An Instrument of Mass Calculation made by Nasṭūlus in Baghdad *ca*. 900

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Keywords: Nasṭūlus - astronomical timekeeping - astronomical instrument - 'Abbāsid astronomy - curves that are neither circles or conics.

Abstract
This remarkable astronomical instrument was made by the Muslim astronomer known as Nasṭūlus, who was active in Baghdad between 890 and 930. Its rediscovery brings our knowledge of the activities in that flourishing scientific centre a substantial step further. This type of instrument was previously not known to exist, although sundials based on the same principle are described in Arabic treatises datable to *ca*. 950 and *ca*. 1280. It is essentially a mathematical device providing a graphic solution to a problem that was of interest to Muslim astronomers, namely, the determination of the time of day as a function of the solar altitude throughout the year, here specifically for the latitude of Baghdad. The instrument reveals a level of mathematical competence and sophistication that is at first sight astounding. However, with a deeper understanding of the scientific milieu from which it came, it can be seen to be fully within the theoretical competence of the scientists of that environment. Nevertheless, the spectacular accuracy of the engraving of the principal curves on the instrument is completely unexpected. The instrument also features the earliest known solar and calendrical scales from the Islamic East; the origin of these was previously thought to be in the Islamic West.

1. Introductory remarks

In 2004-05 I published surveys of Islamic timekeeping and instrumentation.[1] Both necessarily started in Baghdad, and I was moved to label the instruments that I described from 8th, 9th and 10th-century Baghdad as "instruments of mass calculation". I predicted that other such instruments might come to light. Here is one such that took me by complete surprise. It came to light in 2006, and its provenance is unknown. It was offered for auction by Sotheby's of London in a sale of Islamic art in late 2006,[2] and was acquired by the new Museum of Islamic Art in Doha, Qatar, scheduled to open in late 2008. There was no press coverage, even when the instrument passed quietly to its new owners for close to a quarter of a million pounds sterling. The instrument comes from Baghdad and is

1 King, *In Synchrony with the Heavens*, hereafter referred to a *SATMI*. The first volume, dealing with astronomical timekeeping was subtitled *The Call of the Muezzin*, and the second, dealing with instrumentation, was subtitled *Instruments of Mass Calculation*.

2 Sotheby's of London, *Arts of the Islamic World*, October 11, 2006, lot 87 on pp. 76-85.

datable *ca*. 900. Before we look at it closely, we shall first describe its scientific context and then introduce its maker.

2 The historical background

By the 9th century, the new Abbasid capital of Baghdad was the world centre of scientific activity, especially in astronomy and mathematics.[3] Scholars in that flourishing cosmopolitan city had adopted the knowledge they found in Indian, Persian and Greek sources. Also, in the mid 8th century, the Muslims encountered the astrolabe, a two-dimensional representation of the three-dimensional heavens, and by the early 9th century they were devising new types of instruments (several of which are currently thought to have been European inventions of the 15th and 16th centuries). With remarkable rapidity they created a new science that well merits the appellation "Islamic science" for two reasons. Firstly, the principal universal language of serious innovative science from the 9th century until the 15th was Arabic, and secondly, some of the interests of the Muslim scientists were directed towards the complicated problems of (1) regulating the Muslim lunar calendar, (2) calculating the precise times of the five daily prayers that depend on astronomical phenomena and local latitude and vary from one day to the next, and (3) determining the sacred direction (*qibla*) towards the Kaʿba in Mecca. In particular, the times of Muslim prayer are defined in terms of phenomena that depend on the altitude of the sun above or below the local horizon.[4]

From the 8th century to the early modern period, Muslim astronomers were foremost in the science of astronomical timekeeping (*ʿilm al-mīqāt* in Arabic), of which the determination of the times of prayer constitutes a small but highly significant part. The tools of the astronomer were, in addition to the mathematics of spherical and planetary astronomy,

[3] For a bio-bibliographical survey of authors writing in Arabic on mathematics, astronomy and astrology until *ca*. 1050 see Sezgin, *Geschichte des arabischen Schrifttums*, vols. V-VII (1974-1979). For a list of publications specifically on astronomical activity in Baghdad see the website "Astronomy in Baghdad" (2003) at the end of the bibliography, which contains many references to research postdating Sezgin's *magnum opus*.

[4] For brief introductions to the Islamic aspects of Islamic astronomy see the articles "Ru'yat al-hilāl" (sighting the lunar crescent), "Mīḳāt" (astronomical timekeeping), and "Ḳibla" (sacred direction) in the *EI*$_2$. More is in *SATMI*, vol. 1.

handbooks full of astronomical tables and explanatory text,[5] and astronomical observational instruments and mathematical computational devices.[6] In particular, Muslim astronomers produced all sorts of tables relating to timekeeping and instrument construction, some serving specific latitudes and others serving all latitudes. The tables for timekeeping that are discussed below are mainly extant in unique manuscripts that have survived the vicissitudes of time and are now scattered in libraries around the world. This new mathematical instrument, unique of its genre, was surely made using one of these tables, specifically for Baghdad, which has not survived. Two such tables are, however, known from 13th-century Yemen, computed for Sanaa and Taiz. Also, sundials based on the same principle as the new Baghdad instrument, with similar curves, are known from treatises compiled by al-Ṣūfī (Shiraz, *ca*. 950) and al-Marrākushī (Cairo, *ca*. 1280).

