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The Life and Works of Luca Pacioli (1446/7–1517), Humanist Educator

Accounting has few heroes, but one that most acknowledge as worthy of that accolade is Luca Pacioli, the man who published the first printed exposition of double entry bookkeeping in 1494. This was the publication that led to the development of the accounting systems we use today. However, if we consider our literature on Pacioli, it is found to be not just confusing and contradictory, but undecided as to who he was and what he did, or did not do. This paper addresses this ambiguity and uncertainty by adopting a critical hermeneutic analysis of the literature in order to present a more authentic perception of the life and works of Luca Pacioli. Archival discoveries of the past 20 years are embraced with Pacioli's own writings, and those of others who have sought to identify the man and his ambition. The view it reveals shows Pacioli as far more than the peripatetic friar and journeyman teacher of mathematics described in our literature; and it fills a much larger gap than the contradictions and confusion implied in our knowledge of the man who many consider to be the 'father of accounting'.

Key words: Biography; Double entry bookkeeping; Father of accounting; Hermeneutics; Humanism; Pacioli.

> Our account of the flow of human action eventuating in historical change unavoidably needs to take into account the institutional and situational environment in which these actions take place. Part of the topography of a period of historical change is the ensemble of institutions that exist more or less stably in the period: property relations, political institutions, family structures, educational practices, religious and moral values. (*Stanford Encyclopedia of Philosophy*, 2016)

Accounting has few heroes, but one that most acknowledge as worthy of that accolade is Luca Pacioli, the man who published the first printed exposition of double entry bookkeeping in 1494. This was the publication that formed the basis for manuals on bookkeeping for at least 100 (Hatfield, 1924, p. 245) or 150 years (Littleton, 1933, p. 4), standardized the method, and led to the development of the accounting systems we use today.[1]

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[1] Like all other manuals and textbooks on bookkeeping and accounting, Pacioli's lacks the reality of a functioning business: they simplify the complex, explain the method, not its purpose or interpretation; and their tendency to periodize the activities of the firm into a single year loses

FIGURE 1

PACIOLI'S *MANICULA* AND PACIOLI'S PORTRAIT

Note: This portrait is from the previously mentioned painting attributed to Jacopo de'Barbari: *Ritratto di Luca Pacioli*, currently in the Museo Capodimonte, Napoli.

Contrary to the view in the literature initiated by Fabio Besta that Pacioli could not have authored his manual (1909, pp. 363–5), and in support of all the subsequent literature that queries (e.g., Yamey, 1994, p. 23), hesitates (e.g., Hernández-Esteve, 1994a, pp. 72–3), or refutes Besta's claims (e.g., Penndorf, 1933, pp. 63–8; Stevelinck, 1994, pp. 5–6), the pedagogy within the bookkeeping manual and its similarity with how he taught algebra makes it clear that Pacioli wrote it (Sangster, 2018, pp. 306–11). He also left a sign that may indicate that he did so without borrowing text from other writings, something he has been accused of overdoing, particularly in the second volume of *Summa Arithmetica* (1494) and in the geometry part of *De Divina Proportione* (1509). The capital letter ('L') at the start of his bookkeeping treatise presents an image of a friar pointing to a page in an open book, an image that resonates with his portrait in Naples, in which he is doing exactly that (Figure 1). The sparse use throughout the book of this image—it is only used five times in over 600 pages[2]—identifies it as a *manicula*, a sign indicating something important. In this case, that would either have been the content, or that

contact with the reality of time, giving a very misleading picture of business activity, complexity, speed, and practice (Pilla, 1974; Lane, 1977). However, they do convey the method to use, which is the point of their existence.

[2] Pacioli uses this *manicula* at the beginning of the main text (f.1r); at the beginning of *Distinctio* 8 (f.111v); the beginning of *Distinctio* 9 (f.150r); at the beginning of *Tractatus* 6 of *Distinctio* 9 (f.183v); and at the beginning of the bookkeeping treatise (f.198v) (McCarthy, 2013). Supporting this hypothesis, the mathematical historian Argante Ciocci believes that *Distinctio* 9 is Pacioli's original work (Ciocci, 2015).

he wrote it himself or, most likely in this instance—because he would have viewed all the sections in his book as being important—both.

Sangster (2018, p. 307) identified how Pacioli adopted the same approach when teaching bookkeeping as he had done in his teaching of algebra; and removed the mystery of double entry by explaining how it works *mathematically* in five generalizable statements that form the foundation of double entry.

1. All transactions involve two elements: an item exchanged and a form of settlement.
2. All forms of settlement can substitute for each other.
3. One element is debit and the other is credit.
4. The amount of the debit = the amount of the credit.
5. The entries in the money column are to be in one currency only.

It was an axiomatic[3] teaching approach, but Pacioli does not indicate that he is presenting a series of axioms of double entry. That terminology had not yet been invented. He had identified the generalizability of these statements, and he keeps telling his readers that he need say no more for anyone to be able to do double entry. He declared how to do it, and he expected them to do what he prescribed. If they did, they would correctly record a debit and a credit for every entry. As is also evident in how he teaches algebra in *Summa Arithmetica*, Pacioli had realized that learning does not require an explanation. It only requires a desire to learn and a means to do so. Over 300 years later, in 1818, Joseph Jacotot reached the same conclusion:[4]

> One could learn by oneself and without a master explicator when one wanted to, propelled by one's own desire or by the constraint of the situation … There is nothing to understand. Everything is in the book. One has only to recount it—the form of each sign, the adventures of each sentence, the lesson of each volume … soon you will be able to say everything (Rancière, 1987, pp. 12, 23–4).

It was for others to develop Pacioli's first theorization of algebra and double entry into axiom-based theory. The algebraists of the 16th and 17th centuries did so and developed his axiomatic approach to teaching algebra into axiom-based theory (Heeffer, 2012, p. 39). The accounting masters did not, preferring to continue with an exemplar-based approach that was augmented with rules in the 16th century, firstly by Manzoni (1540) and then by those that followed, up to the present day.

Many have investigated Pacioli's life, his works, and his motivation, including English biographies by Knight (1857), Taylor (1942), Jayawardene (1971, 1998),

[3] In an axiomatic system, 'the assumptions are of such a basic and intuitive nature that their truth can be accepted without qualms. Yet axioms must be strong enough, or true enough, that other basic statements can be proved from them.' (Klarreich and Moncrief, 2002)

[4] Jactot was a teacher of French literature at the University of Louvain in Belgium. He spoke no Flemish, and most of his students spoke no French. To teach them French, he required them to write in French what they had learnt from reading a recent bilingual edition of Fénelon's Télémaque (1699), comparing the French to the Flemish.

and Rankin (1992); a Spanish biography by Hernández Esteve (1994b); and Italian biographies by Baldi (1589), Boncompagni (1879), Vianello (1896), Ricci (1940), Masotti Biggiogero (1960), Ulivi (1994, 2009) and, most recently, Ciocci (2003, 2009, 2011, 2017) but, scholars tend to limit their sources to those in their own discipline, and none has succeeded in bringing everything together in one place.

This insularity is most evident in the accounting literature on Pacioli where, despite publication of scores of papers, there is a marked lack of use of wider sources combined with a narrow selection of secondary material that has resulted in this body of literature being both confusing and contradictory. This manifests itself in a variety of statements and conclusions that range from the relatively unimportant year of his birth and his name, to where he was educated as a boy, who taught him, where he obtained his degree, where he lived, what he taught, where he taught, at what level, and whether or not he wrote the treatise that gave rise to his recognition as the 'father of accounting'. In brief, if you read very much of this extensive literature, you will find that you are not sure what or who Luca Pacioli was, or what he did or did not do.

