Astronomical Handbooks and Tables from the Islamic World (750-1900): an Interim Report

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with a contribution by Bernard R. Goldstein

To Ted Kennedy, in appreciation

Contents

1 Introduction

The purpose of this paper[1] is to present an overview of the current state of research on the medieval Islamic astronomical handbooks known as *zījes* in particular,[2] and astronomical tables in general. It is intended as a supplement to E. S. Kennedy's groundbreaking survey of Islamic *zījes* published in 1956,[3] incorporating the numerous categories of tables not contained in *zījes*,[4] and an attempt has been made to mention most of the research that has been done since that time.[5] It is also intended as an interim overview

1 The main authors (D.A.K. and J.S.) are grateful to François Charette, Benno van Dalen, Bernard Goldstein and David Pingree for their comments on earlier versions of this paper, and are alone responsible for any remaining errors and misinterpretations. We are particularly pleased that Bernard Goldstein was willing to contribute an important chapter on Hebrew *zījes*. Bibliographical abbreviations for the basic reference works for the history of Islamic astronomy and mathematics and other frequently-cited works are listed at the end of this paper.

2 The term *zīj* is traditionally derived from Pahlevi *zīk*, originally meaning 'thread' or 'cord' and already used in Pahlevi with the meaning 'astronomical tables', presumably by extension of the notion of a thread to the warp of a fabric and hence to the orthogonal framework of an astronomical table (Kennedy, "*Zīj* Survey", p. 123b, citing C. A. Nallino). However, recent research suggests that the term was originally used in Arabic to denote an astronomical text in verse, as in Sanskrit *tantra*, without any tables: see further R. Mercier, "From Tantra to Zīj", in *Kunitzsch Festschrift*, pp. 451-460.

3 Kennedy, "*Zīj* Survey". See also Samsó, "Tablas astronómicas".

4 A preliminary survey was presented in King, "Islamic Astronomical Tables".

5 The first version of this paper was prepared in answer to a request for an article "Zīdj" for the *EI*². It turned out to be far longer than the Editors could accept, so we decided to prepare this version, reformatting the first for this journal, suppressing all of the quirky features of the *EI*² (notably, the transliteration), but retaining most of the cross-references to other articles on specific topics in that monumental work (here capitalized, as, for example: ZĪDJ). We have, on the other hand, suppressed references to articles on individual astronomers in *EI*² and *DSB*. We assume on the part of the reader a certain familiarity with these works, as well as with the standard bio-bibliographical sources, namely (in chronological order): Suter, *MAA*; Renaud, "Additions à Suter"; Krause, "Stambuler Handschriften"; Storey, *PL*; Sezgin, *GAS*; Matvievskaya & Rosenfeld, *MAMS*; *Cairo ENL Survey*; and İhsanoğlu, ed., *Ottoman Astronomy* and *Ottoman Mathematics*. Several of the studies listed in this article are reprinted in various volumes

anticipating various significant advances in the field in the not-too-distant future.[6]

Most of the relevant astronomical and astrological concepts mentioned below are clearly explained in al-Bīrūnī's *Tafhīm*.[7] Kennedy's numbering 1-109 for *zīj*es and X200-X220 for related works (see below) is used in this article, with each number preceded by a K and the Xs suppressed. An asterisk is used for those *zīj*es singled out by Kennedy for treatment in greater detail than the rest. The notation K∅ means the work was not listed by Kennedy.

such as: Goldstein, *Studies*; Hartner, *Studies*, A-B; Kennedy *et al.*, *Studies*; Kennedy, *Studies*; King, *Studies*, A-C; Kunitzsch, *Studies*; Langermann, *Studies*; Lorch, *Studies*; Millás Vallicrosa, *Estudios*, A-B; Nallino, *Scritti*, V; Sabra, *Studies*; Saliba, *Studies*; Samsó, *Studies*; Schoy, *Beiträge*; Suter, *Beiträge*; Vernet, *Estudios*, A-B; Vernet, ed., *Estudios*, A-B; Wiedemann, *Aufsätze* and *idem*, *Schriften*. Also important for the study of transmission of Islamic tables to Byzantium and Europe are Tihon, *Studies*, and Poulle, *Studies*. Other studies are published in collected works, such as *EHAS*; *Kennedy Festschrift*; *Kunitzsch Festschrift*; *North Festschrift*; *Sayılı Memorial Volumes*; and *Vernet Festschrift*. Certain conferences have produced useful publications, in particular: Aleppo 1976; Istanbul 1977; Istanbul 1981; New Delhi 1985; Istanbul 1986; Istanbul 1987; Istanbul 1991 and 1994; Norman (Oklahoma) 1995; Strasbourg 1995; Kyoto 1997; Paris 1998 (Fatimids); and Cambridge, Ma. (Dibner Institute) 1998.

The monumental reprint series *Islamic Mathematics and Astronomy* recently published by the indefatigable F. Sezgin in Frankfurt in over 100 volumes contains numerous articles and books (texts and studies) relevant to this study and generally difficult to access. Suffice it to mention here that materials relating to al-Khwārizmī feature in vols. 1-7; al-Farghānī 9-10, 68; al-Battānī 11-13; Thābit ibn Qurra 21-22; Ibn Yūnus 25; al-Ṣūfī 26; Ibrāhīm ibn Sinān 27, 101; Abū Naṣr 28, 37; al-Bīrūnī 29-36, 74; Ibn al-Zarqālluh 39-40; al-Marrākushī 41-42; Ibn al-Bannā' 43-44; 'Umar al-Khayyām 45-46; al-Ṭūsī *et al.* 47-51; Ulugh Beg 52-55; al-Kāshī 56, 84; Ibn al-Haytham 57-58, 75; Abu 'l-Wafā' 60-61; al-Sijzī 66; Maimonides 67; Alfonso X 98-99; calendrics 64-65; astronomical instruments 85-96; Sayılı's *Observatory in Islam* 97; Nallino's *Ta'rīkh 'ilm al-falak* 100.

6 In Barcelona research on Andalusī and Maghribī tables (by J. Samsó and his team) continues apace; in Frankfurt research on *zīj*es in general (B. van Dalen) and various categories of other tables (D. A. King and F. Charette) is in progress.

7 Listed as al-Bīrūnī, *Tafhīm*; see also Kennedy, "*Zīj* Survey", pp. 139-145. Also recommended to the reader interested in ancient and medieval astronomy is J. Evans, *The History and Practice of Ancient Astronomy*, New York & Oxford 1998.

1.1 The scope of the zīj *literature*

In the 13th century the Yemeni astronomer Muḥammad ibn Abī Bakr al-Fārisī was able to cite the names of 28 *zījes*, and in the 16th-century Indian encyclopædist Abu 'l-Faḍl al-ʿAllāmī listed in his *Ā'īn-i Akbarī* the titles of 86 works of this genre. In 1956 E. S. Kennedy presented information on some 125 *zījes*. We now know that over 225 *zījes* were compiled in the Islamic world during the period from the 8th to the 19th century. They constitute a major source for our understanding of the development and application of mathematical astronomy in the medieval period. Of these works just less than one-half are lost and known only by references to their titles or their authors, but enough survive to convey a very clear impression of the scope and variety of the activities of the Muslim astronomers in this field and to reveal some of their most outstanding contributions.[8]

Zījes are intended to serve a single locality, in the sense that a terrestrial longitude underlies the solar, lunar and planetary tables and a terrestrial latitude underlies the tables for spherical astronomy. Some of the more important *zījes* were the results of serious observational programmes.[9] However, many *zījes* were simply rehashings of earlier ones, with minor variations, such as a change of meridian for the planetary tables, or a new set of spherical astronomical tables for a different latitude. Such modified versions can be of singular historical importance if the original works are no longer extant. Not all *zījes* contain the extensive explanations of the astronomical and mathematical background typical of, say, Ptolemy's *Almagest*. Furthermore, there are numerous extensive sets of tables which do not constitute a *zīj* or which are not found in any *zīj*.

8 The best survey of Islamic astronomy and astrology in general remains the article by C.A. Nallino in the J. Hastings' *Encyclopædia of Religion and Ethics*, XII (1921), pp. 88-101, shortened from the Italian version in Nallino, *Scritti*, V, pp. 2-87; see also the same author's articles ASTROLOGY and ASTRONOMY in EI^1. Two recent overviews are G. Saliba, "Astronomy / Astrology, Islamic", in *idem*, *Studies*, no. 2 (first published in the *Dictionary of the Middle Ages* in 1982), and King, "Mathematical Astronomy in Islamic Civilisation", in *Astronomy across Cultures*, pp. 585-613; a third by F. J. Ragep is currently in preparation.

9 See Sayılı, *The Observatory in Islam*, and also the EI^2 article MARṢAD.

1.2 The purpose of a zīj

The purpose of a *zīj* was to provide astronomers with all that they needed in the way of theory and tables for such tasks as calculating the positions (longitudes and latitudes) of the Sun, Moon and five naked-eye planets and, the time of day or night from solar or stellar altitudes. In addition the astronomer could use a *zīj* to determine the possibility of lunar crescent or planetary visibility. Stellar positions he could simply take from the star-catalogue. Calculations for meridians other than that underlying the tables could be modified for the longitude differences apparent from the geographical tables. The astronomer could calculate the duration of twilight and the altitude of the Sun at midday or at the time of the afternoon prayer. He could apply the mathematical procedures outlined in the *zīj* to specific geographical data and compute the *qibla* of any locality. He could also determine the ascendant at a given time and the longitudes of the astrological houses, and having calculated the positions of the Sun, Moon and planets, he could set up a horoscope: it could be argued that this was the main purpose of *zīj*es, but there is precious little historical evidence how these works were used in practice. In any case, there are a host of other useful operations one can learn from any *zīj*. But a *zīj* was only part of the astronomer's equipment. Other sets of tables were compiled which were not usually contained in *zīj*es, and which greatly facilitated some of the above tasks. In addition, various instruments were available, notably for solving problems relating to spherical astronomy and timekeeping.[10]

1.3 Regional schools of astronomy

In any discussion of Islamic astronomy it is important to keep in mind that after the 10th century regional schools of astronomy developed in the Islamic world, with different authorities, different interests and specialities.[11]

[10] On Islamic astronomical instrumentation see the *EI*^{2} articles ASṬURLĀB, RUBʿ, SHAKKĀZIYYA, ṬĀSA and MIZWALA; King, "Astronomical Instruments between East and West", in *Kommunikation zwischen Orient und Okzident*, H. Kühnel ed., Vienna 1994, pp. 143-198; and J. Vernet and J. Samsó, eds., *El legado científico andalusí*, Madrid 1992. A new overview is to appear as *SATMI*, VIII.

11 On regional developments in Islamic astronomy see first Sayılı, *The Observatory in Islam*. The following regional surveys are available:
Early Islamic (mainly al-ʿIrāq and Iran): D. Pingree, "The Greek Influence on Early Islamic Mathematical Astronomy", *JAOS* 93 (1973), pp. 32-43; *idem*, "Indian Influence on Sassanian and Early Islamic Astronomy and Astrology", in *The Journal of Oriental Research* (Madras), 34-35 (1964-66), pp. 118-126.
Iraq: al-ʿAzzāwī, *Ta'rīkh ʿilm al-falak fī 'l-ʿIrāq* ..., Baghdad 1958 (to be used with caution).
Egypt, Syria and Yemen: D. A. King, "Aspects of Fatimid Astronomy ... ", in *Paris 1998 Fatimid Colloquium Proceedings*, pp. 497-517; *idem*, "The Astronomy of the Mamluks", *Isis* 74 (1983), pp. 531-555, repr. in *idem*, *Studies*, A-III; *idem*, "L'astronomie en Syrie à l'époque islamique", in *Syrie, mémoire et civilisation*, S. Cluzan *et al.* eds., Paris 1993, pp. 386-395, 432-443 and 480; and *idem*, *Mathematical Astronomy in Medieval Yemen – A Bio-Bibliographical Survey*, Malibu, Ca. 1983.
al-Andalus: Millás Vallicrosa, *Estudios*, A-B; Vernet, *Estudios*, A-B; Vernet, ed., *Estudios*, A-B; J. Samsó, *Las ciencias de los antiguos en al-Andalus*, Madrid 1992; numerous other publications of the Barcelona school; and L. Richter-Bernburg, "Ṣāʿid, the *Toledan Tables*, and Andalusī Science", in *Kennedy Festschrift*, pp. 373-401. See also G. Toomer, "A Survey of the *Toledan Tables*", *Osiris* 15 (1968), pp. 5-174; and J. D. North, "Just whose were the Alphonsine Tables?", in *Vernet Festschrift*, I, pp. 453-476.
The Hebrew tradition: B. R. Goldstein, "The Survival of Arabic Astronomy in Hebrew", *JHAS* 3 (1979), pp. 31-39, repr. in *idem*, *Studies*, XXI; *idem*, "Astronomy in the Medieval Spanish Jewish Community", in *North Festschrift*, pp. 225-241; Y. T. Langermann, "Science in the Jewish Communities of the Iberian Peninsula: an Interim Report", in *idem*, *Studies*, I, and *idem*, "Hebrew Astronomy ... ", in *Astronomy across Cultures*, pp. 555-584.
The Maghrib: D. A. King, "On the History of Astronomy in the Medieval Maghrib", in *Études philosophiques et sociologiques dédiées à Jamal ed-Dine Alaoui*, Fez 1998, pp. 27-61; J. Samsó, "An Outline of the History of Maghribī *Zījes* from the End of the Thirteenth Century", *JHA* 29 (1998), pp. 93-102.
Iran and Central Asia: E. S. Kennedy, "The Exact Sciences in Iran under the Seljuqs and Mongols", in *The Cambridge History of Iran*, vol. V, Cambridge 1968, pp. 659-679, and "The Exact Sciences in Iran under the Timurids", in *The Cambridge History of Iran*, VI, Cambridge 1986, pp. 568-580; H. J. J. Winter, "Persian Science in Safavid Times", *ibid.*, pp. 581-609; T. Heidarzadeh, "From the Maragha School to the Darolfonun – a Historical Review of Astronomy in Iran from the 13th to the 19th Century", paper presented at the Istanbul 1994 Symposium but not included in the *Proceedings*; King, *World-Maps* (cited in n. 43 below), pp. 128-134; and the EI^2 article ṢAFAWIDS, IV.III.
The Byzantine tradition: A. Tihon, "Tables islamiques à Byzance", *Byzantion – Revue Internationale des Études Byzantines* 55 (1990), pp. 401-425, repr. in *eadem*, *Studies*, VI, and numerous studies by D. Pingree mentioned below.

They also achieved different levels of sophistication and had different fates, both with regard to their own internal development and to the nature of their encounter with the new Western science from the 16th century onwards. In the past few years some attention has been paid to these regional schools, and the main studies are listed below. In particular, each regional school had its favorite *zīj* or *zījes*.

In the early period, in the central lands of Islam the *Mumtaḥan* tradition was important, together with al-Battānī and Abu 'l-Wafā' al-Būzajānī. In Iran and Central Asia numerous early *zījes* eventually gave way to the *Zīj-i Īlkhānī* of Naṣīr al-Dīn al-Ṭūsī. In Egypt the *Ḥākimī Zīj* of Ibn Yūnus was never surpassed. In Syria the *zīj* of Ibn al-Shāṭir was particularly important. In both Egypt and Syria the institution of the *muwaqqit*, the professional astronomer responsible for the regulation of the calendar and times of prayer, led to new and impressive developments, mainly, but not only, in astronomical timekeeping.[12] In al-Andalus the *zījes* of al-Khwārizmī and al-Battānī, based respectively on the Indo-Persian and Hellenistic traditions, played a role perhaps greater than either deserved, and the contributions of Ibn al-Zarqālluh were highly significant. In the Maghrib, the *zīj* of Ibn Isḥāq and a mini-version thereof prepared by Ibn al-Bannā' dominated the scene. In the late period, the only *zīj* to achieve any kind of supremacy all over the Islamic world was the *Zīj-i Sulṭānī* of Ulugh Beg.

Ottoman Turkey: E. İhsanoğlu, "Introduction of Western Science to the Ottoman World: a Case Study of Modern Astronomy (1660-1860)", in *Istanbul 1987 Symposium Proceedings*, pp. 67-120; and *idem*, ed., *Ottoman Astronomy* and *Ottoman Mathematics*.

India: S. A. Khan Ghori, "Development of *Zīj* Literature in India", *IJHS* 20 (1985), pp. 21-48, 438-441 (notes) and 480-481 (bibl.); S. M. R. Ansari, "On the Transmission of Arabic-Islamic Astronomy to Medieval India", *AIHS* 45 (1995), pp. 273-297; D. Pingree, "Islamic Astronomy in Sanskrit", *JHAS* 2 (1978), pp. 315-330, and *idem*, "Indian Reception of Muslim Versions of Ptolemaic Astronomy", in *Norman 1995 Conference Proceedings*, pp. 471-485.

12 King, "On the Role of the Muezzin and the Muwaqqit in Medieval Islamic Society", in *Norman 1995 Conference Proceedings*, pp. 285-346, repr. as *SATMI*, V.

1.4 The historiography of modern research on zījes

Already in the 19th century, extracts dealing with observation accounts from the *zīj* of Ibn Yūnus had been published by Caussin de Perceval, and the introduction to the *zīj* of Ulugh Beg had been published by L.-A. Sédillot (*fils*). Before 1950 only two *zīj*es had been published in their entirety, namely, those of al-Battānī by C. A. Nallino and of al-Khwārizmī by H. Suter *et al.* (the latter is extant only in a form substantially different from the original). In 1952 J. Vernet published the canons of the *zīj* of Ibn al-Bannā'. The *zīj* of al-Bīrūnī was published in 1954-56, in an edition originally prepared by M. Krause. In 1956 E. S. Kennedy published his survey of some 125 *zīj*es, and laid the foundation for serious study of this genre of literature. In 1962 O. Neugebauer published an English translation of the text of the *Zīj* of al-Khwārizmī in the form in which it survives for us, together with a commentary and an analysis of the tables. The year 1985 saw the publication of a Byzantine recension of the *zīj* of al-Fahhād by D. Pingree. Many studies of parts of other *zīj*es, of specific topics in several *zīj*es, or of individual tables of particular interest have been published during the past 50 years. Also certain medieval treatises on the concepts underlying *zīj*es, the *kutub 'ilal al-zījāt*,[13] have attracted attention – we shall have occasion to mention the works of al-Hāshimī, Ibn al-Muthannā, Ibn Masrūr, Ibn 'Ezra and Muḥammad ibn Abī Bakr al-Fārisī. Others are lost, (including those by al-Farghānī, al-Sarakhsī, Thābit ibn Qurra and al-Bīrūnī, or are currently being studied, notably that of Samaw'al al-Maghribī.[14]

In the 1970s the first author (D.A.K.) identified various categories of tables not found in *zīj*es, although some of the larger sets are mistitled or miscatalogued as *zīj*es.[15] Such tables will also be treated here: see Section 4 below.[16]

13 See the *EI*² article 'ILLA.

14 Sezgin, *GAS*, VI, pp. 65-66.

15 See already n. 4.

16 The *EI*² article <u>DJ</u>ADWAL deals mainly with magical arrangements of letters and symbols.

2 The contents of *zījes*

2.1 Sexagesimal alphanumerical notation

The entries in the tables in *zījes* and other corpora of tables are expressed sexagesimally, that is, to base 60, although integers are invariably expressed decimally. (In the modern notation standard in the history of the exact sciences a number expressed in the form $a,b;c,d$ stands for $a \times 60 + b + c/60 + d/3600$.) The entries are written in Arabic alphanumerical notation,[17] with the attendant traps for the careless copyist and the trusting reader. One of the challenges to modern investigators, indeed, for some, the spice of their lives, is the restoration of original values from carelessly-copied entries. The standard errors are inevitable or careless or compound. 'Inevitable' refers to those cases where the omission of a diacritical point or two in one copy of a table invites an ambiguous interpretation in the next copy (thus, 14 ↔ 54 or 59 ↔ 19 or 80 ↔ 100). 'Careless' refers to situations where the sloppy rendition of one letter or ligature leads to its misinterpretation as another (thus, 0 ↔ 5, 14 ↔ 15, 40 ↔ 47, 50 ↔ 7, 20 or 21 ↔ 9, 38 ↔ 18, 44 ↔ 47, or 18 ↔ 70). 'Compound' refers to a combination of the previous two (thus 14 ↔ 15 ↔ 55 or 58 ↔ 18 ↔ 13 or 150 ↔ 87).

2.2 Chronology and calendar conversion

All *zījes* begin with one or more chapters and sets of tables devoted to the definition of the various eras and calendars in use at the time and place of writing, to methods of converting dates from one calendar to another, and to the problem of determining the *madkhal*, that is, the day of the week corresponding to the first day of a given year and month in a given calendar. The most common are the lunar Hijra calendar and various solar calendars,

[17] See the EI^2 article ABDJAD; R. A. K. Irani, "Arabic Numeral Forms", *Centaurus* 4 (1955), pp. 1-12, repr. in Kennedy *et al.*, *Studies*, pp. 710-721; also the comments, based on experience, often bitter, with medieval tables of one kind or another, in Kennedy & Kennedy, *Islamic Geographical Tables*, p. x; Kunitzsch, *Sternkatalog des* Almagest (cited in n. 44 below), I, pp. 19-21; and King, *World-Maps* (n. 43), pp. 161-163.

including the Seleucid (Alexander), the Coptic (Diocletian), and the Persian (Yazdijird), and, in the West, the Julian (A.D. and Spanish Era). The Persian calendar, using the Egyptian year of 365 days and no intercalation, is particularly convenient for astronomical purposes. Less commonly treated calendars are the Jewish, Syrian, Malikī, Śaka and Chinese-Uighur.[18] A few *zījes* treat the lunar mansions and the Arab system of dividing the year according to the mansions.[19]

2.3 *Trigonometry*

All *zījes* contain trigonometric tables, usually of at least the sine (*al-jayb*) and the cotangent (*al-ẓill*) functions. The sine, first used by Indian astronomers, replaced the Ptolemaic chord function (*al-watar*) amongst the Muslims. The argument of the sine was an arc (rather than an angle) θ of a circle of radius R units, where R is a base, usually 60, occasionally 1. In the Indian tradition R was taken as 120, 150, 1000, 3270, 3438, *etc.*; in the Hellenistic tradition it was taken as 60. The medieval sine function is denoted by $\mathrm{Sin}\,\theta$ and is related to Ptolemy's chord function and the modern function by: $\mathrm{Sin}\,\theta = \frac{1}{2}\,\mathrm{Ch}(2\theta) = R\sin\theta$. In timekeeping the versed sine (*al-sahm*) was also used ($\mathrm{Vers}\,\theta = R - \mathrm{Cos}\,\theta = R\,(1-\cos\theta)$), and occasionally also the cosecant function (*quṭr al-ẓill*) ($\mathrm{Csc}\,\theta = R^2/\mathrm{Cos}\,\theta = R\csc\theta$). The earli-

18 See F. K. Ginzel, *Handbuch der mathematischen und technischen Chronologie*, 3 vols., Leipzig 1906, especially vol. I; Kennedy, "Zīj Survey", p. 139; various articles in Kennedy *et al.*, *Studies*, pp. 652-709; B. van Dalen, E. S. Kennedy and M. K. Saiyid, "The Chinese-Uighur Calendar in Ṭūsī's *Zīj-i Īlkhānī*", *ZGAIW* 11 (1997), pp. 111-152, and also the EI^2 article TA'RĪKH (2: Era chronology in astronomical handbooks) by B. van Dalen.

19 See the EI^2 articles ANWĀ' and MANĀZIL; and also Samsó, "Calendarios populares y tablas astronómicas"; M. Forcada, "Books of *Anwā'* in al-Andalus", in *The Formation of al-Andalus*, Part 2: *Language, Religion, Culture and the Sciences*, M. Fierro and J. Samsó eds., Aldershot (Ashgate-Variorum) 1998, pp. 305-328; D. M. Varisco, *Medieval Folk Astronomy and Agriculture in Arabia and the Yemen*, Aldershot & Brookfield, Vt. (Variorum) 1997; *idem*, *Medieval Agriculture and Islamic Science – The Almanac of a Yemeni Sultan*, Seattle, Wa. 1993; and *idem*, "Islamic Folk Astronomy", in *Astronomy across Cultures*, pp. 615-650.

est sine tables, from the 9th century, gave values to three sexagesimal places for each 1° of argument. By the 15th century accurate tables were available displaying the function to five places for each minute of argument. This was achieved by first deriving a very precise value of Sin 1° and utilizing a clever method of second-order interpolation.[20] The cotangent function invariably had the solar altitude as argument, and used a base equal to the length of a gnomon, so that the function measures the length of the horizontal shadow cast by the gnomon. The units for the gnomon length used were 12 digits (*iṣba*ʿ, *aṣābi*ʿ), or 7 feet (*qadam*, *aqdām*), although other values were also used. Al-Bīrūnī has a detailed discussion in his treatise *On Shadows* (*Ifrād al-maqāl fī amr al-ẓilāl*). The cotangent function was also first tabulated in the 9th century.[21] Trigonometric fuctions were occasionally tabulated independently, that is, not in *zīj*es – see 4.2. Various procedures were used for interpolation in tables.[22]

2.4 Spherical astronomical functions

The study of spherical astronomy – the mathematics of the celestial sphere and of the apparent daily rotation of the sphere – was of prime concern to Muslim astronomers, not least because of the importance of astronomical time-keeping.[23] The formulæ for deriving time from solar or stellar altitude

20 A. Aaboe, "al-Kāshī's Iteration Method for the Determination of Sin 1°", *Scripta Mathematica* 29 (1954), pp. 24-29.

21 See further Kennedy, "*Zīj* Survey", pp. 139-140; Schoy, "Beiträge zur arabischen Trigonometrie (Originalstudien nach unedierten arabisch-astronomischen Manuscripten)", *Isis* 5 (1923), pp. 364-399, and *Die Gnomonik der Araber*, Bd. I:F of *Die Geschichte der Zeitmessung und der Uhren*, E. von Bassermann-Jordan ed., Berlin & Leipzig 1923, both repr. in *idem*, *Beiträge*, II, pp. 448-483 and 351-447; al-Bīrūnī, *On Shadows*, pp. 71-80; and also J. L. Berggren, *Episodes in the Mathematics of Medieval Islam*, New York, *etc.* 1986, pp. 127-156.