3 Nasṭūlus

The Muslim astronomer who called himself Nasṭūlus has caused scholars in the Middle Ages and in recent times some problems. His signatures on all three of his surviving instruments are always without a diacritical point on the first letter, which could in theory be read as an "n" or a "b". His name appears to be related to Nasṭūrus, "Nestorius", which suggests a Christian (Nestorian) connection.[7] Yet the medieval Arabic textual sources call him Muhammad ibn Muhammad or Muhammad ibn ʿAbdallāh, "known as Nasṭūlus".[8] No biographical information on him is available. Until 2005, he was known to us from a small number of

5 On the scope of the Arabic and Persian literature on mathematical astronomy see King & Samsó, "Astronomical Handbooks and Tables from the Islamic World", with a summary in the article "Zīdj" in EI_2. The new survey of Islamic *zīj*es by Dr. Benno van Dalen is nearing completion.

6 On the earliest surviving texts on instrumentation see now Charette & Schmidl, "Al-Khwārizmī and Practical Astronomy in Ninth-Century Baghdad".

7 On the Nestorians see the article "Nasṭūriyyūn" in the *Encyclopaedia of Islam*.

8 On Nasṭūlus see Sezgin, *Geschichte des arabischen Schrifttums*, VI, pp. 178-179 and 288, and three articles by Brieux & Maddison, King & Kunitzsch, all summarized in *SATMI*, XIIIc-5.

citations in later Arabic scientific literature and two standard astrolabes signed by him that are now preserved in Kuwait and Cairo,[9] both of which are engraved in the same distinctive *kūfī* script and with the standard alphanumerical notation for numbers.[10] A third standard astrolabe by Nasṭūlus came to light in 2007 (see below). Nasṭūlus is further credited with the invention of two of the non-standard types of astrolabe that Muslim scholars developed in the 9th and 10th centuries, partly because the standard astrolabe, brilliant in conception as it was, was considered somewhat mundane.[11]

Nasṭūlus' "Kuwait" astrolabe, dated 315 Hijra [= 927/28], is a standard astrolabe but more precisely made than the earliest surviving astrolabe with inscriptions in Arabic, which dates from late-8th-century Baghdad.[12] Nevertheless, it lacks the spectacular decoration and mathematical complexity of the astrolabe made by the astronomer al-Khujandī in Baghdad in 374 Hijra [= 984/85].[13] Nasṭūlus provided his instrument with astrolabic markings for latitudes 33º (Baghdad) and 36º. His "Cairo" astrolabe is missing the rete and the plates but includes a geographical gazetteer on the mater and quadrants for trigonometric calculations on the back, such as were invented in Baghdad in the early 9th century.[14] There are additional markings by an astronomer in Mamluk Egypt. The third astrolabe by Nasṭūlus, which is undated, is missing the rete and the alidade, but "shadows" of these – less oxidized than the rest of the surfaces – are to be found on one of the plates and the back, respectively, and these are in the same form as the rete and alidade on the Kuwait

9 Both instruments are described in detail in *SATMI*, II, pp II, XIIIb-3.

10 On this see the article Irani, "Arabic Numeral Forms", and the article "Abdjad" in the *Encyclopaedia of Islam*.

11 On these see most recently Charette, *Mathematical Instrumentation*, pp. 63-83, and *SATMI*, X-5.1.

12 This is analyzed in *SATMI*, XIIIb.

13 See *SATMI*, XIIIc-9.

14 On these see *SATMI*, X-6, and now also Charette & Schmidl, "Al-Khwārizmī and Practical Astronomy in Ninth-Century Baghdad".

piece, hence showing the forms of the lost original parts. The two plates serve latitudes 31º, 32º, 33º (Baghdad) and 34º.[15]

From a textual reference we know that Nasṭūlus constructed a plate of his own invention for computing the times and stages of eclipses in 280 Hijra [= 893/94],[16] so we may wonder why he was making a standard astrolabe as late as 927/28.

In an early-13th-century manuscript in a private collection that was studied for the first time in 2005, a text that we can confidently attribute to Nasṭūlus describes a universal sundial for finding the time of day in seasonal hours for any latitude.[17] Several such sundials have survived from Greek, Roman and Byzantine cultures, but no texts are known.[18] The underlying formula relating time to solar altitude is approximate, but has the advantage that it is universal and that it is reasonably accurate for Mediterranean latitudes. The same manuscript contains Nasṭūlus' description of a luni-solar gear mechanism such as is found inside some of the surviving sundials. Nasṭūlus' account appears to be based on his correct understanding of some Byzantine instrument that he actually saw, although he does not mention this. In any case, his treatise is certainly not a translation from any Greek text. His account of this kind of sundial seems to have borne no fruit, except in the form of the simple universal sundials on the alidades of astrolabes, the use of which is based on the same principle.[19] It remains to be published.

The great scientific polymath al-Bīrūnī (Ghazna, Afghanistan, *ca*. 1025) mentioned that Nasṭūlus had written a treatise on a gear mechanism for reproducing the apparent relative motions of the sun and moon, and al-Bīrūnī presented his own mechanism with a different set of gears.[20] This has been published in some detail, together with the only surviving

[15] This instrument is currently in the possession of the Ahuan Gallery, London. I am grateful to David Sulzberger for showing it to me.

[16] See Sezgin, *op. cit.*, VI, p. 288.