This paper addresses this ambiguity and uncertainty by applying critical hermeneutic analysis to the literature in order to present a more authentic perception of the life and works of Luca Pacioli with an emphasis on what is most important for an accounting audience. It identifies and brings together the influences and activities that culminated in his publication of his bookkeeping treatise. This analysis was informed by archival discoveries of the past 20 years, Pacioli's own writings, and those of others who have sought to identify the man and his ambition, along with contextual features of his time and place. In doing so, it complements Sangster's (2018) hermeneutic analysis-based interpretation of Pacioli's treatise that discounted previous interpretations and exposed, for the first time, Pacioli's theoretical interpretation of double entry. This paper extends and completes the background material on Pacioli's life and career contained in that paper. It reveals that Pacioli was far more than the peripatetic journeyman friar and teacher of mathematics our literature describes. In doing so, it fills a much larger gap than most have realized exists in our knowledge of the man who many consider to be the 'father of accounting'.

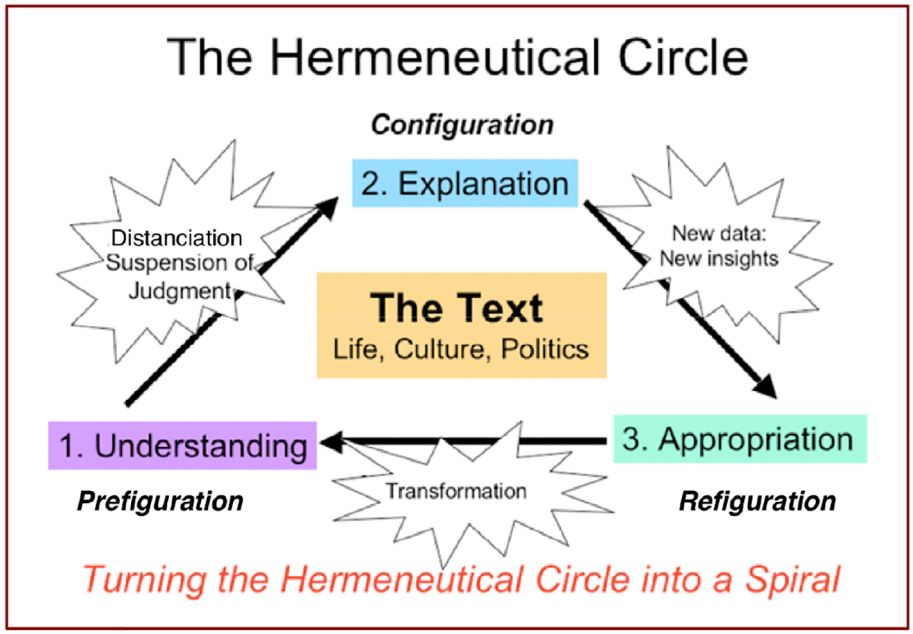
METHODOLOGY

As mentioned above, the methodology adopted in this paper is a critical interpretation of literature viewed through a hermeneutic frame. Context is crucial to that process:

> Any text has an envisioning historical and cultural context and that the context of a text is itself not simply textual—not something that can be played out solely and wholly in the textual domain. This context of the texts that concern us constrains and limits the viable interpretations that these texts are able to bear. The process of

FIGURE 2

THE HERMENEUTICAL CIRCLE



Source: Grassie (2008)

> deconstruction—of interpretatively dissolving any and every text into a plurality of supposedly merit-equivalent construction—can and should be offset by the process of reconstruction which calls for viewing texts within their larger contexts. After all, texts inevitably have a setting—historical, cultural, authorial—on which their actual meaning is critically dependent. (Rescher, 1997, p. 201)

As shown in Figure 2, hermeneutics, or the theory and practice of interpretation, involves a circular spiral of analysis whereby an artefact, in this case one from the late 15th century and what has been written about it and the individual who wrote it, is interpreted in the context of their time and place (i.e., the second half of the 15th century) as revealed through what is known of the contemporary environment: society, life, culture, education, religion, politics, and so on of that time. It is a qualitative approach that

> (a) seeks understanding rather than explanation; (b) acknowledges the situated location of interpretation; (c) recognizes the role of language and historicity in interpretation; (d) views inquiry as conversation; and (e) is comfortable with ambiguity (Kinsella, 2006, p. 1).

Through its adoption, 'hermeneutic analysis enables you to elicit an in-depth understanding of meanings of, for example: human practices, culture, works of art and texts' (Koppa, 2010).

These definitions are complementary. When combined they present an overall description of the hermeneutic process: this study did what Rescher describes, what Kinsella describes, and what Koppa describes. The deconstruction described by Rescher is what the hermeneutic circle of Grassie supports and enables; and the output of the process is as described by Kinsella.

The focus of this study is both Pacioli's own writings and the literature on Pacioli, which was critically analyzed in the context of Pacioli's time and place. This process embraced, not just the culture, society, education, political, religious, and economic knowledge we have of the man and his environment, but also material overlooked in Pacioli's own writing and archival discoveries made over the past 20 years. This process revealed inconsistencies and non-sequiturs across the broad spectrum of the English language literature, much of which is founded upon one major work, a biography of Pacioli, *No Royal Road*, by R. Emmett Taylor (1942). During this analysis, Taylor's book was revealed as substantially founded in speculation ill-informed by the history of the period in which Pacioli lived. That is, it lacks necessary awareness[5] of what was known of this period at the time when it was written. It is also a victim of the time of its creation, which predates many archival discoveries that further refute the validity of accepting assumptions and statements presented within it as facts.

This study finds that the constructed reality we see in the English language literature on Pacioli is far removed from its essence. In part, this is because these more recent archival discoveries have not been identified by those subsequently writing about Pacioli; but, also, because most have failed to appreciate the lack of foundation in Taylor's perception of Pacioli's social, cultural, educational, economic, religious, and political environment. This has resulted in Taylor's portrayal continuing to be taken as fact, leading to many misinterpretations and misunderstandings dominating the English language literature up to the present day. The view this literature presents is, at best, an understated characterization of the reality of the man it portrays, and quite distant from the facts, so far as they can be established 500 years after his death.

The rest of this paper focuses on presenting the findings of this study. At times, the image it portrays aligns with common perceptions but, in many instances, it reveals differences, ranging from the subtle to the extreme. Rather than dwelling on the minutiae of each error or misinterpretation in the extant literature, it focuses on presenting a more valid perspective, including some factual material that has been omitted or overlooked elsewhere. It begins by presenting a brief overview of Pacioli and his works.

BACKGROUND

> Luca Pacioli lived … at a time of great discord and dissension [in Italy] … when leading families sought to establish themselves firmly, and the struggles among the

[5] See the quote from the *Stanford Encyclopedia of Philosophy* at the beginning of this paper.

> families and the city-states were likely to arise at any time to upset tranquillity. The shifting alliances among leading families and duchies, and cities … were difficult to follow. The Popes … often took a firm stand in political as well as spiritual matters. The French … more than [once occupied the North], especially … Milan, and the Spanish … at times [occupied the South]. The Turks … had … control of many of the trade routes. (Taylor, 1956, p. 175)

Luca Pacioli was the son of Bartholomeo Pacioli and Maddalena di Francesco di Matteo Nutti (Ulivi, 2009, pp. 59–60). He was born in Sansepolcro, Tuscany between 23 October 1446 and 19 June 1447. These dates have never previously been combined in this way, their sources coming from two separate events, the first of which has been known for 40 years: Pacioli died at the age of 70, on 19 June 1517 (Nakanishi, 1979, pp. 54–5). The second was first discovered 11 years ago: he was 'older than 18 and younger than, 20' when he sold the family house in Sansepolcro on 20 and 22 October 1466 (Ulivi, 2009, pp. 28–9).