22 See J. Hamadanizadeh, "A Survey of Medieval Islamic Interpolation Schemes", in *Kennedy Festschrift*, pp. 143-152, and the *EI*² article TAʿDĪL BAYN AL-SAṬRAYN.

23 See now King, *SATMI*, especially I-II on tables for timekeeping and the regulation of the times of Muslim prayer. A summary is already in the *EI*² survey article MĪḲĀT.

were known from the 8th century onwards and are discussed in every *zīj*. We can distinguish between several groups of functions that were regularly tabulated in *zījes*:[24]

- The solar declination (*mayl al-shams* or *al-mayl*) as a function of solar longitude (*darajat al-shams*). Underlying such tables was a value for the obliquity of the ecliptic (*al-mayl al-aʿẓam*), a parameter which changes slowly with time. The second declination (*al-mayl al-thānī*) was used in celestial coordinate transformations.[25]

- The half length of daylight (*niṣf qaws al-nahār*) for different latitudes, as a function of solar longitude, in equatorial degrees and minutes, or in hours and minutes. Sometimes the latitude-dependent tables would be presented for the seven climates (*iqlīm*) of Antiquity.[26] The tangent of the declination was often tabulated; it is an auxiliary function useful in the determination of the length of daylight for any latitude (see below).

- The right ascensions (*al-maṭāliʿ fi 'l-falak al-mustaqīm*) as a function of ecliptic longitude (*al-ṭūl*), defining the rising time of a given arc of the ecliptic (measured from the vernal point) over the horizon at the equator, and the oblique ascensions (*al-maṭāliʿ al-baladiyya*), defining the corresponding times for the horizons of different localities. Often the latter would be tabulated for a series of latitudes.[27]

24 On basic tables for spherical astronomy see Kennedy, "Zīj Survey", pp. 140-141; D. A. King, *Astronomical Works of Ibn Yūnus* (cited in n. 115 below); al-Marrākushī, *Mabādiʾ wa-ghāyāt*; J. L. Berggren, "Spherical Astronomy in Kūshyār ibn Labbān's *Jāmiʿ Zīj*", in *Kennedy Festschrift*, pp. 15-33; and E. S. Kennedy, "Spherical Astronomy in al-Kāshī's *Khāqānī Zīj*", *ZGAIW* 2 (1990), pp. 1-46, repr. in *idem*, *Studies*, VII.

25 See the *EI*[2] articles MAYL, MINṬAḲA and SHAMS.

26 See the *EI*[2] article IḲLĪM, and on the importance of the climates see King, "Astronomical Instruments" (n. 10), pp. 152, 168-169, and *idem*, "Bringing Astronomical Instruments Back to Earth: The Geographical Data on Medieval Astrolabes (to *ca.* 1100)", in *North Festschrift*, pp. 3-53, esp. pp. 6-9.

27 On ascensions see the *EI*[2] article MAṬĀLIʿ.

- The solar meridian altitude (*irtifāʿ al-shams li-niṣf al-nahār*), and less frequently the rising amplitude of the Sun (*saʿat al-mashriq*) and the solar altitude in the prime vertical (*al-irtifāʿ alladhī lā samt lahu* or *al-irtifāʿ al-ʿadīm al-samt*), all for specific latitudes. Likewise the solar altitude in the azimuth of the *qibla* (*irtifāʿ al-shams idhā marrat bi-samt al-qibla*) for specific localities, also tabulated for each degree of solar longitude.

- Certain functions with no immediate astronomical significance were also tabulated on account of their utility in the computation of other functions. We may mention as examples such 'auxiliary' functions as the tangent of the declination (labelled *fuḍūl al-maṭāliʿ li-'l-arḍ kulliha*), the sine of the right ascension (*jayb al-maṭāliʿ*), and the product of the cosines of the declination and the terrestrial latitude (*al-aṣl al-muṭlaq*).[28]

Numerous other minor tables are found in corpora of tables relating to astronomical time-keeping, independent of *zīj*es – see 4.8-9. These collections also contain some more extensive tables for time-keeping. The only variety of these occasionally found in *zīj*es is a table displaying the time (T) since rising of the Sun or any star as a function of the meridian altitude (H) and the instantaneous altitude (h), for a specific latitude: these tables are trapezoidal in shape since $h < H$ and are called *zīj al-ṭaylasān*, after the name of a shawl. A *ṭaylasān* table for Maragha is contained in the *Īlkhānī Zīj*.[29] We should also mention the extensive tables for spherical astronomy in the compendium on astronomical instrumentation by al-Marrākushī (Cairo *ca.* 1280).[30]

28 See King, *SATMI*, I-6,7,8.

29 Kennedy, "*Zīj* Survey", p. 161b.

30 al-Marrākushī, *Mabādi' wa-ghāyāt*, A. See also King, *SATMI*, I-4.3.2, and II-6.7.

2.5 *Planetary mean motions, equations and latitudes*

These constitute the hard core of all *zījes*.[31] Extensive tables display the epoch positions and the mean motions (*wasaṭ*, pl. *awsāṭ*) of the 'planets' (*al-kawākib al-sayyāra* or *al-mutaḥayyara*), that is, the Sun, Moon and five naked-eye planets. The tables are intended for a specific terrestrial longitude, usually that of the locality where the *zīj* was compiled. They can easily be modified to the meridian of another locality, if required: sometimes subtables of longitude corrections are provided. The motions for a given number of completed years, months, days and hours are to be added to the epoch positions (Arabic *aṣl*, Latin *radix*) to derive the actual mean positions. These then needed to be modified by equations (*ta'dīl*, pl. *ta'ādīl*) to derive the true ecliptic positions (sometimes called *al-muḥkam* or *al-mu'addal*). The operation of finding the true positions is called *taqwīm* or *ta'dīl*, the former expression also used for ephemerides (see 4.3), and the positions *muqawwam* or *mu'addal*. The professional astronomer who did this could be called *muqawwim*, but only one example of this usage is known, namely, in a signature on an astrolabe made in Damascus in 1222/23.[32] In the Ptolemaic tradition the equations are calculated by successive applications of a series of auxiliary trigonometric functions for each planet.[33] The more extensive double-argument tables for the equations sometimes found in *zījes* are discussed in 3.8 and 4.6. The apogees (*awj*, pl. *awjāt*) of the planets

31 Kennedy, "*Zīj* Survey", pp. 141-142; *idem* and H. Salam, "Solar and Lunar Tables in Early Islamic Astronomy", *JAOS* 87 (1967), pp. 492-497, repr. in Kennedy *et al.*, *Studies*, pp. 108-113; *idem*, "Two Medieval Approaches to the Equation of Time", *Centaurus* 31 (1988), pp. 1-8, repr. in *idem*, *Studies*, VIII; B. van Dalen, "al-Khwārizmī's Tables Revisited: Analysis of the Equation of Time", in *Vernet Festschrift*, I, pp. 195-252, *idem*, "A Table for the True Solar Longitude in the *Jāmi' Zīj*", in *Frankfurt IGN Festband*, pp. 171-190; and G. Van Brummelen, "Mathematical Methods in the Tables of Planetary Motion in Kūshyār ibn Labbān's *Jāmi' Zīj*", *HistMath* 25 (1998), pp. 265-280. On practical solutions of the various operations involving Sun, Moon and planets, down to eclipses, see various studies based on al-Kāshī's treatise on the equatorium reprinted in Kennedy *et al.*, *Studies*, pp. 448-491.

32 *Sayılı Memorial Volumes*, II, pp. 730-731.

33 See the *EI*[2] article SHAMS, ḲAMAR and TA'DĪL.

need to be considered in such calculations, and their positions, which vary with time, were also tabulated. Occasionally, especially, but not only, in Andalusī and Maghribī *zīj*es, we find tables relating to trepidation (*al-iqbāl wa-'l-idbār*), the presumed oscillation of the equinoxes – see further 3.7. Additional auxiliary tables enabled the computation of the planetary latitudes (*al-ʿarḍ*).[34] In the case of the Moon, a single table would suffice, the argument being the nodal distance, derived from the lunar longitude and the position of the ascending node, which was tabulated along with the mean motions. In the calculation of solar and lunar eclipses (see 2.7) also the equation of time,[35] that is, the difference between true and mean solar time, with a maximum of around 30 minutes, had to be considered; tables for this function were likewise standard in *zīj*es. All of the necessary instructions for using these various tables are found in the typical *zīj*. See also 2.12 on modern methods of analyzing medieval planetary tables.

2.6 *Planetary stations and visibility*

Additional tables enabled the investigation of the direct and retrograde motions of the planets, their stations, and their visibility (depending on their apparent elongation from the Sun).[36]

2.7 *Solar and lunar parallax and eclipses*

In *zīj*es we also find tables for calculating the parallax of the Sun and Moon (*ikhtilāf al-manẓar*), preparatory to the prediction of eclipses (*kusūf* for the Sun and *khusūf* for the Moon[37]). This would be achieved by means of tables

34 E. S. Kennedy and W. Ukashah, "al-Khwārizmī's Planetary Latitude Tables", *Centaurus* 14 (1969), pp. 86-96, repr. in Kennedy *et al.*, *Studies*, pp. 125-135; M. Viladrich, "The Planetary Latitude Tables in the *Mumtaḥan Zīj*", *JHA* 19 (1988), pp. 257-268; and B. van Dalen, "Planetary Latitude Tables in the *Huihui li* (II)" (cited in n. 108 below).

35 See the *EI*² article TAʿDĪL AL-ZAMĀN, and also the studies of E. S. Kennedy and B. van Dalen cited in n. 31.

36 E. S. Kennedy and M. Agha, "Planetary Visibility Tables in Islamic Astronomy", *Centaurus* 7 (1960), pp. 134-140, repr. in Kennedy *et al.*, *Studies*, pp. 144-150.

of the times of syzygies, of true solar and lunar motions in small critical periods of time, of apparent solar and lunar radii, and others.[38] There is no survey of Islamic procedures for the computation of eclipses,[39] but this is currently being undertaken by B. van Dalen.

2.8 Lunar visibility

Particular attention was paid by Muslim astronomers to the prediction of the visibility of the lunar crescent on the first evening after a conjunction of the Sun and Moon.[40] From the 9th century onwards tables were prepared to facilitate such predictions, underlying which were limiting conditions on various functions based on the apparent positions of the Sun and Moon relative to each other and to the local horizon. Numerous such tables, of varying sophistication and complexity, are found in various *zījes*.[41] See also 4.5.

37 On various not-too-technical aspects of the determination of eclipses see the *EI*[2] article KUSŪF.

38 E. S. Kennedy, "*Zīj* Survey", pp. 143-144; *idem*, "Parallax Theory in Islamic Astronomy", *Isis* 47 (1956), pp. 33-53, repr. in *idem et al.*, *Studies*, pp. 164-184; also B. R. Goldstein, "Lunar Velocity in the Middle Ages: a Comparative Study", in *Vernet Festschrift*, I, pp. 181-194.

39 See already E. S. Kennedy and N. Faris, "The Solar Eclipse Technique of Yaḥyā ibn Abī Manṣūr", *JHA* 1 (1970), pp. 20-38, and J. A. As-Saleh, "Solar and Lunar Distances and Apparent Velocities in the Astronomical Tables of Ḥabash al-Ḥāsib", in *al-Abhath* (Beirut) 23 (1970), pp. 129-177, repr. in *idem et al.*, *Studies*, pp. 185-203 and 204-252; and a study by Kennedy on eclipse calculations in the *Sanjufīnī Zīj* cited in n. 109 below.

40 See the *EI*[2] article RU'YAT AL-HILĀL.

41 E. S. Kennedy, "The Lunar Crescent Visibility Theory of Ya'qūb ibn Ṭāriq", *JNES* 27 (1968), pp. 126-132, repr. in *idem et al.*, *Studies*, pp. 157-163; D. A. King, "Some Early Islamic Tables for Determining Lunar Crescent Visibility", in *Kennedy Festschrift*, pp. 185-225, repr. in *idem*, *Studies*, C-II; E. S. Kennedy and M. Janjanian, "The Crescent Visibility Table in al-Khwārizmī's *Zīj*", *Centaurus* 11 (1965), pp. 73-78, repr. in *idem et al.*, *Studies*, pp. 151-156 (deals with an Andalusī table included in the Latin version); Morelon, *Thābit ibn Qurra*, pp. xciii-cxviii, 93-112, and 230-259 (includes materials due to Thābit preserved only in the *Sanjarī Zīj*); J. P. Hogendijk, "Three Islamic Lunar Crescent Visibility Tables", *JHA* 19 (1988), pp. 29-44; D. A. King, "Ibn

2.9 *Geographical tables*

Tables displaying longitudes and latitudes of numerous localities are standard in *zījes*. They have been published in various formats, that is, according to locality, source, increasing longitudes and increasing latitudes, by E. S. and M. H. Kennedy, who provided a valuable research tool that is currently being extended by M. Comes of Barcelona.[42] These geographical coordinates were included in *zījes* in order to facilitate the use of planetary tables for other meridians, for which the longitude difference has to be taken into consideration, and also for computing the *qibla*. A minority of tables also display the *qibla* for each locality. Such tables are often engraved on astronomical instruments, especially those from Safavid Iran.[43] See also 4.10 on tables displaying the *qibla* for ranges of longitudes and latitudes.

2.10 *Star catalogues*

In a typical *zīj* we find a table displaying the ecliptic or equatorial coordinates of selected stars, sometimes for a few dozen, sometimes for several hundred. Procedures for coordinate conversion are also described. The star catalogue in the Arabic *Almagest* has been published by P. Kunitzsch,[44] and

Yūnus on Lunar Crescent Visibility", *JHA* 19 (1988), pp. 155-168, repr. in *idem*, *Studies*, C-III; and Kennedy & Hogendijk, "Two Tables from ... Tibet" (cited in n. 109 below), pp. 238-242.

42 Kennedy & Kennedy, *Islamic Geographical Coordinates*; also E. S. Kennedy, "Mathematical Geography", in *EHAS*, I, pp. 185-201. On the Andalusī tradition represented by Ibn al-Zayyāt see also M. Comes, "Islamic Geographical Coordinates: al-Andalus' Contribution to the Correct Measurement of the Size of the Mediterranean", in *Istanbul 1991 and 1994 Symposia Proceedings*, pp. 123-138, and the same author's study mentioned in n. 72.

43 King, *World-Maps for Finding the Direction and Distance to Mecca ...*, Leiden & London 1999, pp. 71-89, 149-186, and App. B.

44 P. Kunitzsch, *Der* Almagest – *Die* Syntaxis Mathematica *des Claudius Ptolemäus in arabisch-lateinischer Überlieferung*, Wiesbaden 1974; *idem*, *Claudius Ptolemäus – Der Sternkatalog des* Almagest – *Die arabisch-mittelalterliche Tradition*, 3 vols., Wiesbaden 1986-1991; also *idem*, *Studies*.

some early Islamic tables have been investigated by D. Girke (alas unpublished).[45] More research is necessary to establish the relationships between individual star catalogues, not all of which are related in a trivial way to that in the *Almagest*. Thus, for example, the tables in the *Kitāb Ṣuwar al-kawākib*, "On Constellations", by ʿAbd al-Raḥmān al-Ṣūfī, and those in the *zīj* of Ulugh Beg, are essentially Ptolemaic. Independent catalogues are found in the *Mumtaḥan Zīj*, the *Ḥākimī Zīj*, the *Huihui li* (a Chinese translation of an independent *zīj*), *etc.*[46]

2.11 Tables for mathematical astrology

*Zīj*es usually contain tables useful for astrological purposes,[47] notably, for drawing up a horoscope for a certain moment or for a series of such moments, such as each year in the life of an individual. Given the horoscopus or ascendant, that is, the point of the ecliptic instantaneously rising over the

45 D. Girke, "Drei Beiträge zu den frühesten islamischen Sternkatalogen mit besonderer Rücksicht auf Hilfsfunktionen für die Zeitrechnung bei Nacht", Frankfurt am Main, Johann Wolfgang Goethe-Universität, Institut für Geschichte der Naturwissenschaften, Preprint Series no. 8 (1988).

46 E. B. Knobel, *Ulughbeg's Catalogue of Stars*, Washington, D.C. 1917; M. Y. Shevchenko, "An Analysis of the Errors in the Star Catalogues of Ptolemy and Ulugh Beg", *JHA* 21 (1990), pp. 187-201; K. Krisciunas, "A More Complete Analysis of the Errors in Ulugh Beg's Star Catalogue", *JHA* 24 (1993), pp. 269-280; and P. Kunitzsch, "The Astronomer al-Ṣūfī as a Source for Ulugh Beg's Star Catalogue (1437)", in *Strasbourg 1995 Colloquium Proceedings*, pp. 41-47.
On the influence of Islamic star catalogues in medieval Europe see P. Kunitzsch, *Typen von Sternverzeichnissen in astronomischen Handschriften des zehnten bis vierzehnten Jahrhunderts*, Wiesbaden 1966, and various contributions to the *Kunitzsch Festschrift*. On star lists in Hebrew that depend on Islamic sources, see B. R. Goldstein, "Star Lists in Hebrew", *Centaurus* 28 (1985), pp. 185-208; K. A. F. Fischer, P. Kunitzsch and Y. T. Langermann, "The Hebrew Astronomical Codex Ms. Sassoon 823", *Jewish Quarterly Review* 78 (1988), pp. 253-292, repr. in Langermann, *Studies*, X; and Goldstein & Chabás, "Ibn al-Kammâd's Star List" (cited in n. 156 below). See also B. van Dalen, "A Non-Ptolemaic Islamic Star Table in Chinese", in *Kunitzsch Festschrift*, pp. 147-176.

47 On the concepts see al-Bīrūnī, *Tafhīm*, *passim*; Kennedy, "Zīj Survey", pp. 144-145; and the *EI*² article NUDJŪM, AḤKĀM AL-.

horizon,[48] one needs to determine the positions of the astrological houses and to assign the Sun, Moon and five naked-eye planets to the appropriate house, and then to investigate the supposed significance of their positions relative to each other. Some *zījes* contain tables displaying the ecliptic longitudes of the cusps of the houses as a function of the longitude of the horoscopus, for a fixed latitude. Occasionally we find tables displaying the longitude of the horoscopus as a function of the altitude of the Sun throughout the year or of various fixed stars, also for a fixed latitude. We may also find tables of the positions of the elusive, not least because fictitious, astrological body *al-Kayd*.[49] Other tables in *zījes* serve the astrological notions of the 'projections of the rays' (*maṭāriḥ al-shuʿāʿāt*), the 'year transfers' (*taḥāwīl al-sinīn*), the 'excess of revolution' (*faḍl al-dawr*), and the duration of gestation (*makth al-mawlūd fī baṭn ummih*).[50] See also 4.13 on some other

48 See the *EI*[2] article ṬĀLIʿ.

49 E. S. Kennedy, "Comets in Islamic Astronomy and Astrology", *JNES* 16 (1956), pp. 44-51; O. Neugebauer, "Notes on al-Kaid", *JAOS* 77 (1957), pp. 211-215; W. Hartner, "Le problème de la planète Kaïd" (published 1955), repr. in *idem*, *Studies*, A, pp. 268-286; Kennedy, "*Zīj* Survey", pp. 145; and also AL-KAYD in *EI*[2].

50 On various categories of tables see E. S. Kennedy and H. Krikorian-Preisler, "The Astrological Doctrine of Projecting the Rays", *al-Abhath* 25 (1972), pp. 3-15, repr. in Kennedy *et al.*, *Studies*, pp. 372-384; several papers in Kennedy *et al.*, *Studies*, pp. 311-384, and *idem*, *Studies*, XV-XVIII; *idem*, "The Astrological Houses as Defined by Medieval Islamic Astronomers", in *Vernet Festschrift*, II, pp. 535-578, repr. in Kennedy, *Studies*, XIX; and J. P. Hogendijk, "The Mathematical Structure of Two Islamic Astrological Tables for Casting the Rays", *Centaurus* 32 (1989), pp. 171-202), and *idem*, "Mathematical Astrology in the Islamic Tradition" (dealing with houses, rays and progressions), in *Cambridge Dibner Institute 1998 Conference Proceedings*, to appear; J. D. North, *Horoscopes and History*, London 1986 (includes an analysis of the various ways to establish the cusps of the astrological houses, called by North 'domification'); and M. Yano and M. Viladrich, "*Tasyīr* Computation of Kūshyār ibn Labbān", *HistSci* 41 (1991), pp. 1-16. On tables for finding the time from conception to birth, see J. Vernet, "Un tractat d'obstetrícia astrològica", *Boletín de la Real Academia de Buenas Letras de Barcelona* 22 (1949), pp. 69-96, repr. in *idem*, *Estudios*, A, pp. 273-300; Chabás & Goldstein, "Andalusian Astronomy" (cited in n. 156 below), p. 37; and *eidem*, *Astronomy in the Iberian Peninsula* (cited in n. 177 below), pp. 86-87 and 150-153. See also Samsó, "Horoscopes and History" (cited in n. 163 below); *idem* & Berrani, "World Astrology in the 11th Century" (cited in n. 153 below); E. S. Kennedy,

astrological tables not found in *zījes*. Of all the aspects of medieval Islamic astronomy, mathematical astrology is the least researched. New insights are to be anticipated from the current investigations of B. van Dalen.

2.12 Analysis of tables and parameters

Already in the early 1960s Kennedy applied the electronic computer to facilitate the analysis of medieval tables, an activity that was continued by his students, notably, D. A. King, G. Saliba and J. Samsó, and independently by R. Mercier.[51] A third generation of younger researchers is currently active: statistical techniques have been applied to individual tables and groups of tables, notably by B. van Dalen and G. Van Brummelen.[52] The former has reactivated a file of over 2000 parameters found in *zījes* which was started by Kennedy: more information is available on the Internet at the address http://www.rz.uni-frankfurt.de/~dalen.

"Astronomical Events from a Persian Astrological Manuscript" (with an appendix by O. J. Gingerich), *Centaurus* 24 (1980), pp. 162-180; and L. P. Elwell-Sutton, *The Horoscope of Asadullāh Mīrzā – a Specimen of Nineteenth-Century Astrology*, Leiden 1977.

51 On modern methods of investigating medieval tables with computers and the way in which our control over such tables has been enhanced see E. S. Kennedy, "The Digital Computer and the History of the Exact Sciences", *Centaurus* 12 (1968), pp. 107-113, repr. in *idem et al.*, *Studies*, pp. 385-391.

52 B. van Dalen, *Ancient and Mediæval Astronomical Tables: Mathematical Structure and Parameter Values*, Utrecht 1992; *idem*, "A Statistical Method for Recovering Unknown Parameters from Medieval Astronomical Tables", *Centaurus* 32 (1989), pp. 85-145; *idem*, "A Table for the True Solar Longitude in the *Jāmi' Zīj*" (n. 31); *idem*, "al-Khwārizmī's Astronomical Tables Revisited ... " (also n. 31); G. Van Brummelen, "The Numerical Structure of al-Khalīlī's Auxiliary Tables", in *Physis* 28 (1991), pp. 667-697; *idem*, "A Survey of the Mathematical Tables in Ptolemy's *Almagest*", in *Frankfurt IGN Festband*, pp. 155-170; and H. Mielgo, "A Method of Analysis for Mean Motion Astronomical Tables", in *Vernet Festschrift*, I, pp. 159-180.

