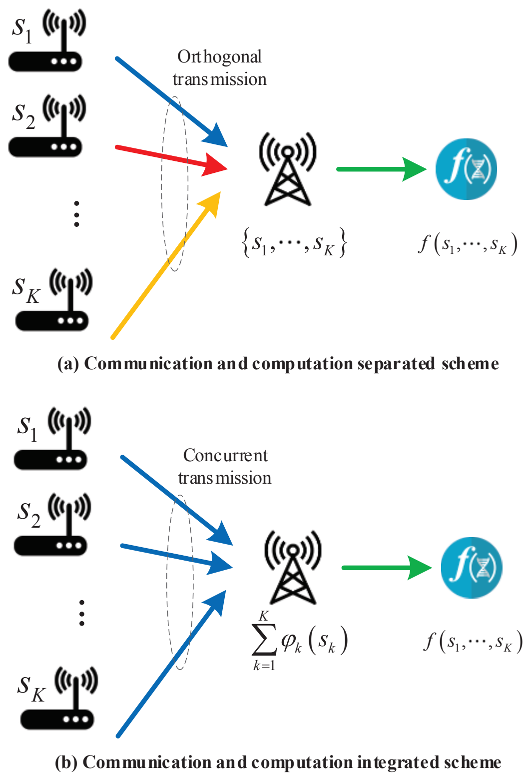


Figure 1. System model of CoMAC scheme.

expectation. $\mathcal{F}_q$ denotes the finite field of size q with the corresponding subset of the integers $\{0,\cdots,q-1\}$. $\bigoplus$ stands for the modulo summation over the finite field. $|\mathbf{x}|$ denotes the Euclidean norm of a complex vector $\mathbf{x}$.

II. System Model

The wireless network is composed of K nodes indexed by $k\in\{1,\cdots,K\}$ and an FC. The target of the FC is to recover a Nomografic function, i.e.,

$$f=\psi\left[\sum_{k=1}^{K}\varphi_k\left(s_k\right)\right], \tag{1}$$

where $\psi(\cdot)$ is the post-processing function of FC, and $\varphi_k(\cdot)$ is the pre-processing function of the node k. Given different $\psi(\cdot)$ and $\varphi_k(\cdot)$, some common multivariable functions can be also computed over the MAC, e.g., arithmetic mean $f=\sum_{k=1}^{K}s_k/K$ given $\varphi_k=s_k$ and $\psi=(\cdot)/K$, geometric mean $f=(\prod_{k=1}^{K}s_k)^{\frac{1}{K}}$ given $\varphi_k=\log(s_k)$ and $\psi=[\exp(\cdot)]^{\frac{1}{K}}$, and euclidean norm $f=(\sum_{k=1}^{K}s_k^2)^{\frac{1}{2}}$ given $\varphi_k=s_k^2$ and $\psi=(\cdot)^{\frac{1}{2}}$[1].

As illustrated in Fig. 1, there are two schemes to recover the target function. One is communication and computation separated scheme, i.e., traditional scheme. The nodes first transmit their readings $\{s_k\}$ through orthogonal MAC, and then the FC computes the target function f thereof. It incurs a high latency by adopting orthogonal MAC such as time division multiple access (TDMA). The other is communication and computation integrated scheme, i.e., CoMAC scheme. The nodes concurrently transmit their pre-processed readings, and then the FC receives the summation part of the target function $\sum_{k=1}^{K}\varphi_k\left(s_k\right)$. It utilizes the superposition property of the wireless channel and harnesses the inter-node interference rather than avoiding it.

However, the above summation of the wireless channel is too ideal in practical situation. Thus, channel coding should be adopted to combat the non-uniform fading and the received noise of the CoMAC scheme. The channel coding process of the node k can be given by

$$\begin{array}{ll}\text{Pre-processing:} & \{s_{k,i}\}\to\{\varphi_k\left(s_{k,i}\right)\}\\ \text{Quantization:} & \{\varphi_k\left(s_{k,i}\right)\}\to\{\mathbf{v}_{k,i}\}\\ \text{Mapping:} & \{\mathbf{v}_{k,i}\}\to\mathbf{q}_k\\ \text{Channel-coding:} & \mathbf{q}_k\to\mathbf{x}_k\end{array},\forall k \tag{2}$$

where $s_{k,i}$, $i\in\{1,\cdots,T\}$ is the i-th reading of the node k, T is the number of the readings, $\mathbf{v}_{k,i}\in\mathcal{B}^N$ is a length-N binary vector, $\mathbf{q}_k\in\mathcal{F}_q^m$ is a length-m q-ary vector, and $\mathbf{x}_k\in\mathcal{C}^M$ is a length-M channel codeword with the power constraint $\|\mathbf{x}_k\|^2\le MP_0$. After all nodes' concurrent transmission, the received signal of the FC can be given by

$$\mathbf{y}=a\sum_{k=1}^{K}h_k b_k\mathbf{x}_k+\mathbf{z}, \tag{3}$$

where $b_k\in\mathcal{C}$ is the transmitter scalar of the node k, $a\in\mathcal{C}$ is the receiver scalar of the FC, $\mathbf{z}$ is additive white Gaussian noise (AWGN) with each element distributed as $\mathcal{CN}\left(0,\sigma_z^2\right)$, and $h_k\in\mathcal{C}$ is channel between the node k and the FC. Finally, the recovery process of the FC can be expressed as

$$\begin{array}{ll}\text{Channel-decoding:} & \mathbf{y}\to\sum_{k=1}^{K}\mathbf{q}_k\\ \text{Remapping:} & \sum_{k=1}^{K}\mathbf{q}_k\to\{\mathbf{u}_i\}\\ \text{Quantization-recovery:} & \{\mathbf{u}_i\}\to\{\sum_{k=1}^{K}\varphi_k\left(s_{k,i}\right)\}\\ \text{Post-processing:} & \{\sum_{k=1}^{K}\varphi_k\left(s_{k,i}\right)\}\to\{f_i\}\end{array}, \tag{4}$$

where $\mathbf{u}_i=\sum_{k=1}^{K}\mathbf{v}_{k,i}$, and $f_i\left(s_{1,i},\cdots,s_{K,i}\right)$ is the i-th target function of the FC.

Ignoring the quantization error, we adopt the achievable function rate as the performance metric, which specifies how many functions can be computed per channel use given the length of the quantization vector, i.e.,

Definition 1. (Achievable function rate) Given the length-N quantization vector $\{\mathbf{v}_{k,i}\}\in\mathcal{B}^N$, R_{C} is achievable, if for every rate $R=T/M\le R_{\mathrm{C}}$ and every $\delta>0$, the decoding error ε during the decoding process satisfies $\Pr\left(\cup_{i=1}^{T}\{|\hat{\mathbf{u}}_i-\mathbf{u}_i|>\varepsilon\}\right)<\delta$ with sufficiently large channel uses M, where $\mathbf{u}_i=\sum_{k=1}^{K}\mathbf{v}_{k,i}$ and $\hat{\mathbf{u}}_i$ is the estimate of $\mathbf{u}_i$ at the FC.