Pacioli was a Franciscan friar, author of at least ten books, schoolteacher, university teacher, mathematician, conjurer, calligrapher, linguist, advisor to dukes, generals, kings, and popes. He was a close friend and tutor of Leonardo da Vinci, tutor of Albrecht Dürer, a chess expert, a humanist educator, and a visionary. In terms of his influence or impact on future generations, it has been suggested that he had 'much more influence on human life than has Dante or Michelangelo' (Muller, 1952, p. 257); and 'had more effect on history than Marx's Communist Manifesto' (Strathern, 2001, p. 328).

In the context of the impact of his treatise on double entry bookkeeping, its publication came at a time when the spoken language had seldom appeared in print. The influence of its presentation of that language went far beyond accounting, into the heart of commerce itself:

> His work systematised accounting. It spread and was translated into the major languages throughout Europe and played a huge cultural role that goes beyond purely linguistic meanings … The fact that Pacioli stabilized the main accounting rules meant that the friar of Borgo San Sepolcro become the central point of the periodization of Italian economic language in its 'pre-scientific' period (Sosnowski, 2006, p. 29, translated by the author).

Pacioli was not simply the summarizer of other people's work, as widely assumed in the English language literature (e.g., Parshall, 2008, p. 99). He was an educator whose innovative pedagogy revolutionized the way his main subject, mathematics, was taught. This can be seen in his *Summa Arithmetica* (1494), in which his treatise on bookkeeping was published. *Summa Arithmetica* is identified as the most comprehensive and most elegant business school text of the Renaissance (Maccagni and Giusti, 1994, p. 18; Rowland, 1995, p. 701; Burton, 2011, p. 219). But this was only one aspect of the text and is an oversimplification on two counts. The book was not intended for teachers: it was intended to be used to self-teach

its readers without the aid of a teacher; and it also embraced the first of a two-part serialization of Euclid's *Elements* that was completed with the printing of *De Divina Proportione* (1509) (Ciocci, 2011a, p. 279).[6] These two texts set down an approach to instruction that led to the development of algebra into a theoretical subject in Europe in the 16th century. They radically transformed the role and image of mathematics (Ciocci, 2011a, p. 285) and influenced all branches and applications of the subject at that time, and for many years thereafter.

The Educator

Pacioli was first and foremost an educator, one inspired by a humanist-driven ability to grasp an opportunity to enhance learning and understanding for a large audience. In adopting the spoken language in his teaching, he adhered to a more 'democratic' view of the ideals of humanism as espoused, for example, by Leon Battista Alberti (1404–1472), than by some of his other fellow humanists, such as Marsilio Ficino (1433–1499), who viewed it as an essentially elitist phenomenon, which affected a relatively small group within the princely courts (Nauert, 2006, p. 72). Pacioli also recognized the benefits of facilitating self-instruction in his text beyond provision of the written word. In 1498, his manuscript of *De Divina Proportione* was accompanied by a set of wooden figures intended to reinforce learning. Prepared for Ludovico Sforza, Duke of Milan, Pacioli not only wrote the text and convinced Leonardo da Vinci to provide original drawings of 60 regular and irregular shapes that formed the appendix to the manuscript,[7] he also constructed them in wood. Each drawing and each figure was individually numbered and labelled and marginal notes were inserted in the text to indicate which one to consult. He made it clear in the text that he intended the book, the drawings, and the wooden figures to be used together (Pacioli, 1498, f.LXXXVr).

His inventiveness continued in the printed version of *De Divina Proportione* (1509), in which he presented a design of the alphabet that has been described as, 'a creation of pure artistic form' (Smedresman, 2006, p. 21). His section on architecture in the same volume was instrumental in the development of a new form of architecture thought (Pérez-Gómez, 2004); and, contrary to what many assume given how well he knew the leading exponent of the art, Leon Battista Alberti (Mancini, 1882), his approach to architecture was from Vitruvius, not Alberti (Daly Davis, 1977, p. 5). Neither of these sections were part of the original manuscript book. The penultimate part of the printed book contains a translation

[6] *Summa Arithmetica* covers the first ten of Euclid's *Elements*, which relate to practical mathematics relevant to merchants and craftsmen, on plane geometry, ratios, and proportions. *De Divina Proportione* covers the final three on more theoretical mathematics of spacial geometry (Ciocci, 2011a, p. 279)

[7] There are 60 diagrams, but only 59 were used in the book. The other was labelled, 'superfluous due to an error'. There were, however, 60 wooden figures. Pacioli also presented a copy of the book and the figures to his sponsor, Galeazzo Sanseverino in Milan; and made another set in Florence that he gave to Piero Soderini, *gonfalonier* for life (i.e., permanent ruler) of that city (Pacioli 1509, f.28v), for which Soderini paid him 52 lire 9 soldi in August 1502 (Ciocci, 2009, p. 137).

from Latin into the spoken language of the day of a treatise on geometry by the mathematician and artist, Piero della Francesca, *Libellus de quinque corporibus regularibus* (c. 1483).

Plagiarism

Pacioli did not mention that this was Piero's text. This has puzzled scholars virtually since it was published and has given rise to claims of plagiarism from the 16th century to the present day (Picutti, 1989, p. 72; Antinori, 1959, p. 402; Ciocci, 2017, pp. 90–92). However, if we set aside the hyperbole in the literature surrounding this act, there is sufficient evidence to conclude that Pacioli either wrote or helped Piero write his treatise (Ciocci, 2009, p. 96). A similar point is made by Cavazzoni (2010, p. 29) concerning another of Piero's texts, *Del abaco* (c.1470). In Cavazzoni's opinion, Pacioli may have been the author. He was well-equipped to have written the text, or to have helped Piero write it in 1471. He had just spent six years studying theoretical mathematics in Venice at the *Scuola di Rialto* and teaching practical mathematics (*abaco)*, the mathematics used by merchants, craftsmen, architects, engineers, and artists. It is very unlikely that Pacioli believed he was doing anything wrong when he included the geometry treatise in *De Divina Proportione*.

His Later Unpublished Work

Pacioli had intended to print two more books: *De Viribus Quantitus* [On the Powers of Numbers] (1496–1508), a book 'written to enliven the evenings at court and to sharpen the wits to solve mysterious problems and arithmetic and geometric puzzles' (Ciocci, 2009, p. 179); and *De Ludo Scachorum* [On the Game of Chess] (c. 1500), a book of chess puzzles intended to teach the latest form of the game. Originally, these books were to have been included in one volume, *'De ludis in genere cum illicitorum reprobatione spetialmente di quello de schachi in tuti modo'*, or *'Schifanoia'* [Escape from Boredom] (Pacioli 1496–1508, f.2r). Both were written for the princely courts and were meant to teach as well as entertain, enabling them also to benefit from knowledge of the secrets of mathematics, just as the mercantile class, artisans, and artists had from *Summa Arithmetica*, and learned society had from *De Divina Proportione.*

Pacioli and Art

Paciolo's reputation as a mathematician and, in particular, his material on proportion and perspective in *Summa Arithmetica* and *De Divina Proportione* had an incalculable impact on art, leading to his being sought-after by leading artists procuring insights into these mysteries. Most notably, he taught Albrecht Dürer (Galitzia, 2011) and instructed Leonardo da Vinci in powers and roots (Ulivi, 2009, p. 43) and 'in the foundation of all the arts and all knowledge: the theory of proportions' (Ciocci, 2010, p. 164). Leonardo reciprocated by gifting Pacioli the diagrams that illustrate his *De Divina Proportione*. Dürer may have done so by appearing beside Pacioli in the portrait of Pacioli currently to be found

in the Museo Capodimonte in Naples (Mackinnon, 1993).[8] This painting is attributed to the painter Jacopo di Barberi, another artist who sought Pacioli's assistance with the mysteries of perspective (Brion, 1964, p. 192). However, this attribution is currently under challenge with evidence apparently revealing Leonardo as the artist (Barca n.d.). For the time being, no definite conclusion can be drawn, either of the artist or of the second figure in the painting (Ciocci, 2011b, pp. 111–15).