[17] The manuscript was in 2005 in the possession of Sam Fogg, Islamic art dealer in London.

[18] See Field & Wright, "Gears from the Byzantines".

[19] See King, *op. cit.*, XI, Appendix A.

[20] al-Bīrūnī's treatise is published in Hill, "Al-Bīrūnī's Mechanical Calendar".

example of such a mechanism made by a Muslim craftsman in Isfahan in 614 Hijra [= 1223/24] that is now preserved in the Museum of the History of Science at Oxford.[21] al-Bīrūnī also mentioned that Nasṭūlus had written on a sundial but that its underlying principle was very incorrect (*fāsid*). The sundial described by Nasṭūlus is in fact highly ingenious and works well for latitudes in Iraq and Iran.

From the available textual references, it is clear that Nasṭūlus was an extremely innovative astronomer-mathematician, and from his three surviving astrolabes, it is also clear that he was a highly competent craftsman.

It is signed by Nasṭūlus on the back of the throne in the same way as on his three surviving astrolabes, without diacritical points, simply:

صنعه نسطولس , "constructed by Nasṭūlus".

4 Description of Nasṭūlus' new instrument

The instrument is in the form of an astrolabe, a circular plate with a throne at the top fitted with its original suspensory apparatus. It is made of brass and is slightly corroded. Its diameter is 19.2 cm, and its total height is 26.7 cm. The main markings, to be used together with a radial rule, are on the front. The alidade or sighting device on the back can rotate over an altitude scale.

Around the rim of the front plate there are concentric solar and calendrical scales, on which the equinox is on the right hand side and the scales are arranged counter-clockwise. The names of the signs of the zodiac are standard,[22] and the months are the standard ones of the Syrian / Julian calendar (Ādhār = March, *etc.*).[23] Each set is divided and labelled for each 5° of each zodiacal sign or for each 5 days (with appropriate adjustments for months with more or less than 30 days), with subdivisions for each 1° and each day. The vernal equinox is at March 15, and the other equinox and two solstices are at the middle of the appropriate months, so our maker has introduced a slight approximation here (see further below).

[21] First described in Gunther, *Astrolabes of the World*, I, pp. 118-120. On the gear mechanism see more recently Field & Wright, "Gears from the Byzantines"; and *SATMI*, II, pp. 65-68.

[22] See the article "Minṭaḳat al-burūdj" (zodiac) in EI_2.

[23] See the article "Ta'rīkh, I" (dates and eras) in EI_2.

Fig. 1: The front of Nasṭūlus' instrument and the main inscription indicating its use.
[Images courtesy of Sotheby's of London.]

Within these scales is *a family of six lemon-shaped rings* engraved for the seasonal hours from 1 on the innermost one to 6 on the outermost one in words using the classical Arabic forms:

ساعة ساعتان ثلاث أربع خمس ستّ

The space near the circumference of the circular disc at the base of this may have been labelled *ufuq*, "horizon", but corrosion has apparently eliminated this. A radial rule is engraved uniformly for each 5º from the horizon circle up to 80º, with subdivisions for each 1º.

The back bears no markings beyond an altitude scale marked and labelled for each 5º of solar altitude, subdivided for each 1º. The alidade or sighting device is unduly wide, but accurately aligned. It is original, as can be seen from the alidade on the Kuwait astrolabe, and the "shadow" of the alidade on the back of Nasṭūlus third astrolabe.

The function of the instrument is explained in the inscription on the front, which reads:

لمعرفة موضع الشمس ولمعرفة ما مضا وما بقى من النهار من ساعة زمانية
وأيضاً لمعرفة ارتفاع الساعات الزمانية وارتفاع نصف النهار لمدينة السلام
وحيث يكون العرض لج

"for finding the longitude of the sun and for finding how many seasonal hours have elapsed (since sunrise) or remain (until sunset) and also for finding the solar altitude at the seasonal hours and its altitude at midday throughout the year for the "City of peace" (*i.e.*, Baghdad), and wherever the latitude is 33º."

The appellation "City of peace" was common for Baghdad, even though life there in the 9th and 10th centuries was not always free from political and religious strife.[24]

Glen Van Brummelen has made a reconstruction of the principal markings – see Fig. 3. He, like myself, was astonished that Nasṭūlus could have made such an instrument. As he expressed it in his new book on the history of trigonometry, Nasṭūlus "was plotting contour curves of a double-argument function in polar coordinates".[25]

[24] See the EI_2 article "Baghdād", especially pp. 897b and 900.

[25] Van Brummelen, *History of Trigonometry*. I am grateful to the author for a copy of his manuscript before publication.

Figs. 2a: The back of the instrument

Figs. 2b-c: The inscription and the signature

5 Some preliminary considerations

This extraordinary instrument raises all sorts of technical questions, but also some other issues.

The name of the instrument

This is not an instrument suited for timekeeping as such, in spite of what Nasṭūlus wrote in his inscription. To find the time from the solar altitude one would have to interpolate between the hour curves and lose all control of accuracy.

For that purpose various varieties of astrolabes and quadrants and sundials were available, in addition to all sorts of tables, as well as a mysterious one known only by the name *lawḥ al-sāʿāt*, "board for the hours".[26] It is rather an *excès de délicatesse*, in the sense that it provides information that one does not really need to know. It is a mathematician's delight, and proves that its maker had the wits and the time to conceive it and to make it well. We do not know what Nasṭūlus called it, possibly *ṣafīḥat al-sāʿāt*, "plate for the hours". And we may well wonder what else he made.