In this work, the time axis is divided into several time blocks, and the channel is assumed to be block-fading, which keeps constant during the transmission of the codeword $\mathbf{x}_k$. According to the distribution of the channel, the networks can be categorized into homogeneous networks and heterogeneous

[1]General multivariable functions can be also computed over the MAC through Nomograhfic function approximation.

networks. The homogeneous networks model the scenario that the distances between the nodes and the FC have little differences, where the nodes experience independent and identically distributed (i.i.d.) Rayleigh fading with the same average power. And the heterogeneous networks model the scenario that the distances between the nodes and the FC have large differences, where the nodes experience independent and not-identically distributed (i.ni.d.) Rayleigh fading with different channel statistics.

III. Proposed CoMAC Scheme

A. Traditional CoMAC Scheme

Nested lattice coding is adopted as the channel coding, which can achieve reliable computation of modulo sum. The corresponding achievable rate can be given as follows.

Lemma 1. [38, Theorem 1] (Achievable rate to decode modulo sum) Considering the nested lattice codewords $\mathbf{x}_k \in \mathcal{L}^M$, their summation modulo the coarse lattice r_c, i.e., $\sum_{k=1}^{K} \mathbf{x}_k \mod r_c$ also takes values on the codebook $\mathcal{L}^M$. Thus, considering the received signal (3) of the FC, the decoding process is $\mathcal{D}(\mathbf{y}) = \bigoplus_{k=1}^{K} \alpha_k \mathbf{q_k}$. Then, the corresponding achievable rate is

$$R_{\mathcal{L}} = {\log_2}^+ \left(\frac{P_0}{\sum\limits_{k=1}^{K} |a h_k b_k - \alpha_k|^2 P_0 + |a|^2 \sigma_z^2} \right), \quad (5)$$

where ${\log_2}^+(\cdot) = \max\{\log_2(\cdot), 0\}$, and $|\mathbf{x}_k| \le M P_0$.

Different from modulo sum, the target function of the CoMAC scheme is the sum of transmitted messages. Thus, the message vector $\mathbf{q}_k$ should be carefully designed to avoid the wrapping around of modulo sum, and the corresponding achievable function rate of the CoMAC scheme can be given as follows.

Proposition 1. (Achievable function rate of the CoMAC scheme) The achievable function rate in Definition 1 can be derived as

$$R_{\mathrm{C}} = \frac{1}{N + \log_2 K} {\log_2}^+ \left(\frac{P_0}{\sum\limits_{k=1}^{K} |a h_k b_k - 1|^2 P_0 + |a|^2 \sigma_z^2} \right). \quad (6)$$

Proof. Refer to Appendix A. □

The uniform-forcing transceiver is adopted to compensate the non-uniform fading to the uniform level, i.e.,

$$a = \frac{1}{\sqrt{\kappa}}, \ \ b_k = \frac{\sqrt{\kappa} h_k^*}{|h_k|^2}, \ \forall k \quad (7)$$

where κ is the uniform channel power gain. Considering the transmit power constraint, one has $|b_k| \le 1$, $\forall k$, and then $\kappa \le \min_k |h_k|^2$. The corresponding achievable function rate of the CoMAC scheme in (6) can be rewritten as

$$R_1 = \frac{1}{N + \log_2 K} {\log_2}^+ \left(\frac{P_0 \min_k |h_k|^2}{\sigma_z^2} \right). \quad (8)$$

B. ARQ-aided CoMAC scheme

There are two challenges for the traditional CoMAC scheme. First, considering the practical channel power gain, the uniform-forcing transmitter designed in (7) requires the global CSI of all nodes, and its aggregation will also incur extensive network latency. Then, the performance of the CoMAC scheme in (8) is limited by the channel power gain of the worst node, i.e., $\min_k |h_k|^2$, which deteriorates as the number of nodes increases.

In order to cope with these challenges, we introduce ARQ into the CoMAC scheme, where the uniform-level is predefined to avoid the global CSI aggregation and the CoMAC performance can be improved through rate-delay tradoff. First, let us provide the traditional ARQ-aided communication scheme.

ARQ-aided communication scheme: ARQ has been widely adopted in the communication scheme to avoid the global CSI aggregation. In order to avoid inter-node interference, each node transmits in an orthogonal way through round-robin scheduling[2]. Then, the ARQ-aided communication scheme can be given as follows.

- The FC defines the channel power gain threshold ν and broadcasts it to all nodes.
- In the time slot t, the FC broadcasts pilots to all nodes, and the node k estimates its CSI $h_{k,t}$ based on the channel reciprocity.
- Each node transmits in a round-robin way. When the node k is scheduled, if $|h_{k,t}|^2 < \nu$, it does not transmit codewords but transmits a signal indicating NACK. If $|h_{k,t}|^2 \ge \nu$, the node k transmits codewords and it finishes the transmission.
- When all nodes have been scheduled and no NACK is detected by the FC, the ARQ-aided communication scheme is over.

When the node k is scheduled and $|h_{k,t}|^2 \ge \nu$, the effective received signal at the FC will be regarded as

$$\mathbf{y} = \sqrt{P_0 \nu} \mathbf{x}_k + \mathbf{z}, \quad (9)$$

and the corresponding achievable function rate can be calculated as each nodes finishes its length-N message transmission, i.e.,

$$R_2 = \frac{1}{N \sum\limits_{k=1}^{K} D_k} \log_2 \left(1 + \frac{P_0 \nu}{\sigma_z^2} \right) \quad (10)$$

where D_k is the number of the transmission time slots for the node k.

[2]Although the capacity region is larger than that achieved by time-division multiplexing, the improvement in capacity may not be sufficient to warrant the increased complexity.

ARQ-aided CoMAC scheme: Applying the idea of ARQ to the CoMAC scheme, the proposed ARQ-aided CoMAC scheme can be given as follows.