His Status

Pacioli was also prominent in his order, head of his own Convent in Sansepolcro, and was invited in 1504 (Ulivi, 2009, p. 35), but declined, to be head of all Franciscan monasteries and convents in the province of Romagna. In complete contrast, he was consultant on military tactics to advisors to the King of Naples; and, as a mathematician and teacher, he was recognized at the highest level of society by being appointed astrologer (i.e., mathematician) to Pope Leo X (Taylor, 1942, p. 143). He achieved these heights of fame and prestige from very humble beginnings.

PACIOLI'S EARLY YEARS

Pacioli's father died when Pacioli was 11 or 12, in January 1459. Almost certainly an orphan, and with his two surviving brothers already Franciscan friars, he was taken into the care of the wealthy merchant and local politician, Folco di Giovanni di Canti Bofolci (Ulivi, 2009, pp. 29–30). His career indicates that he had an *abaco* schooling in practical mathematics, and was taught in the spoken language.[9] This is confirmed by his handwriting; but he also acquired an education in Latin, and Latin and *abaco* were not learned in the same place.

In Pacioli's day, there were two other forms of school education in Italy: the traditional scholastic grammar schools and the relatively recently established humanist grammar schools (Grendler, 1989, pp. 110, 133; Sangster *et al*., 2008, p. 117), both of which sought to teach the future leaders of society and those that aided them, for example, secretaries and lawyers (Grendler, 1989, p. 311). As summarized in Sangster et al. (2008), the scholastic grammar schools focused on learning from medieval texts, such as Aesop's fables (12th century) and poetic classics, such as those of Virgil (70–20BCE), from which they learnt Latin, grammar, and Christian morality; and they learnt the theory and practice of letter writing from Cicero's *De Inventione* (Grendler, pp. 111–15). Poetry dominated the curriculum and 'Classical Latin prose, such as in the letters,

[8] There is an opposing and, arguably, more justifiable view, that the second person in this portrait is Guidobaldo da Montefeltro, the Duke of Urbino, see, for example, Ciocci (2011a, pp. 161–2).

[9] For a detailed summary of the syllabus of the *abaco* schools, see Van Egmond's (1981) summary of c. 200 extant medieval *abaco* manuscripts (pp. 21–26).

orations, and philosophical treatises of Cicero, was conspicuous in its absence' (Grendler, p. 117).

In contrast, the humanist grammar schools focused on ancient writers of prose, of which the dominant source used was Cicero, most of whose letters and orations were rediscovered between the early 14th and the early 15th centuries. Students at the grammar schools studied grammar, rhetoric, poetry, history, moral philosophy, and ancient Greek and Latin. In the late 14th century, Greek began to enter the curriculum, though a shortage of humanists fluent in the language made it more the exception than the rule (Grendler, pp. 117–125) They rarely included mathematics in the curriculum, and the texts used were in Latin (Grendler, p. 50). Black (2001, p. 24) viewed the impact of the change in education brought about by the humanists as significant. He outlined that it stopped the creation of individuals who were able to think and argue for themselves to, instead, creating docile and potentially more malleable individuals better suited to the emerging absolutist regimes of the period. However, the fact remains that the focus of instruction shifted from medieval texts to classical texts from antiquity, both Roman and Greek.

Teachers were often untrained and moved to wherever they could find employment, either from a commune or from parents wishing their children to be educated (Grendler, 1989, pp. 36–7). The only school in Sansepolcro was a humanist grammar school funded by the commune (town council) (Banker, 2003, p. 59). It is where Pacioli learned basic Latin and, possibly, some Greek. Despite his clearly also having had an *abaco* education, there is no record of an *abaco* master having worked there in the 15th century (Banker, 2003, p. 88). His *abaco* instruction may have come from the town's *agrimensores* (surveyors) (Banker, p. 87) who were often also teachers of *abaco* (Ulivi, 2015, p. 661), which they taught privately in their spare time (Biagioli, 1989, p. 43), but this would have required payment. Otherwise, he may have learned *abaco* from his merchant guardian; but, his most likely *abaco* tutor was his kinsman, the mathematician and artist, Piero della Francesca (1414–1492).

Piero was absent from Sansepolcro during much of Pacioli's early years, only returning in 1459 when Pacioli would have been between 11 and 13. He stayed there for most of the following three years. It is difficult to conceive of him not having tutored Pacioli during that period, not only in the *abaco* tradition, but also in Euclid's *Elements*, a subject Piero is believed to have been studying in Rome before his return to the town (Banker, 2009, p. 207). Lending support to this hypothesis, in the manuscript book that Pacioli wrote in Perugia in the late 1470s, he uses the same unique symbolism for powers as Piero (Ulivi, 2015, p. 663). He could only have learnt this from him in Sansepolcro, or from his manuscript *abaco* treatise, *De Abaco* (c.1470).

Pacioli's *abaco* education awakened his interest in mathematics; and the humanist element of his grammar school education sowed both the seeds for his interest in ancient mathematical texts and his passion for the pursuit and dissemination of knowledge. In 1464, he left Sansepolcro to continue his mathematical studies in Venice. While there, it is likely that he also continued to

study Latin, not just to prepare to become a friar and to enable him to teach in university where 'all lectures, disputations, and texts were in Latin' (D'Elia, 2004, p. 4), but also so that he could read and use the Latin manuscripts he consulted in his study of mathematics.

Venice

When Paciolo arrived in Venice in 1464, he was employed by a leather merchant, Antonio Rompiasi, with whom he stayed until 1470. While there, he continued his education in theoretical mathematics by attending public lectures at the *Scuola di Rialto*[10] under the renowned mathematician, Domenico Bragadin.[11] Pacioli did not, however, remain in Venice throughout that period. For example, in April 1466, he was in Ferrara (Banker, 2009, p. 208),[12] presumably on behalf of his employer; and Pacioli himself says that he went on voyages on behalf of Rompiasi (Stevelinck, 1994, p. 4). However, his trips within Italy were not always for his employer. In October 1466, he was in Sansepolcro to sell the Pacioli family house (Banker, 2009, p. 208; Ulivi, 2009, p. 29). We do not know the details of what else Pacioli did while employed by Rompiasi but, when he left Venice in 1470 he dedicated his first manuscript book, on arithmetic and algebra (i.e., *abaco*), to Rompiasi's three sons (Baldi 1589, f.180r), something he could not have written had he not himself been taught *abaco* before he arrived in Venice; and would not have written, had he not taught them the subject. However, given that he was between 16 and 18 in 1464, he would not have acted as their *magister* (teacher) but in the role of a *ripetitore*, someone who either assisted a *magister* in his school or, more to the point in this case, coached children at home (Grendler, 1985, p. 198).

It is not unreasonable to speculate that an opportunity to study theoretical mathematics under Bragadin, probably on the recommendation of Piero, would have been the reason why Pacioli went to Venice; and that his appointment with Rompiasi as assistant/apprentice and *ripetitore* was how he funded it. This does seem more likely than the widely accepted belief that he went to Venice primarily to work for Rompiasi in the role of either companion or tutor for his three sons (e.g., Weis and Tinius, 1991, p. 55; Yamey, 1994, p. 13; Giroux, 2017, p. 28).

In employing Pacioli, Rompiasi not only provided the means for Pacioli to fund his studies, he also provided him with other opportunities, including learning

[10] Although there was no university in Venice at that time, the City had pretensions to having its own university and the teaching at the *Scuola di Rialto* was of topics that would be taught in universities—philosophy, logic, and theology (Ross, 1976, p. 557), which included mathematics (Ciocci, 2017, p. 60).