3 Overview of the *zīj* literature

3.1 The Indian-Iranian tradition

3.1.1 The Zīj al-Shāh

The first Islamic *zīj*es were part of an Indian-Iranian tradition which has a pre-Ptolemaic Greek origin. This is the case of the *Zīj al-Arkand* (K214), written in Arabic in 735 probably in the Sind region, which is related to the *Khaṇḍakhādyaka*, composed in Sanskrit by Brahmagupta in 665. The same main influence appears in the *Zīj-i Shahriyār*, translated into Arabic as the *Zīj al-Shāh* (K30) *ca.* 790 probably by a certain Abu 'l-Ḥasan 'Alī ibn Ziyād al-Tamīmī, although this work was known to Muslim astronomers in the Pahlevi original already before that translation: it was used by an anonymous astrologer who, soon after 679, computed a series of horoscopes illustrating the early history of Islam. The *Zīj al-Shāh* was also used by Māshā'allāh (see below), Nawbakht, 'Umar ibn Farrukhān al-Ṭabarī and al-Fazārī to compute the horoscope for the foundation of Baghdad on 30 July 762. Although neither the Pahlevi nor the Arabic texts of the *Zīj al-Shāh* are extant, its main features are known from indirect sources studied by E. S. Kennedy and D. Pingree,[53] who established, for example, the parame-

53 E. S. Kennedy, "The Sasanian Astronomical Handbook *Zīj-i Shāh* and the Astrological Doctrine of 'Transit' (*mamarr*)", *JAOS* 78 (1958), pp. 246-262, repr. in *idem et al.*, *Studies*, pp. 319-335; B. L. van der Waerden, "Ausgleichspunkt, 'Methode der Perser' und indische Planetenrechnung", *AHES* 1 (1961), pp. 107-121; Kennedy and van der Waerden, "The World-Year of the Persians", *JAOS* 83 (1963), pp. 315-327, repr. in Kennedy *et al.*, *Studies*, pp. 338-350; Kennedy, "Ramifications of the World-Year Concept in Islamic Astrology", in *Actes du dixième Congrès international d'histoire des sciences, Ithaca, 26* VIII *1962* – 2 IX *1962*, H. Guerlac ed., Paris *ca.* 1964, pp. 23-45, repr. in *idem et al.*, *Studies*, pp. 351-371; D. Pingree, "The Persian 'Observation' of the Solar Apogee in *ca.* A.D. 450", *JNES* 24 (1965), pp. 334-336; J. J. Burckhardt and B. L. van der Waerden, "Das astronomische System der Persischen Tafeln I", *Centaurus* 13 (1968-69), pp. 1-28; van der Waerden, "The 'Babylonians' and the 'Persians' ", in *Hartner Festschrift*, pp. 431-440.
Much of van der Waerden's interpretation depended on his acceptance of Abū Ma'shar's claim that his system, with a *yuga* of 360,000 years, was Persian; Pingree, in his *The Thousands of Abū Ma'shar*, London 1968, showed that Abū Ma'shar's claim was that

ters of the mean motions of Saturn, Jupiter and the lunar nodes using the collection of historical horoscopes written by the Jewish astrologer of Basra Māshā'allāh (d. *ca.* 815)[54] who computed them with this *zīj*. Both the *Zīj al-Shāh* and the *Zīj al-Arkand*, as well as, probably, other early *zīj*es, were the channel through which the astrology of great conjunctions was introduced in Islam.[55] These concepts were first expounded in Arabic in the *Kitāb al-Qirānāt wa-'l-duwal* of Zarādusht, translated from the Pahlevi by Sa'īd ibn Khurasānkhurrah *ca.* 750.[56] An important source for the early history of Islamic *zīj*es is 'Alī ibn Sulaymān al-Hāshimī's *Kitāb fī 'Ilal al-zījāt*.[57]

3.1.2 The Sindhind

The third great *zīj* within the Indian-Iranian tradition was the *Sindhind* (K28).[58] This text seems to derive, indirectly, from the Sanskrit *Brahmāsphuṭasiddhānta* composed in 629 by Brahmagupta, through a work the title of which was probably *Mahāsiddhānta*. According to tradition, this *Mahāsiddhānta* was introduced in Baghdad in 771 or 773 by an Indian embassy sent to the court of the Caliph al-Manṣūr. One of the members of this embassy was an astronomer Kanaka who is not necessarily to be identified with the Indian astrologer Kanka or Kanaka, who worked in Baghdad *ca.* 775-820.[59] With his help, the astronomers Ya'qūb ibn Ṭāriq and al-Fazārī

his system was antediluvian, which is obvious nonsense, and that, in fact, it was derived from Indian material by Abū Ma'shar himself. Pingree's *Thousands* also reconstructs Abū Ma'shar's *Zīj al-Hazārāt* (K63), in so far as that is possible.

54 On Māshā'allāh's horoscopes see Kennedy and Pingree, *The Astrological History of Māshā'allāh*, Cambridge, Ma. 1971.

55 See the *EI*² article ḲIRĀN.

56 D. Pingree, *From Astral Omens to Astrology, from Babylon to Bībāner*, Rome 1997, p. 45.

57 Published with translation and commentary by Kennedy and Pingree with F. I. Haddad as *The Book of the Reasons behind Astronomical Tables*, New York 1981.

58 See the *EI*² article SINDHIND.

59 See D. Pingree's article "Kanaka" in *DSB*, and, more recently, *idem*, *From Astral Omens*

translated the Sanskrit text of the *Sindhind* into Arabic. Both Yaʿqūb and al-Fazārī compiled several *zījes* based on the *Sindhind* system, which was used by them to compute the tables of the mean motions of the planets, although the planetary equations were taken from the *Zīj al-Shāh*. At the same time, the extant scattered indirect quotations from these *zījes* bear witness to the first limited appearance of Ptolemaic elements in Islamic astronomy.[60]

3.1.3 The Zīj *of al-Khwārizmī*

This tendency was strengthened in the revision of the *Sindhind* prepared by al-Khwārizmī (*ca.* 825) (K21*),[61] the first Islamic *zīj* which is extant, and which has been published and translated, albeit in a form substantially different from the original. Most of al-Khwārizmī's original text has not been preserved, but we do possess one complete Latin translation, made by Adelard of Bath (*fl.* 1116-1142) of a revision of the *zīj* by the Andalusī astronomers Maslama al-Majrīṭī (d. *ca.* 1010) and Ibn al-Ṣaffār (d. 1034/35).[62] A second Latin translation, probably by the Spanish Jew

to Astrology (n. 56), pp. 51-62.

60 See D. Pingree, "The Fragments of the Works of Yaʿqūb ibn Ṭāriq", *JNES* 27 (1968), pp. 97-125; and *idem*, "The Fragments of the Works of al-Fazārī", *ibid.* 29 (1970), pp. 103-123; as well as the papers by Kennedy and Hogendijk on the lunar visibility table of Yaʿqūb ibn Ṭāriq (n. 41). On this topic see also Pingree, "Greek Influence on Early Islamic Mathematical Astronomy" (n. 11).

61 The relation of al-Khwārizmī's *Sindhind* tradition to Brahmagupta's *Brāhmasphuṭa-siddhānta* is shown in detail in D. Pingree, "The Indian and Pseudo-Indian Passages in Greek and Latin Astronomical and Astrological Texts", in *Viator* 7 (1976), pp. 141-195, esp. 151-169.

62 See the edition of H. Suter and the translation and commentary of O. Neugebauer listed under al-Khwārizmī, *Zīj*, as well as B. R. Goldstein's edition of the Hebrew text of Ibn al-Muthannā's commentary listed as Ibn al-Muthannā, *Commentary on al-Khwārizmī*. More recent studies include B. van Dalen, "al-Khwārizmī's Tables Revisited ... " (n. 31), of singular importance for differentiating between original material and later additions; M. Castells and J. Samsó, "Seven Chapters of Ibn al-Ṣaffār's Lost *Zīj*" (cited in n. 149 below); and also D. Pingree, "Indian Astronomy in Medieval Spain", in *Vernet Festschrift*, I, pp. 39-48.

Petrus Alfonsi (Mosheh Sefardi), who seems to have been responsible for the introduction of the *zīj* in England towards the beginning of the 12th century, is extant in two manuscripts. MS Oxford Corpus Christi College 283 was edited and commented upon by O. Neugebauer in his study of the tables of al-Khwārizmī. A new manuscript has recently appeared, namely, Lambeth Palace Library 67.[63] There exists a third Latin translation which might be closer to al-Khwārizmī's original work than Adelard's version.[64] Further Khwārizmian materials can be recovered from commentaries such as those written by Ibn al-Muthannā, extant in Latin and Hebrew translations,[65] by ʿAbdallāh ibn Masrūr,[66] and by the Spanish Jew of the 12th century Abrahām ibn ʿEzra.[67] Material from al-Khwārizmī's *zīj* was also used in an 11th-century Byzantine text.[68]

It is sometimes difficult to establish which of the tables in the later translation correspond to al-Khwārizmī's original work or are to be considered interpolations by Maslama or by the subsequent Andalusī tradition. However, research has been able to establish a few instances in which materials in the *zīj* are clearly Andalusī as in the case of the table for lunar crescent

63 J. Casulleras, "Las Tablas astronómicas de Pedro Alfonso", in *Estudios sobre Pedro Alfonso de Huesca*, M. J. Lacarra ed., Huesca 1996, pp. 349-366.

64 R. Mercier, "Astronomical Tables in the Twelfth Century", in *Adelard of Bath: An English Scientist and Arabist of the Early Twelfth Century*, C. Burnett ed., London 1987, pp. 87-118, esp. 101.

65 E. Millás Vendrell, *El comentario de Ibn al-Muṯannà a las tablas astronómicas de al-Juwarizmi – Estudio y edición crítica del texto latino en la versión de Hugo Sanctallensis*, Madrid & Barcelona 1963; and B. R. Goldstein, *Ibn al-Muthannā's Commentary on al-Khwārizmī* (n. 62).

66 *Cairo ENL Survey*, no. B37.

67 J. M. Millás Vallicrosa, *El libro de los fundamentos de las tablas astronómicas de R. Abraham ibn ʿEzra*, Barcelona 1947.

68 A. Jones, *An Eleventh-Century Manual of Arabo-Byzantine Astronomy*, Amsterdam 1987.

visibility;[69] the tables for the astrological houses;[70] the very extensive astrological tables for 'casting the rays';[71] or the introduction of a new prime meridian placed 17;30° to the West of the Canary Islands producing a significant reduction in the size of the Mediterranean, which acquires proportions much closer to the actual ones.[72] It seems clear that, apart from the prevalent Indian-Iranian tradition which fits well with what we know of al-Fazārī and Yaʿqūb ibn Ṭāriq's *Sindhind zījes*, and the obvious Andalusī additions, Ptolemaic elements have become much more relevant. This is not surprising since al-Khwārizmī wrote his *zīj* under Caliph al-Maʾmūn (813-833) during whose reign two Arabic translations of Ptolemy's *Almagest* were made. Some one thousand years later the original form of al-Khwārizmī's *zīj* was still being used in Samaria.[73]

3.2 al-Maʾmūn and the earliest Ptolemaic **zījes**

3.2.1 The Arabic Almagest

The first translation of Ptolemy's *Almagest* was into Syriac. The date is not certain, but the Syriac translation clearly preceded the first Arabic one, prepared either by a certain al-Ḥasan ibn Quraysh for the Caliph al-Maʾmūn (according to Ibn al-Ṣalāḥ) or somewhat earlier by other translators for Yaḥyā ibn Khālid ibn Barmak (d. 805), *wazīr* of Hārūn al-Rashīd (according

69 King, "Early Islamic tables for Lunar Crescent Visibility" (n. 41), pp. 192-197, and Hogendijk, "Three Islamic Lunar Crescent Visibility Tables" (also n. 41), pp. 32-35.

70 Van Dalen, "al-Khwārizmī's *Zīj* Revisited" (nn. 31 and 62), p. 209.

71 Kennedy & Krikorian-Preisler, "The Astrological Doctrine of Projecting the Rays (n. 50); and Hogendijk, "Two Islamic Astrological Tables for Casting the Rays" (also n. 50).

72 M. Comes, "The 'Meridian of Water' in the Tables of Geographical Coordinates of al-Andalus and North Africa", *JHAS* 10 (1994), pp. 41-51.

73 B. R. Goldstein and D. Pingree, "The Astronomical Tables of al-Khwārizmī in a Nineteenth-Century Egyptian Text", *JAOS* 98 (1978), pp. 96-99; Pingree, in a review of S. Powels, *Der Kalender der Samaritaner*, Berlin 1977, in *Bibliotheca Orientalis* 38 (1981), p. 564; and *idem*, "al-Khwārizmī in Samaria", *AIHS* 33 (1983), pp. 14-21.

to Ibn al-Nadīm). Neither of these translations is extant. The new translation from the Greek was achieved by al-Ḥajjāj ibn Yūsuf ibn Maṭar in 827/28. This survives in one complete manuscript and another fragmentary one. A related translation from the Greek is by Isḥāq ibn Ḥusayn, prepared for the *wazīr* Abu 'l-Ṣaqr ibn Bulbul (d. *ca.* 890). Shortly thereafter this was reworked by Thābit ibn Qurra, whose version survives in ten manuscripts, only two of which are complete, the version of Isḥāq being available only in the form of quotations (by Ibn al-Ṣalāḥ). The Isḥāq/Thābit version was edited by Naṣīr al-Dīn al-Ṭūsī in 1274, whose version survives in several copies. According to P. Kunitzsch, Jābir ibn Aflaḥ used both of the translations of al-Ḥajjāj and Isḥāq in al-Andalus in the 12th century, and this is confirmed by the Latin translation of Gerard of Cremona, also from that century.[74] But the Muslim astronomers were concerned from the outset to test and to improve what they found in the astronomical traditions to which they were heirs.

3.2.2 The Mumtaḥan Zīj *and the Zījes of Ḥabash*

It is clear that al-Ma'mūn's reign marks a turning point in the development of Islamic astronomy. Although al-Nihāwandī (*fl. ca.* 790) had made observations of his own in Jundishapur, the first systematic program of astronomical observations took place between 828 and 833 in the Shammāsiyya quarter of Baghdad (828, led by Yaḥyā ibn Abī Manṣūr) and in the monastery of Dayr Murrān in Mount Qāsiyyūn, near Damascus (833, led by Khālid ibn ʿAbd al-Malik al-Marwarrūdhī). Al-Ma'mūn may have sponsored these observations for astrological reasons (like his predecessor al-Manṣūr, he seems to have been a keen believer in astrology), although it seems probable that the main purpose of the programme was to reach definite conclusions about the problem of contradictory parameters and geometrical models in use in early Is-

74 This account is based on Kunitzsch, *Der Sternkatalog des* Almagest (n. 44), I, pp. 2-3. See also the *EI*[2] articles BAṬLAMIYŪS and TARDJAMA, ii.; as well as G. Saliba, "The Role of the *Almagest* Commentaries in Medieval Arabic Astronomy: a Preliminary Survey of Ṭūsī's Redaction of Ptolemy's *Almagest*", *AIHS* 37 (1987), pp. 3-20, repr. in *idem*, *Studies*, no. 7.

lamic astronomy, resulting from the simultaneous application of three main known traditions: Indian, Persian and Greek. The programme seems to have concentrated mainly on solar, lunar and stellar observations but it also determined new parameters for the mean planetary motions. Several *zījes* were written as a result of these observations: the official one was compiled by Yaḥyā ibn Abī Manṣūr and bore the title of *al-Zīj al-Mumtaḥan* (K51*), of which a later recension from Mosul survives in a unique manuscript in the Escorial.[75] Other astronomers who were directly or indirectly associated with the observatories of Baghdad and Damascus also wrote their own *zījes*: among these works, the *zīj* of Ḥabash al-Ḥāsib is the most prominent. This is extant in two manuscripts in Istanbul and Berlin. Only the former (K16) has been properly investigated, although it has not been published.[76] The latter (K15) is a later recension. Whereas Ḥabash (d. *ca.* 864-874) does not seem to have taken an active part in the Ma'mūnī observations, he used their results for the compilation of his *zīj*. An analysis of both Ḥabash's and Yaḥyā's *zījes* shows that they are Ptolemaic in character and that they improved certain parameters, especially the solar ones.[77] The determination (made in 829) of the position of the solar apogee at 82;39° from the vernal equinox, when compared with the value obtained by Hipparchus and Ptolemy (65;30°) destroyed the Ptolemaic belief in its immobility. Several more or less successful attempts were made to improve Ptolemy's value for the length of the tropical year. The obliquity of the ecliptic was observed as being 23;33° (a parameter used by Yaḥyā) or 23;35° (used by Ḥabash) while Ptolemy had 23;51,20°. A new parameter for the precession of equinoxes

75 Kennedy, "Zīj Survey", pp. 145-147; J. Vernet, "Las *tabulæ probatæ*", in *Homenaje a Millás-Vallicrosa*, II, Barcelona 1956, pp. 501-522, repr. in *idem*, *Estudios*, A, pp. 191-212. The Escorial manuscript has been published (Frankfurt am Main (IGAIW), 1986) in a 'facsimile', in some respects unfaithful to the original.

76 M. Th. Debarnot, "The *Zīj* of Ḥabash al-Ḥāsib: a Survey of MS Istanbul Yeni Cami 784/2", in *Kennedy Festschrift*, pp. 35-69. The literature on Ḥabash is growing fast: see now E.S. Kennedy, P. Kunitzsch and R. P. Lorch, *The Melon-Shaped Astrolabe in Arabic Astronomy*, Stuttgart 1999 (see also n. 271); King, *World-Maps* (n. 43), pp. 345-359; and the paper by Charette and Schmidl in this volume (on which see also n. 283).

77 See also As-Saleh, "Solar and Lunar Distances and Apparent Velocities ... " (n. 39).

was determined: 1° in $66\frac{2}{3}$ years, which might have an Indian origin,[78] instead of the Ptolemaic 1° in 100 years. The Moon's maximum latitude was established as 4;46° (Ptolemy: 5°) and Yaḥyā, whose tables for the computation of lunar longitude derive from those in the *Handy Tables*, added a supplementary table which introduced a correction based on the fact that lunar motion does not take place on the ecliptic but on the lunar orbit inclined to it. Finally, an important improvement was introduced by Ḥabash in his lunar equation tables for he seems to have been the first Muslim astronomer to use 'displaced' functions – on which more will be said below – for lunar equations.[79]

On the other hand both *zīj*es seem to keep a certain number of elements from the Indian-Iranian tradition: the solar model used in both *zīj*es is Ptolemaic with new parameters (a maximum equation of 1;59°, instead of the Ptolemaic 2;23°) in spite of the fact that several sources attribute to both astronomers the use of pre-Ptolemaic techniques, with an Indian-Iranian origin, for the computation of the solar equation. This is also the case for the model underlying the tables for the computation of planetary latitudes in Yaḥyā's *zīj*,[80] but the most conspicuous Indian-Iranian influence is seen in the model implied, in both *zīj*es, for Venus. Ḥabash (and probably Yaḥyā) as well as, later on, al-Battānī (d. 929) follow the *Zīj al-Shāh* when they establish that the mean motion of Venus equals that of the Sun, and that the positions of their apogees and the values of their maximum equations of the centre are the same.[81] This means that they use, implicitly, a kinematic

78 D. Pingree, "Precession and Trepidation in Indian Astronomy before A.D. 1200", *JHA* 3 (1972), pp. 27-35

79 See E. S. Kennedy and A. Muruwwa, "Bīrūnī on the Solar Equation", *JNES* 17 (1958), pp. 112-121, repr. in Kennedy *et al.*, *Studies*, pp. 603-612; H. Salam and E. S. Kennedy, "Solar and Lunar Tables in Early Islamic Astronomy", *JAOS* 87 (1967), pp. 492-496, repr. in Kennedy *et al.*, *Studies*, pp. 108-113; E. S. Kennedy, "The Solar Equation in the *Zīj* of Yaḥyā ibn Abī Manṣūr", in *Hartner Festschrift*, pp. 183-186, repr. in *idem et al.*, *Studies*, pp. 136-139; and B. van Dalen, "A Table for the True Solar Longitude in the *Jāmiʿ Zīj*", in *Frankfurt IGN Festband*, pp. 171-190.

80 Viladrich, "Planetary Latitude Tables in the *Mumtaḥan Zīj*" (n. 34).

81 B. R. Goldstein, "Remarks on Ptolemy's Equant Model in Islamic Astronomy", in *Hart-*

model in which the centre of the epicycle of Venus lies along the direction of the mean Sun. (The Indians also made the centre of Venus' epicycle, and that of Mercury, lie in the direction of the mean Sun.)

After al-Ma'mūn, Islamic astronomy followed along the same lines. Observations continued almost without interruption either in small private observatories or in more or less organised institutions with official support. These resulted in the compilation of new *zījes* in which the old Indian-Iranian tradition seems to be forgotten (except in al-Andalus – see below), and the tradition of the *Almagest* and the *Handy Tables* was followed. The authors accepted Ptolemy's kinematic models but used new parameters and destroyed some dogmatic beliefs of the *Almagest* such as the invariability of the obliquity of the ecliptic, the constant character of the precession of equinoxes, the immobility of the solar apogee, and the impossibility of annular solar eclipses.[82] Some of the changes introduced are improvements, whilst others are understandable mistakes resulting from the fact that Muslim astronomers relied excessively on the accuracy of observations made in Antiquity and in their own times. In any case, it is obvious that Ptolemaic astronomy was not always followed unquestioningly. At the same time, these astronomical tables – starting with Ḥabash – introduced a new spherical trigonometry which had remote Indian roots but which was mainly an Islamic creation: while the only trigonometrical tool known to Ptolemy was Menelaos' Theorem, which established the relation between the six arcs generated by a transversal of the three sides of a spherical triangle (that is, a 'complete' spherical triangle), the new spherical trigonometry developed a series of new theorems which related four elements of a simple spherical triangle.[83]

ner Festschrift, pp. 165-181.

82 On an observation of an annular solar eclipse by al-Irānshahrī in 873 A.D., see B. R. Goldstein, "Medieval Observations of Solar and Lunar Eclipses", *AIHS* 29 (1979), pp. 101-156 (repr. in *idem*, *Studies*, XVII), esp. p. 101.

83 See, for example, al-Bīrūnī, *Maqālīd*.

3.3 *The major Eastern Islamic* zījes

It is possible to mention here only a limited number of later *zījes* from the Eastern Islamic world, arranged more or less according to the geographical location where they were compiled.

3.3.1 *Some important* zījes *lost for posterity*

Confident that the glass is more full than empty, we begin with a sample of some important *zījes* which are not known to have survived:

- Some four *zījes* (K8, 67, 78, 79, also 90 in the Indian tradition) are attributed to the Banū Amājūr (Baghdad *ca.* 900), none extant. Some of their observations are recorded by Ibn Yūnus (see 3.3.11). These *zījes* would surely have been works of supreme interest.
- *al-Zīj al-'Aḍūdī* (K70) by Ibn al-A'lam (Baghdad *ca.* 960), produced for the Buwayhid ruler 'Aḍūd al-Dawla, influential in later astronomy in al-'Irāq and Iran, notably in the *Zīj-i Īlkhānī* (see 3.3.9) and in Byzantine astronomy, not extant but some details have been reconstructed.[84]
- A *zīj* (K107) by 'Abd al-Raḥmān al-Ṣūfī of Shiraz (*ca.* 975), lost in the original but now known through extracts preserved in medieval Italian tables from Pisa recently investigated for the first time.[85]
- *al-Majisṭī al-Shāhī* (K77) of Abu Naṣr ibn 'Irāq (Khwārazm *ca.* 1000), not extant except for a small extract.[86]

84 E. S. Kennedy, "The Astronomical Tables of Ibn al-A'lam", *JHAS* 1 (1977), pp. 13-23; A. Tihon, "Sur l'identité de l'astronome Alim", *AIHS* 39 (1989), pp. 3-21, repr. in *eadem*, *Studies*, IV; and R. Mercier, "The Parameters of the *Zīj* of Ibn al-A'lam", *AIHS* 39 (1989), pp. 22-50.

85 See R. Mercier, "The Lost *Zīj* of al-Ṣūfī in the Twelfth-Century Tables for London and Pisa", in *Lectures from the Conference on al-Ṣūfī and Ibn al-Nafīs ..., 5-8 October, 1987*, Amman (University of Jordan), pp. 38-74.

86 Sezgin, *GAS*, VI, pp. 242-245.

- *al-Zīj al-Malikshāhī* (K22) by ʿUmar al-Khayyām (Nishapur *ca.* 1090), alas not extant.

3.3.2 The tradition of al-Battānī

- *al-Zīj al-Ṣābiʾ* (K55*) of al-Battānī (Raqqa *ca.* 900) is a respectable and orderly work, if lacking the originality of the *zīj*es of Ḥabash. This was one of the most important works for the transmission of Ptolemaic astronomy to Europe but it enjoyed far less influence in the Islamic world (not least because there it had more competition).[87] The canons were translated into Castilian under the auspices of King Alfonso X of Castile (13th century).[88] There were two Hebrew versions of the *zīj* of al-Battānī: the first by Abraham Bar Ḥiyya (12th century), and the second by Immanuel ben Jacob Bonfils of Tarascon (14th century): see further 3.5.

- Two *zīj*es by Kūshyār ibn Labbān (*ca.* 1000), entitled *al-Zīj al-Jāmiʿ* and *al-Zīj al-Bāligh* (K9 and K7), are extant in several copies of varying content. The contents of the two *zīj*es have yet to be sorted out. It is certain that the expression *al-Zīj al-Jāmiʿ wa-'l-bāligh* refers to two separate works because Kūshyār himself refers to them using the phrase *kitābān sammaytuhumā* ... .[89] Kūshyār calculated accurate mean motion parameters from al-Battānī's data, but made slight adjustments for Mars. He applied a different, less accurate type of Ptolemaic interpolation for his planetary equations, which were systematically of the 'displaced' type.[90]

87 The edition of the Arabic text with a Latin translation and a commentary, also in Latin, by C. A. Nallino is listed under al-Battānī, *Zīj*.