- The FC determines the pre-defined uniform channel power gain η and broadcasts it to all nodes.
- In the time slot t, the FC broadcasts pilots to all nodes, and the node k estimates its CSI $h_{k,t}$ based on the channel reciprocity.
- All nodes transmit in a concurrent way. The nodes with $|h_{k,t}|^2 < \eta$ do not transmit codewords and they transmit a signal indicating NACK. The other nodes with $|h_{k,D_k}|^2 \geq \eta$ transmits $\mathbf{x}_k$ with
$$b_{k,D_k} = \frac{\sqrt{\eta} h_{k,D_k}^*}{|h_{k,D_k}|^2}. \tag{11}$$
concurrently, and then they finishes the transmission.
- The FC decodes the sub-function in (13) at the end of the time slot t.
- When there is no NACK detected by the FC, the FC combines all the decoded sub-functions to recover the target function as (14), and the ARQ-aided CoMAC scheme is over.

Assuming the set of the nodes transmitting in the time slot t is Φ_t, the received signal of the FC in the time slot t can be given by

$$\mathbf{y} = \sum_{k \in \Phi_t} \sqrt{P_0 \eta} \mathbf{x}_k + \mathbf{z}, t \in \{1, \cdots, D\}, \tag{12}$$

where D is the number of the time slots for the ARQ-aided CoMAC scheme, and $\cup_{t=1}^{D} \Phi_t = \{1, \cdots, K\}$. The decoding is done at the end of each time slot, and the following sub-function can be recovered, i.e.,

$$g_t = \sum_{k \in \Phi_t} \varphi_k(s_k), \tag{13}$$

When all sub-functions have been recovered, i.e., $\{g_t\}, t \in \{1, \cdots, D\}$, the target function can be given by

$$f = \psi\left[\sum_{t=1}^{D} \sum_{k \in \Phi_t} \varphi_k(s_k)\right] = \psi\left[\sum_{k=1}^{K} \varphi_k(s_k)\right]. \tag{14}$$

Then, the corresponding achievable function rate of the ARQ-aided CoMAC scheme can be given by

$$R_3 = \frac{1}{D(N + \log_2 K)} \log_2^+ \left(\frac{P_0 \eta}{\sigma_z^2}\right). \tag{15}$$

The signaling procedures of the traditional CoMAC scheme, the ARQ-aided communication scheme and the ARQ-aided CoMAC scheme are illustrated in Fig. 2. And the corresponding signaling complexity can be compared as follows.

Remark 1. (Signaling complexity) Assuming each node's CSI is c_1 bits, the signaling complexity of the traditional CoMAC scheme is $c_1 K$ bits. Assuming the pilot and the NACK are c_2 bits and c_3 bits, the signaling complexity of the ARQ-aided communication scheme is $(c_2 + c_3) \sum_{k=1}^{K} D_k$ bits, and the signaling complexity of the ARQ-aided CoMAC scheme is $(c_2 + c_3) D$ bits, which does not linearly increase with number of the nodes K.

IV. Performance in Homogeneous Networks

In this section, we discuss the performance of the proposed ARQ-aided CoMAC scheme in homogeneous networks. The cumulative distribution function (CDF) of the node k's channel power gain can be given by

$$\rho = \Pr\left\{|h_k|^2 < x\right\} = 1 - \exp\left(-\frac{x}{\gamma_0}\right). \tag{16}$$

A. Traditional CoMAC Scheme

According to (8), the average achievable function rate of the traditional CoMAC scheme in homogeneous networks can be given by

$$\begin{aligned} \mathrm{E}(R_1) &= \frac{1}{N + \log_2 K} \int_{\sigma_z^2/P_0}^{\infty} \log_2\left(\frac{P_0 x}{\sigma_z^2}\right) f_{\min_k |h_k|^2}(x)\, dx \\ &= \frac{1}{\ln 2 (N + \log_2 K)} \mathrm{E}_1\left(\frac{\sigma_z^2 K}{P_0 \gamma_0}\right), \end{aligned} \tag{17}$$

where

$$f_{\min_k |h_k|^2}(x) = \frac{K}{\gamma_0}\left[\exp\left(-\frac{x}{\gamma_0}\right)\right]^K, \tag{18}$$

$\mathrm{E}_1(\cdot)$ is the exponential integral function defined as $\mathrm{E}_1(x) = \int_x^{\infty} (e^{-u}/u) du$ and the integral is based on [39, Eq. 4.331.2].

B. ARQ-aided communication scheme

The CDF of the number of the time slots for the node k can be derived as

$$\Pr\{D_k < T\} = 1 - \left[\Pr\left\{|h_{k,t}|^2 < \nu\right\}\right]^{T-1} = 1 - \rho^{T-1}, \tag{19}$$

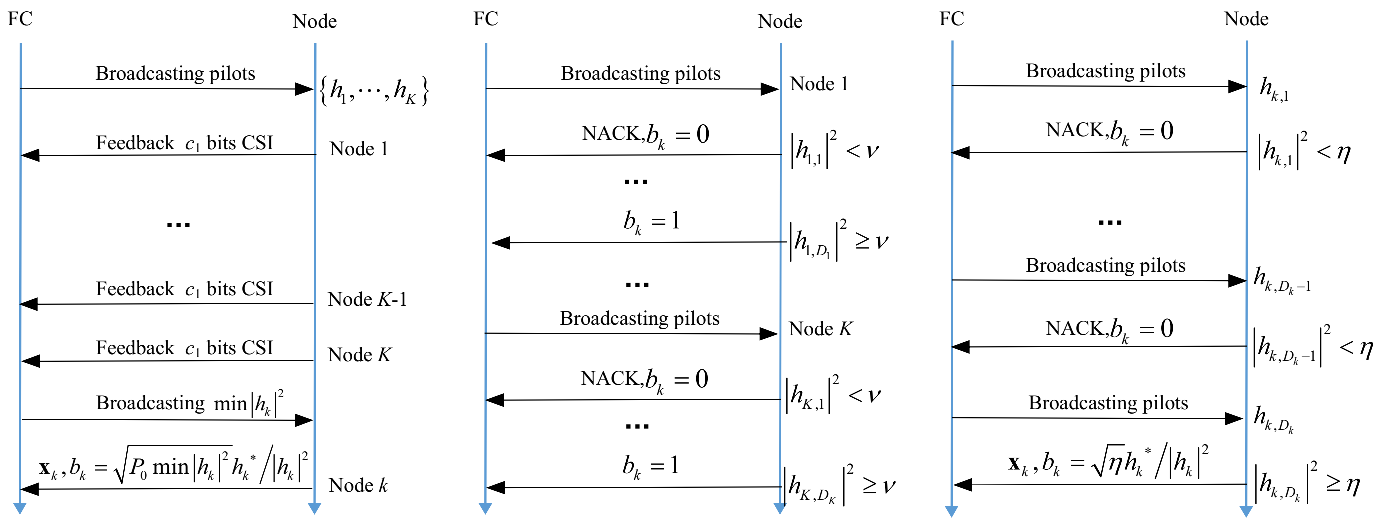
and the average number of the time slots for the node k is

$$\mathrm{E}(D_k) = \sum_{T=1}^{\infty} \Pr\{D_k \geq T\} = \frac{1}{1 - \rho}. \tag{20}$$