[11] Bragadin would not have taught practical arithmetic and algebra. These were branches of mathematics taught in the *abaco* schools, not in the universities. Universities did not begin to teach algebra until the 19th century (Høyrup, 2013). He would have taught theoretical mathematics, leaving Piero as the most likely source of Pacioli's tuition in *abaco*.

[12] Banker (2009) speculates that this may have been part of his journey to sell the family home in October the same year, but these two events are six months apart, which makes that extremely unlikely.

bookkeeping—an obvious task to give someone good at mathematics, especially if he acted, as he did, as the merchant's representative on journeys and sea voyages (Taylor, 1942, p. 55): accounts would have been maintained throughout the trip. His experience in Rompiasi's business also developed his skills in mercantile mathematics, building on what he had already learned from his guardian and from Piero della Francesca. During this period, Pacioli is also likely to have become acquainted with elements of notarial practice, which he used when he prepared his bookkeeping treatise. This is consistent with his knowledge of Venetian business practice not being entirely up-to-date when *Summa Arithmetica* was published in 1494 (see, Saporetti 1898, pp. 105–6). During this period, he would also have become aware of the varying customs, weights, measures, currencies, moneys of account, taxes, and fees for various services that any merchant needed to know, either through his work for his employer or from one or more of the several merchant manuals in circulation (Lane, 1977, p. 183).

Leon Battista Alberti

It was not just his education and his work experience that influenced the young Pacioli. Another major inspiration came from his friendship with the leading humanist educator, Leon Battista Alberti (1404–1472). Upon leaving the employment of Rompiasi in 1470, he spent many months with Alberti in Rome, but left for Sansepolcro before the end of February 1471 not, as has been assumed in the literature (e.g., Taylor, 1942, p. 170; Gleeson-White, 2011, p. 61), after Alberti's death in 1472.[13] Alberti had an enormous influence upon Pacioli's development as a teacher. One of the leaders of the humanist education movement, he promoted the use of the spoken language of the day in order that the maximum number of people could access what was written (Santayana, 1930, p. 96). This was something Pacioli did consistently in his educational writing and he is known to have used reasoning similar to Alberti's to justify his doing so (see, for example, Mancini 1882, footnote 1, p. 146). With particular relevance to *Summa Arithmetica*, the primary focus of which is *abaco* mathematics, Alberti also advocated the inclusion of *abaco* in everyone's education (Grendler, 1989, p. 310), so bringing a practical element to humanist educational ideals.

PACIOLI THE FRIAR

Pacioli had three brothers. Two of them,[14] Ginepro and Ambrogio, were Franciscan friars. Ginepro was *Guardiano* (head) of the Convent in Sansepolcro between 1472 and 1473 (Ulivi, 2009, p. 20). When Pacioli left Alberti in late 1470/early 1471, he followed his brothers into the Franciscan Order of the Friars Minor

[13] Pacioli writes in *De Divina Proportione* (1509) that he spent many months staying with Alberti in the time of Pope Paul (Tavernor, 1999, p. 116). Pope Paul II died on 26 July 1471. Pacioli was in Sansepolcro in late February 1471 (Ulivi, 2009, p. 30). Alberti died on 25 April 1472.

[14] Pacioli's other brother, Antonio, died young (Ulivi, 2009, p. 20).

Conventual. Baldi (1589, f.180r) suggested that this happened soon after he left Venice in 1470 but that his mathematical studies delayed the obtaining of Pacioli's master of theology degree. To some extent, that was correct, but it was the time he spent in Rome with Alberti that caused the delay. As to when he began his period as a novitiate, records show that Pacioli attended meetings in the town dressed in the robes of a friar on 26 February 1471 (Ulivi, 2009, p. 30[15]).

It took between one and three years to complete this training. By the time he left Sansepolcro for Perugia in 1475, Pacioli had been ordained as a friar and had received an education to prepare him for further study. The goals of this Franciscan education were in four-parts: the establishment of the centrality of God to the pursuit of wisdom and the primacy of theology over all other sciences; the qualities of a teacher; how teaching and learning should take place; and an extended course in the Arts, including grammar, poetics, dialectic, and rhetoric; arithmetic, music, geometry, and astronomy; physics, ethics, magic, and mechanics (Roest, 2000, pp. 250–71). Proceeding thereafter to the nearest university to continue his studies was not only the next step for Franciscan friars with exceptional talent (Roest, 2000, p. 250), it was necessary if he were to be allowed to teach in universities (Verger, 1992, p. 36; Grendler, 2002, p. 172).

PACIOLI THE STUDENT AND THE UNIVERSITIES OF THE 15TH CENTURY

Pacioli studied mathematics and philosophy at the University of Perugia (Boncompagni 1879, p. 382) in the Faculty of Theology. He supported his studies by teaching *abaco* classes in the town.[16] In 1476, he started work on a new book (Pacioli 1494, f.67v[17]), a 396-folio (i.e., 792-page) manuscript *abaco* treatise that he dedicated to the youth of Perugia: *Tractatus mathematicus ad discipulos perusinos* (Pacioli, 1480). In modern translation (Calzoni and Cavazzoni, 2007), it amounts to 650 typeset pages. Contrary to what is generally believed (e.g., Taylor, 1942, p. 128; Pin, 1993, p. 164), he did not prepare the final version in four-and-a-half months.[18] He was still adding to it in Perugia in April 1480 when he was sent an

[15] To eliminate confusion with the variable dates of the year-end at that time, Ulivi uses modern dating with the year ending on 31 December. That convention is also adopted in this paper.

[16] Pacioli writes in *Summa* that he went to Perugia in 1475 and that he stayed there three years. One of those statements is incorrect, because he was employed to teach in Perugia by the Commune for three years starting in November 1477. However, as he also states in *Summa* that he prepared his Perugian *abaco* manuscript in 1476, it seems more likely that it was the period he stayed there that was incorrect, rather than the year in which he went there.

[17] All references to Pacioli (1494) are to Volume 1 of *Summa Arithemtica*, except where Volume 2 is indicated in the citation. Conventionally, both volumes are treated as part of one book, but the pagination restarts in Volume 2, hence the need to distinguish between them when referring to specific pages.

[18] His writing has been interpreted as saying that he wrote it between 13 December 1477 and 29 April 1478 (Taylor, 1942, p. 118).

advanced problem and asked for his solution by Giovanni del Sodo, reputed at that time to be the finest *abaco* master in Florence (Ulivi, 2015, p. 661). His appointment in Perugia ended in June 1480 and on July 11 he was back in Sansepolcro. A note inserted at the end of his manuscript dated 10 December 1480 reveals that he had returned to Perugia where, like Piero in Rome before him, he was studying Euclid's *Elements* (Pin, 1993, p. 164).

In 1481, he was in Zara, then the capital of Dalmatia and part of the Venetian Republic, now part of Croatia. It has often been said that he was teaching (e.g., MacKinnon, 1993, p. 130; Yamey, 1994, p. 13; Fischer, 2000, p. 301; Ciocci, 2017, p. 70), but of that there is no evidence. It has also been suggested that he was there to recuperate from overwork (Taylor, 1942, p. 156), but a more plausible explanation is that he went there to contemplate on how to use what he had learned from Euclid's *Elements* to change the way he taught or, possibly, to prepare for the final stages of his degree, or both. This would be consistent with the fact that, while there, he wrote a third, more advanced book on mathematics with more subtle and stronger cases. This may have been his first attempt to select an appropriate example to illustrate each of the generalizable algebraic statements, or 'keys'[19] he developed for teaching that subject, and may have been the first draft of that part, *Distinctio* 9, of *Summa Arithmetica*. Unfortunately, this book, like his first, is lost.