The lack of markings of religious significance

None of the surviving astrolabes from the 9th and 10th century, mostly from Baghdad, bear any markings relating to the times of prayer. Nasṭūlus' new instrument conforms to this tradition. It would have involved rather little additional work to have included a curve for the time of the *ʿaṣr* prayer, though this would have had a shape that was different from that of the hour curves. It has been incorrectly argued that Islamic astronomy developed simply because of the importance of astronomy for

[26] It may be that this refers to a highly sophisticated prototype of an instrument for timekeeping by the sun for any latitude that was known in 14th-century England as the *navicula de Venetiis*, "little ship of the Venetians", and from 15th-century Vienna as Regiomontanus' *Uhrtäfelchen*, "little board for finding the hours". The latter was extremely popular in Europe for several centuries thereafter. It has recently been shown that Ḥabash invented a more complicated device than this for timekeeping by the stars: see Charette & Schmidl, "Ḥabash's Universal Plate". Also, the medieval English tradition of the *navicula* demonstrates a suspicious lack of familiarity with the correct way to make the instrument: see *SATMI*, XIIb.

religious purposes. Here we see a Muslim astronomer devising yet another instrument purely for scientific purposes, which include mathematical experimentation.

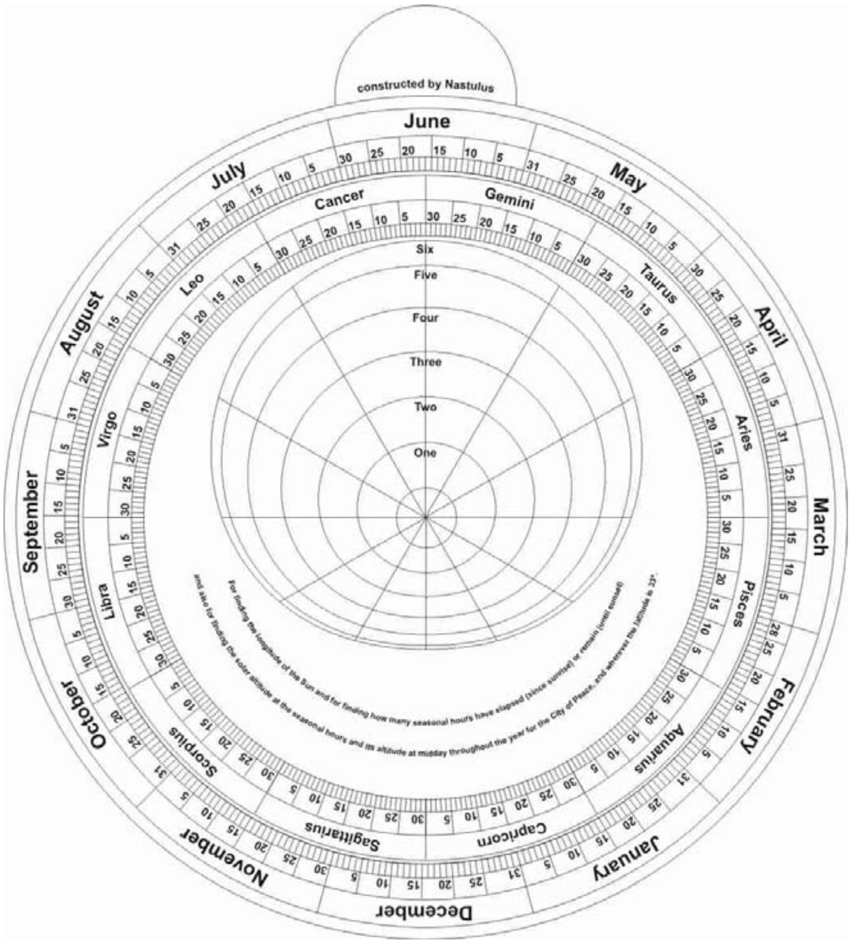


Fig. 3. A reconstruction of the markings on Nasṭūlus' instrument by Professor Glen Van Brummelen, reproduced with permission.

The latitude of Baghdad

In the early 9th century Muslim scholars, as part of their geodetic measurements, had determined the latitude of Baghdad (accurately 33º20'). The first group derived 33º, which is one value that remained popular, especially in the instrument tradition of al-Khwārizmī. Other astronomers preferred 33º25'.[27] (Readers should bear in mind that scientific knowledge was not cumulative in pre-modern science; one professional group or regional school might favour a certain tradition and sustain it regardless of the opinions of others.)

The date of the equinox

In the 830s Muslim astronomers had observed the vernal equinox on March 17, and knew perfectly well that the four seasons were not equal in length.[28] Nasṭūlus has not unreasonably simplified his scales – with the equinoxes and solstices each at 15 full days of the appropriate month – to make them easier to engrave.[29]