Then, the average achievable function rate of ARQ-aided communication scheme in homogeneous networks can be given by

$$\begin{aligned} \mathrm{E}(R_2) &= \frac{1}{NK\mathrm{E}(D_k)} \log_2\left(1 + \frac{P_0 \nu}{\sigma_n^2}\right) \\ &= \frac{1}{NK} \exp\left(-\frac{\nu}{\gamma_0}\right) \log_2\left(1 + \frac{P_0 \nu}{\sigma_n^2}\right). \end{aligned} \tag{21}$$

$\mathrm{E}(R_2)$ is a nonlinear fractional function, and it is non-concave on variable ν. Thus, we prove the pseudoconvexity of $-\mathrm{E}(R_2)$, which is a kind of generalized convexity. The judgement and property of pseudoconvexity can be provided by the following Lemma.



(a) Traditional CoMAC (b) ARQ-aided communication (c) ARQ-aided CoMAC

Figure 2. Signaling procedures of the traditional CoMAC scheme, the ARQ-aided communication and the ARQ-aided CoMAC schemes.

Lemma 2. [40, Theorem 6.6 & 6.9] (Judgement and property of pseudoconvexity) Consider nonlinear fractional functions. Let X be a convex set, let $g_1(x)$ and $g_2(x)$ be real-valued function on X, and let

$$g(x) = \frac{g_1(x)}{g_2(x)}. \tag{22}$$

If $g_1(x)$ is nonpositive and convex on X and $g_2(x)$ is positive and convex on X, $g(x)$ is pseudoconvex on X. For pseudoconvex function $g(x)$ on X, suppose that $g'(x^*) = 0$ for some $x^* \in X$, x^* is the global minimum of $g(x)$ over X.

Thus, the global optimal ν^* can be given as follows.

Proposition 2. (Optimal channel power gain threshold) $-\mathrm{E}(R_2)$ is pseudoconvex function of ν, and the global maximum of $\mathrm{E}(R_2)$ in (21) can be derived as

$$\nu^* = \left[W\left(\exp\left(\frac{P_0\gamma_0}{\sigma_n^2}\right)\right) - 1\right]\frac{\sigma_n^2}{P_0}, \tag{23}$$

where $W(\cdot)$ is Lambert-W function.

Proof. Refer to Appendix B. □

C. ARQ-aided CoMAC scheme

Based on the CDF of the number of the time slots for the node k, the CDF of the number of the time slots for the network can be derived as

$$\Pr\{D < T\} = \left[\Pr\{D_k < T\}\right]^K = \left(1 - \rho^{T-1}\right)^K, \tag{24}$$

and the average number of the time slots of the network is

$$\mathrm{E}(D) = \sum_{T=1}^{+\infty} \Pr\{D \geq T\} = \sum_{T=0}^{+\infty}\left[1 - \left(1 - \rho^T\right)^K\right]. \tag{25}$$

Then, the average achievable function rate of ARQ-aided CoMAC scheme in homogeneous networks can be given by

$$\begin{aligned}\mathrm{E}(R_3) &= \frac{1}{\mathrm{E}(D)} \frac{\log_2\left(P_0\eta/\sigma_z^2\right)}{[N + \log_2(K)]} \\ &= \frac{1}{\sum\limits_{T=0}^{+\infty}\left[1 - \left(1 - \rho^T\right)^K\right][N + \log_2(K)]}\log_2\left(\frac{P_0\eta}{\sigma_z^2}\right),\end{aligned} \tag{26}$$

where ρ is given by (16).

Remark 2. (Rate-delay tradeoff) According to (26), the achievable function rate with the uniform level η increases with η. However, the CDF of the node's channel power gain ρ in (16) is also an increasing function of η. And a large η will cause a large average transmission delay of the CoMAC scheme in (25). Thus, there is a tradeoff between the achievable function rate and the transmission delay, which can be achieved by the pre-defined uniform level.

Using (26), we can optimize η to maximize the average achievable function rate of ARQ-aided CoMAC scheme in homogeneous networks. In order to avoid exhaustive search, we investigate the asymptotic performance by resorting to the extreme value theory.

Lemma 3. [41, Theorem 8.3.3 & 8.3.4] (Sufficient conditions for convergence in distribution) Let $X_{\max} = \max_k X_k$, where X_k is a set of i.i.d. random variables with CDF F_X and PDF f_X. If F_X be an absolutely continuous CDF and let h_X be the hazard function, i.e., $h_X = f_X/[1 - F_X]$. If

$$\lim_{x \to F_X^{-1}(1)} \frac{d}{dx}\left\{\frac{1}{h(x)}\right\} = 0, \tag{27}$$

there exists constants a_K and b_K such that

$$\lim_{K \to \infty} \frac{X_{\max} - a_K}{b_K} \xrightarrow{d} \text{Gumbel distribution}, \tag{28}$$

where $\xrightarrow{d}$ means converging in distribution. a_K and b_K can be given by

$$a_K = F_X^{-1}\left(1-\frac{1}{K}\right), \tag{29}$$

and

$$b_K = [h_X(a_K)]^{-1}, \tag{30}$$

respectively.

Proposition 3. (Asymptotic performance of ARQ-aided CoMAC scheme in homogeneous networks) For ARQ-aided CoMAC scheme in large-scale homogeneous networks, the CDF of D_k lies in the domain of attraction of the Gumbel distribution for maxima. That is

$$\frac{\max\limits_k D_k + \log_\rho K - 1}{-\log_\rho(e)} \xrightarrow{d} \text{Gumbel distribution}. \tag{31}$$

The average number of the time slots of the network can be approximated as

$$\mathrm{E}(D) \xrightarrow{d} 1 - \log_\rho K - \xi \log_\rho e, \tag{32}$$

and the average achievable function rate in (26) can be approximated as

$$\mathrm{E}(R_3) \xrightarrow{d} \frac{\exp(-\eta/\gamma_0)}{(N+\log_2 K)(\ln K + \xi)} \log_2\left(\frac{\eta P_0}{\sigma_z^2}\right), \tag{33}$$

where $\xi \approx 0.5772$ is Euler-Mascheroni constant.

Proof. Refer to Appendix C. □

Thus, the asymptotically optimal η^* can be given as follows.

