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Superconductor and Lenz's law

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Abstract: A superconductor has the unique properties of zero resistance and Meissner effect. Lenz's law is a fundamental law of physics. People have occasionally brought up the question that if a superconductor abides by Lenz's law. There has been lack of an explicit answer to this question so far. Recently, we carried out experiments with superconductor coils and a magnet in search of an answer to this question. We find out that the interacting behavior between a superconducting coil and a magnet does not comply with one of the primary interpretations of Lenz's law: the current induced in a circuit due to a change or a motion in a magnetic field is so directed as to exert a mechanical force opposing the motion. Our experimental results show that the induced current in the superconducting coil do not always oppose the motion of magnet during their interaction. Instead, in a certain portion of the interaction the induced current aids the motion of the magnet. This finding may require the aforementioned interpretation of Lenz's law to be revised as superconductors are involved.

1. INTRODUCTION

Lenz's law, named after the Russian physicist Heinrich Friedrich Emil Lenz who deduced it in 1834 [1,2], a fundamental law of physics, states that the direction of current induced in a conductor by a changing magnetic field due to induction is such that it creates a magnetic field that opposes the change that produced it. Since its establishment, there have been a number of interpretations bonded to Lenz's law, involving not only electromagnetism, but also mechanical motion, conservation of energy, etc [3-6]. A superconductor has zero electrical resistance and perfect diamagnetism (Meissner effect) [7,8], fundamentally different from conventional conductors such as copper and aluminum. Questions on the inconsistency between Meissner effect and Lenz's law or whether superconductors abide by Lenz's law have been sometimes brought up [9-13], but there has been no a well agreed explicit answer yet. Studies on the interaction behavior between a magnet and a superconducting ring have been mainly focused on the properties of journal bearing constructed with a magnet and a high temperature superconductor bulk or ring [14-16], form which outcomes are never able to answer the above question.

In 2006, J. E. Hirsch published a paper titled "Do superconductors violate Lenz's law?" [13]. In his paper, he expounded the gyromagnetic effect of a superconducting body and postulated a theory of hole superconductivity to explain the behavior of superconductors under a magnetic field to save the validity of Lenz's law for superconductors. However, a well agreed conclusion with a solid experimental

verification on the relationship between superconductor and Lenz's law has been absent in literature so far. To investigate this question and try to find more direct evidence able to answer it, we designed and carried out experiments based on the principle described in references [3,5,18,19].

2. APPARATUS AND PRINCIPLE OF EXPERIMENT

The apparatus built for this study is demonstrated in Fig. 1. It mainly consists of a bar NdFeB magnet attached to a dynamometer through a thin aluminum rod, a displacement gauge, a sample holding platform, and a lifting structure manipulated by a controllable drive motor. The dimensions of the NdFeB magnet are diameter = 20 mm and height = 20 mm. The maximum surface magnetic flux density is 0.35 T. The dynamometer used in this work is DS2-5N digital dynamometer with precision of 0.001 N (product of Dongguan City Intelligent Precision Instrument Co., Ltd). FLUKE 319 Clamp Meter with precision of 1.5% is used to measure the current in the superconductor coil. This system is



Figure 1 Experiment apparatus.

programmed to realize continuous and automatic control through Lab-VIEW. The measurement results can be recorded and displayed in real time on the screen graphically.

Fig. 2 demonstrates the principle of the experiment. When the bar magnet co-axially moves downward to approach a conducting ring (suppose being made of normal conductor), the amount of magnetic flux inside the ring will increase, inducing

an electrical current in the ring. If the pole configuration of the magnet is as shown in Fig. 2, the direction of the induced current in the ring is marked in the figure according to Lenz's law, clockwise looking toward the ring from the top. Under such circumstances, the magnet and the conducting ring will interact with each through Ampere force. other The characteristic of this interaction is analyzed in the paragraphs below.

The magnetic flux density B surrounding the conductor ring can be decomposed into two components, radial component B_1 and axial component B_2 .



Figure 2 Schematic of the interaction between a bar magnet and a conductor ring.

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Since the flux \emptyset and flux density **B** have an identical direction, B_1 points to the center of the ring and B_2 points to vertically downward before the geometrical center of the magnet reaching the geometrical center of the ring. After the center of the magnet passes the center of the ring, both the directions of B_1 and B_2 will reverse.

If the ring having a circumference of l, it is subjected to an axial force,

$$F_1 = I \times B_1 I$$

where I is the induced current in the conductor ring. Considering the direction of I, the direction F_1 should be vertically downward. Meanwhile, the magnet receives a counterforce with the same magnitude but opposite direction, i.e. vertically upward. The magnitude of this force is proportional to the values of I and B_1 . I and B_1 vary during this course, so the value of F_1 changes with time.