6 The underlying mathematics

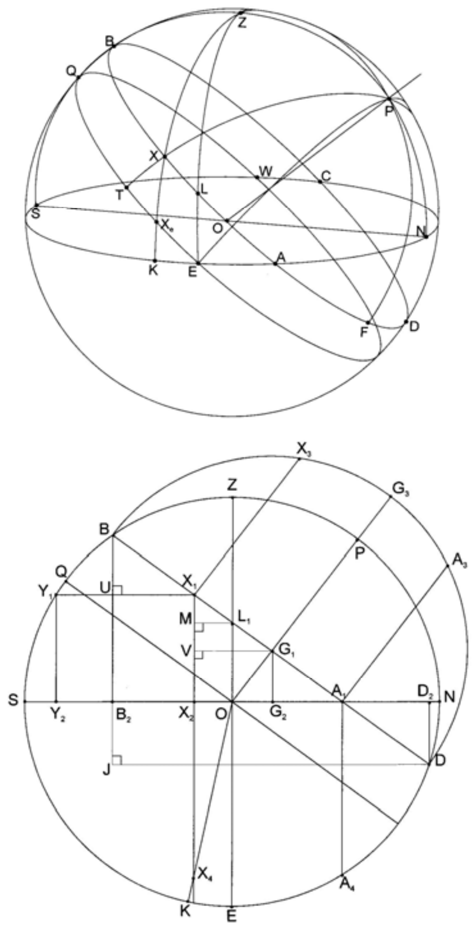
The astronomy and mathematics necessary to understand Nasṭūlus' instrument is not trivial. Fig. 4 shows the three-dimensional celestial sphere as the Muslim astronomers thought of it in mathematical terms. The sphere is considered to be of unit radius. The observer is at O with his horizon NESW. The celestial axis is OP and the celestial equator is EQW. The local latitude is equal to the arc NP. The sun is shown here at an arbitrary position in the morning, X, in the summer. Its declination is XT, and the limits of this are roughly +23½º in summer and -23½º in winter

27 On early values of the latitude of Baghdad (and Mecca) see King, "Too Many Cooks ... ", pp. 225-228. The basic reference work on such values is Kennedy & Kennedy, *Geographical Coordinates of Localities from Islamic Sources*.

28 See Caussin de Perceval's edition of the first few chapters of Ibn Yūnus' *Ḥākimī Zīj*, pp. 130-131.

29 On the problems associated with dating instruments by the date of the equinox shown on their scales see Turner, "Dating Astrolabes". In this case, we are in the fortunate situation of being able to date the activity of the maker.

(this quantity is known as called the obliquity of the ecliptic). The sun appears to rise at A, culminate at B and set at C. The length of daylight is the arc AC, that is, twice the arc AB. The altitude of a general position of the sun at X above the horizon is the arc XK. The problem is to find the varying altitude of the sun at the five divisions of the arc AB throughout the year, the sun being at B at the sixth seasonal hour.



Figs. 4-5: The celestial sphere dressed for solving many of the problems discussed by medieval astronomers, and the same reduced to two dimensions. (From King, *In Synchrony with the Heavens*, I, pp. 28 and 34.)

Fig. 5 shows the same scene reduced to the meridian plane by applications of the procedure known as the analemma, adopted by the Muslims from Greek sources.[30] It was such a procedure that was adopted by the earliest Muslim astronomers, before they applied spherical trigonometry to solve the same problems in completely different ways, albeit with equivalent results.

In this discussion, we shall use medieval versions of the relevant formulae, which are actually simpler than the modern versions. However, we shall adopt the modern trigonometric functions, which differ from the modern ones only in that they are to base 1 rather than 60. In all medieval Arabic texts the formulae are written out in words, although abbreviations and special terms were used.

We use the following notation, and the corresponding Arabic terms:

λ	solar ecliptic longitude (*ṭūl or mawḍiʿ al-shams*)
δ	solar declination (*al-mayl*)
ε	obliquity of the ecliptic (*al-mayl al-aʿẓam*)
φ	local latitude (*ʻarḍ al-balad*)
h	instantaneous solar altitude ($h < H$) (*irtifāʿ al-shams*)
H	solar meridian altitude (*irtifāʿ niṣf al-nahār*)
D	time in equatorial degrees from sunrise to midday or from midday to sunset (*niṣf al-nahār*)
T	time in equatorial degrees since sunrise or time remaining until sunset ($< D$) (*al-dā'ir min al-falak*)
$^\circ$	equatorial degrees, where 360° is equivalent to 24 hours, that is, 1° is equivalent to 4 minutes) (*darajāt*)
sdh	seasonal day hours, that is, the length of daylight 2D divided by 12 (*sāʿāt zamāniyya*)
vers	the versed sine function,[31] where vers q = 1 - cos q (*al-sahm*)

In Fig. 4, arc XK = h and arc BS = H. Also, arc XT = arc BQ = δ and arc PN = φ. Further, arc AB = D and arc AX = T. In Fig. 5, $X_1X_2 = Y_1Y_2$ = sin h, and BB_2 = sin H. Further, since $G_1B = \cos \delta$, X_1B = vers D x cos δ

30 On the analemma in Islamic mathematical astronomy see the studies listed in *SATMI*, I, p. 27, n. 40.

31 See the article "Sahm" (versed sine) in the *EI_2*.

and X_1B = vers (D-T) x cos δ. With such relations we can derive the formula relating T and D with H and h.