Proposition 4. (Asymptotically optimal uniform level) $-\mathrm{E}(R_3)$ is pseudoconvex function of η, and the global maximum of $\mathrm{E}(R_3)$ in (33) can be derived by letting its first order derivative equal to 0, and we have

$$\eta^* = \frac{\sigma_z^2}{P_0} \exp\left[W\left(\frac{\gamma_0 P_0}{\sigma_z^2}\right)\right], \tag{34}$$

where $W(\cdot)$ is Lambert-W function.

Proof. Refer to Appendix D. □

V. Performance in Heterogeneous Networks

In this section, we consider heterogeneous networks, where all nodes experience independent and not-identically distributed (i.ni.d.) Rayleigh fading with different channel statistics due to the path-loss differences. By adopting "point mass approximation", the heterogeneous case can be approximated by multiple homogeneous cases.

As illustrated in Fig. 3, the nodes are uniformly distributed in a circular cell with radius $r_0 < r \le r_J$. Assuming the nodes are uniformly distribution in the circular cell. The CDF of distance of a node can be given by

$$F_r(r) = \frac{r^2 - {r_0}^2}{{r_J}^2 - {r_0}^2}, r \in (r_0, r_J], \tag{35}$$

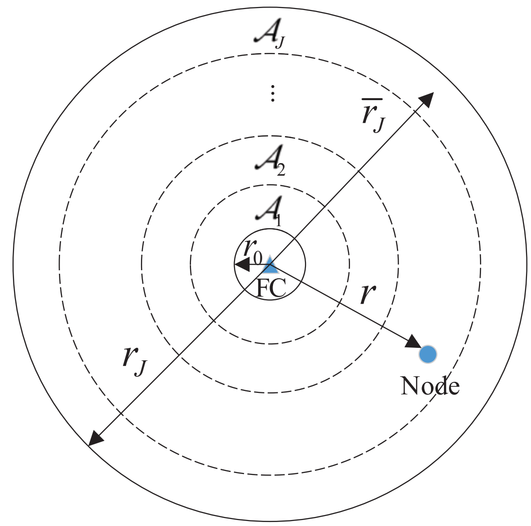


Figure 3. Point mass approximation for heterogeneous networks.

and the PDF of distance between the node k and the FC can be given by

$$f_r(r) = \frac{2r}{{r_J}^2 - {r_0}^2}, r \in (r_0, r_J]. \tag{36}$$

Although a simple geographical model is adopted, an exact analytical expression of the achievable function rate is complicated. Thus, we adopt the "point mass approximation" to simplify the distribution of the node location.

The circular cell is divided into J sub-areas, i.e., $\{\mathcal{A}_1, \mathcal{A}_2, \cdots, \mathcal{A}_J\}$, where sub-area $\mathcal{A}_j$ is bounded by radius $(r_{j-1}, r_j]$. Then, the nodes in the cell are also divided into J sub-groups. Thus, we have the following lemma to approximate the distribution of the nodes' location.

Lemma 4. [42] (Point mass approximation) Based on point mass approximation, the nodes in the sub-area $\mathcal{A}_j$ are assumed to have the same average distance from the FC, which is given by

$$\bar{r}_j = \frac{2\left(r_j^3 - r_{j-1}^3\right)}{3\left(r_j^2 - r_{j-1}^2\right)}, \tag{37}$$

and the probability of the nodes in sub-area $\mathcal{A}_j$ can be derived as

$$p_j = \frac{2j-1}{J^2}. \tag{38}$$

Assuming that the node k is located in sub-area $\mathcal{A}_j$, the CDF of its channel power gain can be given by

$$\rho = \Pr\left\{|h_k|^2 < x\right\} = 1 - \exp\left(-\sum_{j=1}^{J} \frac{p_j x}{\Phi_0 \bar{r}_j^\alpha}\right). \tag{39}$$

A. Traditional CoMAC Scheme

According to (17), the average achievable function rate of traditional CoMAC scheme in heterogeneous networks can be approximated by

$$\mathrm{E}(R_1) = \frac{1}{\ln 2\left(N+\log_2 K\right)} \mathrm{E}_1\left(\sum_{j=1}^{J} \frac{p_j \sigma_z^2 K}{P_0 \Phi_0 \bar{r}_j{}^{\alpha}}\right), \quad (40)$$

where Φ_0 is the path-loss constant of a unit distance, α is the path-loss exponent, $\bar{r}_j$ and p_j and are given by (37) and (38), respectively.

B. ARQ-aided communication scheme

According to (21), the average achievable function rate of ARQ-aided communication scheme in heterogeneous networks can be given by

$$\mathrm{E}(R_2) = \frac{1}{NK} \exp\left(-\sum_{j=1}^{J} \frac{p_j \nu}{\Phi_0 \bar{r}_j{}^{\alpha}}\right) \log_2\left(1+\frac{P_0 \nu}{\sigma_n^2}\right), \quad (41)$$

where Φ_0 is the path-loss constant of a unit distance, α is the path-loss exponent, $\bar{r}_j$ and p_j and are given by (37) and (38), respectively.

Further, the optimal ν^* can be derived as follows.

Proposition 5. (Optimal channel power gain threshold) $-\mathrm{E}(R_2)$ is pseudoconvex function of ν, and the global maximum of $\mathrm{E}(R_2)$ in (41) can be calculated as

$$\nu^* = \left[W_0\left(\exp\left(\sum_{j=1}^{J} \frac{P_0 \Phi_0 \bar{r}_j{}^{\alpha}}{\sigma_n^2 p_j}\right)\right) - 1\right] \frac{\sigma_n^2}{P_0}, \quad (42)$$

Proof. The proof is similar to that of Proposition 2. □

C. ARQ-aided CoMAC scheme

According to (26), the average achievable function rate of ARQ-aided CoMAC scheme in heterogeneous networks can be also derived as

$$\mathrm{E}(R_3) = \frac{1}{\sum\limits_{T=0}^{+\infty}\left[1-\left(1-\rho^T\right)^K\right]\left[N+\log_2(K)\right]} \log_2{}^{+}\left(\frac{P_0 \eta}{\sigma_z^2}\right), \quad (43)$$

where ρ is given by (39).

Further, we optimize η to maximize the average achievable function rate in heterogeneous networks in (43). In order to avoid exhaustive search, we also investigate the asymptotic performance by resorting to extreme value theory. Based on Lemma 3, an approximated achievable function can be provided, and the asymptotically optimal η can be derived as follows.