For a small section of the conductor ring, dl, it bears up a force generated by B_2 ,

$$dF_2 = I \times B_2 dl,$$

whose direction is toward the center of the ring. Taking the symmetry of the ring into account, the total force on the ring inspired by B_2 is zero as the result of counterbalance.

Therefore, the magnet is only subject to an upward resisting force when it approaches the conductor ring. As the magnet moves to the point that its geometrical center is overlap with the geometrical center of the ring, B_1 reduces to zero and the interacting force between the magnet and the ring fades away. It should be pointed out that with the decrease of magnetic flux changing rate as the magnet comes near the center of the ring, the induced current in the ring also diminishes due to the resistance of the ring, becoming zero while the two geometry centers overlapping.

When the magnet continues moving downwards after passing the center of the conductor ring, an induced current with opposite direction (as compared with when the magnet approaches the ring) appears. On the other hand, B_1 reverses its direction, becoming outward. These changes result in that the direction of the force acting on the magnet keeps unchanged, i.e. upwards.

3. TRIAL TESTS WITH AI RING AND Cu RING

We examined the apparatus and the principle of our experiment with an Al ring and a Cu ring. The Al ring and Cu ring have the same size, ID = 30 mm, OD = 105 mm, and height = 10 mm. Fig. 3 is the photo picture of these samples.

First, we placed the Al ring on the sample holding platform, started the magnet moving downward with a constant speed of 10 mm/s at the position about 80 mm from the geometric center of the ring. Measured and recorded the force acting on the magnet continually with a time interval of 180 ms until the magnet came to the point far



Figure 3 Photo picture of the Al ring and Cu ring samples.

enough below the ring where the measured value of the force reduced to zero. Then stopped the motor and finished the test. Then we replaced the Al ring with the Cu ring and repeated the same procedures described above.

Fig. 4 defines the coordination for result analysis. The geometric center of the superconductor coil is set to be the coordinate origin. Both the superconductor coil and the magnet are axial-symmetric and they share a mutual axis, shown as the dotted line. Their geometric centers determine their distance. The force on the magnet is taken as positive if it is upward, and negative on the opposite.

The results of these trial tests are displayed in Fig. 5. In the test with the the Al ring, when the magnet approaches the Al ring with the constant speed, arriving at $x \approx -45$, an upward force is detected. This force reaches



maximum of 0.055 N as the magnet gets to $x \approx -12$. Then this force decreases as the magnet continues advancing but its direction remains unchanged. At x = 0, this force reaches its minimum. It should be pointed out that the minimum value is zero theoretically. The non-zero value measured in this experiment is believed to come from the size effect of the magnet and the Al ring. After passing the origin, the magnet continues to face an upward force. In the figure, the curve portraying the force is almost symmetrical about the vertical line passing x = 0. The force reaches maximum at $x \approx -12$ and $x \approx 12$ respectively because at these positions, $I \times B_1$ gets to the maxima. The observed fact that the magnet is subject to an opposing force during the whole course of its movement in this experiment is in agreement with the interpretation of Lenz's law, "the current induced in a circuit due to a change or a motion in a magnetic field is so directed as to oppose the change in flux and to exert a mechanical force opposing the motion" [3]. We can conclude that this experiment is trustworthy for demonstrating Lenz's law.

In the test with the Cu ring, the pattern of the force acting on the magnet is basically the same as in the test with the Al ring except that the value of the force is larger. The maximum of the force is 0.101 N, about 1.84 times of that in the test with the Al ring. What the maxima of the forces appear at the same position ($x \approx \pm 12$) in both tests is consistent with the fact that the Al ring and the Cu ring have the same geometrical dimensions. The identical geometrical dimensions also result in



Figure 5 The forces on the magnet in the tests with Al ring and Cu ring.

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the emf's induced on Al ring and Cu ring being the same at the matching displacement positions in these tests. The force ratio of 1.84 is fairly close to the reciprocal of the ratio of their resistivity (the room temperature resistivity of copper is $1.75 \times 10^{-8} \Omega \cdot m$ and that of aluminum is $2.83 \times 10^{-8} \Omega \cdot m$). Hence, it can be concluded that the performance of the apparatus and testing procedures satisfy our experiment requirement.

4. TESTS WITH SUPERCONDUCTOR COILS

The superconductor coil specimens for this study are 30 turn double pan-cake HTS coils with ID of 60 mm, OD of 80 mm, and height of 10 mm. One is made of 4.2 mm wide, 0.23 mm thick (Bi,Pb)₂Sr₂Ca₂Cu₃O₁₀ (Bi-2223) tape and the other is made of 4 mm wide, 0.21 mm thick GdBa₂Cu₃O₇ (Gd-123) tape. The I_c (77 K, self field) of the Bi-2223 tape is about 110 A and the I_c of the Gd-123 tape is about 120 A. For each coil, the two ends of the superconductor tapes are jointed with Sn-Bi alloy solder, forming a closed circuit. The joint resistance of the coil is in the range of 10⁻⁷-10⁻⁶ Ω at 77 K. A

specimen coil is placed inside an epoxy resin dewar cooled by LN₂ when executing an experiment. Fig. 6 shows the pictures of the superconductor coils and the dewar.