To start with, we need to calculate for each few degrees of λ, the functions δ (λ) and H(λ, φ) and D(λ, φ), using:

$$\delta(\lambda) = \sin\lambda \sin\varepsilon \; ; \; H(\lambda, \varphi) = 90^{o} - \varphi + \delta(\lambda) \; ; \text{ and}$$
$$D(\lambda, \varphi) = 90^{o} + \text{arc sin} \{ \tan\delta(\lambda) \tan\varphi \}.$$

The formulae that were generally used for calculating the time from the solar altitude[32] were equivalent to:

$$T = D - \text{arc vers} \{ \text{vers } D (1 - [\sin h / \sin H]) \} .$$

Not only did the Muslims know of this formula, which is of Indian origin, already by the 9th century, but in the early 14th century an Egyptian astronomer Najm al-Dīn al-Miṣrī even tabulated T(H,D,h) for each 1º of all three arguments, with h < H and D < max D(H) – a total of about 440,000 entries.[33]

To find for different longitudes the altitude h_n of the sun at the n^{th} hour, they would use a formula equivalent to:

$$\sin h_n(\lambda) = \sin H(\lambda) \text{ x } \{ 1 - \text{vers} [D(\lambda) - T_n(\lambda)] / \text{vers } D(\lambda) \}.$$

Even in the 9th century leading astronomers such as Ḥabash al-Ḥāsib would have prepared a table of such a function as this by first compiling auxiliary tables of the functions sin H(λ) and vers D(λ) and then their quotient. Their trigonometric tables were, of course, adequate to the task. Nevertheless, interpolating in tables of the sine and versed sine that only had values to three sexagesimal digits for each degree of argument was fraught with risk of error.

Now Muslim astronomers also used an approximate formula that worked extremely well for all latitudes from the Yemen to N. Iraq. This was:

$$T^{sdh} = {}^{1}/_{15} \text{ arc sin} \{ \sin h / \sin H \}$$

This apparently simple formula is more complicated that it looks. It is accurate at the equinoxes for any latitude, but expressing the result in seasonal hours stabilizes the result for all latitudes, and the error is

32 Here I mean scholars like Ḥabash, Ibn Yūnus, Abu ‘l-Wafā’ and al-Bīrūnī. On the procedures of the last two, see the studies reprinted in Kennedy *et al*., *Studies in the Islamic Exact Sciences*, pp. 274-283.

33 A detailed analysis is in Charette, “A Monumental Medieval Table for Solving the Problems of Spherical Astronomy for all Latitudes”, and a brief discussion is in *SATMI*, I-2.6.1 and I-9.3.

minimal for the latitudes of Muslim centres of astronomical activity.[34] It is important to realize that this approximation was, in medieval terms, "universal" (medieval Arabic *āfāqī*, "serving all latitudes").[35] Although the formula was adopted by the Muslims from Indian sources, it was also known in Hellenistic astronomy and underlies the universal sundial that was known to Nasṭūlus. This relatively simple formula underlies the universal horary quadrant that was widely used on the backs of astrolabes in both the Islamic world but also, without any evidence of its being understood, in Medieval and Renaissance Europe.[36]

The inverse problem of finding the solar altitude at the seasonal hours is most simply solved using the approximate formula. First we find the length of any seasonal hour $T_n(\lambda, \varphi)$ (n = 1, 2, ... , 6) for latitude φ, in equatorial degrees, using:

$$T_n(\lambda) = n \times D(\lambda, \varphi) / 6 \qquad (T_6 = D)\ ,$$

and then the altitude of the sun at the n^{th} hour is:

$$h_n(\lambda) = \text{arc sin} \{ \sin H \times \sin T_n \}\ .$$

7 Some relevant astronomical tables

Tables of time as a function of solar altitude and solar meridian altitude for each degree of both arguments are known from 10th-century Baghdad, some specifically for the latitude of Baghdad and based on the accurate formula, others for all latitudes and based on an approximate formula. Here we have a graphical representation of the inverse problem: the determination of the solar altitude as a function of time throughout the year, using one or other of these formulae.

The 10th-century astronomer ʿAlī ibn Amājūr compiled a table of T(H,h) with values in equatorial degrees and minutes thereof (1^o = 4 minutes of time) for arguments:

$$H = 21^o, 22^o, \ldots, 84^o \text{ and } h = 1^o, 2^o, \ldots, H.$$

The values, which number *ca*. 3,300, are computed with remarkable accuracy for latitude 33º25', and the limits for H indicate that the table

[34] The history of this formula in the Islamic world and in Europe from 750 to 1900 is documented in *SATMI*, XI. The universal sundials are mentioned in App. C of that study.

[35] On the concept of "universality" in Islamic astronomy see *SATMI*, VIa-b.

[36] See *SATMI*, XIIa, on the universal horary quadrant.

was intended for timekeeping by the stars as well as by the sun.[37] The same astronomer also compiled a table of T(H,h) with values in seasonal hours and minutes for arguments:

$$H = 1^\circ, 2^\circ, \ldots 90^\circ \text{ and } h = 1^\circ, 2^\circ, \ldots , H ,$$

now based on the approximate formula and hence serving all latitudes.[38]

Values of the solar altitude as a function of time (seasonal hours 1-6) at the solstices form part of a set of tables compiled in Baghdad in the early 9th century for the construction of horizontal sundials.[39] These tables, for which each subset serves a specific latitude between 21º and 40º, are attributed to al-Khwārizmī, though Ḥabash al-Ḥāsib is also a candidate. They display not only the solar altitude at the hours, but also the polar coordinates (shadow and azimuth) needed for the construction of the hyperbolae on a sundial. The points given by these coordinates for each hour would then be connected with straight lines to produce the sundial. For the latitude of Baghdad (33º) and Samarra (34º) more values are given, respectively, for each 10 minutes and each 30 minutes. Surely Nasṭūlus had seen these tables, but, for his purposes, they would only have given him only a set of points along the vertical axis of his new instrument.