88 Edited in G. Bossong, *Los cánones de Albateni*, Tübingen 1978.

89 M. Yano, ed. and transl., *Kūšyār ibn Labbān's Introduction to Astrology*, Tokyo 1997, pp. 6-7.

90 J. L. Berggren, "Spherical Astronomy in Kūshyār ibn Labbān's *Jāmiʿ Zīj*" (n. 24); M. Bagheri, "The Persian Version of *Zīj-i Jāmiʿ* by Kūšyār Gīlānī", in *Strasbourg 1995 Colloquium Proceedings*, pp. 25-31; *idem*, "A Chapter from Kushyar's Lost *Zīj*", paper

– *Zīj-i Mufrad* (K65) in Persian by Muḥammad ibn Ayyūb al-Ṭabarī (N. Iran, *ca.* 1230), apparently based on al-Battānī, is extant in the unique copy MS Cambridge Browne O.1, merits detailed study. This manuscript contains a lunar crescent visiblity table attributed to al-Bīrūnī (see below), not known from the works of the master.[91]

3.3.3 The tables of a zīj *engraved on an astronomical instrument*

– The tables of a *zīj* were engraved on the mater and plates of an astrolabe in an instrument labelled *Zīj al-Ṣafā'iḥ* (K200) by Abū Ja'far al-Khāzin (Baghdad *ca.* 950); together with this was an extensive treatise on all manner of astronomical subjects, of sufficient merit to attract the attention of scholars of the calibre of Abū Naṣr ibn 'Irāq and al-Bīrūnī. An incomplete instrument of this kind made in 1120/21 by Hibat Allāh al-Asṭurlābī was deemed lost from a private collection in Munich during World War II; the treatise was until recently deemed lost for all time.[92] However, the instrument was rediscovered in the store-rooms of the Museum für Islamische Kunst in Berlin *ca.* 1997, and a copy of al-Khāzin's treatise has been located by S. M. R. Ansari in a library in India (incomplete copies are in Frankfurt and Barcelona). These materials will present a major challenge to any future researcher.[93] Of particular interest are some planetary tables

presented at the 17th annual conference on the history of Arabic science, Sweida, Syria, in 1993; and Van Brummelen, "Mathematical methods in the *Jāmi' Zīj*" (n. 31).

91 See King, "Early Islamic Tables for Lunar Crescent Visibility" (n. 41), pp. 208-210. On another table of the same kind in this work see *ibid.*, pp. 207-208, and Hogendijk, "Three Tables ... " (n. 41), pp. 35-36. Other material in the *Zīj-i Mufrad* is investigated in E.S. Kennedy, "Applied Mathematics in Eleventh-Century Iran: Abū Ja'far's Determination of the Solar Parameters", *The Mathematics Teacher* 58 (1965), pp. 441-446, repr. in *idem et al.*, *Studies*, pp. 535-540.

92 D. A. King, "New Light on the *Zīj al-Ṣafā'iḥ* of Abū Ja'far al-Khāzin", *Centaurus* 23 (1980), pp. 105-117, repr. in *idem*, *Studies*, B-XI.

93 See already J. Samsó, " 'Al-Bīrūnī' in al-Andalus", in *Vernet Festschrift*, II, pp. 583-612, esp. pp. 594-601.

in the *Sindhind* tradition also found in the manuscript. The Andalusī astronomer Ibn al-Samḥ (d. 1035) also engraved mean-motion tables on the plates for each planet in his equatorium.[94] A table for lunar visibility attributed to Abū Ja'far al-Khāzin is found in the *Dustūr al-munajjimīn* mentioned below.[95]

3.3.4 The tradition of Abu 'l-Wafā'

- A *zīj* entitled *al-Majisṭī* (K73) by Abu 'l-Wafā' al-Būzajānī (*ca.* 970) is partially extant in MS Paris BNF ar. 2494. The tables are indicated but are not found in this copy; nevertheless, the sections on trigonometry and spherical astronomy are extensive and systematic and of considerable historical interest.[96] Abu 'l-Wafā' also compiled another work entitled *al-Zīj al-Waḍīḥ*, which is not extant.

- The later, anonymous *zīj* entitled *al-Zīj al-Shāmil* (K29), was based on that of Abu 'l-Wafā', and was clearly of some influence, with some spherical astronomical tables for the latitude of Mardin. This is extant in several manuscripts,[97] of which a preliminary study has been made by J. P. Hogendijk. The *Athīrī Zīj* (K40 = K56) by Athīr al-Dīn al-Abharī of Mosul, extant in MS Dublin Chester Beatty 4076, is closely related to, if not identical with, the *Shāmil Zīj*.

94 J. Samsó, "Notas sobre el ecuatorio de Ibn al-Samḥ", in Vernet, ed., *Estudios*, B, pp. 105-118, repr. in Samsó, *Studies*, XVI; and M. Comes, *Ecuatorios andalusíes – Ibn al-Samḥ, al-Zarqālluh y Abū-l-Ṣalt*, Barcelona 1991.

95 King, "Early Islamic Tables ... " (n. 41), pp. 208-210, and Hogendijk, "Three Tables ... " (also n. 41), pp. 36-42.

96 B. Carra de Vaux, "L'Almageste d'Abû'l-Wéfa Albûzdjâni", *Journal Asiatique*, 8e sér., 19 (1892), pp. 408-471.

97 See, for example, *Cairo ENL Survey*, no. B100.

3.3.5 *Sundry zījes from al-ʿIrāq and Iran*

- A substantial *zīj* (K3) by Jamāl al-Dīn al-Qāsim ibn Maḥfūẓ al-Baghdādī (Baghdad, 1258) is extant in a unique copy, MS Paris BNF ar. 2486.[98] The planetary tables are computed with the parameters of Ḥabash (see above). However, a set of nine tables for spherical astronomy were taken over from the *Majisṭī* of Abu 'l-Wafā'.[99]

- The *Zīj-i Ashrafī* (K4) of Sayf-i Munajjim (Shiraz, *ca.* 1300) is extant in MS Paris BNF supp. pers. 1488,[100] and merits further investigation. The *qibla*-table is something of a disaster, and the geographical tables were lifted from the *Sanjarī Zīj* (see below).[101]

3.3.6 *al-Bīrūnī*

- *al-Qānūn al-Masʿūdī* (K59*) by al-Bīrūnī (*ca.* 1025), dedicated to Sulṭān Masʿūd of Ghazna, is much more than an ordinary *zīj*, being a great astronomical handbook, full of good sense, containing substantial information about the author's personal astronomical research as well as about the development of astronomy in Islamic countries until the author's time. There is an edition by M. Krause.[102] The *Qānūn* was not as influential as it should have been (the same is true of al-Bīrūnī's other productions): some of the trigonometric tables were taken over in the *Zīj-i Īlkhānī* (see below) and the geographical tables,

98 C. Jensen, "The Lunar Theories of al-Baghdādī", *AHES* 8 (1971-72), pp. 321-328.

99 See van Dalen, *Ancient and Mediæval Astronomical Tables* (n. 52), ch. 4.

100 Storey, *PL*, II:1, pp. 64-65.

101 See n. 264 below, and King, *World-Maps* (n. 43), pp. 66 and 71.

102 This was published as al-Bīrūnī, *al-Qānūn al-Masʿūdī*, 3 vols., Hyderabad 1954-56, albeit without the editor's critical apparatus. See the annotated table of contents in E. S. Kennedy, "al-Bīrūnī's *Masudic Canon*", *al-Abhath* 24 (1971), pp. 59-81, repr. in *idem et al.*, *Studies*, pp. 573-595. A valuable study of part of this work is C. Schoy, *Die trigonometrischen Lehren des persischen Astronomen ... al-Bîrûnî*, Hanover 1927, repr. in *idem*, *Beiträge*, II, pp. 629-746.

albeit in a slightly modified form, as if read from a map, in the *Sanjarī Zīj* (see below). No recensions or modified versions have come to light.

3.3.7 *A* zīj *from Marw (12th century)*

- *al-Zīj al-Sanjarī* (K27*) by ʿAbd al-Raḥmān al-Khāzinī (*fl. ca.* 1120), dedicated to the Seljuq sultan Sanjar ibn Malikshāh, is an imposing work on which much more research needs to be done.[103] Besides the original *Sanjarī Zīj* there exists a shorter version, designated *al-Wajīz*, apparently also by al-Khāzinī. This was translated into Greek by Gregory Chioniades (*ca.* 1290-1300).[104]

3.3.8 *The tradition of al-Fahhād: from Adharbayjān to Constantinople and the Yemen*

- Several *zījes* (K23, 42, 53, 58, 62, 64, 84), including one called *al-Zīj al-ʿAlāʾī*, are associated with al-Fahhād (Shirwān *ca.* 1150), none extant in their original form. The Yemeni astronomer Muḥammad ibn Abī Bakr al-Fārisī (see 3.3.12), as well as various Byzantine astronomers, used al-Fahhād's work and preserve for us substantial portions of it.[105]

[103] See D. Pingree, "Gregory Chioniades and Palæologan Astronomy", in *Dumbarton Oaks Papers* 18 (1964), pp. 135-160, esp. 151-158; and, on the geographical tables, D. A. King, *World-Maps* (n. 43), pp. 71-75, and App. D. The tables for lunar crescent visiblity in the *Sanjarī Zīj* are due to Thābit ibn Qurra and have been published by R. Morelon (see n. 41).

[104] On the relation between the original *zīj*, the abridgement, and the translation, see D. Pingree, "A Preliminary Assessment of the Problems of Editing the *Zīj al-Sanjarī* of al-Khāzinī", in *Editing Islamic Manuscripts on Science*, Y. Ibish ed., London 1999, pp. 105-113. The Byzantine text is being published by J. Leichter of Brown University.

[105] For the Byzantine version see D. Pingree, *The Astronomical Works of Gregory Chioniades*, vol. I: *The* Zīj al-ʿAlāʾī, Amsterdam 1985-86.

3.3.9 The Maragha productions

The *Zīj-i Īlkhānī*, the major production of a group of astronomers working at Maragha under Naṣīr al-Dīn and purportedly based on their observations, is still in the Ptolemaic tradition. The parameters underlying the solar, lunar and planetary tables were, however, taken from Ibn al-Aʿlam and Ibn Yūnus, and the trigonometric tables lifted from Ibn Yūnus and al-Bīrūnī. What is new is the calendrical material. There is no influence of the important modifications of the Ptolemaic planetary models conceived by the Maragha astronomers (see 3.6).

- The *Zīj-i Īlkhānī* (K6*) is generally attributed to Naṣīr al-Dīn al-Ṭūsī (1201-1274).[106] This work has not yet received the attention it deserves. It is, however, already apparent that the authors were indebted to Ibn al-Aʿlam, Ibn Yūnus and al-Bīrūnī. The *Zīj-i Īlkhānī* was later revised and corrected by al-Kāshī in his *Zīj-i Khāqānī* (see below). Two works are based on the new observations made at Maragha:

- A *zīj* for Maragha (K108) by Muḥyī al-Dīn al-Maghribī (*ca.* 1280), distinct from his earlier compilation for Damascus, extant in several copies, including MS Meshed 332/103, merits detailed investigation. We are fortunate to have a kind of notebook in the author's hand.[107]

- A substantial *zīj* by Shams (al-Dīn) al-Munajjim al-Wābiknawī (K35), compiled *ca.* 1320, survives in a unique copy MS Istanbul Aya Sofya 2694. It seems to be based on *Īlkhānī* parameters, but is currently under investigation in Frankfurt.

106 See J. A. Boyle, "The Longer Introduction to the *Zīj-i-Īlkhānī* of Naṣīr-ad-Dīn Ṭūsī", *Journal of Semitic Studies* 8 (1963), pp. 244-254. Further, R. Mercier, "The Greek 'Persian Syntaxis' and the *Zīj-i Īlkhānī*", *AIHS* 34 (1984), pp. 35-60; and, with a different interpretation, D. Pingree, "In Defence of Gregory Chioniades", *ibid.* 35 (1985), pp. 436-438.

107 G. Saliba, "The Observational Notebook of a Thirteenth-Century Astronomer", *Isis* 74 (1983), pp. 388-401, repr. in *idem*, *Studies*, no. 8.

- Another Islamic *zīj* (K∅) from the Mongol period (*ca.* 1275) is available in a Chinese translation printed several times between the late 14th and the 18th century known under the name *Huihui li*, meaning 'Islamic astronomical system'.[108] Whereas Yabuuti assumed a relation with the *Īlkhānī Zīj*, Yano and Van Dalen recently showed that the underlying parameters of the *Huihui li* derive from independent observations presumably made by Muslim astronomers in Yuan China in the late 13th century. Some of the original tables of the work are contained in the *zīj* (K∅) by Abū Muḥammad ʿAṭāʾ ibn Aḥmad ibn Muḥammad Khwāja Ghāzī al-Samarqandī, also called al-Sanjufīnī (Tibet, 1366), extant in MS Paris BNF ar. 6040 and in a manuscript that was obtained in China by the Pulkovo Observatory (near St Petersburg) in the 19th century (present location uncertain). The *zīj* of al-Sanjufīnī is in Arabic and the Paris copy has Mongolian titles of tables, Tibetan transcriptions of month-names, and marks by a Chinese librarian.[109]

108 On the *Huihui li* see K. Yabuuti, "Indian and Arabian Astronomy in China", in *Silver Jubilee Volume of the Zinbun-Kagaku-Kenkyusyo*, Kyoto 1954, pp. 585-603; *idem*, "The Influence of Islamic Astronomy in China", in *Kennedy Festschrift*, pp. 547-559; *idem* (translated and partially revised by B. van Dalen), "Islamic Astronomy in China during the Yuan and Ming Dynasties", *HistSci* 7 (1997), pp. 11-43; M. Yano and B. van Dalen, "Tables of Planetary Latitude in the *Huihui li* (I) and (II)", in *Current Perspectives in the History of Science in East Asia*, Y. S. Kim and F. Bray eds., Seoul 1999, pp. 307-315 and 316-329; B. van Dalen, "Islamic Astronomical Tables in China: The Sources for the *Huihui li*", in *Kyoto 1997 Conference Proceedings* (in press).

109 On the *Sanjufīnī Zīj* see H. Franke, "Mittel-mongolische Glossen in einer arabischen astronomischen Handschrift", *Oriens* 31 (1988), pp. 95-118; E. S. Kennedy, "Eclipse Predictions in Arabic Astronomical Tables Prepared for the Mongol Viceroy of Tibet", *ZGAIW* 4 (1987/88), pp. 60-80; *idem* and J. Hogendijk, "Two Tables from an Astronomical Handbook for the Mongol Viceroy of Tibet", in *A Scientific Humanist: Studies in Memory of Abraham Sachs*, Philadelphia, Pa. 1988, pp. 233-242, repr. in *idem*, *Studies*, XIII.

3.3.10 Syrian zījes *and the major production of the chief* muwaqqit *of the Umayyad Mosque in Damascus*

- An anonymous *zīj* for N. Syria (?) entitled *Dustūr al-munajjimīn* (K∅) is extant in the unique MS Paris BNF ar. 5968.[110] The work contains considerable material attributed to earlier astronomers, such as the lunar visibility table of Abū Jaʿfar al-Khāzin (see above).

- A *zīj* (K89) was prepared by Ibn al-Dahhān (*ca.* 1170), who worked in Damascus in the service of the Ayyubid ruler Ṣalāḥ al-Dīn (Saladin), not extant, although the geographical table is preserved in a Rasūlid Yemeni compendium.[111]

- The *Tāj al-azyāj* (K41) of Muhyī al-Dīn al-Maghribī was compiled for Damascus *ca.* 1250 and is extant in three copies: MS Escorial ár. 932, MS Dublin Chester Beatty 4129, and a manuscript belonging to the Arabic Department of the University of Barcelona.[112] The author moved on to Maragha where he compiled two other *zījes* (see above).

- It is perhaps worth noting that the sole surviving copy of a recension of the *zīj* of Ibn Isḥāq al-Tūnisī (K∅) was copied in Ḥimṣ in the year 1317 (see further 3.4). Otherwise Ibn Isḥāq seems to have been known in the East only in the Yemen.

- *al-Zīj al-Jadīd* (K11*) of Ibn al-Shāṭir (Damascus *ca.* 1350),[113] with equation tables based on his new planetary models, which represent the culmination of the Islamic activities to replace those of Ptolemy:

110 Sezgin, *GAS*, VI, pp. 63-64; Pingree, *The Thousands of Abū Maʿshar* (n. 53), pp. 24-25, *etc.*; and F. W. Zimmermann, "The *Dustūr al-Munajjimīn* of ms. Paris, BN ar. no. 5968", in *Aleppo 1976 Symposium Proceedings*, II, pp. 184-192.

111 King, *Astronomy in Yemen* (n. 11), pp. 37.

112 See Saliba, "Observational Notebook" (n. 107), esp. p. 167; and Samsó, "Maghribī Zījes" (n. 11), pp. 96-97.

113 Various early studies are reprinted in E. S. Kennedy and I. Ghanem, *The Life and Work of Ibn al-Shāṭir* … , Aleppo 1976.

see further 3.6.4 below. This work merits detailed investigation. Several recensions were made for Damascus in later centuries, also one for Algiers, and one for Cairo was made *ca.* 1400 by al-Kawm al-Rīshī (see below).[114] A copy of Ibn al-Shāṭir's *zīj* in Hebrew characters, executed in Aleppo in the mid-19th century, has been preserved: see further 3.5.

- Around 1500, recensions for Damascus were prepared of the *Īlkhānī Zīj* (by Shihāb al-Dīn al-Ḥalabī) and the *Sulṭānī Zīj* (by ʿAbd al-Raḥmān al-Ṣāliḥī).

3.3.11 The tradition of Ibn Yūnus in Egypt and beyond

- A monumental work entitled *al-Zīj al-Ḥākimī 'l-kabīr* (K14), compiled *ca.* 1000 for the Fāṭimid Caliph al-Ḥākim by Ibn Yūnus, some four times as large as the *zīj* of al-Battānī. Substantial parts of the work survive in manuscripts in Leiden, Oxford and Paris (with derivative tables in MS Istanbul Selim Ağa 728/2). Of particular historical interest are the accounts of observations by the author and his predecessors in ʿAbbāsid Baghdad.[115] The *Ḥākimī Zīj* was influential in later Egypt and also the Yemen.[116] Thus, for example, the Geniza astrological almanacs as well as the medieval Yemeni ephemerides (see 4.3) are based on calculations using the mean motion and equation tables of the *Ḥākimī Zīj*. The star-catalogue from the *Ḥākimī Zīj*, not contained in the extant fragments of the *zīj*, is preserved in

[114] *Cairo ENL Survey*, no. C30/2.1.19.

[115] For text and translation of the introduction and the observation accounts see Caussin de Perceval, Le livre de la grande table Hakémite observée par le Sheikh ... ebn Iounis ..., in *Notices et extraits des manuscrits de la Bibliothèque Nationale* 7 (An XII = 1804), pp. 16-240 (separatum paginated 1-224); on the spherical astronomy D. A. King, *The Astronomical Works of Ibn Yūnus*, Ph.D. dissertation, Yale University 1972, available through University Microfilms, Ann Arbor, Mich.

[116] See King, "Astronomy of the Mamluks" (n. 11), pp. 532 and 535-537, and *idem*, *Astronomy in Yemen* (also n. 11), *passim*; and also n. 120 below.

the Yemeni *Mukhtār Zīj* and a collection of tables by the Cairo astronomer Aḥmad ibn Timurbāy *ca.* 1475, as well as in a Byzantine source.[117] Towards the end of the 17th century, Cezmī Zāde, *qāḍī* of Belgrade, wrote that he had come across a copy of the *Ḥākimī Zīj* in a private library in Istanbul, where he had earlier worked sweeping the floor. Perhaps it is this copy that is now preserved as MS Istanbul Selim Ağa 728/2, which contains tables based on, but not actually taken from, the *Ḥākimī Zīj* (although the title indicates that they are original).[118] He can hardly have been referring to the copy now in Leiden, which belonged to Taqī al-Dīn (see 3.3.14) in the late 16th century,[119] because it was bought by Golius *ca.* 1600.

- *al-Zīj al-Muṣṭalaḥ* (K47) was the most popular *zīj* in medieval Egypt, attributed by Ḥājjī Khalīfa to one Muḥammad ibn Muḥammad al-Fāriqī, who is not otherwise known to the literature and whose name has not been found in any medieval sources. The work is extant in two recensions, one Yemeni and the other Egyptian preserved respectively in MSS Paris BNF ar. 2513 and 2520, and contains material from ʿAbbāsid *zīj*es and from the *Ḥākimī Zīj* but nothing original.[120]

- Two *zīj*es were compiled by Ibn al-Lubūdī (*ca.* 1250), *al-Zīj al-Zāhī* (K86) and *al-Zīj al-Muqarrab* (K87) (an unlikely title), the latter apparently based on the *Mumtaḥan* observations, are not extant.[121]

117 King, *Astronomy in Yemen* (n. 11), p. 31; *Cairo ENL Survey*, no. B59/2.1.6; and D. Pingree, "Gregory Chioniades and Palæologan Astronomy" (n. 103), pp. 138-139.

118 First mentioned in İhsanoğlu, "Introduction of Western Science to the Ottoman World" (n. 11), p. 73. A microfilm of this manuscript was kindly shown to us by Dr. Sonja Brentjes.

119 See S. Ünver, *İstanbul rasathanesi*, Ankara 1969, pp. 101-104 and figs. 41-42 on some of the manuscripts from Taqī al-Dīn's library.

120 See King, "Astronomy of the Mamluks" (n. 11), pp. 535-536; *idem*, "Double-Argument Equation Table" (cited in n. 222 below), pp. 145-146; and *idem*, "Ibn Yūnus on Lunar Crescent Visibility" (n. 41), pp. 155-156.

121 See Suter, *MAA*, no. 365. On the problems of the documentation of this interim period

- A *zīj* for the Sun and Moon (K∅) was compiled by the early-13th-century Coptic scholar al-As'ad Ibn 'Assāl, with text in Arabic and tables in Coptic numerical notation, extant in MSS Cairo Dār al-Kutub *mīqāt* 910,1 and Vatican ar. 152, as yet unstudied.[122]

- The work entitled *al-Lum'a fī ḥall al-kawākib al-sab'a* (K∅) by the Cairene *muwaqqit* al-Kawm al-Rīshī *ca.* 1400 is a recension for Cairo of Ibn al-Shāṭir's *al-Zīj al-Jadīd* (see above).[123]

- *Zīj al-Ṣūfī* (K37), an Egyptian recension of the *Sulṭānī Zīj* of Ulugh Beg, was compiled by Ibn Abi 'l-Fatḥ al-Ṣūfī (*ca.* 1460), one of the leading Cairene astronomers of his time. It is extant in several copies, of which the best appears to be MS Tehran Millī 768.[124]

- *al-Zīj al-Mufīd 'alā uṣūl al-raṣad al-jadīd* (K209), by Riḍwān Efendī (Cairo *ca.* 1700), is based on Ulugh Beg but has expanded equation tables; it is extant in several copies.[125]

3.3.12 Some Yemeni zījes *(10th-19th century)*

Altogether some 18 *zīj*es are known from the Yemen. Their historical importance derives not only from the fact that they attest to a serious tradition of mathematical astronomy for close to a millennium but also from the fact that they preserve for us 'Abbāsid and Fāṭimid materials which would otherwise have been lost.[126]

between Ibn Yūnus in the late 10th century and the 13th century, when the *Muṣṭalaḥ Zīj* (see n. 116) was compiled, see King, "Astronomy of the Mamluks" (n. 11), pp. 532-534.

122 *Cairo ENL Survey*, no. C10.

123 *Ibid.*, no. C41.

124 *Ibid.*, no. C98.

125 *Ibid.*, no. D58.

126 See King, *Astronomy in Yemen* (n. 11).

– The *zīj* (K69) of al-Hamdānī (Yemen *ca.* 930), who was familiar with the works of the astronomers of the 8th and 9th centuries, is not extant.[127] The 13th-century historian of science Ibn al-Qifṭī states that it was used in the Yemen in his time (*'alayhi 'timād ahl al-Yaman*), but he was not well informed on Yemeni astronomy. The recovery of this work would be a major breakthrough; the scope of al-Hamdānī's mastery of astronomy is indicated by the surviving fragment of his treatise on mathematical astrology, the *Sarā'ir al-ḥikma.*[128]

– A *zīj* (K∅) by the Yemeni astronomer Muḥammad ibn Abī Bakr al-Kawāshī (*ca.* 1280), based on an 'Irāqī *zīj* in the 'Abbāsid tradition and on Ibn Yūnus' *Ḥākimī Zīj*, is extant in MS Alexandria Baladiyya 5577J.[129]

– *al-Zīj al-Mumtaḥan al-Muẓaffarī* (K54) by Muḥammad ibn Abī Bakr al-Fārisī (Aden *ca.* 1260) is an extensive work in the tradition of al-Fahhād (see 3.3.8) and is extant in MS Cambridge Gg. 3.27, which also contains an anonymous recension.[130] The author quotes no less than 28 earlier *zīj*es, and his remarks constitute the only source for our knowledge of many of them.[131] al-Fārisī also authored a treatise on topics treated in *zīj*es, a work entitled *Ma'ārij al-fikr al-wahīj fī ḥall mushkilāt al-zīj* of the genre *kutub 'ilal al-zījāt* (see 1.1), as well as an important compendium of folk astronomy.[132]

127 *Ibid.*, pp. 19-20.