Proposition 6. (Asymptotic performance of ARQ-aided CoMAC in heterogeneous networks) For ARQ-aided CoMAC scheme in heterogeneous networks, the CDF of D_k lies in the domain of attraction of the Gumbel distribution for maxima. That is

$$\frac{\max\limits_{k} D_k + \log_\rho K - 1}{-\log_\rho(e)} \xrightarrow{d} \text{Gumbel distribution}. \quad (44)$$

Table I
SIMULATION PARAMETERS

Parameter	Value
Number of nodes	$K = 10 \sim 100$
Length of message vector	$N = 10$
Average channel power gain	$\bar{\mu} = 0$ dB
Signal power to noise ratio	$P_0/\sigma_z^2 = 0 \sim 15$ dB
Pre-defined uniform level	$\eta = -5 \sim 15$ dB
Scale of the network	$r_c = 3$ m
Number of sub-areas	$J = 10$
Path loss constant	$\Phi = 0.023568$
Path loss exponent	$\alpha = 3$

The average number of the time slots for the network can be approximated as

$$\mathrm{E}(D) \xrightarrow{d} 1 - \log_\rho K - \xi \log_\rho e, \quad (45)$$

and the average achievable function rate can be approximated as

$$\mathrm{E}(R_3) \xrightarrow{d} \frac{\log_2\left(\eta P_0/\sigma_z^2\right)}{\left(N+\log_2 K\right)\left(\ln K+\xi\right)} \exp\left(-\sum_{j=1}^{J} \frac{p_j \eta}{\Phi_0 \bar{r}_j^{\alpha}}\right), \quad (46)$$

Proof. The proof is similar to that of Proposition 3. □

Proposition 7. $-\mathrm{E}(R_3)$ is pseudoconvex function of η, And the global maximum of $\mathrm{E}(R_3)$ in (46) can be derived by letting its first order derivative equal to 0, and we have

$$\eta^* = \frac{\sigma_z^2}{P_0} \exp\left[W\left(\sum_{j=1}^{J} \frac{p_j P_0}{\Phi_0 \bar{r}_j^{\alpha} \sigma_z^2}\right)\right]. \quad (47)$$

Proof. The proof is similar to that of Proposition 4. □

VI. SIMULATION RESULTS AND DISCUSSION

In this section, we adopt simulation to illustrate the performance of the proposed ARQ-aided CoMAC scheme. Both homogeneous networks and heterogeneous networks are discussed. The parameter settings are listed in Table I unless specified otherwise. All simulation results are obtained using 10^3 Monte-Carlo simulations.

A. Homogeneous Networks

The achievable function rate versus different ARQ level (η or ν) is shown in Fig. 4. Both ARQ-aided CoMAC scheme and ARQ-aided communication scheme are simulated with different numbers of nodes. First, the simulated results based on the Monte-Carlo simulations are almost identical with the theoretical ones, which verifies their correctness. Some deviation for ARQ-aided CoMAC scheme is due to its asymptotic derivation based on the extreme value theory. And the performance of ARQ-aided CoMAC scheme is better than that of ARQ-aided communication scheme, which verifies the superiority of the proposed ARQ scheme. When the number of the nodes increases, the achievable function rate of both schemes decreases. Then, by varying the ARQ level, the Fig.

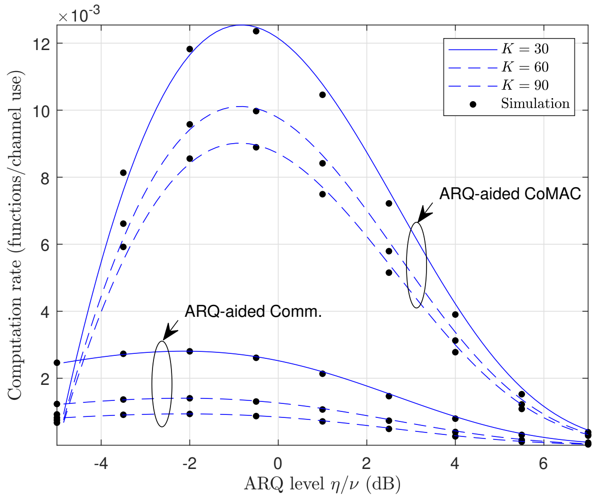


Figure 4. The achievable function rate of different schemes versus different ARQ level in homogeneous networks with different number of nodes.

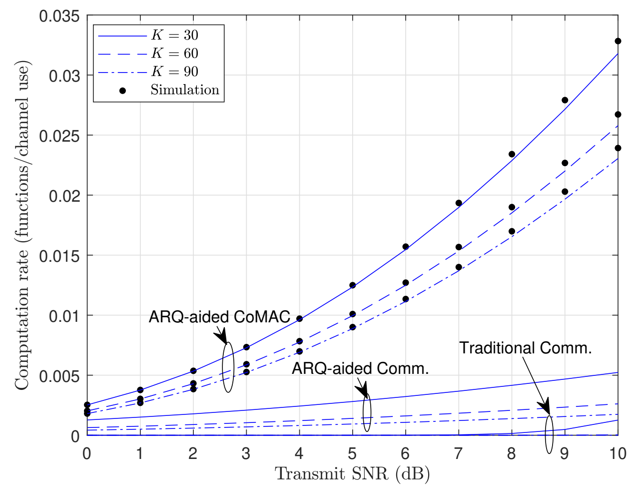


Figure 5. The optimal achievable function rate of different schemes versus different transmit SNR in homogeneous networks with different number of nodes.

4 reveals a tradeoff between the achievable function rate and the transmission delay. And it is shown that the achievable function rate of the ARQ-aided CoMAC scheme is maximized when the ARQ level $\eta \approx -1$ dB, and the achievable function rate of the ARQ-aided communication scheme is maximized when the ARQ level $\nu \approx -2$ dB. It achieves the optimal tradeoff between achievable function rate and transmission delay.

In homogeneous networks, the optimal achievable function rate of the ARQ-aided CoMAC scheme is compared to that of the ARQ-aided communication scheme and the traditional communication scheme. The simulated results found by exhaustive search are almost identical with the theoretical ones, which verifies the effectiveness of the asymptotically optimal ARQ level derived as the closed-form expression. The performance of the ARQ-aided CoMAC scheme is much better than that of the ARQ-aided communication scheme and the traditional communication scheme. When the number of the nodes increases, the achievable function rates of all schemes decrease. When the transmit signal power to noise ratio (SNR) increases, the achievable function rates of all schemes increase. And the increasing rate of the proposed ARQ-aided CoMAC scheme is larger than that of the other two schemes.

In order to illustrate the accuracy of the asymptotic performance and the asymptotically optimal ARQ level, the optimal achievable function rate of the ARQ-aided CoMAC scheme versus different number of nodes are provided in homogeneous networks with different transmit SNR in Fig. 6. The simulated results based on the Monte-Carlo simulations are almost identical with the theoretical ones based on extreme value theory. Thus, the approximation based on extreme value theory is accurate even when the number of the nodes is small. And the derivation between the simulated results and the theoretical ones decreases with the increase of the number of nodes.