A table compiled by al-Khwārizmī displays the function h(T,H) to the nearest degree for each seasonal hour of T and each 1º of H, and this is based on the approximate formula and hence serves all latitudes.[40] To draw the lemon-shaped curves, one could calculate H for each few degrees of λ and then derive the values $h_n(H)$ from the table. If Nasṭūlus had used this rather crude table, I doubt that it would not have given him a set of smooth curves.

It is simpler to hypothesize that somebody compiled a table of the function h(T, λ) for latitude 33º, with values for each seasonal hour, that is, with 1,080 entries for each degree of λ (since the function is symmetric). The underlying value of the obliquity of the ecliptic would have been 23º35', used by Nasṭūlus on his astrolabes (as we can see from the lengths of maximum daylight that he used for various latitudes). We

37 *SATMI*, I-2.3.1.

38 *SATMI*, I-2.5.1.

39 *SATMI*, I-4.1.1.

40 *SATMI*, I-4.3.1.

do know of two such tables from the Yemen, which, particularly in the 13th and 14th centuries, was an important centre of astronomy. These, computed with either the accurate or the approximate formula, probably the latter, serve respectively the latitude of Sanaa by the Rasulid Sultan al-Ashraf himself, and the latitude of Taiz by the court astronomer Abu ’l-ʿUqūl.[41] Nasṭūlus possibly computed the necessary table for Baghdad himself.

With such a table Nasṭūlus could have constructed six sets of 72 or even 360 dots. Any serious mistakes in the table would have been immediately obvious as each new mark was about to be made and the position of the mark could have been corrected. One problem with this proposal is that it assumes that Nasṭūlus could have made the marks using a radial rule that was (a) accurately centred, and (b) accurately divided. Any defects in the radial rule would be reflected in the resulting curves, and these show no obvious traces of error.

8 The lemon-shaped curves

Fig. 6 shows the two sets of curves generated by a modern computer using the exact and the approximate formula.

The agreement with the curves based on the approximate formula seems to be better than with the curves based on the exact formula, but this is not definitively the case: see Fig. 7.

[41] *SATMI*, I-4.2.5 and 4.2.6.

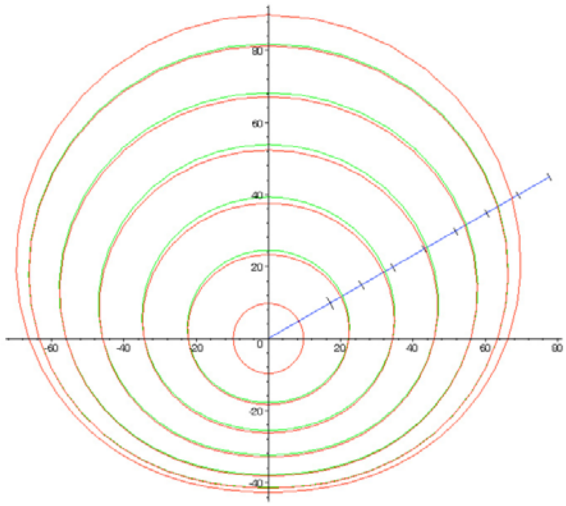


Fig. 6: Computer-generated curves based on the accurate formula (innermost) and on the standard medieval approximate formula (outermost). When these are superposed on an image of Nasṭūlus' curves the agreement is astounding: see Fig. 7. [Courtesy of Prof. Glen Van Brummelen.]

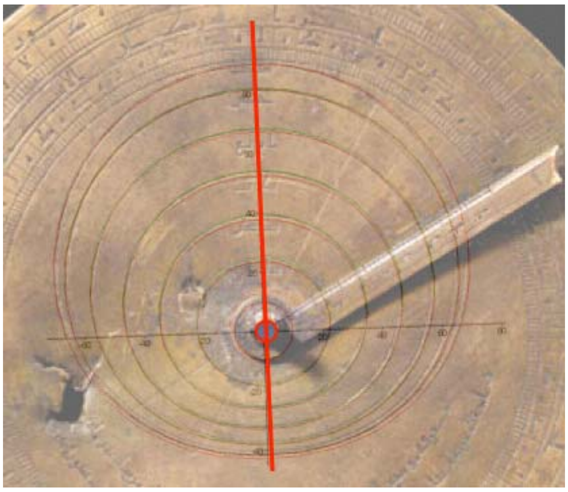


Fig. 7: The same printout laid over the markings on the actual instrument.

9 Some sundials with related markings

The lemon-shaped horary markings are of considerable historical interest, in that they are the earliest known graphical representation of curves on metal or on stone or marble that are *neither* circles nor conics.[42] Their execution is spectacularly accurate. What is particularly remarkable is that the curves are engraved smoothly. The "curves" on one horary quadrant for Baghdad – on the back of al-Khujandī's astrolabe dated 984/85[43] – and one sundial for Cordova datable *ca.* 1000[44] – that were made by leading astronomers are constructed by joining the base points with line segments. It is by no means clear how Nasṭūlus achieved this feat, and it is anticipated that this will prompt some debate. A microscopic analysis would surely yield some clues to the way in which Nasṭūlus engraved his curves.