128 *Cairo ENL Survey*, no. B41.

129 King, *Astronomy in Yemen*, p. 27; also *idem* and O. Gingerich, "Some Astronomical Observations from Thirteenth-Century Egypt", *JHA* 13 (1982), pp. 121-128, repr. in *idem*, *Studies*, A-VII.

130 King, *Astronomy in Yemen*, pp. 23-26.

131 See S. Lee, "Notice on the Astronomical Tables of Mohammed Abibekr Al Farsi ... ", in *Transactions of the Cambridge Philosophical Society* 1 (1822), pp. 249-265 (still useful), quoted in Sezgin, *GAS*, VI, p. 67.

132 Currently under investigation in a doctoral thesis by Petra Schmidl of Frankfurt *thumma* Laer.

- *al-Zīj al-Mukhtār min al-azyāj* (K57) by Abu 'l-ʿUqūl (Taiz, *ca.* 1300), based on a *zīj* of Ibn Yūnus other than the *Ḥākimī* (see above), extant in MS London B.L. Or. 3624. Various tables in later Yemeni sources stated to be *min zīj Abi 'l-ʿUqūl* are from a different work by this author.[133]

- Although they do not constitute a *zīj*, the astronomical parts of a scientific compendium of the Rasulid Yemeni Sulṭān al-Afḍal (d. 1377) (K∅), extant in a manuscript in a private collection in Sanaa, are of considerable historical interest and merit detailed investigation.[134]

- The *zīj* for Sanaa (K212) compiled by ʿAbdallāh ibn ʿAbdallāh (al-Muthannā) al-Sarḥī in 1670, which is mainly in the *Muẓaffarī* tradition, survives in several manuscripts. In the early 1970s the first author (D.A.K.) met several people in the Yemen who could still use it to compute planetary positions.

3.3.13 The productions of the Samarqand school (15th century)

The *Zīj-i Īlkhānī* was revised and corrected by al-Kāshī in his *Zīj-i Khāqānī*, a work completed towards 1420, before the end of the cycle of observations made by the team of astronomers at the Samarqand observatory. These latter observations were, however, used by Ulugh Beg to compile his *Zīj-i Sulṭānī* (completed between 1437 and 1448), which is the last great Ptolemaic *zīj* of the Islamic Middle Ages.

- The *Zīj-i Khāqānī* (K20*), a monumental *zīj* in Persian by Jamshīd al-Kāshī, is extant in several manuscripts, of which the best is perhaps MS Cairo Taymūr *riyāḍa* 149.[135]

133 See King, *Astronomy in Yemen*, pp. 30-32; and, on the importance of some of the contents, King, "Double-Argument Equation Table" (cited in n. 222 below), p. 132, and *idem*, "Ibn Yūnus on Lunar Crescent Visibility" (n. 41), p. 156.

134 King, *Astronomy in Yemen*, p. 37. The entire manuscript is now published in facsimile in *The Manuscript of al-Malik al-Afḍal ... – a Medieval Arabic Anthology from the Yemen*, with an introduction by D. M. Varisco and G. R. Smith, Warminster for the E. J. W. Gibb Memorial Trust, 1998.

- The *Zīj-i Sulṭānī* (K12*) of Ulugh Beg and his collaborators in Samarqand is extant in numerous manuscripts, yet to be sorted.[136] The Persian text and the calendrical tables, as well as the trigonometric tables and the star catalogue, have been published.[137] But the remainder, namely the tables for planetary and spherical astronomy, has still not received the attention it deserves. Recensions were prepared for Damascus (al-Ṣāliḥī), Cairo (Ibn Abi 'l-Fatḥ al-Ṣūfī – K37), Istanbul (Meḥmet Chelebī), Tunis (Sanjaq Dār and ʿAbdallāh Ḥusayn Quṣʿa), and India (Mullā Chānd and Farīd al-Dīn).[138] An anonymous Hebrew translation of Ulugh Beg's *Zīj* (tables only) has been preserved; it is undated but was probably copied *ca.* 1500: see further 3.5.

3.3.14 The productions associated with the Istanbul Observatory and later Turkish zījes *displaying European influence*

- Two *zīj*es for Istanbul by Taqī al-Dīn (*ca.* 1580), entitled *Kharīdat al-durar wa-jarīdat al-fikar* (K∅) and *Sidrat muntaha 'l-afkār fī malakūt al-falak al-dawwār* (K∅), are extant in several manuscripts and both await study.[139]

135 Storey, *PL*, II:1, p. 67; *Cairo ENL Survey*, no. G48. Various studies by E. S. Kennedy have been published, including: "Spherical Astronomy in Kāshī's *Khāqānī Zīj*", *ZGAIW* 2 (1985), pp. 1-46, repr. in *idem*, *Studies*, VII; *idem*, "Kāshī's *Zīj-i Khāqānī*", in *Strasbourg 1995 Colloquium Proceedings*, pp. 33-40; and *idem*, *On the Contents and Significance of the* Khāqānī Zīj *by Jamshīd Ghiyāth al-Dīn al-Kāshī*, (*Islamic Mathematics and Astronomy*, vol. 84), Frankfurt am Main 1998.

136 See, most recently, E. S. Kennedy, "Ulugh Beg as Scientist", and "The Heritage of Ulugh Beg", first published in *idem*, *Studies*, X-XI.

137 See Storey, *PL*, II:1, pp. 67-70, on various early European studies, especially L.-A. Sédillot, *Prolégomènes des tables astronomiques d'Oloug-Beg - Traduction et commentaire*, 2 vols., Paris 1847/1853, for the introductory text; Schoy, *Die trigonometrischen Lehren des al-Bîrûnî* (n. 102), pp. 92-100, for the sine table; and Knobel, *Ulughbeg's Catalogue of Stars* (n. 46).

138 *Cairo ENL Survey*, no. G49; and Khan Ghori, "*Zīj* Literature in India" (n. 11), pp. 33-36.

139 See Ünver, *İstanbul rasathanesi* (n. 119); *Cairo ENL Survey*, no. H12; and İhsanoğlu,

- Turkish recensions of the astronomical tables of Cassini (d. 1756) (K208) and Lalande (d. 1807) (K∅) and other European astronomers are available.[140]

3.3.15 Zījes from India

For these the overview of S. A. Khan Ghori is a useful guide.[141]

- The *Zīj-i Nāṣirī* (K∅) was compiled by Maḥmūd ibn ʿUmar *ca.* 1250 for the Sultan of Delhi Nāṣir al-Dīn Abu ʾl-Muẓaffar Maḥmūd ibn Shams al-Dīn Īltutmish. A copy of this appears to be preserved in a private library in Tabriz but it has never been studied.[142] It could be a work of considerable historical interest.

- The *Tashīl-i Zīj-i Ulugh Beg* (K∅) was prepared for the Mughal Sultan Akbar by Mullā Chānd, and is extant in a unique copy in Jaipur.[143]

- *Zīj-i Shāh-Jahānī* by Farīd al-Dīn Dihlawī (K204), completed in 1629 and also based on the *zīj* of Ulugh Beg, extant in several copies.[144] This was translated into Sanskrit *ca.* 1635.[145]

- The Persian *Zīj-i Jadīd-i Muḥammad Shāhī* (K203) by Jai Singh, completed around 1735, is based on the tables of La Hire (1727), with commentaries and recensions; it is extant in several manuscripts. It

ed., *Ottoman Astronomy*, I, pp. 199-217.

140 *Cairo ENL Survey*, nos. H41, H47, H77 and H78; and İhsanoğlu, "Introduction of Western Science to the Ottoman World" (n. 11).

141 Khan Ghori, "*Zīj* Literature in India" (n. 11).

142 Storey, *PL*, II:1, p. 52; and Khan Ghori, *op. cit.*, pp. 30-31.

143 Khan Ghori, *op. cit.*, pp. 33-34.

144 Storey, *PL*, II:1, p. 89; and Khan Ghori, *op. cit.*, pp. 34-36.

145 D. Pingree, "Indian Reception of Muslim Versions ... " (n. 11); and *idem*, "An Astronomer's Progress", *Proceedings of the American Philosophical Society* 143 (1999), pp. 73-85.

seems certain that, apart from the obliquity of the ecliptic and the local latitude, no measurements made at Jai Singh's various 'stone observatories', notably the one at Jaipur, were incorporated in this work.[146]

3.4 *Andalusī and Maghribī* **zījes**

For al-Andalus and al-Maghrib the history of astronomy has been studied more intensively than for other regions of the Islamic world. It is thus possible to place the activity relating to the compilation of *zījes* in a clearer historical perspective.[147]

3.4.1 The earliest Andalusī zījes

The first *zījes*, probably based on an Indo-Iranian tradition, were introduced in al-Andalus in the time of ʿAbd al-Raḥmān II (822-852): one of these was al-Khwārizmī's *Zīj al-Sindhind*.[148] This *zīj* (see already 3.1.3) was the object of new recensions by Maslama al-Majrīṭī (d. 1007/08) and his disciples Ibn al-Ṣaffār (d. 1035) and Ibn al-Samḥ (d. 1035), as well as by Ibn Ḥayy (d. 1064) and ʿAbdallāh al-Saraqusṭī (d. 1056/57), who wrote a treatise on the errors of the *Sindhind* method. Of all these materials only a disappointing

[146] See Storey, *PL*, II:1, pp. 93-94; Khan Ghori, *op. cit.*, pp. 36-41; D. A. King, "A Handlist of the Arabic and Persian Manuscripts in the Maharaja Mansingh II Library in Jaipur", *JHAS* 4 (1980), pp. 81-86, repr. in *idem*, *Studies*, A-XVI; R. P. Mercier, "The Astronomical Tables of Rajah Jai Singh Sawāʾī", *IJHS* 19 (1984), pp. 143-171; B. van Dalen, "The Origin of the Mean Motion Tables of Jai Singh", *IJHS* 35 (2000), pp. 41-66; D. Pingree, "Sanskrit Translations of Arabic and Persian Astronomical Texts at the Court of Jayasimha of Jayapura", *Suhayl* 1 (2000), pp. 101-106. On various works based on the *Zīj-i Jadīd-i Muḥammad Shāhī* see Ansari, "Transmission of Arabic-Islamic Astronomy" (n. 11), especially pp. 283-284.

[147] For a general history of *zījes* in al-Andalus see J. Samsó, *Las ciencias de los Antiguos en al-Andalus*, Madrid 1992. On Maghribī *zījes* see King, "Astronomy in the Maghrib", and Samsó, "Maghribi Zījes" (both cited in n. 11).

[148] See further D. Pingree, "Indian Astronomy in Medieval Spain", in *Vernet Festschrift*, I, pp. 39-48.

fragment of Ibn al-Ṣaffār's version is extant in Arabic,[149] while Maslama's version has been preserved in the Latin translation of Adelard of Bath and in a recension by Petrus Alfonsi. Adelard's version contains materials derived from al-Khwārizmī's original *zīj*, Maslama's modifications and additions and, probably, other later materials, such as the table, attributed elsewhere to a certain al-Qallās, as yet unidentified, for determining the visibility of the new Moon for a latitude of 41;35° (Saragossa?). Maslama introduced modifications in the chronological and in the mean-motion tables (al-Khwārizmī used the Persian calendar and the era of Yazdijird III, while the extant tables use the Muslim calendar and the beginning of the Hijra). He also adapted the radix positions of the lunar ascending node and of the mean oppositions and conjunctions of the Sun and the Moon to the meridian of Cordova which, in these tables as well as in a passage of Ibn al-Ṣaffār's canons, is placed at a distance of 63° west of Arīn (not 79;40° as in the original Khwārizmī's *Zīj*). This correction appears in a horoscope cast in Cordova and dated in 940, and it has the effect of reducing the length of the Mediterranean to a much more reasonable value than was implied by Ptolemy. Other Maslamian additions can be found in trigonometrical (sine and cotangent) and astrological tables such as those concerned with the equalization of the houses and the projection of rays, computed for a latitude of 38;30°, presumably Cordoba, and much better than the original tables of al-Khwārizmī, preserved in another source.

Al-Khwārizmī's astronomical tradition was never fully abandoned in al-Andalus or in the Maghrib. It was followed by Ibn Muʿādh (d. 1093) in his *Tabulæ Jahen* (that is, the *zīj* for Jayyān = Jaén in al-Andalus), of which only the canons are extant in a Latin translation by Gerard of Cremona.[150] Arabic passages from these canons as well as tabular materials have been discovered in late Maghribī sources.[151]

149 M. Castells and J. Samsó, "Seven Chapters of Ibn al-Ṣaffār's Lost *Zīj*", *AIHS* 45 (1995), pp. 229-262.

150 H. Hermelink, "Tabulæ Jahen", *AHES* 2 (1964), pp. 108-112.

151 J. Samsó and H. Mielgo, "Ibn Isḥāq al-Tūnisī and Ibn Muʿādh al-Jayyānī on the Qibla", first published in Samsó, *Studies*, VI; A. Mestres, "Maghribi Astronomy in the 13th

3.4.2 *Ibn al-Zarqālluh, Ṣāʿid al-Andalusī, and the* Toledan Tables

Far more successful were the *Toledan Tables*, the result of an adaptation of the available astronomical material (mainly al-Khwārizmī and al-Battānī) to the coordinates of Toledo made by a group of Toledan astronomers led by the famous *qāḍī* Ṣāʿid al-Andalusī (d. 1070). Even if the results achieved were not brilliant, the mean-motion tables are original and constitute the result of a programme of observations that must have begun earlier than 1068 and which was continued by Ibn al-Zarqālluh (Azarquiel) (d. 1100),[152] one of the collaborators of *qāḍī* Ṣāʿid, until much later.[153] These tables, like those of al-Khwārizmī, calculated sidereal longitudes but added trepidation tables allowing the calculation of tropical longitudes. The topic of trepidation studied by several members of Ṣāʿid's team (Ṣāʿid himself, Ibn al-Zarqālluh, Abū Marwān al-Istijjī) – together with other theoretical innovations developed by Ibn al-Zarqālluh (cycles that regulate the obliquity of the ecliptic, motion of the solar apogee, solar model with variable eccentricity, corrections in the Ptolemaic lunar model) – became standard in the Andalusī and Maghribī tradition. Ibn al-Zarqālluh also adapted a perpetual *Almanac* (K213) from a Hellenistic work computed by a certain Ammo-

Century" (cited in n. 159 below), pp. 402-403 and 435; Samsó, " 'Al-Bīrūnī' in al-Andalus" (n. 93), pp. 601-610; and *idem*, "Andalusian Astronomy in 14th Century Fez" (cited in n. 162 below), pp. 91-92.

152 The standard work is J. M. Millás Vallicrosa, *Estudios sobre Azarquiel*, Madrid & Granada 1943-50.

153 On the *Toledan Tables* and Ibn al-Zarqālluh's *Almanac* see G. J. Toomer, "A Survey of the *Toledan Tables*", *Osiris* 15 (1968), pp. 5-174; L. Richter-Bernburg, "Ṣāʿid, the *Toledan Tables*, and Andalusī Science", in *Kennedy Festschrift*, pp. 373-401; F. S. Pedersen, "Canones Azarchelis. Some Versions and a Text", *Cahiers de l'Institut du Moyen Age Grec et Latin* (Copenhagen) 54 (1987), pp. 129-218; J. M. Millás Vallicrosa, *Estudios sobre Azarquiel* (n. 152), pp. 72-237; and R. Mercier, "Astronomical Tables in the Twelfth Century" (n. 64), pp. 104-112. On the date of the *Toledan Tables* see J. Samsó and H. Berrani, "World-Astrology in Eleventh Century al-Andalus: the Epistle on *Tasyīr* and the Projection of the Rays by al-Istijjī", *Journal of Islamic Studies* (Oxford) 10 (1999), pp. 293-312. An edition of the *Toledan Tables* has been prepared by F. S. Pedersen. See also D. Pingree, "The Byzantine Version of the *Toledan Tables*: the work of George Lapides?", in *Dumbarton Oaks Papers* 30 (1976), pp. 87-132.

nius (*Awmānyws*) in the 3rd or 4th century A.D., which allowed astrologers to obtain planetary longitudes without all the computation involved in the use of a *zīj*.[154] The tables of Ammonius were described by Stephanus the Philosopher in Byzantium *ca.* 790;[155] according to this description, Ammonius used the Era of Philip and the Egyptian months. Ibn al-Zarqālluh's tables, on the other hand, used the Era of Alexander and the Egyptian months. This kind of table was often used in al-Andalus, the Maghrib and medieval Christian Spain (see below on the tables of Zacut).

3.4.3 The works of Ibn al-Kammād and Ibn al-Hā'im

Ibn al-Kammād (active in Cordova *ca.* 1110) and Ibn al-Hā'im (*fl. ca.* 1200) wrote *zīj*es in the Zarqāllian tradition.[156] The former was probably a disciple of Ibn al-Zarqālluh and composed three *zīj*es (K5, K66, K72) of which only one, *al-Muqtabas* (K66), is extant in a Latin translation, although materials from the other two can be recovered in Castilian translations or in Maghribī sources. Ibn al-Kammād (like Ibn al-Hā'im, Ibn Isḥāq, Ibn al-Raqqām in his *Shāmil Zīj*, and Ibn ʿAzzūz al-Qusanṭīnī, on whom see below) apply the Zarqāllian motion of the solar apogee (1° in 279 Julian years) to that of the apogees of the other planets, which poses the problem of establishing whether this was Ibn al-Kammād's contribution or whether it already appeared in the lost work of Ibn al-Zarqālluh. Apart from Zarqāllian materials, Ibn al-Kammād also used other sources such as Yaʿqūb ibn Ṭāriq (*fl.*

154 Published in Millás Vallicrosa, *Estudios sobre Azarquiel*, pp. 72-237 (see previous note); preliminary analysis in M. Boutelle, "The Almanach of Azarquiel", *Centaurus* 12 (1967), pp. 12-19, repr. in Kennedy *et al.*, *Studies*, pp. 502-510; corrections by N. Swerdlow in *Mathematical Reviews* 41:4 (1971), no. 5149; also Samsó, *Ciencias de los Antiguos*, pp. 166-171.

155 See D. Pingree, "Classical and Byzantine Astrology in Sassanian Persia", *Dumbarton Oaks Papers* 43 (1989), pp. 227-288, esp. p. 238.

156 B. R. Goldstein and J. Chabás, "Andalusian Astronomy: *al-Zīj al-Muqtabis* [*sic*] of Ibn al-Kammād", *AHES* 48 (1994), pp. 1-41; *eidem*, "Ibn al-Kammâd's Star List", *Centaurus* 38 (1996), pp. 317-334; and J. L. Mancha, "On Ibn al-Kammād's Table for Trepidation", *AHES* 52 (1998), pp. 1-11.

2nd half of the 8th century) and the *Mumtaḥan Zīj* of Yaḥyā ibn Abī Manṣūr (d. 832), which seems to have been known to Maslama. Ibn al-Kammād deviated from Zarqāllian orthodoxy in several items such as his trepidation model (in which trepidation of the equinoxes is connected to the oscillation of the obliquity of the ecliptic) and he was strongly criticised by Ibn al-Hā'im, who dedicated his *al-Zīj al-Kāmil fi 'l-ta'ālīm* (K48) to the Almohad Caliph Abū 'Abdallāh Muḥammad al-Nāṣir (1199-1213). This work is not a standard *zīj* as it contains an extremely elaborate set of canons (173 pages in the unique MS Oxford Bodl. Marsh 618), with careful geometrical proofs, but no numerical tables. It also contains a great amount of historical information on the work done by the Toledan school in the 11th century, as well as corrections in the Zarqāllian parameters.[157] The *Muqtabas Zīj* seems to be the main source of the astronomical tables prepared in the 14th century for King Peter IV of Aragon.[158]

3.4.4 The Zīj *of Ibn Isḥāq and its derivatives*

After Ibn al-Hā'im the main development of Western *zījes* took place in the Maghrib. There, already at the beginning of the 11th century, the celebrated astrologer Ibn Abi 'l-Rijāl al-Shaybānī al-Qayrawānī had composed a *zīj* entitled *Ḥall al-'aqd wa-bayān al-raṣd* (K∅), which has not survived. No other *zījes* are extant until *ca.* 1200, when we have the set of tables (K∅) prepared by Abu 'l-'Abbās Ibn Isḥāq al-Tamīmī al-Tūnisī (*fl.* Tunis and Marrakesh *ca.* 1193-1222), which survive, among other materials, in a unique Hyderabad manuscript (copied in Ḥimṣ in 1317). According to Ibn Khaldūn, Ibn Isḥāq's tables were based on observations made by a Sicilian Jew: this does not seem to be true and Ibn Isḥāq's authentic tables seem

157 E. Calvo, "Astronomical Theories Related to the Sun in Ibn al-Hā'im's *al-Zīj al-Kāmil fī 'l-ta'līm*", *ZGAIW* 12 (1998), pp. 51-111; M. Abd al-Rahman, "*Wujūd jadāwil fī Zīj Ibn al-Hā'im*", in *Vernet Festschrift*, I, pp. 365-381; and two new studies: R. Puig, "The Theory of the Moon in the *al-Zīj al-Kāmil fī-l-ta'ālīm* of Ibn al-Hā'im (*ca.* 1205)", *Suhayl* 1 (2000), pp. 71-100, and M. Comes, "Ibn al-Hā'im's Trepidation Model", in *Suhayl* 2 (2001).

158 J. Chabas, "Las Tablas de Barcelona" (cited in n. 174 below).

to derive directly from the Andalusī tradition. But Ibn Isḥāq's *zīj* was left unfinished: it lacked an adequate set of canons and at least four 'editions' of this work were prepared, in the Maghrib, by three different astronomers of the end of the 13th and beginning of the 14th century. One of them was the compiler of the Hyderabad manuscript who, *ca.* 1266-1281, added to the original *zīj* an impressive collection of materials (both canons and numerical tables) in which the predominant influence is clearly Andalusī, but the compilation was enormous and ill-suited to practical use.[159]

Ibn al-Bannā' of Marrakesh (1256-1321) wrote his *Minhāj al-ṭālib fi-ta'dīl al-kawākib* (K∅) with an entirely different structure, mainly a selection of Ibn Isḥāq's tables accompanied by a readily comprehensible collection of canons which makes the *zīj* accessible for the computation of planetary longitudes.[160] This is accompanied by some formal modifications intended to make calculations easier: for the first time in the western Islamic world Ibn al-Bannā' uses 'displaced' equations for the Sun and the planetary equations of the centre, and he applies the Ptolemaic lunar method of calculation to the computation of the equation of anomaly of Saturn and Jupiter which, like the Moon, have small epicycles.

The two other 'editions' of Ibn Isḥāq's *zīj* were prepared by Muḥammad ibn al-Raqqām (*fl.* Tunis and Granada, d. 1315).[161] His two *zīj*es are entitled *al-Zīj al-Shāmil fi tahdhīb al-Kāmil* (K∅) and *al-Zīj al-Qawīm fī funūn*

159 See now A. Mestres, "Maghribi Astronomy in the 13th Century: a Description of Manuscript Hyderabad Andra Pradesh State Library 298", in *Vernet Festschrift*, I, pp. 383-443; and *idem*, *Materials andalusins en el* Zīj *d'Ibn Isḥāq al-Tūnisī* (edited text and tables, with introduction and commentary in English), doctoral thesis, University of Barcelona, 2000.

160 J. Vernet, *Contribución al estudio de la labor astronómica de Ibn al-Bannā'*, Tetuán 1952; J. Samsó and E. Millás, "Ibn al-Bannā', Ibn Isḥāq and Ibn al-Zarqālluh's Solar Theory", first published in Samsó, *Studies*, no. X; and *eidem*, "The Computation of Planetary Longitudes in the *Zīj* of Ibn al-Bannā'", *ASP* 8 (1998), pp. 259-286.

161 E. S. Kennedy, "The Astronomical Tables of Ibn al-Raqqām, a Scientist of Granada", *ZGAIW* 11 (1997), pp. 35-72. A partial edition of, and commentary on the *Shāmil Zīj* is M. 'Abd al-Raḥmān, *Ḥisāb aṭwāl al-kawākib fi 'l-Zīj al-Shāmil fī tahdhīb al-Kāmil li-Ibn al-Raqqām*, doctoral dissertation, University of Barcelona 1996.

al-ta'dīl wa-'l-taqwīm (K∅). The former was composed in Tunis in 1280/81 by copying, word for word, the canons of Ibn al-Hā'im's *Kāmil Zīj* (K48) but omitting all the careful geometrical demonstrations. To this he added the numerical tables of Ibn Isḥāq. The *Qawīm Zīj* seems to contain a simplified rewording of the canons of the *Shāmil Zīj* but it adds a few tables adapted to the geographical coordinates of Granada – after Ibn al-Raqqām's arrival in this city under Muḥammad II (1273-1302) – for which the author uses a latitude of 37;10°, identical to the modern value. A third *zīj* by Ibn al-Raqqām, *al-Zīj al-Mustawfī* (K∅), is extant in Rabat and Tunis, but the relation between it and the *zīj* of Ibn Isḥāq has not yet been studied.