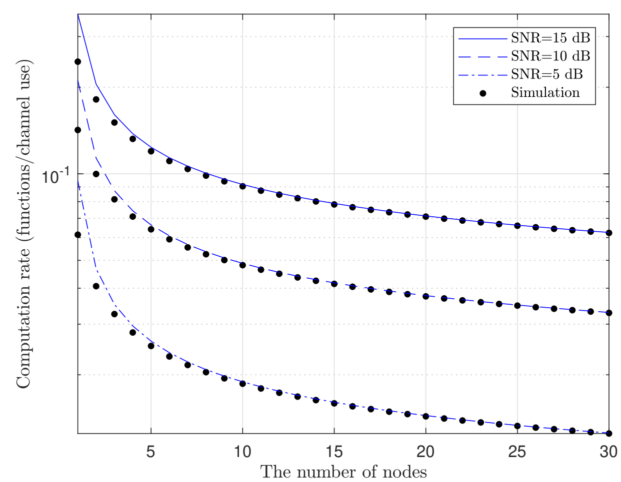


Figure 6. The optimal achievable function rate of the ARQ-aided CoMAC scheme versus different number of nodes in homogeneous networks with different transmit SNR.

B. Heterogeneous Networks

The accuracy of "point mass approximation" is evaluated in Fig. 7, where the optimal achievable function rate is illustrated for different numbers of approximation sub-areas. A larger number of sub-areas yields a more accurate approximation. When the number of approximation sub-areas is larger than 10, the performance of "point mass approximation" is very close to that of simulation results based on Monte-Carlo simulations.

The achievable function rate of the ARQ-aided CoMAC scheme and the ARQ-aided communication scheme versus different ARQ level is illustrated in Fig. 8 for heterogeneous networks with different number of nodes. The simulated results are also almost identical with the theoretical ones, which verifies their correctness. In the simulation results, the achievable function rate of the proposed scheme is maximized when the ARQ level $\eta \approx 0$ dB for the ARQ-aided CoMAC

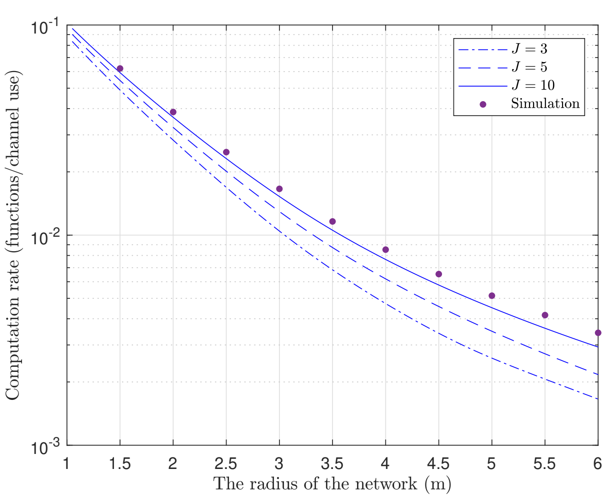


Figure 7. Performance of "point mass approximation" with different numbers of sub-areas for heterogeneous networks.

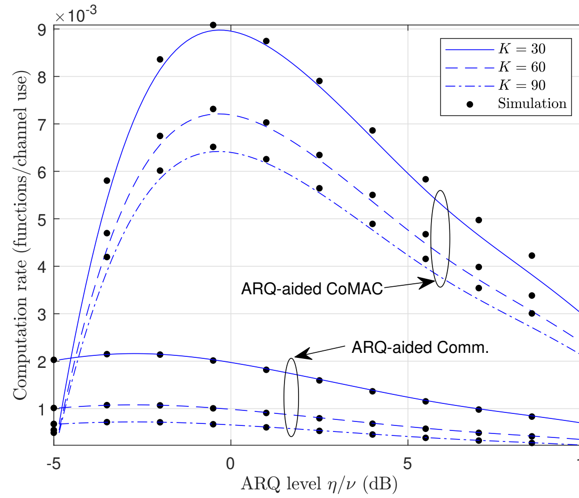


Figure 8. The achievable function rate of different schemes versus different ARQ level in heterogeneous networks with different number of nodes.

scheme and $\nu \approx -1$ dB for the ARQ-aided communication scheme. Also, the performance of the ARQ-aided CoMAC scheme is better than that of the ARQ-aided communication scheme, and the achievable function rate decreases with the increase of the number of nodes. The optimal ARQ level is almost independent of the number of nodes.

Furthermore, the optimal achievable function rate of the ARQ-aided CoMAC scheme is compared to that of the ARQ-aided communication scheme and the traditional communication scheme in Fig. 9. The performance of the proposed ARQ-aided CoMAC scheme is greatly improved compared to that of the ARQ-aided communication scheme and the traditional communication scheme. Also, when the number of the nodes increases, the achievable function rates of all schemes decrease. When the transmit SNR increases, the achievable function rates of all schemes increase, and the increasing rate of the proposed ARQ-aided CoMAC scheme is larger than that of the other two schemes.

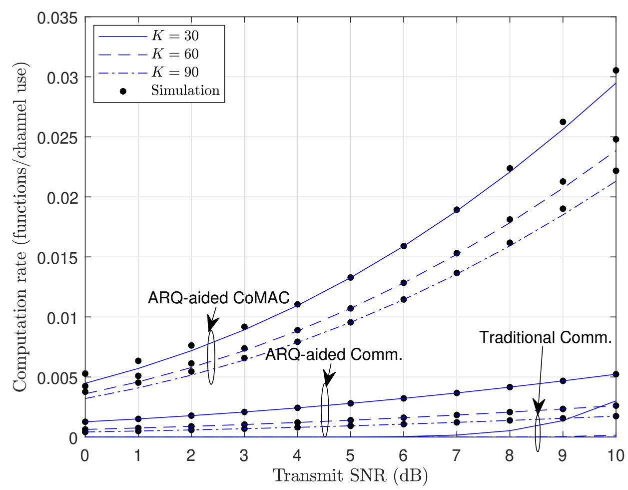


Figure 9. The optimal achievable function rate of different schemes versus different transmit power to noise ratio in heterogeneous networks with different number of nodes.

VII. Conclusion

In this work, we have proposed an ARQ-aided CoMAC scheme to avoid massive CSI aggregation and improve the limited performance. Both the transmitter design and the signaling procedure have been provided. We have derived the performance of the achievable function rate for the proposed ARQ-aided CoMAC scheme and the traditional ARQ-aided communication scheme in both homogeneous networks and heterogeneous networks. Then the achievable function has been maximized by optimizing the ARQ level to achieve the optimal tradeoff between rate and delay Asymptotic closed-form results have been given by means of extreme value theory and point mass approximation. This work has provided a tradeoff between achievable function rate and transmission delay for the CoMAC scheme and achieved an improved performance without massive CSI aggregation.