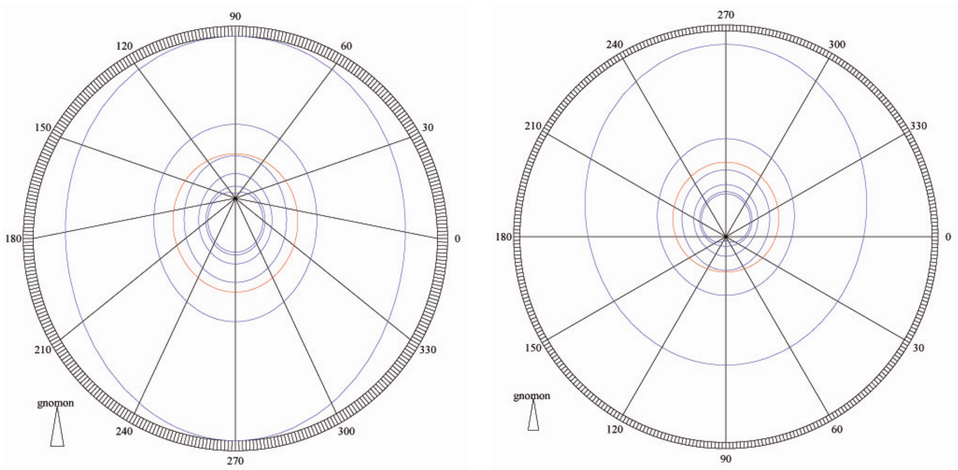
Similar curiously-shaped markings found on later sundials are graphical representations in the same format of the *shadows* at the hours, and the outer lemon-shaped ring is much larger than all of the others, so that the instruments look rather strange. There are no surviving examples of these sundials, and the diagrams in the manuscripts in which they are illustrated are not drawn with any great precision.

Fig. 8a: The diagram of a sundial in a 13th-century copy of al-Ṣūfī's astrolabe treatise from p. 469 of the Frankfurt facsimile, with the hour curves completely distorted by the copyist.

[42] I recall seeing a spiral on a Roman mosaic (?) in the Museo Nazionale Romano delle Terme in Rome.

[43] See *SATMI*, XIIIb-9 and Fig. 9b.

[44] *SATMI*, Fig. X-7.2 and Fig. 7.2.1.



Figs. 8b-c: Reconstructions of the markings (8a) and those on the later sundial of al-Marrākushī (from Charette, *Mathematical Instrumentation*, pp. 154-155, with kind permission).

Horizontal sundials based on the same principle were described by ʿAbd al-Raḥmān Ṣūfī (Shiraz, *ca*. 950), without a table, and al-Marrākushī (Cairo, *ca*. 1280), with a table,[45] and although these represent functional instruments giving the time from the gnomon shadow, there were so many more standard varieties available that, until the rediscovery of the present piece, one might have doubted that such complex sundials were ever actually constructed.

10 An old dispute partly resolved

Finally, this instrument is important for the history of instrumentation for another reason: it partly resolves the question of the origin of the solar / calendrical scales on Islamic instruments.[46] Julio Samsó has favoured an Andalusī origin.[47] Direct evidence from Late Antiquity of scales of this

[45] See Sezgin's facsimile editions of al-Ṣūfī's astrolabe treatise, p. 469, and of al-Marrākushī's encyclopaedic work on timekeeping, pp. 228-229, and on Najm al-Dīn's instrument see Charette, *Mathematical Instrumentation*, pp. 153-155.

[46] This is discussed in *SATMI*, X-4.7, *etc*. (see vol. 1, p. 1026 of the index).

[47] See, for example, Samsó, "Andalusian Astronomy", pp. 4-5, and *idem*, *Las ciencias de los antiguos en al-Andalus*, p. 33.

kind from either end of the Mediterranean is not available. I have pointed out that they occur on various Eastern Islamic instruments that are independent of any Andalusī influence, but I have nevertheless succumbed to accepting the possibility of an Andalusī origin for these too,[48] or even a hypothetical Roman origin for the Western Islamic tradition[49] or an equally hypothetical Graeco-Roman origin for the Eastern one. We know that the Andalusī Abu ’l-Ṣalt wrote about them in his astrolabe treatise, compiled whilst he was in prison in Alexandria *ca.* 1100, and this treatise was influential in both the Eastern and Western Islamic worlds. However, now we have an earlier example of them from Baghdad that is certainly without any Andalusī influence whatsoever.

11 Concluding remarks

Nasṭūlus’ instrument is compelling evidence of the sophistication of scientific and technical achievements in Baghdad about 1,100 years ago. It is an “instrument of mass calculation” *par excellence* and certainly never did anybody any harm. As Ted Kennedy used to say of Muslim astronomers and mathematicians as competent as Nasṭūlus, “their hearts were in the right place”.

Acknowledgements

It is a pleasure to thank Sotheby’s of London for the images of the new instrument, and David Sulzberger of the Ahuan Gallery in London for images of the third Nasṭūlus astrolabe. Likewise I thank Prof. Glen Van Brummelen of Quest University (Squamish, BC, Canada) for computer printouts of various sets of hour curves. Dr. François Charette provided all of the other graphics and read a preliminary version of this paper, thus saving me from several pitfalls.

[48] See, for example, *SATMI*, vol. 2, pp. 52-53 and 368.

[49] *SATMI*, II, p. 383, also *idem*, “Earliest Known European Astrolabe”, pp. 384-385.

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Schmidl: see Charette & Schmidl.

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