3.4.5 Two zījes *from Fez*

The Andalusī tradition was also developed by two astronomers from Constantine who were active in Fez in the 14th century. One of them is Ibn 'Azzūz al-Qusanṭīnī (d. 1354) who compiled his *al-Zīj al-Muwāfiq* (K∅) correcting the mean motion parameters in Ibn Isḥāq's *zīj* on the basis of observations made in Fez *ca.* 1345,[162] later corrected using a peculiar 'experimental' method: the mean motions were adjusted for casting horoscopes which could fit the historical reality of well-known events of the past, such as the battle of Faḥṣ Ṭarīf (El Salado, 1340).[163] Other materials in this *zīj* also derive from Andalusī sources, mainly Ibn al-Kammād, but it also contains interesting information such as tables of planetary velocities (also attested in the Alfonsine tradition), and the oldest mention of a lunar cycle of 11325 days which can be used for the computation of lunar longitudes using almanac techniques. Ibn 'Azzūz included in his *Zīj* a table for planetary velocities that is also found in many Latin copies; the original compiler of this table has not been determined but he was almost certainly Andalusī.[164]

162 J. Samsó, "Andalusian Astronomy in 14th Century Fez: *al-Zīj al-Muwāfiq* of Ibn 'Azzūz al-Qusanṭīnī", *ZGAIW* 11 (1997), pp. 73-110.

163 *Idem*, "Horoscopes and History: Ibn 'Azzūz and his Retrospective Horoscopes related to the Battle of El Salado (1340)", in *North Festschrift*, pp. 101-124.

164 Samsó, "*al-Zīj al-Muwāfiq*" (n. 162), pp. 88-89 and 104-105; B. R. Goldstein, J. Chabás and J. L. Mancha, "Planetary and Lunar Velocities in the Castilian Alfonsine Tables",

Another person of the same origin, Abu 'l-Ḥasan ʿAlī al-Qusanṭīnī, compiled a small *zīj* (K∅) the canons of which were written in verse so that they could easily be learnt by heart.[165] This work is the only known Western Islamic document extant in Arabic in which the planetary theory is Indian and not Ptolemaic. In addition to this material ultimately due to al-Khwārizmī, this *zīj*, however, also shows the influence of Ibn Isḥāq and Ibn al-Bannāʾ.

3.4.6 Later Maghribī zījes

The *zīj*es derived from Ibn Isḥāq were used in the Maghrib until the 19th century, for they allowed the computation of sidereal longitudes which were used by astrologers. We have, however, a limited amount of information about observations made in the Maghrib in the 13th and 14th centuries which established that precession exceeded the amounts fixed in 'Andalusī' trepidation tables and that the obliquity of the ecliptic had fallen below the limits of Ibn al-Zarqālluh's model and tables. This explains the introduction of eastern *zīj*es in the Maghrib from the 14th century onwards. In them, mean motions were tropical, trepidation was replaced by constant precession and there were no tables to compute the obliquity of the ecliptic. The *Tāj al-azyāj* (K41) of Ibn Abi 'l-Shukr al-Maghribī (d. 1283) and the *al-Zīj al-Jadīd* (K11) of Ibn al-Shāṭir (d. 1375) were known in Tunis from the late 14th century onwards, while the *Zīj-i Sulṭānī* (K12) of Ulugh Beg (1393-1449) was known in the Maghrib towards the end of the 17th, and it became very popular during the next two centuries. There were, at least, two Tunisian recensions of this *zīj* prepared by Muḥammad al-Sharīf, called Sanjaq Dār al-Tūnisī, and by ʿAbdallāh Ḥusayn Quṣʿa al-Tūnisī: the former was produced in the late 17th century and it contains, for the first time in the Maghrib, double-argument tables which combine the equation of the centre with the equation of the anomaly.[166] The transmission was not all one

Proceedings of the American Philosophical Society 138 (1994), pp. 61-95.

165 E. S. Kennedy and D. A. King, "Indian Astronomy in Fourteenth-Century Fez: the Versified *Zīj* of al-Qusunṭīnī [*sic*]", *JHAS* 6 (1982), pp. 3-45, repr. in King, *Studies*, A-VIII.

166 J. Samsó, "On the Lunar Tables in Sanjaq Dār's *Zīj al-Sharīf*", to appear in *Cambridge*

way: for example, the Maghribī astronomer Abū 'Alī al-Marrākushī, who was active in Cairo *ca.* 1280 (see 2.4), mentioned Ibn al-Zarqālluh and Ibn al-Kammād in his monumental book on instrumentation. Also, the unique Hyderabad manuscript of the *zīj* of Ibn Isḥāq was copied in Syria, and fragments attributed to the same author are found in various Yemeni sources.

3.4.7 The tables of Zacut

The change of mentality represented by these eastern *zīj*es also reached the Maghrib in the 16th century through a different channel. The Jewish astronomer of Salamanca Abraham Zacut (see also 3.5) left Portugal in 1496 and lived in Fez, Tlemcen and Tunis until at least 1505. In one of these cities he compiled a new set of astronomical tables (1501) and his perpetual *Almanac* (K215) was translated from the printed Castilian version of 1496 into Arabic in the Maghrib in the early-17th century by Aḥmad ibn Qāsim al-Ḥajarī.[167] The new Arabic tables were the object of several commentaries. The *Almanac* represented not only a renewal of the old Andalusī tradition but also the introduction in the Maghrib of Alfonsine astronomy and of the astronomical research made in Southern France by Levi ben Gerson in the 14th century.[168] It was still used in Morocco in the 19th century.

Dibner Institute 1998 Conference Proceedings.

[167] P. S. van Koningsveld, Q. al-Samarrai, and G. A. Wiegers (ed. and transl.), *Aḥmad ibn Qāsim al-Ḥajarī*, Kitāb Nāṣir al-Dīn 'alā 'l-qawm al-kāfirīn *(The Supporter of Religion against the Infidel)*, Madrid 1997; and J. Samsó, "Abraham Zacuto en el Magrib: sobre la presunta cristianización del astrónomo judío y la islamización de su discípulo Jose Vizinho", *Anuari de Filologia* (Barcelona) 21-E-8 (1998-99), pp. 155-165.

[168] See B. R. Goldstein, "The Hebrew Astronomical Tradition" (cited in n. 184 below); *idem*, "Abraham Zacut and the Medieval Hebrew Astronomical Tradition" (cited in n. 175 below); and also *Cairo ENL Survey*, nos. F31, F33 and F50, on various Maghribī recensions. For an analysis of the tables of Zacut (both in Hebrew and in Latin), see Chabás & Goldstein, *Astronomy in the Iberian Peninsula* (cited in n. 177 below).

3.5 *Zījes in Hebrew [by B. R. Goldstein]*

The Hebrew tradition of *zījes* was dependent on Islamic sources and, in addition to translations and adaptations, there were original *zījes* composed in Hebrew. Moreover, there are copies of some Arabic *zījes* preserved in Hebrew characters. The earliest *zīj* in Hebrew was composed in Spain in the 12th century, and a strong tradition continued in the Iberian peninsula until the end of the 15th century. A related tradition developed in southern France in the 13th century and continued to the end of the 15th century. Other Hebrew *zījes* were composed in the 15th century in Sicily, in northern Italy, and in Byzantine territory. There was also a tradition in the Ottoman lands from the 15th century to the 19th century. Finally, a tradition developed in Yemen that continued into the 20th century.

In Spain the earliest *zīj* was compiled by Abraham bar Ḥiyya (12th century), and it is largely based on the *zīj* of al-Battānī (see 3.3.2) that was particularly popular among Andalusī astronomers.[169] In the 13th century two Jews were responsible for the Castilian canons of the *Alfonsine Tables*, Judah ben Moses ha-Cohen and Isaac ben Sid;[170] however, if they composed a *zīj* in Hebrew, it has not survived. In the 14th century several *zījes* were compiled, including: the *zīj* of Isaac Israeli;[171] the *zīj* of Joseph ben Waqār (in which Ibn al-Kammād is mentioned);[172] the *zīj* of Jacob ben David Bon-

169 J. M. Millás Vallicrosa, *La obra Séfer Ḥešbón mahlekot ha-kokabim de R. Abraham Bar Ḥiyya ha-Bargeloni*, Barcelona 1959.

170 See, e.g., B. R. Goldstein, "Astronomy in the Medieval Spanish Jewish Community", in *North Festschrift*, pp. 225-241.

171 Isaac Israeli, *Liber Jesod olam seu Fundamentum mundi*, ed. by B. Goldberg and L. Rosenkranz, 2 vols., Berlin 1846-48.

172 M. Castells, "Notas astrológicas y astronómicas en el manuscrito médico árabe 873 de El Escorial", *al-Qanṭara* 12 (1991), pp. 19-59; and *eadem*, "Una tabla de posiciones medias planetarias en el *Zīj* de Ibn Waqār (Toledo, *ca.* 1357)", in *Vernet Festschrift*, I, pp. 445-452. Some of the tables in this *zīj* are discussed in J. Chabás and B. R. Goldstein, "Computational Astronomy: Five Centuries of Finding True Syzygy", *JHA* 28 (1997), pp. 93-105.

jorn (also known as Jacob Poel);[173] the *Tables of Barcelona*;[174] a Hebrew translation of the *zīj* of Juan Gil (that depends largely on Ibn al-Kammād's *zīj*); and the *zīj* of Judah ben Asher II of Burgos.[175] A *zīj* in Arabic written in Hebrew characters (probably composed in the 14th century in Spain) derives from the Latin text of Campanus of Novara (13th century) that in turn is based on the *zīj* of al-Battānī.[176] In the 15th century two important *zīj*es appeared: the *zīj* of Judah ben Verga of Lisbon (*ca.* 1470), and the *zīj* of Abraham Zacut of Salamanca, composed in Hebrew in 1478 (see also 3.4.7); versions of Zacut's work were published in Latin and Castilian in Leiria, Portugal, in 1496.[177]

In Southern France the almanac of Jacob ben Makhir (also known as Profatius Judæus) with radix 1300, and the *zīj*es of Levi ben Gerson and Im-

173 The Catalan text of the canons and the tables have been edited with commentary by J. Chabás (in collaboration with A. Roca and X. Rodríguez), *L'Astronomia de Jacob ben David Bonjorn*, Barcelona 1992. See also *idem*, "Une période de récurrrence de syzygies au XIVe siècle: le cycle de Jacob ben David Bonjorn", *AIHS* 38 (1988), pp. 243-251; *idem*, "L'influence de l'astronomie de Lévi ben Gershom sur Jacob ben David Bonjorn", in *Studies on Gersonides* (cited in n. 188 below), pp. 47-54; and *idem*, "The Astronomical Tables of Jacob ben David Bonjorn", *AHES* 42 (1991), pp. 279-314.

174 See J. M. Millás Vallicrosa, *Las tablas astronómicas del Rey Don Pedro el Ceremonioso*, Madrid & Barcelona, 1962; and J. Chabás, "Astronomia andalusí en Cataluña: Las Tablas de Barcelona", in *Vernet Festschrift*, I, pp. 477-525.

175 On Juan Gil see B. R. Goldstein, "Scientific Traditions in Late Medieval Jewish Communities", in *Les Juifs au regard de l'histoire: Mélanges en l'honneur de M. Bernhard Blumenkranz*, G. Dahan ed., Paris 1985, pp. 235-247, esp. p. 237; and MS Jews College, London, Heb. 135: note that Burgos is mentioned repeatedly in the headings of the tables in this manuscript. On Judah ben Asher II see B. R. Goldstein, "Abraham Zacut and the Medieval Hebrew Astronomical Tradition", *JHA* 29 (1998), pp. 177-186, esp. pp. 179ff; see also MS Vatican, Heb. 384, fols. 284a-384b.

176 B. R. Goldstein, "The Survival of Arabic Astronomy in Hebrew", *JHAS* 3 (1979), pp. 31-39 (repr. in *idem*, *Studies*, XXI), esp. pp. 34f.

177 B. R. Goldstein, "The Astronomical Tables of Judah ben Verga", *Suhayl* 2 (2001); and J. Chabás and B. R. Goldstein, *Astronomy in the Iberian Peninsula: Abraham Zacut and the Transition from Manuscript to Print*, in *Transactions of the American Philosophical Society*, vol. 90.2, Philadelphia 2000; A. Zacut, *Tabule tabularum celestium motuum astronomi zacuti*, Leiria 1496.

manuel ben Jacob Bonfils of Tarascon were composed in the 14th century. Bonfils compiled a popular *zīj* (judging from the number of extant copies) for the motions of the Sun and the Moon, called *The Six Wings*, that was translated into Latin and into Byzantine Greek.[178] Moreover, Bonfils composed another *zīj* (preserved, for example, in MS Munich Staatsbibliothek 386, fols. 8b-38b) adapted from the *zīj* of al-Battānī that is quite different from the version produced by Abraham bar Ḥiyya. A *zīj*, called the *Paris Tables*, with radix 1368, was translated from Latin into Hebrew by Solomon ben Davin de Rodez; no Latin title or author is cited, but these tables derive from Batecombe's *Oxford Tables* of 1348.[179] The Parisian *Alfonsine Tables* were translated into Hebrew by Moses ben Abraham de Nîmes in 1460.[180]

From Sicily we have a *zīj* by Isaac al-Ḥadib, a refugee who arrived from Spain at the end of the 14th century, and a *zīj* by Isaac ben Elia ha-Cohen at the end of the 15th century. Al-Ḥadib specifically mentions Ibn al-Kammād (3.4.3) and Ibn al-Raqqām (3.4.4).[181] In Northern Italy in the 15th century Mordecai Finzi composed a *zīj*, based on the *Oxford Tables* of 1348 that in turn were adapted from the Parisian *Alfonsine Tables*, and another *zīj* that survives in a manuscript now in the Bodleian Library, Oxford, in his own hand.[182]

In the Byzantine world there was a *zīj*, called the *Persian Tables*, translated from Greek into Hebrew by Solomon ben Eliyahu of Saloniki (14th century). This *zīj* ultimately depends on the *Sanjarī Zīj* of al-Khāzinī (3.3.7)

178 P. Solon, "The Six Wings of Immanuel Bonfils and Michael Chrysokokkes", *Centaurus* 15 (1970), pp. 1-20.

179 On Batecombe's tables, see North, "The *Alphonsine Tables* in England" (cited in n. 223 below), and B. R. Goldstein, "The Role of Science in the Jewish Community in Fourteenth Century France", *Annals of the New York Academy of Sciences* 314 (1978), pp. 39-49 (repr. in *idem*, *Studies*, XX), esp. p. 47.

180 Chabás & Goldstein, *Astronomy in the Iberian Peninsula* (n. 177), p. 22.

181 Goldstein, "Scientific Traditions in Late Medieval Jewish Communities" (n. 174), p. 239.

182 Y. T. Langermann; "The Scientific Writings of Mordekhai Finzi", *Italia* 7 (1988), pp. 7-44, repr. in *idem*, *Studies*, IX.

and the *'Alā'ī Zīj* of al-Fahhād (3.3.8). In the Ottoman lands, in the 15th century, there was a *zīj* by Mordecai Comtino who also wrote a commentary on the *Persian Tables*; and another *zīj* in the 16th century by Abraham ben Yom Tov Yerushalmi of which only the canons seem to survive (MS New York, Jewish Theological Seminary of America, 5516). An anonymous undated Hebrew translation of Ulugh Beg's *zīj* (3.3.13) without the canons is extant in a unique manuscript that, based on paleographical evidence, dates from about 1500 (MS Paris BNF heb. 1091). The paper on which this manuscript is written has a watermark indicating that it was produced in Venice *ca.* 1500, but it is not clear that the translator was in Italy – another possibility is that he lived in Istanbul.[183]

In about 1512 Abraham Zacut arrived in Jerusalem where he composed a *zīj* using the Hebrew calendar, rather than the Christian calendar that he had used in his *zīj* composed in 1478 in Salamanca. The works of Abraham Zacut continued to be consulted by Jews in Syria and Iraq in the 16th and 17th centuries, e.g., in 1696 Simon ben Jonah Mizraḥi of Baghdad composed a *zīj* in which he cites Levi ben Gerson, Immanuel Bonfils, and Abraham Zacut.[184] Finally, *al-Zīj al-Jadīd* by Ibn al-Shāṭir (3.3.10) survives in Hebrew characters in MS New York, Jewish Theological Seminary of America 2580, copied in the mid-19th century in Aleppo.[185]

In Yemen Joseph ben Yefet Halevi (14th century) composed a text in Arabic on the motions of the Sun and the Moon that contains some tables (extant in a manuscript in Hebrew characters together with a Hebrew translation of it: MS London British Library Or. 4104).[186] Copies in Hebrew characters of al-Fārisī's *Muẓaffarī Zīj* (3.3.12) and his *Ma'ārij al-fikr al-wahīj fī ḥall mushkilāt al-zīj*, as well as of Kūshyār ibn Labbān's *Jāmi' Zīj* (3.3.2), are preserved in a number of Yemeni manuscripts.[187]

183 *Idem*, "Survival of Arabic Astronomy in Hebrew" (n. 176), pp. 36-39.

184 *Idem*, "The Hebrew Astronomical Tradition: New Sources", *Isis* 72 (1981), pp. 237-251, repr. in *idem*, *Studies*, XXII.

185 Goldstein, "Survival of Arabic Astronomy in Hebrew" (n. 176), reprint version only, p. 38.

186 *Ibid.*, pp. 31-32.

Levi ben Gerson composed the most original *zīj* in Hebrew. He depended on Ptolemy and al-Battānī, but then went on to construct new models for the motions of planets and new tables based on them with parameters derived from his own observations. His tables for the planets are not extant, but the rest of the tables survive and have been published.[188]

3.6 The hay'a *tradition and the 'New* Zīj' *of Ibn al-Shāṭir*

Ptolemaic astronomy has traditionally been considered as a system of purely mathematical models, the purpose of which was "to save the phenomena",[189] that is to furnish the mathematical tools which enable the astronomer to compute accurately future astronomical events. Doubts were cast on this interpretation by the discovery of some previously unknown chapters of the Arabic translation of Ptolemy's *Planetary Hypotheses*.[190] Ptolemy appeared in them defending an astronomical system which had physical reality and was not a purely mathematical construction, since his

187 Goldstein, "Scientific Traditions in Late Medieval Jewish Communities" (n. 171), p. 243; and Y. T. Langermann, "Arabic Writings in Hebrew Manuscripts: A Preliminary Relisting", *ASP* 6 (1996), pp. 137-169, esp. 147, 151.

188 Goldstein, *The Astronomical Tables of Levi ben Gerson*, in *Transactions of the Connecticut Academy of Arts and Sciences* 45, New Haven, Ct. 1974; *idem*, "A New Set of Fourteenth Century Planetary Observations", in *Proceedings of the American Philosophical Society* 132 (1988), pp. 371-399; and *idem*, "Levi ben Gerson's Contributions to Astronomy", in *Studies on Gersonides: A Fourteenth-Century Jewish Philosopher-Scientist*, G. Freudenthal ed., Leiden 1992, pp. 3-19.

189 This section was originally intended to expand and update the part of the EI^2 article 'ILM AL-HAY'A dealing with 'the School of Maragha' and modifications to Ptolemaic models. In this version due attention is paid to the Western Islamic tradition. The appellation 'School of Maragha' as a designation of the whole of Muslim activity in this field should now be dropped since it is clear that *hay'a* was a topic treated by numerous authors from the 8th to the 17th century.

190 W. Hartner, "Medieval Views on Cosmic Dimensions and Ptolemy's *Kitāb al-Manshūrāt*", in *idem, Studies*, A, pp. 319-348; B. R. Goldstein, *The Arabic Version of Ptolemy's Planetary Hypotheses*, in *Transactions of the American Philosophical Society* (Philadelphia, Pa.), N.S., 57:4 (1967).

geometrical models became three-dimensional and were used to compute distances and sizes of planets.

3.6.1 Hay'a *in the early Islamic East*

The interest in a physical system of the world appeared quite early in Islamic astronomy and we find Yaʿqūb ibn Ṭāriq dealing, as early as the 8th century, with the problem of the size of the Universe.[191] Another early text of this kind was Māshā'allāh's *De scientia motus orbis*.[192]

This tendency led to the development of *hay'a* (theoretical astronomy and cosmology), the origins and early development of which are not well known. It clearly had a greater importance than scholars used to believe a few years ago. Thus, the Toledan astronomer and historian of the 11th century Ṣāʿid al-Andalusī in his *Ṭabaqāt al-umam* makes a careful distinction between *ʿilm hay'at al-aflāk* (science of the physical structure of spheres) and *ḥarakāt al-nujūm* (mathematical astronomy which deals with the motion of celestial bodies) and ascribes to several Andalusī astronomers of the 10th and 11th centuries an interest in *hay'a* which has left very few traces in the sources known up to the present date.

Sizes and distances of the planets were an important topic of early *hay'a*.[193] Other developments soon appeared: the celebrated physicist Ibn

191 Pingree, "Yaʿqūb ibn Ṭāriq" (n. 60).

192 D. Pingree, "Māshā'allāh: Some Sasanian and Syriac Sources", in *Essays in Islamic Philosophy and Science*, G. F. Hourani ed., Albany, N.Y. 1975, pp. 5-14.

193 See the doctoral dissertation by N. Swerdlow, *Ptolemy's Theory of the Distances and Sizes of the Planets. A Study of the Scientific Foundations of Medieval Cosmology*, Yale University 1968, available through University Microfilms, Ann Arbor, Mich., no. 69-8442); G. Saliba, "Early Arabic Critique of Ptolemaic Cosmology: A Ninth-Century Text on the Motion of the Celestial Spheres", *JHA* 25 (1994), pp. 115-141; N. Swerdlow, "al-Battânî's Determination of the Solar Distance", *Centaurus* 17 (1972), pp. 95-105; Y. T. Langermann, "The Book of Bodies and Distances of Ḥabash al-Ḥāsib", *Centaurus* 28 (1985), pp. 108-128; B. R. Goldstein and N. Swerdlow, "Planetary Distances and Sizes in an Anonymous Arabic Treatise Preserved in Bodleian Ms. Marsh 621", *Centaurus* 15 (1970-71), pp. 135-170 (on the theory of the 13th-century astronomer al-ʿUrḍī), repr. in Goldstein, *Studies*, VI; and B. R. Goldstein, "Levi ben Gerson's Theory

al-Haytham (965-*ca.* 1040) made a serious attempt in his *Maqāla fī Hay'at al-ʿālam* to reinterpret the geometrical models of the *Almagest* in physical terms.[194] But beware: recent investigations have posed the question whether there might have been two scholars known as Ibn al-Haytham.[195] The same methodology was applied by a man named Ibn al-Haytham to the problem of the oscillation of the epicycle in Ptolemy's planetary latitude theory.[196] This led him very soon to the criticism of Ptolemy which appears in his *Doubts on Ptolemy*.[197] In this work Ibn al-Haytham discusses Ptolemy's omissions, in the *Hypotheses*, to justify physically all the motions described in his *Almagest*, as well as certain aspects of the geometrical models of this latter work which Ibn al-Haytham considers to be physically impossible. The most important of these criticisms concerns the equant point (the centre of mean motion in longitude of Ptolemy's planetary models), a device which clearly violated the principle that any celestial motion must be a combination of uniform circular motions. The problem of the equant point became crucial in all attempts to create a physically admissible astronomical system: in the 11th century Ibn Sīnā boasted of having discovered a solution for the equant problem, and one of his students, Abū ʿUbayd al-Juzjānī, made an unsuccessful attempt to design planetary models without equant.[198]

of Planetary Distances", *Centaurus* 29 (1986), pp. 272-313.

194 Y. T. Langermann, *Ibn al-Haytham's* On the Configuration of the World, New York & London 1990.

195 A. I. Sabra, "One Ibn al-Haytham or Two? An Exercise in Reading the Bio-Bibliographical Sources", *ZGAIW* 12 (1998), pp. 1-50; and R. Rashed, "Ibn al-Haytham, mathématicien de l'époque fatimide", in *Paris 1998 Fatimid Colloquium Proceedings*, pp. 527-536.

196 A. I. Sabra, "Ibn al-Haytham's Treatise: Solution of Difficulties Concerning the Movement of *Iltifāf*", *JHAS* 3 (1979), pp. 388-422.

197 *al-Shukūk ʿalā Baṭlamyūs*, ed. by A. I. Sabra and N. Shehaby, Cairo 1971.

198 G. Saliba, "Ibn Sīnā and Abū ʿUbayd al-Juzjānī: the Problem of the Ptolemaic Equant", *JHAS* 4 (1980), pp. 376-403, repr. in *idem*, *Studies*, pp. 85-112.