The recorded

Figure 6 Pictures of B1-2223 coil (left), Gd-123 coil (middle), and the epoxy resin dewar (right).

force curve in the experiment with the Bi-2223 coils is plotted in Fig. 7. It shows that an interacting force appears as the

magnet moves towards the superconductor coil and reaches x \approx -65, indicting the superconductor coil starts to be subject to the magnet's magnetic field. The force reaches its maximum at $x \approx -15$, being about 4 N, which is almost 40 limes of the maximum force recorded in the test with the Cu ring, implying a fairly large induced current in the superconducting coil. It can be attributed to the zero resistance of the superconducting coil



Figure 7 Force on the magnet in the test with the Bi-2223 coil.

It is truly unanticipated that the force changes its direction from upwards to downwards after the magnet passes the origin. This is completely different from the results of the experiments with the Al and Cu significantly, rings. More this phenomenon violates the statement, "the current induced in a circuit due to a change or a motion in a magnetic field is so directed as to oppose the change in flux and to exert a mechanical force opposing the motion", widely a accepted interpretation of Lenz's law.





After the test with the Bi-2223 coil, we repeated the same test procedures with the Gd-123 coil. Fig. 8 shows the result. The Gd-123 coil interacts with the magnet with almost identical behavior to that of the Bi-2223 coil except that the force is slightly greater as compared with the result in Fig. 7.

5. DISCUSSIONS AND ADDITIONAL EXPERIMENTS

According to the above mentioned interpretation of Lenz's law, the magnet should experience an upward resisting force in the whole course of motion, which has been clearly demonstrated in the tests with Al ring and Cu ring. However, the fact observed in the experiments with superconductor coils is that the force on the magnet changes its direction from upwards to downwards after the magnet passes the origin. It means that after passing the origin, the magnet is subject to a repelling force instead of a resisting one.

The force curves in Figs. 7 and 8 are essentially symmetrical about the origin, (0,0), revealing that the forces are approximately equal but in opposite directions at any two matching displacement positions on the two sides of the origin. This also indicates that when the magnet at every matching displacement positions, the value and direction of the current in the superconducting coil is essentially the same. The opposite directions of the force are the result of the reverse of magnetic field direction above and below the origin.

Taking into the consideration of all the factors and results involved in the experiments done so far, it can be speculated that the direction of the current in the superconducting coil was unchanged in the entire experiment as soon as it is created. We then hypothesize that the HTS coils are possibly capable of carrying a persistent current at 77 K. To examine the reliability of the hypothesis and further investigate the characteristic of the induced current in the superconducting coil in such kind of experiment, we devised another test with the Bi-2223 coil. In this test, the magnet moved from the top with the same operating parameters as those in the previous tests, stopped at the origin. The current in the coil was measured and recorded from the start

to about 180 seconds after the magnet stopped. The recorded current curve is plotted

in Fig. 9. It shows that the current increases very fast when the magnet being pushed to approach the superconducting coil. In about 7 seconds, the magnet arrives at the origin and the induced current reaches its maximum of 567 A. Then the current starts to decrease with time. Three minutes after the magnet stops, the current reduces to 379 A, approximately 67% of the maximum value.

The result indicates that the current in the superconducting coil is not perfectly persistent but with



Figure 9 Current in the Bi-2223 coil in the test magnet stopped at the origin.

some attenuation. We suspect two possible factors responsible for the attenuation. The first is flux jumping from inside to outside of the coil and the second is the energy dispassion caused by the finite joint resistance on the coil.

The maximum surface magnetic flux density is about 350 mT for this NdFeB magnet. We are certain that at the very beginning phase of the interaction, the strength of the magnetic field around the superconducting coil is smaller than the lower critical field H_{c1} of HTS materials, which is anisotropy and not greater than a few tens of mT at 77 K [20-24]. Thus, the superconducting coil is in the Meissner state and there is no magnetic flux in the region inside the coil. Meanwhile, a sufficiently large current is generated on the coil. As the magnet continues to move downwards, the magnetic flux density in the vicinity of the superconducting coil will exceed its H_{c1} . Then the superconducting coil is at the mixed state and some magnetic fluxes squeeze into the inside region even if the resistance of the coil is still zero. With the further movement of the magnet and the increase of the flux intensity inside the coil, some fluxes may start to jump out the coil. At the moment that the magnet arrives at the origin, the geometrical centers of the magnet and the coil are overlapping, the flux density inside the coil attains the maximum. Afterwards, the jumping out of magnetic fluxes keeps happening, so the current is dropping.