The final stage of Pacioli's formal education occurred when he obtained his degree in theology in Perugia between mid-1480 and mid-1484: on 20 September 1484 he was recorded as having the title 'Magister Professor' (Ulivi, 2009, p. 35). As he was also by then *Guardiano* of the Franciscan Convent in Sansepolcro, it seems likely that the award of the degree in Perugia occurred nearer the beginning of that period than the end. During the four years after he left Zara, Pacioli appears to have spent most of his time in Sansepolcro, the last date when we know he was there being 15 May 1485.

Florence

While he may have visited Florence during the previous two years, it was at this point that Pacioli began to spend a lot of time in the city studying ancient and contemporary texts, principally at the Dominican Order's Library of San Marco. This was where he found much of the mathematics he used in his *Summa*, in particular the Campanus (1482) printing of Euclid's *Elements*.[20] He also consulted various studies of perspective, and the teachings of Piero della Francesca; and discussed art and perspective with Botticelli and Pollaiolo, and artists from the workshop of Verrocchio (Ciocci, 2009, pp. 104–6).

[19] For a discussion of these keys and their use in Pacioli's teaching of algebra, see Heeffer (2010, p. 15, 2012, pp. 38–9, 2013).

[20] Apart from the San Marco library in Florence, Pacioli found some of his sources for *Summa* in the Duke's library in Urbino and in the Marciana Library in Venice (Ciocci, 2009, p. 54). He would have found most of the *abaco* manuscripts he consulted in the libraries of merchants (Heeffer, 2017).

Most notably, it was in Florence that he found *Tractato di praticha di geometria*, a translation of Leonardo Pisano (Fibonacci)'s Latin treatise, *Pratica geometrie* (1220), written by an anonymous Florentine disciple of the abacist Domenico di Agostino Vaiaio (Ulivi, 2015, pp. 658–9). It was this text that was the main source used by Pacioli in *Summa Arithmetica* and the one from which he copied the statement he makes in *Summa Arithemtica* that anything to which he does not attribute a source comes from Leonardo Pisano (Picutti, 1989, p. 76).

The main reference text for the second to the eighth *distinctio* in *Summa*, on arithmetic and algebra, is Leonardo's *Liber Abaci* (1202). It is thought that he may have seen that text in Venice (Ciocci, 2009, p. 156) in the 1460s. However, it was not Leonardo who inspired Pacioli in his treatment of algebra. That accolade belongs to Antonio de' Mazzinghi (Heeffer, 2017), who is acclaimed as the outstanding algebraist of the 14th and 15th centuries (Franci, 1988, pp. 240–241). In Naples c.1489/91, Pacioli consulted a copy of Euclid's *Elements* in its original Greek acquired by the Duke of Urbino. In particular, the 5th book on proportions of magnitude, which he had translated 'bit by bit' by Giovan Jacomo (Pacioli 1494, Vol. 2, f.2v), presumably to check the 1482 printed version of the Campanus (1220–96) translation in Latin against the original Greek source.

THE TEACHER

The Abaco Level

As mentioned above, Pacioli taught *abaco* privately in Perugia: in 1475 and 1476 and for the Perugia Commune from November 1477 until June 1480 (Boncompagni 1879, pp. 432–437).[21] His other priorities then caused a break from teaching until November 1486, when he was once again appointed by the Commune to teach *abaco* (*abicuz e arsmetricam*) in Perugia. The appointment was for one year. However, at that time he was in Florence studying proportion in the Dominican Order's library of San Marco and did not take up the teaching position until May 1487 (Ciocci, 2009, p. 104). This was when he started writing *Summa Arithmetica* (Ciocci, 2009, p. 107). He left the following April and was to return one more time to teach in Perugia, in November 1510.

The Commune records relating to this last appointment reveal that he asked to be appointed to teach *abaco* and geometry and that: 'During that year he will offer classes in the art of geometry and abaco with no other salary[22] as others before him and he himself had done and intends to do' (Boncompagni 1879, p. 437). This was not the only other time he taught *abaco*. In 1491, under the threat of excommunication, he was commanded by his Order to stop teaching and to close the secular school in his Convent in Sansepolcro (Ciocci, 2009, p. 117),

[21] It has been suggested that Pacioli spent some time teaching *abaco* in Naples in 1472 (Taylor, 1942, p. 170), perhaps following the death of Alberti on 25 April that year. However, resolving his having done so with his contemporaneously being a novitiate appears problematic at best.

[22] That is, with no additional payment from his pupils.

presumably one he had founded. In addition, Pacioli taught *abaco* arithmetic, proportions, and perspective to members of many trades and crafts: painters, sculptors, masons, architects, engineers, and surveyors, typically in their place of work, many of them in Sansepolcro (Ciocci, 2009, p. 170). *Abaco* was sometimes taught one-to-one, especially when teaching the more advanced secrets of the trade (Heeffer, 2011). His most famous pupils were Leonardo da Vinci and Albrecht Dürer, both of whom sought him out after the publication of *Summa Arithmetica*, the former a year or so after its publication; it is likely that Dürer did so in Bologna in 1506.[23]

The University Level

Until he was awarded his degree, all Pacioli's teaching had been at the *abaco* level but, even after graduating, he delayed teaching at the university level, preferring to spend more time teaching in Perugia while working on his manuscript of *Summa Arithmetica*. When he did teach in university, Pacioli taught theoretical mathematics, principally Euclid's *Elements*.

Pacioli's first university appointment was in Rome, where he obtained a public lectureship at the *Studio della Sapienza* in 1488 (Baldi, 1589, f.181r; Taylor, 1942, p. 161; Grendler, 2002, pp. 59–60). The next was in Naples (Antinori, 2000, p. 13), where he was enrolled as a public lecturer, probably between July 1489 and October 1490[24] (the university year at that time started in November). He remained there in 1490–1491 (Biagioli, 1989, p. 92), but was in Sansepolcro in late June 1491 and, again, at the end of September, and appears to have stayed there throughout 1492. He was still in Sansepolcro during Lent (March/April) 1493 where he delivered the Lenten sermon on March 12 (Ricci, 1940, p. 13). On 27 May that year, he withdrew the large sum of 550 lire from his savings (Ulivi, 2009, p. 33), possibly to contribute towards the printing of *Summa Arithmetica*.