3.6.2 Hay'a *in al-Andalus*

The historical development of the *hay'a* tradition leads us now to the group of 'Aristotelian' scholars who flourished in al-Andalus in the 12th century.[199] Their efforts do not seem to be related to the *Iṣlāḥ al-Majisṭī* ("Correction of the *Almagest*") of Jābir ibn Aflaḥ, whose criticisms of Ptolemy seem to be based on mathematical, rather than on physical, grounds.[200] Ibn Ṭufayl, Ibn Bājja and Ibn Rushd tried to solve the problem by a total or partial abandonment of the Ptolemaic system but they limited themselves to stating general principles.[201] The only serious attempt to create an alternative astronomical system was made by al-Biṭrūjī in his *Kitāb fī 'l-Hay'a* written *ca.* 1190: here he tried to revive the old Eudoxian-Aristotelian system of homocentric spheres, combining it with the later developments of Islamic astronomy.[202] According to another interpretation, his models are independent from those of Eudoxus and derive from Ibn al-Zarqālluh's third model of trepidation.[203] Al-Biṭrūjī's system was a complete failure from the point of view of mathematical astronomy but it is interesting to note

199 A. I. Sabra, "The Andalusian Revolt against Ptolemaic Astronomy – Averroes and al-Biṭrūjī", in *Transformation and Tradition in the Sciences*, E. Mendelsohn ed., Cambridge, Ma. 1984, pp. 133-153, repr. in Sabra, *Studies*, XV.

200 R. P. Lorch, "The Astronomy of Jābir ibn Aflaḥ", *Centaurus* 19 (1975), pp. 85-107, repr. in *idem*, *Studies*, VI; N. M. Swerdlow, "Jābir ibn Aflaḥ's Interesting Method for Finding the Eccentricities and Direction of the Apsidal Line of a Superior Planet", in *Kennedy Festschrift*, pp. 501-512.

201 See F. J. Carmody, "The Planetary Theory of Ibn Rushd", *Osiris* 10 (1952), pp. 556-586; H. Hugonnard-Roche, "L'épitomé du *De cælo* d'Aristote par Averroès: Questions de méthode et de doctrine", *Archives d'Histoire Doctrinale et Littéraire du Moyen Age* 52 (1985), pp. 7-39, and "Remarques sur l'évolution doctrinale d'Averroès dans les commentaires au *De cælo*. Le problème du mouvement de la terre", in *Mélanges de la Casa de Velázquez* 13 (1977), pp. 103-117; J. Lay, "L'Abrégé de l'*Almageste*: un inédit d'Averroès en version hébraïque", *ASP* 6 (1996), pp. 23-61; and M. Forcada, "La ciencia en Averroes", in *Averroes y los averroismos*, J. M. Ayala Martínez ed., Saragossa 1999, pp. 49-102.

202 B. R. Goldstein, *al-Biṭrūjī: On the Principles of Astronomy*, 2 vols., New Haven, Ct. & London 1971, also E. S. Kennedy in *Speculum* 29 (1954), pp. 246-251, and *idem*, "Alpetragius' Astronomy", *JHA* 4 (1973), pp. 134-136.

that he used Neo-Platonic (not Aristotelian) dynamics to explain how the first motor placed in the ninth sphere transmits to the other spheres below it two different motions, in opposite directions to each other.[204] His ideas were influential among philosophical circles of Western Europe in the 13th century.[205]

3.6.3 The Maragha School

Although al-Biṭrūjī's *Kitāb fī 'l-Hay'a* was known in the East (it had probably been introduced in Egypt by Maimonides towards the end of the 12th century – in any case, the Escorial manuscript was copied by an Egyptian Christian in 1281), it is Ibn al-Haytham who had a strong influence on the development of the new non-Ptolemaic astronomical theories in the East from the 13th century onwards. These efforts to revive the *hay'a* tradition, to create an astronomical system having a physical reality, and to improve on Ptolemy's results by reaching a greater coherence – for example, models without equant – and, sometimes (Ibn al-Shāṭir), a better agreement between geometrical models and observation – even in those cases in which Ptolemy's models failed – were made by a group of astronomers who worked in the Maragha observatory and, thus, the label 'the Maragha school' has often been applied to them.[206] The first formulation of the new

203 B. R. Goldstein, "On the Theory of Trepidation according to Thābit ibn Qurra and al-Zarqālluh and its Implications for Homocentric Planetary Theory", *Centaurus* 10 (1964), pp. 232-247.

204 J. Samsó, "On al-Biṭrūjī and the *hay'a* Tradition in al-Andalus", first published in *idem*, *Studies*, XII.

205 R. S. Avi-Yonah, "Ptolemy *vs.* al-Biṭrūjī. A Study of Scientific Decision-making in the Middle Ages", *AIHS* 35 (1985), pp. 124-147; A. Cortabarria, "El astrónomo Alpetragio en las obras de S. Alberto Magno", *La Ciudad de Dios* 193 (1980), pp. 505-535, and *idem*, "Deux sources de S. Albert le Grand: al-Biṭrūjī et al-Battānī", in *Mélanges de l'Institut dominican d'études orientales du Caire* 15 (1982), pp. 31-52.

206 See n. 189. General surveys of this topic are in E. S. Kennedy, "Planetary theory. Late Islamic and Renaissance", *Awrāq* (Madrid: Instituto Hispano-Árabe de Cultura) 5-6 (1982-83), pp. 19-24, repr. in *idem*, *Studies*, XII; N. M. Swerdlow and O. Neugebauer, *Mathematical Astronomy in Copernicus's* De revolutionibus, New York, *etc.* 1984,

astronomical system was made, however, before the Maragha Observatory was built in 1259, either by Mu'ayyad al-Dīn al-ʿUrḍī (d. 1266) in his *Kitāb al-Hay'a*, or by Naṣīr al-Dīn al-Ṭūsī (1201-1274) in his *Ḥall-i mushkilāt-i Muʿīniyya.* The *Kitāb al-Hay'a* describes non-Ptolemaic models for the superior planets, Mercury and the Moon: in them al-ʿUrḍī, like the rest of the members of the school, succeeds in justifying planetary motions by using linkages of vectors of constant length rotating at uniform speed, and he obtains results which can be compared with those of Ptolemy's models.[207] Al-ʿUrḍī also formulated the first of two important mathematical tools which were used in the new models: "ʿUrḍī's lemma", a development of the theorem of Apollonius which allows for the transformation of eccentric models to epicyclic ones.[208] The second theorem (featuring the so-called 'Ṭūsī couple', an expression coined by E. S. Kennedy) was discovered by al-Ṭūsī and it states that the combination of two circular motions can produce rectilinear motion.[209] Both theorems were known to Copernicus who used them in a way which suggests influence. On the other hand, al-Ṭūsī built a new non-Ptolemaic lunar model which, like those of al-ʿUrḍī

pp. 41-48; G. Saliba, "The Role of Maragha in the Development of Islamic Astronomy: A Scientific Revolution before the Renaissance", *Revue de Synthèse* 108 (1987), pp. 361-373, *idem*, "The Astronomical Tradition of Maragha: a Historical Survey and Prospects for Future Research", *ASP* 1 (1991), pp. 67-99, and *idem*, "Arabic Astronomy and Copernicus", *ZGAIW* 1 (1984), pp. 73-87 , repr. in *idem*, *Studies*, nos. 13-15; also *idem*, "Arabic Planetary Theories after the Eleventh Century A.D.", in *EHAS*, I, pp. 58-127 (French transl., I, pp. 71-138).

207 Edited by G. Saliba as *The Astronomical Works of Mu'ayyad al-Dīn al-ʿUrḍī*, Beirut 1990. See also various papers by the same author, including: "The First Non-Ptolemaic Astronomy at the Maraghah School", *Isis* 70 (1979), pp. 571-576, "The Original Source of Quṭb al-Dīn al-Shīrāzī's Planetary Model", *JHAS* 3 (1979), pp. 3-18, and "A Medieval Arabic Reform of the Ptolemaic Lunar Model", *JHA* 20 (1989), pp. 157-164, repr. in *idem*, *Studies*, nos. 4-6.

208 *Idem*, "Arabic Astronomy and Copernicus" (n. 206), esp. pp. 77-81.

209 See, amongst the recent literature, F. J. Ragep, "The Two Versions of the Ṭūsī Couple", in *Kennedy Festschrift*, pp. 329-356; G. Saliba and E. S. Kennedy, "The Spherical Case of the Ṭūsī Couple", *ASP* 1 (1991), pp. 285-291; and Ragep, "The Persian Context of the Ṭūsī Couple", in *Tehran 1997 Conference Proceedings*, pp. 113-130.

and al-Shīrāzī, keeps Ptolemy's extreme values in the geocentric distance of the Moon (and, therefore, does not correct the well-known defect in his lunar model) and gives the same longitudes as Ptolemy, but it does not use the eccenter and the centre of the prosneusis employed in the *Almagest*: it is interesting to remark that al-Ṭūsī stated that the centre of the epicycle of the Moon describes a non-circular curve.[210] He also designed analogous models for the Sun, the superior planets and Venus, but not for Mercury: these models, announced in the *Ḥall*, reached a definitive form in al-Ṭūsī's masterwork, the *Tadhkira fi 'ilm al-hay'a*.[211] Further mathematical research along the same lines was done by al-Ṭūsī's disciple Quṭb al-Dīn al-Shīrāzī (1236-1311), who added new models for the Moon and Mercury, the latter described by E. S. Kennedy as "the apex of the techniques developed by the Maragha school".[212]

3.6.4 The planetary models of Ibn al-Shāṭir

The work of the Maragha astronomers was continued by the Damascus astronomer Ibn al-Shāṭir (*ca.* 1305 – *ca.* 1375) who not only developed this kind of theoretical research in his *Nihāyat al-su'l* but also computed his *al-Zīj al-Jadīd* ("The New *Zīj*") according to his own planetary models and based on the observations he made in Damascus. We have, thus, the first non-Ptolemaic set of astronomical tables according to models which strongly recall Copernican ones. For the Sun Ibn al-Shāṭir uses a deferent and a double epicycle. His lunar model, qualitatively analogous to the solar one and identical to that of Copernicus, is clearly superior to those of Ptolemy, al-Ṭūsī and al-Shīrāzī, because his second epicycle reduces the

210 W. Hartner, "Naṣīr al-Dīn al-Ṭūsī's Lunar Theory", in *Physis* 11 (1969), pp. 287-304, repr. in *idem*, *Studies*, B, pp. 166-183.

211 Critical edition, translation and commentary in F. J. Ragep, *Naṣīr al-Dīn al-Ṭūsī's* Memoir on Astronomy *(al-Tadhkira fi 'ilm al-hay'a)*, 2 vols., New York *etc.* 1993. See also Kennedy, "Two Persian Astronomical Treatises by Naṣīr al-Dīn al-Ṭūsī", *Centaurus* 27 (1984), pp. 109-120; and G. Saliba, "*Almagest* Commentaries" (n. 74).

212 G. Saliba, "al-Qūshjī's reform of the Ptolemaic model for Mercury", *ASP* 15 (1993), pp. 161-203. See also the EI^2 article 'UṬĀRID specifically on models for Mercury.

variation of the geocentric distances of the Moon to a tolerable level. In his model for Venus and the superior planets, Ibn al-Shāṭir succeeds in eliminating the Ptolemaic equant by using a deferent and a triple epicycle. Finally his Mercury model achieves results analogous to those of Ptolemy's movable deferent by using a combination of four epicycles.[213] On the whole, the work of Ibn al-Shāṭir as well as that of his immediate predecessors shows a remarkable level of geometric imagination and a line of research which is very similar to that of Copernicus, although there is no mention of heliocentrism. Today it seems beyond doubt that Copernicus knew, somehow, about the achievements of the Maragha school; he probably became acquainted with it during his stay at Padua in 1501-1503. There he might have obtained, directly or indirectly, information from Byzantine manuscripts such as MS Vatican gr. 211, a translation from an unidentified Arabic source made by Gregory Chioniades (*ca.* 1290-1300), which contains al-Ṭūsī's lunar model as well as the famous 'Ṭūsī couple'.[214]

3.6.5 Other developments in hay'a *after Maragha*

Less important is the work of Ibn al-Shāṭir's contemporary, the polymath ʿUbayd Allāh ibn Masʿūd, known as Ṣadr al-Sharīʿa al-Thānī (d. 1347), who worked at Bukhara and Herat and wrote an encyclopædia of the exact sciences which includes a *Taʿdīl fī hay'at al-aflāk* ("The Adjustment of the Configuration of the Celestial Spheres").[215] In this work the author studies carefully the models for the motion of the Moon and the planets created by Ptolemy, al-Ṭūsī and al-Shīrāzī, and then proceeds to give his own solution

[213] The most significant studies on Ibn al-Shāṭir's planetary theory and tables by E. S. Kennedy and former colleagues have been reprinted twice: in Kennedy *et al.*, *Studies*, pp. 50-83, and in Kennedy & Ghanem, *Ibn al-Shāṭir* (n. 113). See now also G. Saliba, "Theory and Observation in Islamic Astronomy: the Work of Ibn al-Shāṭir of Damascus", *JHA* 18 (1987), pp. 35-43, repr. in *idem*, *Studies*, no. 12.

[214] Swerdlow and Neugebauer, *Mathematical Astronomy in Copernicus's* De revolutionibus (n. 206), pp. 47-48.

[215] A. Dallal, *An Islamic Response to Greek Astronomy* – Kitāb Taʿdīl hay'at al-aflāk *of Ṣadr al-Sharīʿa (edition with translation and commentary)*, Leiden 1995.

which is not very clear in the case of the Moon and follows previous models (created by al-ʿUrḍī and Shīrāzī) in the case of the superior planets.

The story does not end here, because there is a barely explored continuation of the Maragha tradition which lasts at least until the beginning of the 17th century when scholars were still discussing such problems in a creative spirit. An individual of prime importance in this late tradition is Shams al-Dīn al-Khafrī (d. 1550), whose works are currently under investigation by G. Saliba.[216]

3.7 *On the notion of trepidation*

The theory of trepidation (*al-iqbāl wa-'l-idbār*), that is, the supposed oscillation of the equinoxes relative to the fixed point Aries 0°, aims to justify two purported facts known to Muslim astronomers since the time of al-Ma'mūn: that the obliquity of the ecliptic decreases slowly, and that the motion of precession is not constant. We mention it here not least because the Islamic sources often have tables relating to trepidation. The notion of trepidation, as formulated by Muslim astronomers, had clear predecessors both in classical Antiquity, and in the echoes which Greek astronomy had in India.[217] These early formulations established merely that the equinoctial and solstitial points had a very slow motion forwards and backwards along a limited arc of the ecliptic, but no geometrical model justifying such a motion was known. Only in the first half of the 10th century did Ibrāhīm ibn Sinān design the first known trepidation model.[218] Either his formulation of the theory or a different one was introduced in al-Andalus and known to *qāḍī* Ṣāʿid who probably dealt with the topic of trepidation, which was

216 See Saliba, "A Sixteenth-Century Arabic Critique of Ptolemaic Astronomy: the Work of Shams al-Dīn al-Khafrī", *JHA* 25 (1994), pp. 15-38; and *idem*, "The Ultimate Challenge to Greek Astronomy: *Ḥall mā lā yanḥall* of Shams al-Dīn al-Khafrī (d. 1550)", in *Kunitzsch Festschrift*, pp. 490-505.

217 See Neugebauer, *HAMA*, II, pp. 631-634; and D. Pingree, "Precession and Trepidation in Indian Astronomy before A.D. 1200", *JHA* 3 (1972), pp. 27-35.

218 See his *Kitāb fī Ḥarakat al-shams*, ed. by A. S. Saʿīdān, in *Rasāʾil Ibn Sinān*, Kuwait 1983, 274-304.

one of the main concerns of the Toledan astronomers. The famous *Liber de motu octave spere* ("Book on the Motion of the Eighth Sphere"), traditionally ascribed to Thābit ibn Qurra (d. 901), could be the work of one of the members of Ṣā'id's group.[219] There is, in any case, a clear link between the *Liber de motu* and Ibn al-Zarqālluh's treatise on the motion of the fixed stars (*ca.* 1085), extant in a Hebrew translation, where we find an elaborate description of three different trepidation models, the third of which was an improvement, from a practical point of view, on that of the *Liber de motu*.[220] The notion of trepidation introduced into Latin astronomy through the *Toledan Tables* was extremely influential in Europe until the Scientific Revolution.[221]

219 The text has been published several times by J. M. Millás Vallicrosa: see, for example, his *Estudios sobre Azarquiel* (n. 152), pp. 496-509; English translation and commentary by O. Neugebauer, "Thābit ibn Qurra 'On the Solar Year' and 'On the Motion of the Eighth Sphere' ", *Proceedings of the American Philosophical Society* 106 (1962), pp. 290-299, based on corrupt Latin translations; French translation of the Arabic text and commentary on the same two treatises in Morelon, *Thābit ibn Qurra*, pp. xlvi-lxxix, 26-82, 189-221 (Morelon shows that the first work was probably due to the Banū Mūsā).
On the geometrical model of the *Liber de motu* see B. R. Goldstein, "On the Theory of Trepidation according to Thābit ibn Qurra and its Implications for Homocentric Planetary Theory", *Centaurus* 10 (1964), pp. 232-247; J. Dobrzycki, "Teoria precesji w astronômii sredniowiecznej", in *Studia i Materialy Dziejow Nauki Polskiej*, Seria Z.Z., 11 (1965), pp. 3-47 (in Polish with a long summary in English); J. D. North, "Thebit's Theory of Trepidation and the Adjustment of John Maudith's Star Catalogue", in *idem*, *Richard of Wallingford ...* , 3 vols., Oxford 1976, III, pp. 155-158; R. Mercier, "Studies in the Medieval Conception of Precession", *AIHS* 26 (1976), pp. 197-220, and 27 (1977), pp. 33-71; *idem*, "Accession and Recession: Reconstruction of the Parameters", in *Vernet Festschrift*, I, pp. 299-347; and F. J. Ragep, "al-Battānī, Cosmology and the History of Trepidation in Islam", *ibid.*, I, pp. 267-298.

220 See Millás Vallicrosa, *Estudios sobre Azarquiel*, pp. 250-343; Goldstein, "Trepidation" (n. 219); and J. Samsó, "Sobre el modelo de Azarquiel para determinar la oblicuidad de la eclíptica", in *Homenaje al Prof. Darío Cabanelas O.F.M.*, Granada 1987, II, pp. 367-377, repr. in *idem*, *Studies*, IX. See also Samsó, "Trepidation in al-Andalus in the 11th Century", in *idem*, *Studies*, VIII; and M. Comes, "The Accession and Recession Theory in al-Andalus and the North of Africa", in *Vernet Festschrift*, I, pp. 349-364, and *eadem*, "Ibn al-Hā'im's Trepidation Model" (n. 157).

3.8 Other innovations

The *zījes* often introduce an important set of improvements which make them more accurate and easier to handle than standard Ptolemaic tables.

1. There is a steady evolution towards increasing precision which can be easily appreciated in trigonometrical tables: for example, both Ḥabash in the 9th century and al-Battānī in the early 10th century present in their *zījes* sine tables with argument difference 1°, while the function is calculated to the third sexagesimal fraction; in the 15th century the sine table extant in Ulugh Beg's *zīj* is computed with argument difference 1 minute, while the function is approximated to five sexagesimal places (see 2.3). Further simplified computation is achieved by the use of:

2. Double-argument equation tables which simplify the complicated Ptolemaic procedures for the computation of planetary longitudes, involving the successive application of various auxiliary functions. The arguments are to be taken directly from the mean-motion tables. A particularly ingenious table with some 34,000 entries defining the lunar equation for each degree of mean anomaly and each degree of double elongation is attributed to Ibn Yūnus. Such tables are found frequently from the 13th century onwards.[222] As this paper was in press, an extensive set of double-argument tables for each of the planets based on the parameters of Ibn Yūnus came to light (MS Istanbul Selim Ağa 728/2): these were compiled in Cairo by the 13th-century astronomer Yūsuf ibn Ismāʿīl al-Damīrī (perhaps the lunar equation

[221] B. R. Goldstein, "Historical Perspectives on Copernicus's Account of Precession", *JHA* 25 (1994), pp. 189-197.

[222] D. A. King, "A Double Argument Table for the Lunar Equation Attributed to Ibn Yūnus", *Centaurus* 18 (1974), pp. 129-146, repr. in *idem*, *Studies*, A-V; C. Jensen, "The Lunar Theories of al-Baghdādī", *AHES* 8 (1971-72), pp. 321-328; M. J. Tichenor, "Late Medieval Two-Argument Tables for Planetary Longitudes", *JNES* 26 (1967), pp. 126-128, repr. in Kennedy *et al.*, *Studies*, pp. 122-124; and G. Saliba, "The Double-Argument Lunar Tables of Cyriacus", *JHA* 7 (1976), pp. 41-46.

tables attributed to Ibn Yūnus are also by him?). In Europe, such tables appear for the first time in the *Tabulæ Magnæ* of Jean de Lignères (*fl.* 1320-1335).[223]

3. 'Displaced' tables (in Kennedy's terminology) in which a constant has been added to the values of the function in such a way that it has become 'displaced' and its values are always positive. In standard Ptolemaic tables these values can be positive or negative and the computer has to remember complex rules which tell him whether he should add or subtract: the first known table of this kind was computed by Ḥabash for the lunar equation and displaced tables are fairly common from the 10th century onwards.[224]

4. Planetary velocity tables. While tables for the velocities of the Sun and Moon are relatively common, there are very few tables for planetary velocities. One such table has been found in a *zīj* from the Maghrib and the same table appears in Latin and Hebrew manuscripts, beginning in the 13th century. This table is also described in the Castilian canons of the *Alfonsine Tables*.[225]

223 J. D. North, "The *Alfonsine Tables* in England", in *Hartner Festschrift*, pp. 269-301. Batecombe's *Oxford Tables* of 1348 in Latin contain double-argument planetary-equation tables, and there were two versions of them in Hebrew: see Chabás & Goldstein, *Astronomy in the Iberian Peninsula* (n. 177), p. 22. For another two sets of double argument planetary equation tables in Hebrew, see Goldstein, "The Astronomical Tables of Judah ben Verga" (n. 177), and an anonymous *zīj* for epoch 1400 AD preserved in MS Vatican, Heb. 384, ff. 263a-277a.

224 On 'displaced' tables see the papers by Kennedy and Tihon on the *zīj* of Ibn al-A'lam mentioned in n. 84 above, and also G. Saliba, "The Planetary Tables of Cyriacus", *JHAS* 2 (1978), pp. 53-65, and *idem*, "Computational Techniques in a Set of Late Medieval Astronomical Tables", *JHAS* 1 (1977), pp. 24-32. Other references to displaced equations are to be found in papers already cited such as Kennedy, "al-Bīrūnī's *Masudic Canon*" (n. 102); King, "Double-Argument Table for the Lunar Equation" (n. 222); Jensen, "The Lunar Theories of al-Baghdādī" (n. 222); and Van Brummelen, "Mathematical Methods in the *Jāmi' Zīj*" (n. 31).

225 J. A. as-Saleh, "Solar and Lunar Distances and Apparent Velocities" (nn. 39 and 77), pp. 141-163; al-Battānī, *Zīj*, II, p. 88; al-Khwārizmī, *Zīj*, pp. 175-180; Chabás &

Next, there was a distinct tendency in Islamic astronomy to work towards universal solutions, that is, solutions which would serve all terrestrial latitudes.[226] The Arabic expressions occasionally used for such solutions are *li-jamīʿ al-ʿurūḍ*, "for all latitudes", and *āfāqī*, "for all horizons", from *ufq* or *ufuq*, pl. *āfāq*, "horizon". The tables compiled were of two main kinds:

5. Tables for all latitudes. Tables of various functions for each of the climates are found already in Hellenistic astronomy. But Muslim astronomers took virtually all of the various topics in spherical astronomy and mathematical geography to their natural 'universal' conclusion. In his 1956 *zīj* survey Kennedy noted the tables of oblique ascensions for each degree of terrestrial latitude, and more recent research has revealed the existence of universal tables for lunar crescent visibility, for the times of prayer (latitudes 21°-41°, serving localities between the latitudes of Mecca and Istanbul), for the duration of twilight, for the effect of refraction at the horizon, and for constructing markings on astrolabes and sundials. In the case of the *qibla* universal solutions in the form of tables would serve all (reasonable) longitudes and latitudes. Furthermore, various instruments serving all latitudes, or, in the case of the *qibla*, serving all latitudes and longitudes in the 'inhabited world', have been studied.

6. Auxiliary tables which were employed to solve diverse problems of spherical astronomy, usually for all latitudes. The first ones known appear in Ḥabash's *zīj* in the 9th century, while the most impressive

Goldstein, "Andalusian Astronomy" (n. 156), pp. 10-13; Goldstein, "Lunar Velocity in the Middle Ages" (n. 38); B. R. Goldstein, J. Chabás and J. L. Mancha, "Planetary and Lunar Velocities in the Castilian *Alfonsine Tables*", *Proceedings of the American Philosophical Society* 138 (1994), pp. 61-95; J. Samsó, "Andalusian Astronomy in 14th Century Fez" (n. 162), pp. 88-91 and 104-105; Goldstein, "Abraham Zacut and the Medieval Hebrew Astronomical Tradition" (n. 175), p. 179.

226 King, "Universal Solutions in Islamic Astronomy" (published 1987), and *idem*, "Universal Solutions to Problems of Spherical Astronomy from Mamluk Egypt and Syria" (published 1988), repr. in *idem*, *Studies*, C-VI and VII, and again in *SATMI*, VI; also *idem*, *World-Maps* (n. 43), pp. 329-332 and 351-359.

ones are those of Najm al-Dīn al-Miṣrī (Cairo *ca.* 1325) and al-Khalīlī (Damascus *ca.* 1365), but these do not occur in *zījes* – see further 4.7.