At this point, we are not sure which of these two factors contributes more than the other to the current attenuation. More or less, these causes make the current in the superconducting coil less persistent. Now, we may call the current "quasi persistent" due to the attenuation is slow. In principle, the hypothesis of the existence of persistent current in the superconducting coil can still stand up, nevertheless.

Finally, we performed another test with the Bi-2223 coil. The procedures for this test was the same as the previous force measurement tests except that this time both the force on the magnet and the current in the superconducting coil were measured simultaneously. The recorded force curve is almost identical to the curve in Fig. 7. The recorded current curve is plotted in Fig. 10.

The current curve demonstrates that as soon as the interaction between the magnet and а superconducting coil starts, the induced current in the coil appears. The current increases as the magnet moves towards the geometrical center of the coil and reaches the maximum at the center. After the magnet departs from the center and continually moves downwards, the current in the coil attenuates but keep the same direction until the magnet's displacement is a little bit beyond



45 mm, at where the current curve intersects the x-axis.

In this test, some 570 A of the current exhausts in less than 5 seconds. In the previous test, the current reduction is approximately 1% in the same time interval and the current decreases from 567 A to 379 A 3 minutes after the magnet stops at the center of the Bi-2223 coil, suggesting a much slower attenuation rate. The considerable different outcomes between the two current tests imply that "flux jumping" and "joint resistance" are anything but the main cause of the current attenuation.

The current curve in Fig. 10 is essentially symmetrical about the vertical line of x = 0. Considering the fact that the magnitudes of the forces on the magnet are approximately equal at each matching displacement positions on the two sides of the origin (refer Figs. 7 and 8), we believe the predominating cause of the current reduction in such cases is the work done in driving the magnet in its course of moving downwards. On the other side, the small deviations of the current curve in symmetry about the line of x = 0 in Fig. 10 and the force curve in symmetry about the origin are caused by some minor factors, e.g. flux jumping and/or joint resistance of the coil.

Based on the characteristic of the current in the superconducting coil observed in the last experiment, we may make another hypothesis, i.e. even though the magnetic field switches its direction with respect to the coil after the magnet passes the origin, there is no reversed electromotive force (emf) established on the superconducting coil as the consequence of its zero resistance. Otherwise, the shape of the current curve in Fig. 10 and the shape of the force curves in Figs. 7 and 8 as well will be fundamentally different. It is equivalent to say that the quasi persistent current prevent the magnet and the superconducting coil from further electromagnetic induction or the quasi persistent current screens the coil from the magnet after the magnet passes the origin in these experiments. All the experimental results also tell us that the superconducting coil never quenches during the experiments, conforming that the magnetic field suffered by the coil is below the upper critical field H_{c2} of the HTS tapes all the time during the tests.

It may need to be mentioned that in Fig. 10 a negative current appears at the very

end of the test. We suppose that it is due to electromagnetic induction between the magnet and the superconducting coil after the quasi persistent current fides out, i.e. the screening effect ceases.

The unique properties of a superconductor, i.e. zero resistance and Meissner effect, determine the outcome of our experiment. Meissner effect initiates a current in the superconducting coil as the magnet's approaching. Then, the induced current reaches its maximum as the magnet arrives at the center of the coil. Afterwards, a quasi persistent current will maintain in the superconducting coil for a period of time. The quasi persistent current makes the interaction behavior between a magnet and a superconducting coil fundamentally different from the interaction behavior between a magnet and a normal conductor.

In our experiments, work is done when forcing the magnet into a superconducting coil against the magnetic effect of the induced current. The energy created from this work is converted into the form of electrical energy carried by the current in the coil. The electrical energy is largely utilized in pushing the magnet in the second halves of these experiments.

We have carried out more experimental investigations on the interaction behavior between a magnet and a superconducting coil, including the impact of field cooling, the influence of starting position of the magnet and the geometry of the magnet and superconductor coil. The results of those experiments will be reported in our future publications.

6. CONCLUSION

When Lenz's law was formulated in 1834, it was a qualitative law that specified the direction of induced current in electromagnetic inductions. A few explanations and interpretations have been added to the term since, forming a comprehensive law of physics. In our experiment, the direction of the initially induced current in the superconductor coil is in consistent with the original term of Lenz's law. Considering the overall results of our experiment, however, we can conclude that superconductors do not abide by Lenz's law (the expanded term).

As a matter of fact, after the forming of the quasi persistent current in the superconducting coil, the electromagnetic induction between a magnet and a superconducting coil halts, or we can say that the quasi persistent current screens the coil from the magnet. Therefore, the superconducting coil no longer instantaneously responds to the change of external magnetic field and Lenz's law becomes not applicable.

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