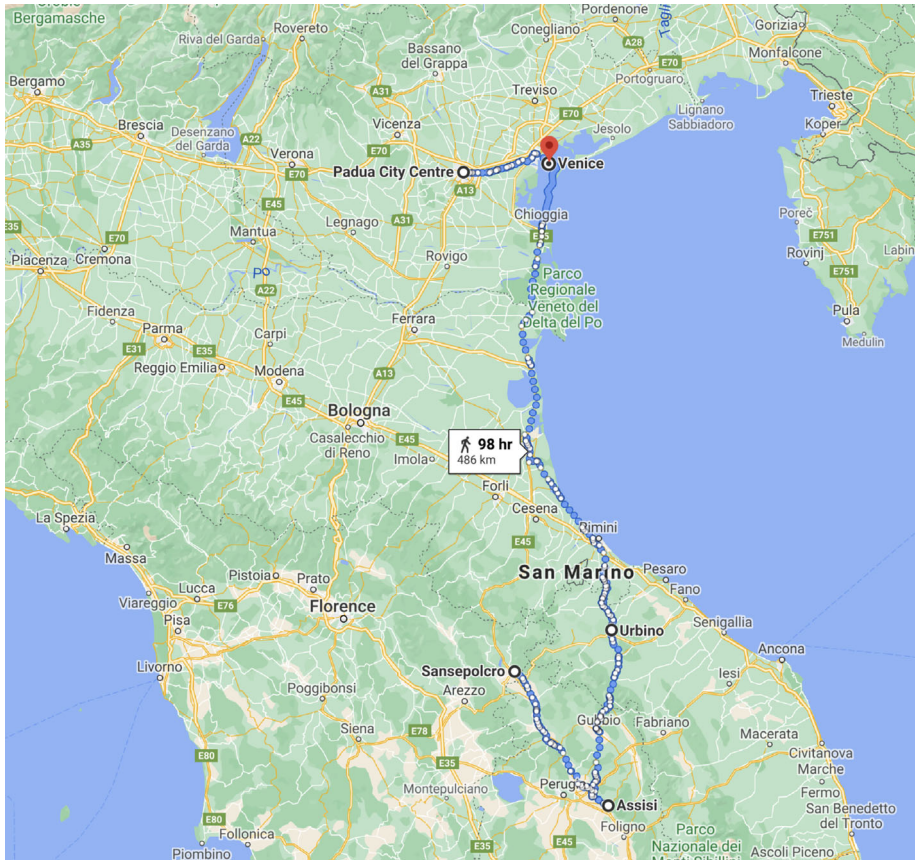
Later that summer (Ricci, 1940, p. 13), he travelled to Padua where he taught arithmetic and geometry at the university (Saporetti 1898, p. 81). According to Ricci, he then visited Assisi and Urbino before going to Venice for the publication of his book in 1494. This visit to Urbino in 1493 was the first time we know he was there (Ciocci, 2009, p. 120), but his knowledge of the buildings and the library of Urbino, which he describes in *Summa Arithmetica*, and his gratitude for the corrections made to his manuscript by the Urbino Court mathematician, Paolo de Middelburgo (Pacioli 1494, f.2r-v), indicates that he visited there before he finished writing the book. That is, before December 1493, when printing of the book is believed to have begun (Sangster, 2007, pp. 138, 142).

[23] Moore (1905) recounts how Albrecht Dürer wrote of his plans to travel to Bologna for 8–10 days in 1506 to meet a man who is 'willing to teach me the secrets of the art of perspective'. No candidate for this unknown person has ever been suggested other than Pacioli.

[24] Ulivi (2009, p. 36) suggests that Pacioli was in Naples either between April 1488 and April 1489 or between July 1489 and October 1490, but the earlier date is not likely if he was working in Rome between 1488 and 1489.

FIGURE 3

AN ALTERNATIVE VIEW OF PACIOLI'S TRAVELS IN 1493



While the sequence of that journey as described by Ricci is plausible, Figure 3 shows a more likely scenario: that he travelled first to Assisi, then Urbino to collect his manuscript from Paolo de Middelburgo, then to Venice to deposit his manuscript, and then on to Padua where he finished his bookkeeping treatise (Sangster, 2012, p. 106) and then took it with him to Venice. Would he really have travelled to Padua first, only to retrace his steps to Assisi, especially as the printer started printing *Summa Arithmetica* in December, 1993, when Pacioli was in Padua?

Following the publication of an estimated 2,000 copies of *Summa Arithmetica* in November 1494 (Sangster, 2007, p. 141), Pacioli's next teaching post was in Milan. He travelled there immediately after settling his affairs in Sansepolcro on 9 September 9 (Ulivi, 2009, p. 33). In Milan, he taught Leonardo da Vinci and was employed as a public lecturer in mathematics between 1496 and 1499 in the *Scuole Palatine* (Ulivi, p. 38), the most prestigious higher school in Milan, while employed by the University of Pavia (Grendler, 2002, p. 87).

Teaching was not the only thing Pacioli was doing in Milan. On 10 February 1498 he participated in a disputation, a scientific duel or debate, which took place in front of Ludovico Sforza, the Duke of Milan, and his Court. With Leonardo at his side, he confronted leading astronomers and astrologers, challenging the traditional hierarchy of the disciplines that ranked the university subjects, astrology and astronomy, above the technical, typically school-level subjects of arithmetic and geometry. Pacioli pleaded for the nobility of mathematics, arguing that astrology and astronomy, the mathematics of the university, were inferior as they both depended on the true principles of geometry, and that mathematics is essential for a range of other disciplines, including the art of war, theology, philosophy, and law. (Azzolini, 2004, pp. 115, 123) This debate formed the catalyst for *De Divina Proportione*, which he completed in December 1498 and published in print in 1509.[25] The argument he presented during the disputation was also included by Pacioli in the dedicatory letters in both *Summa Arithmetica* and *De Divina Proportione* (Ciocci, 2009, pp. 205–6).

Pacioli spent the winter of 1499 in Sansepolcro, possibly part of it in Mantua, Venice, and Perugia. His next teaching post was in Florence, where he taught for the University of Pisa while it was temporarily located in Florence. He did so for six years, from the beginning of the academic year on 1 November 1500 until the end of October 1506. The municipal records in Florence for appointments between 1500 and 1504 indicate that Pacioli, teacher of mathematics, was employed to teach Euclid (Boncompagni 1879, p. 438). While there, on 25 November 1502 Pacioli paid a visit to the office of the father of Leonardo, Piero di Antonio da Vinci. The records of the meeting reveal that Pacioli was staying in the Franciscan Convent of Santa Croce. Three years later, he became an official member of the Convent, on 28 July 1505 (Ulivi, 2009, p. 49) and it is likely that he stayed there throughout these years he spent in Florence. It is also where the record is kept of his death in 1517 (Nakanishi, 1979, pp. 54–5).

During 1501–1502, he was also listed as a teacher of mathematics at the University of Bologna. Antinori (2000, p. 14) expressed some doubt that he ever took up the appointment, though having two concurrent appointments 100 kilometres apart would not have been impossible and there is no evidence that he did not. In 1504, while he was still working in Florence, *La Scuola Perfetta dei Mercanti*, an abbreviated version of the mercantile parts of *Summa Arithmetica* with different bookkeeping material including many examples, was published in Venice (Yamey, 1974). No copy has survived of this book, but it was not prepared by Pacioli. Had he done so, he would have mentioned it in the list of his publications when seeking copyright protection for his work in 1508.[26] Further refutation of his authorship of this book lies in the incompatibility of the concept of padding out his text with a large number of examples, compared with the relatively example-free pedagogy he presents in his bookkeeping treatise, *De*

[25] Pacioli presents details of this debate in *De Divina Proportione*, which he began writing upon the suggestion of Ludovico Sforza following this debate. He completed the book on 14 December 1498.

[26] *See* Taylor (1942, p. 339) for the list of titles Pacioli presented in his application.

Computis et Scripturis (Sangster, 2015, pp. 10, 23, 2018, p. 302), and in his coverage of algebra in *Summa Arithmetica* (Heeffer, 2010, p. 15).

During this period in Florence, he did not remain constantly in that city but made many visits to Sansepolcro on Convent business (Ulivi, 2009, p. 33), a fact that lends support to the practicality of his having concurrently taken up his appointment at the University of Bologna during 1501–1502. Between November 1506 and early August 1508, his whereabouts are unknown (Ciocci, 2017, p. 48), though it has been suggested that he was in Rome (Green, 1968, p. 40; Fischer, 2000, p. 302). As indicated earlier, there is a strong possibility that he was in Bologna at some point in 1506 and that he taught Albrecht Dürer perspective while there; and he may also have spent some of this and the following two years in Mantua (Ulivi, 2009, p. 45).

In 1508, Pacioli returned to Venice where he worked on the printing of his reworking of the Campanus Latin translation of Euclid's *Elements* and a translation into Italian of the same work. He was also teaching (Bagni, 2009, p. 21) at the *Scuola di Rialto* (Ceci *et al.*, 2010, p. 78). However, Ross (1976) does not list Pacioli among those identified from the available records as having done so, suggesting that these were invited guest lectures, a view supported by his having presented a public lecture on the 5th book of Euclid in the *Chiesa de San Bartolomeo* in the Rialto on 11 August 1508 (Tamborini, 1999, p. 43). The topic was proportions.