Appendix A
Proof of Proposition 1

The proof is similar to that of [25, Theorem 5], where the additional term $N + \log_2 K$ in the denominator of (6) compared to (5) is the penalty for avoiding wraparounds of modulo sum. Specifically, the achievable function rate in [25, Theorem 5] was given by

$$\hat{R}_C = \frac{1}{2}\frac{\log_2{}^+\left(P_0/\sigma_z^2\right)}{N + \log_2 K}. \tag{48}$$

where the received signal of the FC is assumed to be

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{x}_k + \mathbf{z}, \tag{49}$$

where $\mathbf{x}_k \in \mathcal{R}^M$, and each element of $\mathbf{z}$ in (49) distributes as $\mathcal{N}\left(0, \sigma_n^2\right)$.

While, there are two differences in our work. First, considering the non-uniform fading of MAC and the designed transceiver, the received signal of the FC is given by (3) in this work, and the additional term $\sum_{k=1}^{K}\left|ah_kb_k-1\right|^2P_0$ of (6) is the penalty for the distortion caused by the non-uniform fading. Then, the achievable function rate was given for a real-valued network in [25]. We consider a complex-valued network in this work, where $\mathbf{x}_k\in\mathcal{C}^M$, and each element of $\mathbf{z}$ in (3) distributes as $\mathcal{CN}\left(0,\sigma_n^2\right)$. Thus, the achievable function rate in this work is twice the expression of [25] according to Lemma 1. Above all, it completes the proof.

Appendix B
Proof of Proposition 2

According to (21),

$$-\operatorname{E}(R_2)=-\frac{1}{NK}\exp\left(-\frac{\eta}{\gamma_0}\right)\log_2\left(1+\frac{P_0\eta}{\sigma_n^2}\right). \quad (50)$$

where $-\log_2\left(1+\mathrm{P}_0\eta/\sigma_\mathrm{n}^2\right)$ is nonpositive and convex and $\exp\left(\eta/\gamma_0\right)$ is positive and convex. Thus, $-\operatorname{E}(R_2)$ is pseudo-convex function of η. Calculating $\partial\mathrm{E}(\mathrm{R}_2)/\partial\eta=0$, one has

$$\eta^*=\left[W\left(\exp\left(\frac{P_0\gamma_0}{\sigma_n^2}\right)\right)-1\right]\frac{\sigma_n^2}{P_0}, \quad (51)$$

which completes the proof.

Appendix C
Proof of Proposition 3

The CDF and PDF of D_k can be calculated as

$$F_{D_k}(T)=\Pr\left\{D_k<T\right\}=1-\rho^{T-1}, \quad (52)$$

and

$$f_{D_k}(T)=-\ln(\rho)\,\rho^{T-1}, \quad (53)$$

where ρ is given in (16). And the corresponding hazard function is

$$h_{D_k}(T)=\frac{-\ln(\rho)\,\rho^{T-1}}{\rho^{T-1}}=-\ln(\rho)\,, \quad (54)$$

which is a constant and satisfies (27). Further,

$$a_K={F_{D_k}}^{-1}\left(1-\frac{1}{K}\right)=1-\log_\rho K \quad (55)$$

and

$$b_K=\left[h_{D_k}(a_K)\right]^{-1}=\left[Kf_{D_k}(a_K)\right]^{-1}=-\log_\rho e,. \quad (56)$$

Thus,

$$\frac{\max\limits_k D_k+\log_\rho K-1}{-\log_\rho(e)}\xrightarrow{d}\text{Gumbel distribution}, \quad (57)$$

whose mean is $a_K+\xi b_K$. And the average number of the time slots of the network can be approximated as

$$\mathrm{E}\left[D\right]\xrightarrow{d}1-\log_\rho K-\xi\log_\rho e. \quad (58)$$

According to (26), the average achievable function rate can be approximated as

$$\begin{aligned}\mathrm{E}(R_3)&\xrightarrow{d}\frac{-\ln\rho}{\left(N+\log_2 K\right)\left(\ln K+\xi-\ln\rho\right)}\log_2\left(\frac{\eta P_0}{\sigma_z^2}\right)\\&=\frac{-\ln\left[1-\exp\left(-\eta/\gamma_0\right)\right]}{\left(N+\log_2 K\right)\left(\ln K+\xi-\ln\rho\right)}\log_2\left(\frac{\eta P_0}{\sigma_z^2}\right)\\&\overset{(a)}{=}\frac{\exp\left(-\eta/\gamma_0\right)\log_2\left(\eta P_0/\sigma_z^2\right)}{\left(N+\log_2 K\right)\left(\ln K+\xi\right)}+o\left[\exp\left(-\frac{\eta}{\gamma_0}\right)\right],\end{aligned} \quad (59)$$

where the procedure (a) is due to the Taylor expansion of $\ln(1-x)$ and ignores the higher order terms for small x. It completes the proof.

Appendix D
Proof of Proposition 4

According to (33),

$$-\mathrm{E}(R_3)=\frac{1}{\left(N+\log_2 K\right)\left(\ln K+\xi\right)}\frac{-\log_2\left(\eta P_0/\sigma_z^2\right)}{\exp\left(\eta/\gamma_0\right)}, \quad (60)$$

where $-\log_2\left(\eta P_0/\sigma_z^2\right)$ is nonpositive and convex and $\exp\left(\eta/\gamma_0\right)$ is positive and convex. According to Lemma 2, $-\mathrm{E}(R_3)$ is pseudoconvex function of η. Calculating $\partial\mathrm{E}\left(\mathrm{R}_3\right)/\partial\eta=0$, one has

$$\eta\ln\left(\frac{\eta P_0}{\sigma_z^2}\right)=\gamma_0. \quad (61)$$

Assuming $\ln\left(\eta P_0/\sigma_z^2\right)=x$, (61) can be rewritten as

$$e^x x=\frac{\gamma_0 P_0}{\sigma_z^2}, \quad (62)$$

where $x=W\left(\gamma_0P_0/\sigma_z^2\right)$. Then η^* can be calculated as (34) and $\eta^*\geq\sigma_\mathrm{z}^2/P_0$, which completes the proof.

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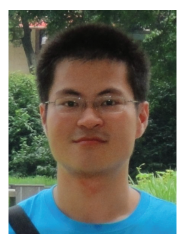

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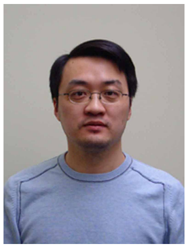

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