His Final Years

In 1509, *De Divina Proportione* was published in Venice. That same year, his version of Euclid's *Elements* in Latin, and his Italian translation were also published. No copies have survived of the latter book. He then returned to Sansepolcro where he fell-out with his fellow friars to the extent that the General of his Order had to intervene. Perhaps because of this, he returned to Perugia to teach *abaco* in 1510–1511. However, it is clear that the problems he had in Sansepolcro were initially resolved when, while in Perugia in 1510, he was appointed *Commissario* of his Convent in Sansepolcro (Ricci, 1940, p. 19). In August 1511, the problems flared-up again when Pacioli returned from Perugia and was refused the keys of the Convent by one of the other friars, backed-up by some of his companions. It was only with the intervention of Florentine magistrates (after Pacioli had requested the assistance of the *Capitano* of Sansepolcro) and the *Gonfaloniere* of Florence, Piero Soderini, (after the intervention of the provincial *Ministro* of the Franciscan Order) that this was resolved in January 1512 (Ciocci, 2009, p. 137).

We do not know what Pacioli was doing between 1511 and 1513 though, given his appointment as *Commissario* of his Convent and the ongoing dispute concerning the keys, he was probably in Sansepolcro for much of this period. He signed his third and last will in Sansepolcro, in November 2013 (Ciocci, 2017, p. 48). His final university teaching appointment was in Rome, where his name is on the university roll for 1514–1515. In 1515, the Commune of Sansepolcro wrote to him in Rome (Rankin, 1992, pp. 121–2), which confirms, contrary to what some have suggested (e.g., Weis and Tinius, 1991, p. 56; Macve, 1996, p. 10), that he did take-up the appointment. He was in Sansepolcro on some occasions during these

TABLE 1

WHERE AND AT WHAT LEVEL PACIOLI TAUGHT[31]

When	*Where*	*Level*
1464–70	Venice	*Abaco*
1475–80	Perugia	*Abaco*
1487–88	Perugia	*Abaco*
1488–89	Rome	University
1489–91	Naples	University
1491	Sansepolcro	*Abaco*
1493	Padua	University
Summa Arithmetica published, November 1494		
1496–99	Milan (University of Pavia)	University
1500–06	Florence (University of Pisa)	University
1501–02	Bologna	University
1507–08	Venice	University
De Divina Proportione published, June 1509		
1510–11	Perugia	*Abaco*
1514–15	Rome	University

two years but none of the dates when he is known to have been there would have conflicted with a 1-year appointment which ran from the beginning of November 1514 until the end of October 1515.

In 1516 and 1517, Pacioli's whereabouts are unknown and, in March 1517, someone else was occupying his room in the Sansepolcro Convent (Ulivi, 2009, p. 57). On 15 April 1517, the Commune of Sansepolcro wrote to the *Commissario* of the Province of Assisi asking that Pacioli be elected minister of the province (Ulivi, 2009), the equivalent of a ceremonial post in Romagna that he had turned down some years before. However, he died before this could be ratified, on 19 June 1517 (Nakanishi, 1979, pp. 54–5). Where he died is unknown, but he had many family in Sansepolcro and may have returned there after his teaching in Rome was completed, moving from the Convent to live with them as his health deteriorated, which would explain why his room in the Convent was occupied by someone else, and might also explain the Commune's desire for him to recognized in this way.

Pacioli's contribution to mathematics is without question, as is his devotion to his Church. Two other activities stand out: his teaching and his treatise on double entry.

PACIOLI'S CAREER AS A TEACHER

As described above, Pacioli worked in six of the then 12 Italian universities,[27,28] and he also did some teaching at the 'incomplete'

[27] The other six were in Siena, Perugia, Florence, Turin, Ferrara, and Catania (Grendler, 2002, p. 2).

[28] Biagioli (1989, p. 94) believed that the University of Perugia created a chair, *ad docendum aritmeticham seu abicum et geometrium*, before 1412 and that Pacioli was appointed to that chair in 1486 to teach 'arithmetics' (*abaco*). Ulivi (2015, p. 660) believes that this was *abaco* level teaching,

university[29] at the *Scuola di Rialto* in Venice. He taught at the *abaco* level in Venice, Perugia, and Sansepolcro. To this can be added his tuition of Leonardo da Vinci in Milan and of Albrecht Dürer, plus many artisans and craftsmen, including his pupils in Sansepolcro. His career as a teacher is summarized in Table 1.

We know how Pacioli taught from his writing (Sangster, 2018, p. 306); and, as shown in the Introduction to this paper and in the comparison between how he taught double entry and algebra, we can see that he was the author of his treatise on double entry bookkeeping.

CONCLUSION

Luca Pacioli followed his aspirations with the full support of his Church, making many lasting contributions to society, not least to business and accounting. He is recognized as the 'father of accounting' because he established the foundation of modern accounting by taking the bookkeeping method of Venice, improving it, and presenting it to the world as 'the best approach' in a printed book written for those who kept account day-by-day: merchants. He lived a long and full life, mixed with the great and soon to be great and was unquestionably their equal. His background as a teacher of mathematics, schooled in the Franciscan education system, who had spent six years learning the mercantile craft before he began training for life as a friar prepared him for this task. And his belief in God and humanism fuelled his vocation to teach others to learn the secrets of mathematics in the spoken language through the relatively new medium of the printed book.

Aside from his well-documented contribution to the theorization of mathematics,[31] Pacioli's contribution to the discipline of accounting was greater than has been recognized, extending beyond a simplistic view of him as 'the messenger' to a recognition that he was the first to begin to theorize the still fledgling method of double entry. It was a book written for its time and place, one where double entry was about maintaining records of obligations and facilitating

> a view consistent with what is known of where *abaco* was taught. Arithmetic was part of the *quadrivium*, the last four of the seven liberal arts that formed the basis of medieval university education, but it was not practical arithmetic. It was considered a science of pure numbers comprising the theory or philosophy of numbers, their relationships and meanings, largely based on Boethius, *De institutione arithmetica* (c. 500). It was only after the middle ages that 'arithmetic' was designated an elementary discipline of counting and calculation. (Masi, 1983, p. 11)

[29] There were six 'incomplete' universities. They offered university level instruction, but none had sufficient professors to be recognized as 'universities'. The others were at Arezzo, Modena, Piacenza, the second Venetian School of San Marco, and the *Casa della Sapienza* in Pistoia.

[30] This table updates and corrects an earlier version that appeared in Sangster and Scataglinibelghitar (2010, p. 428).

[31] This contribution to mathematics was driven by his belief that knowledge of mathematics brought anyone closer to God because mathematics were the tools with which God made the world (Sangster, 2018, p. 306).

control over agents, when adjusting entries were a rarity. His method encapsulated what the merchants required (Chatfield, 1996): a foolproof method for recording transactions, and nothing else. In the simplicity of his theorization, we can see where the true boundaries of double entry lie and those of mathematics-based adjustments begin. Pacioli's treatise was accounting in its purest form, entirely fit for purpose, and sufficient.

The contribution of this paper lies in its synthesis of what we know of Pacioli so as to eliminate misconception and mistaken beliefs about the man and present a clear and unambiguous account, founded in primary sources interpreted in the context of his time and place. It is hoped that accounting scholars will now see him for what he was: the first to develop a theoretical base for accounting; and a committed educationalist whose mission in life was to bring knowledge to those who would most benefit, but within the wider context of benefiting society as a whole. Few would argue that this was what he achieved.

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