4 Categories of tables not contained in *zījes*

The varieties of tables mentioned here rarely occur in *zījes*, sometimes because the individual tables were even more voluminous than a typical *zīj*, occasionally because they are more suited to inclusion in treatises on instruments, or simply because they formed part of a corpus of tables for timekeeping for a specific latitude.

4.1 Sexagesimal multiplication tables

Tables of sexagesimal products $m \times n$ ($m, n = 1, 2, \ldots, 60$) are common in the manuscript sources (*al-jadwal al-sittīnī*); the earliest known is from the arithmetic of Kūshyār ibn Labbān (*ca.* 1000) (see 3.3.2). These invariably contain some 3,600 entries. Less common were larger tables for $m = 0;1, 0;2, \ldots 59;59$ and $n = 1, 2, \ldots, 60$ containing some 216,000 entries (aptly called *al-jadwal al-sittīnī al-kabīr*). These were of use in extensive sexagesimal calculations. A single table of quotients m/n (m and n from 1 to 120) is known.[227]

4.2 Trigonometric tables

In Mamluk Egypt the sine and cotangent functions for each minute of argument were tabulated separately.[228] Such tables, to greater accuracy, were found in the *Sulṭānī Zīj* of Ulugh Beg (see above), and these were also copied separately.

[227] D. A. King, "On Medieval Islamic Multiplication Tables", *HistMath* 1 (1974), pp. 317-323, and *idem*, "Supplementary Notes … ", *ibid.* 6 (1979), pp. 405-417, repr. in *idem*, *Studies*, A-XIV and XV.

[228] *Cairo ENL Survey*, no. C137.

4.3 Ephemerides

Ephemerides displaying positions of the Sun, Moon and five planets for each day of a given year[229] were compiled already in Baghdad in the 9th century and the production continued in various centres until the 19th century. Thābit ibn Qurra (Baghdad *ca.* 900) called them *daftar al-sana*, and al-Bīrūnī gives a short extract from one. Fragments of various Egyptian astrological almanacs from the 12th century have been preserved in the Cairo Geniza; these are in Arabic, mostly written in Hebrew characters, but some are in Arabic characters. The Geniza almanacs give only the daily positions of the Moon, together with its planetary aspects.[230] The earliest surviving complete ephemerides are from Rasulid Yemen, namely, MS Cairo Dār al-Kutub *mīqāt* 817,2 for the year 727 Hijra (= 1326/27) and MS Cairo Taymūr *riyāḍa* 274 for 808 Hijra (= 1405/06).[231] The main tables display the daily positions of the Sun, Moon and five planets for each month on the right-hand page and the planetary aspects of the Moon for each day with the appropriate prognostications on the left-hand page. Considerable additional astrological information is appended, mainly in tabular form, occasionally schematically in diagrams. A Byzantine almanac for the year 1336 includes the planets.[232] Numerous such ephemerides survive from the later period, mainly from Cairo, Istanbul and various centres in Iran.[233]

229 See the *EI*² article TAḲWĪM.

230 B. R. Goldstein and D. Pingree, "Astrological Almanacs from the Cairo Geniza", *JNES* 38 (1979), pp. 153-175, pp. 231-256; *eidem*, "More Horoscopes from the Cairo Geniza", *Proceedings of the American Philosophical Society* 125 (1981), pp. 155-189; *eidem*, "Additional Astrological Almanacs from the Cairo Geniza", *JAOS* 103 (1983), pp. 673-690; and *eidem*, "Astronomical Computations for 1299 from the Cairo Geniza", *Centaurus* 25 (1982), pp. 303-308.

231 *Cairo ENL Survey*, no. E11, and King, *Astronomy in Yemen* (n. 11), pp. 33 and 39. These ephemerides are currently under investigation by M. Hofelich of Frankfurt.

232 R. Mercier, *An Almanac for Trebizond for the Year 1336*, Louvain-la-Neuve 1994.

233 See, for example, *Cairo ENL Survey*, no. H78, and İhsanoğlu, ed., *Ottoman Astronomy*, II, pp. 885-939.

4.4 Auxiliary tables for compiling ephemerides

Since the motions of the Sun, Moon and planets are cyclical, a given set of tables defining one complete cycle can be used to calculate positions by simply plugging into the table at the right place for the beginning of a given year: the positions for the entire year can then be derived with facility. This was recognized already in Antiquity, and some Islamic tables reflect this. We have already mentioned the tables of Ibn al-Zarqālluh and Zacut, which could be used to calculate individual positions. Two other sets of such auxiliary tables specifically for generating ephemerides are known, but there are surely more:

1. An anonymous set of such tables for the Sun and Moon, extant in MS Oxford Bodl. Marsh 374, was compiled in Iran at the end of the 11th century.[234]

2. *al-Durr al-yatīm*, an extensive set of auxiliary tables for computing ephemerides (K36), compiled by the 15th-century Cairo astronomer Ibn al-Majdī, extant in several manuscripts.[235]

4.5 Tables for determining lunar crescent visibility

In addition to the tables in *zījes* (see 2.8) and independent treatises such as those of Thābit ibn Qurra and Ibn al-Bannā', we also find occasional sets of calculations for visibility over a series of months, or lists of minimum apparent distances between the Sun and Moon to assure visibility, with values given to the nearest degree for each zodiacal sign.[236]

[234] E. S. Kennedy, "A Set of Medieval Tables for Quick Calculation of Solar and Lunar Ephemerides", *Oriens* 18/19 (1967), pp. 327-334, repr. in *idem et al.*, *Studies*, pp. 114-121.

[235] *Cairo ENL Survey*, no. C62; analyzed in E. S. Kennedy and D. A. King, "Ibn al-Majdī's Tables for Calculating Ephemerides", *JHAS* 4 (1980), pp. 48-68, repr. in King, *Studies*, A-VI.

[236] King, "Lunar Crescent Visibility Predictions in Medieval Islamic Ephemerides" (published in 1991), repr. in *idem*, *Studies*, C-IV.

4.6 Double-argument planetary equation tables

In the case of the lunar equation tables attributed to Ibn Yūnus, these are not presented in a *zīj*, although the unique manuscript bears the spurious title added in a later hand: *Zīj ḥabṭaq al-shams wa-'l-qamar*, the curious term *ḥabṭaq* being derived from Greek *epaktoi* (in the tables of Copt Ibn al-'Assāl mentioned in 3.3.11 we find the more convincing equivalent, *'bqṭy*). On other tables of this kind in *zīj*es see 3.8.

4.7 Auxiliary tables for solving spherical astronomical problems for all latitudes

Muslim astronomers compiled several sets of tables of trigonometric functions with no specific significance but so conceived that ordered applications of them could lead to the solution of problems of spherical astronomy. Their progress in mathematical methods is well reflected in these tables, a dozen of which are known.[237] The four most significant examples are:

1. The *Jadwal al-taqwīm* of Ḥabash al-Ḥāsib (Baghdad and Samarra, 9th century) with five functions (450 entries).[238]

2. The *Jadwal al-daqā'iq* of Abū Naṣr ibn 'Irāq (Central Asia, *ca.* 1000) with five functions (225 entries).[239]

3. The *Jadāwil al-dā'ir al-āfāqī* of Najm al-Dīn al-Miṣrī (Cairo, *ca.* 1300), serving both as a table for finding the time of day or night from the altitude of the Sun or any non-circumpolar star and also as a universal auxiliary table, with a single main function for three independent arguments and a grand total of *ca.* 420,000 entries![240]

237 Surveyed in King, *SATMI*, I-9.

238 R. A. K. Irani, *The* Jadwal al-taqwīm *of Ḥabash al-Ḥāsib*, unpublished master's thesis, American University of Beirut 1956.

239 C. Jensen, "Abū Naṣr's Approach to Spherical Astronomy as Developed in his Treatise *The Table of Minutes*", *Centaurus* 16 (1971), pp. 1-19.

240 F. Charette, "A Monumental Medieval Table for Solving the Problems of Spherical

4. *al-Jadwal al-āfāqī* of Shams al-Dīn al-Khalīlī (Damascus, *ca.* 1360) with three main functions (*ca.* 14,000 entries).[241]

4.8 Tables for time-keeping by the Sun and stars

Extensive corpora of tables of the hour-angle t (*faḍl al-dā'ir*) and the time since rising T (*al-dā'ir*), as well as the azimuth a (*al-samt*), for specific latitudes were compiled. All of these tables have been investigated in a recent study.[242] The arguments were the instantaneous altitude h and either the solar longitude λ or the solar meridian altitude H. The functions tabulated might include $t(h,\lambda)$ or $T(h,\lambda)$, as well as $a(h,\lambda)$ (each with some 10,000 entries), or $T(H,h)$ (with some 3,000 entries). As noted above, the universal auxiliary table of Najm al-Dīn al-Miṣrī with its three arguments (h, H and half the arc of visibility) serves to find the time since rising of the Sun or any star from its altitude, for any latitude. The main corpuses were:

1. The so-called *Zīj al-Ṭaylasān* (K201), comprising two tables for time-keeping by the Sun (one universal and the other for the latitude of Baghdad) by Abu 'l-Qāsim ʿAlī ibn Amājūr (Baghdad *ca.* 910), extant in MS Paris BNF ar. 2486.[243] A later table of the same kind for a specific latitude, based on an approximate formula and very corrupt, is extant in MS Leiden Or. 199,3.[244] Numerous tables of the same

Astronomy for all Latitudes", *AIHS* 48 (1998), pp. 11-64. On the use of these tables in instrumentation see the same author's thesis cited in n. 277 below.

241 D. A. King, "al-Khalīlī's Auxiliary Tables for Solving Problems of Spherical Astronomy", *JHA* 4 (1973), pp. 99-110, repr. in *idem*, *Studies*, A-XI; also G. Van Brummelen, "The Numerical Structure of al-Khalīlī's Auxiliary Tables", *Physis* 28 (1991), pp. 667-697. In 2001 an earlier set of universal auxiliary tables by al-Khalīlī for determining the solar azimuth was located in a manuscript in Bursa: these new tables confirm Van Brummelen's hypothesis about the order in which the subtables in al-Khalīlī's main set were compiled. See further King, *SATMI*, I-9.4.

242 See King, *SATMI*, I, and the survey article MĪḲĀT in EI^2.

243 Sezgin, *GAS*, VI, p. 178.

244 Storey, *PL*, II:1, p. 117; analyzed in B. R. Goldstein, "A Medieval Table for Reckoning Time from Solar Altitude", *Scripta Mathematica* 27 (1964), pp. 61-66, repr. in Kennedy

kind are found in the Islamic sources.

2. A table displaying the longitude of the horoscopus as a function of the altitude of 25 stars for each degree in the east and west up to the maximum was compiled for the latitude of Qandahar apparently in the 10th century and survives in MS Berlin Ahlwardt 5751, appended to a copy of the *zīj* of Kūshyār (see 3.3.2).

3. A corpus for Cairo, latitude 30;0°, started by Ibn Yūnus (*ca.* 1000), completed by al-Maqsī (*ca.* 1280) (see also 4.12.3) *et al.*, used until the 19th century, extant in numerous copies, of which the best is MS Dublin Chester Beatty 3673.[245]

4. A corpus for Taiz, latitude 13;37°, by Abu 'l-ʿUqūl (*ca.* 1300) (see 3.3.12), the largest single corpus for a single latitude from the medieval period, extant in the unique copy MS Berlin Ahlwardt 5720.[246]

5. A corpus for Damascus, latitude 33;30°, by Shams al-Dīn al-Khalīlī (Damascus, *ca.* 1360) (see also 4.10.3), used until the 19th century, extant in numerous copies, of which the best is MS Paris BNF ar. 2558.[247]

6. A corpus for Jerusalem, latitude 32;0°, by Zayn al-Dīn al-Karakī (*ca.* 1350), extant in the unique MS Leipzig Universitätsbibliothek 808, based on earlier tables by Shams al-Dīn Ibn al-Rashīdī, of which only a fragment survives.[248]

et al., *Studies*, pp. 293-298. For an explanation of the substantial errors in this table see now King, *SATMI*, VII-4.2.

245 Analyzed in King, "Ibn Yūnus' *Very Useful Tables* for Reckoning Time by the Sun", *AHES* 10 (1973), pp. 342-394, repr. in *idem*, *Studies*, A-IX. On the attribution see especially *idem*, *SATMI*, II-5.

246 King, *Astronomy in Yemen* (n. 11), pp. 31-32.

247 *Idem*, "Astronomical Timekeeping in Fourteenth-Century Syria", in *Aleppo 1976 Symposium Proceedings*, II, pp. 75-84, repr. in *idem*, *Studies*, A-X, esp. pp. 80-84.

248 *Ibid.*, p. 80.

7. An anonymous corpus for Tunis, latitudes 36;40° and 37;0°, anonymous, extant in a single copy, MS Berlin Ahlwardt 5724.[249]

8. An anonymous corpus for Alexandria, latitude 31;0°, extant in a single copy, MS Cairo Taymūr *riyāḍa* 354.[250]

9. An enormous table (*ca.* 250,000 entries) for timekeeping by the stars in Istanbul, latitude 41;0°, compiled by Muḥammad ibn Kātib Sinān (*ca.* 1500), extant in two copies, MSS Istanbul Aya Sofya 2710 and Topkapı AIII 3515.[251]

10. A *ṭaylasān* table for Istanbul, latitude 41;0°, by Taqī al-Dīn (*ca.* 1580) (see 3.3.14), extant in a single copy, MS Istanbul Kandilli Observatory 208.[252]

11. A substantial corpus for Istanbul, latitude 41;0°, by Ṣāliḥ Efendī (*ca.* 1775), extant in several copies, of which the best is MS Princeton Yahuda 353.[253]

Several tables of this kind for specific latitudes were compiled in Europe during the Middle Ages and the Renaissance, and from the 18th to the 20th century numerous corpuses of tables for different latitudes were compiled.[254]

[249] King, "Astronomy in the Maghrib" (n. 11), pp. 38-39.

[250] *Cairo ENL Survey*, no. C143.

[251] King, "Astronomical Timekeeping in Ottoman Turkey", in *Istanbul 1977 Symposium Proceedings*, pp. 245-269, repr. in *idem*, *Studies*, A-XII, esp. pp. 247-248.

[252] *Ibid.*, pp. 248-249.

[253] *Ibid.*, pp. 250-251.

[254] King, *SATMI*, I-10.1-2; and C. H. Cotter, "The Development of Nautical Astronomical Inspection Tables in the Period from 1770 to 1919", *Vistas in Astronomy* 20 (1976), pp. 245-247.

4.9 Tables for regulating the times of Muslim prayer

Numerous corpuses of tables serve the regulation of the astronomically-determined times of Muslim prayer.[255] The first tables of this kind were compiled in the 9th century: already al-Khwārizmī (see 3.1.3) prepared a table displaying the solar altitude at the time of the *ʿaṣr* prayer for each 6° of solar longitude and for the latitude of Baghdad,[256] and Ḥabash prepared tables of the duration of morning and evening twilight for the same city. Ibn Yūnus tabulated various functions with religious significance: the solar altitude at the *ʿaṣr* and when the Sun is in the azimuth of the *qibla*, and the durations of morning and evening twilight. In each of the corpora of tables listed above we find sets of tables relating to the prayer-times. For some localities we have sets of 'prayer-tables' without the more extensive tables for timekeeping by the Sun: examples include Fez, Marrakesh, Crete, Damietta, Rosetta, Aleppo, Lattakia, an unspecified location in Central Anatolia (with latitude 38°), Isfahan, and Yarqand. Some of these were used until the early modern period. In passing we mention the lists of shadow lengths at the *ẓuhr* (and sometimes also at the *ʿaṣr*) for each zodiacal sign or each month of the solar year found in treatises on folk astronomy and the sacred law.[257]

[255] These are surveyed in King, *SATMI*, II. See also the *EI*² articles MĪḲĀT and S͟HAFAḲ (on twilight).

[256] King, "al-Khwārizmī and New Trends in Mathematical Astronomy in the Ninth Century", *Occasional Papers on the Near East* (Hagop Kevorkian Center for Near Eastern Studies, New York University), 2 (1983), pp. 7-9, and pl. IX.1 in the *EI*² article MĪḲĀT. Not all of the works treated in that study are by al-Khwārizmī, even though they are all attributed to him; they are, however, all from 9th-century Baghdad.

[257] King, "A Survey of Medieval Islamic Shadow Schemes for Simple Time-reckoning", *Oriens* 32 (1990), pp. 191-249, repr. in *SATMI*, III; and M. Forcada, "Esquemes d'ombres per determinar el moment de les pregàries en llibres d'*anwāʾ* i calendaris d'al-Andalus", in *I Trobades d'història de la ciència i de la tècnica*, J. M. Camarasa, H. Mielgo and A. Roca eds., Barcelona, 1994, pp. 107-117.

4.10 *Tables for finding the qibla*

Already in the 9th century two astronomers in Baghdad produced two different tables displaying the *qibla* q,[258] measured from the local meridian, as a function of longitude difference ΔL and latitude difference $\Delta\phi = \phi - \phi_M$ from Mecca, both for each degree of the two arguments from 1° to 20°. One was based on an approximate formula, the other on an accurate formula, although the table in the form in which it survives is also approximate (!). Each is known from several manuscripts.[259] Several later Muslim scholars turned their attention to producing tables of this kind, including the following:

1. The celebrated Ibn al-Haytham (Cairo, *ca.* 1025) (see 3.6.1) states in his autobiography that he compiled a table for finding the *qibla* but this is not extant.[260]
2. The early 12th-century astronomer ʿAbd al-Raḥmān al-Khāzinī of Marw (see 3.3.7) compiled a table of $q(\Delta L, \Delta\phi)$ for each 1° of ΔL up to 60° and each 1° of $\Delta\phi$ up to 30°. This is no longer extant, but al-Khāzinī appears to have used it to compute the *qibla* values for the 250-odd localities in his geographical tables, and the results are not impressive.[261]
3. Shams al-Dīn al-Khalīlī (see 4.8.5) tabulated $q(\phi, \Delta L)$ for ϕ = 10°, 11°, ..., 50° and ΔL = 1°, 2°, ..., 60°, with remarkable accuracy, extant in three copies, of which the best is MS Paris BNF ar. 2558.[262]

258 See the *EI*[2] article ḲIBLA (ii. Astronomical Aspects), repr. with corrections in King, *Studies*, C-IX.

259 King, "The Earliest Islamic Mathematical Methods and Tables for Finding the Direction of Mecca", *ZGAIW* 3 (1986), pp. 82-149 (repr. in *idem*, *Studies*, C-XIV), esp. pp. 107-129; R. P. Lorch, "The Qibla Table attributed to al-Khāzinī", *JHAS* 4 (1980), pp. 259-264, repr. in *idem*, *Studies*, XIV; and King, *World-Maps* (n. 43), pp. 64-65.

260 King, "Earliest Islamic Mathematical Methods and Tables" (n. 259), p. 133.

261 *Ibid.*, pp. 134-136, and *idem*, *World-Maps* (n. 43), pp. 71-75 and App. D.

4. ʿAlāʾ al-Dīn ibn Ṭaybughā (Aleppo *ca.* 1350) tabulated $q(\Delta L, \phi)$ and also the distance to Mecca $d(\Delta L, \phi)$ for each 1° of ΔL up to 12° and each 1° of ϕ from 16° to 44°.[263]

5. A *qibla*-table in the *zīj* (K4) of Sayf al-Munajjim (Shiraz *ca.* 1400) (see 3.3.5) is a hodge-podge based on an earlier table by an incompetent.[264]

We have noted that the *qibla*-values in the geographical table of al-Khāzinī are somewhat inaccurate, as a result of his having used a crude interpolation procedure in a defective *qibla*-table. Some 300 years later, apparently in Kish near Samarqand, an anonymous Timurid astronomer computed the *qiblas* and distances from Mecca for some 275 localities, with values mainly accurate to a few seconds. This was achieved by calculating each from scratch rather than using any *qibla*-tables.[265]

4.11 Tables for constructing astrolabes and astrolabic quadrants

Al-Khwārizmī (see 4.9) was apparently the first to address in trigonometric terms the problem of the calculation of the size and position of the various markings on the astrolabe.[266] One needs the distance from the centre of the astrolabe to the centre of a given circle and the radius of that circle for a given altitude or azimuth, calculated for a given latitude. Al-Khwārizmī presented a table of the basic trigonometric auxiliary function for facilitating these calculations.[267] One of his contemporaries and several later

262 King, "al-Khalīlī's Qibla Table", *JNES* 34 (1975), pp. 81-122, repr. in *idem*, *Studies*, A-XIII.

263 King, "Earliest Islamic Mathematical Methods", pp. 139-140, and *idem*, *World-Maps* (n. 43), pp. 66-67.

264 King, "Earliest Islamic Mathematical Methods", p. 138; and J. P. Hogendijk, "The Qibla Table in the *Ashrafī Zīj*", in *Frankfurt IGN Festband*, pp. 81-94.

265 See further King, *World-Maps* (n. 43), pp. 149-168.

266 See n. 10 above.

267 King, "al-Khwārizmī" (n. 256), pp. 23-27.

astronomers presented more extensive sets of tables for a whole range of latitudes:

1. al-Farghānī (Baghdad *ca.* 825) presented a more useful auxiliary function and an entire corpus of tables showing the centre-distance and radius for each degree of altitude and azimuth, serving each degree of latitude from 15° to 50°. These tables, which contain over 8000 entries, are contained in several manuscripts, of which MS Berlin Ahlwardt 5790 is perhaps the best.[268]

2. The Rasulid Yemeni Sultan al-Ashraf (Taiz *ca.* 1295) compiled similar tables for the latitudes of various localities in the Yemen as well as Mecca and Medina (extant in Cairo and Tehran).[269]

3. The Egyptian astronomer al-Bakhāniqī (Cairo and the Yemen *ca.* 1350) extended the tables of al-Farghānī to cover each degree of latitude from 1° to 90°, extant in a unique copy in Dublin.[270]

Finally we note that Ḥabash al-Ḥāsib (see 4.7.1) tabulated coordinates for constructing the non-circular markings on a non-standard astrolabe of his own invention.[271] A survey of all such tables in the Islamic sources by the first author (D.A.K.) with F. Charette is in preparation.

4.12 Tables for constructing sundials

To mark the intersections of the hour-curves with the equinoctial and solstitial shadow-traces on a sundial,[272] it is convenient to have a set of tables

[268] King, "Islamic Astronomical Tables", pp. 53-55.

[269] King, *Astronomy in Yemen* (n. 11), pp. 28-29, and *idem*, "The Medieval Yemeni Astrolabe in the Metropolitan Museum of Art in New York", *ZGAIW* 2 (1985), pp. 99-122, repr. in *idem*, *Studies*, B-II (also *Studies*, C, Addenda, p. 6), with addenda in *ZGAIW* 4 (1987-88), pp. 268-269.

[270] King, *Astronomy in Yemen*, pp. 34-35.

[271] Kennedy, Kunitzsch & Lorch, *Melon Astrolabe* (n. 76), pp. 78-89. The original tables were much more accurate than the surviving manuscript and the new edition would lead one to suppose.

at hand displaying the coordinates of these points. From the 9th to the 19th century numerous Muslim astronomers compiled such tables. Some of the most significant are:

1. The tables of Ḥabash al-Ḥāsib (see 4.7.1), giving the radial coordinates (shadow lengths and azimuths) of these points for horizontal sundials serving latitudes 15°, 18°, ..., 33°, then 38° and 40°, and also Samarra (34°) and the equator (0°), extant in the unique copy, MS Istanbul Aya Sofya 4830, fols. 231r-235r, incorrectly attributed to al-Khwārizmī (see 3.1.3).[273]

2. The tables in a treatise on gnomonics by one or other of the two 10th-century Baghdad astronomers Ibn al-Ādamī or Saʿīd ibn Khafīf al-Samarqandī, including a set of auxiliary tables for generating the rectangular ("Cartesian") coordinates of these intersections on vertical sundials inclined to the meridian at any latitude. The treatise is extant in a unique copy, MS Paris BNF ar. 2506, not studied yet; the copyist, the late-15th-century Cairo astronomer Ibn Abi 'l-Fatḥ al-Ṣūfī, was unsure of the authorship.[274]

3. A corpus of tables by Shihāb al-Dīn al-Maqsī (Cairo *ca.* 1280) (see also 4.8.3) giving the 'Cartesian' coordinates of these intersections for vertical inclined sundials for each degree of inclination and for the latitude of Cairo, 30°. Extant in several copies.[275]

4. The tables in the extensive treatise on instrumentation by Abū ʿAlī al-Marrākushī (Cairo *ca.* 1280) (see 2.4) serve all manner of tables

[272] On Islamic sundials see the survey article MIZWALA in EI^2.

[273] See King, "al-Khwārizmī" (n. 256), pp. 17-22; the edition with numerous misreadings in B. A. Rosenfeld *et al.*, eds., *Muḥammad ibn Musa al-Khorezmi*, Moscow 1983, pp. 221-234; and, on the attribution, King, *World-Maps* (n. 43), pp. 349-350.

[274] Sezgin, *GAS*, VI, pp. 180 and 217.

[275] *Cairo ENL Survey*, no. C15, and King, "Astronomy of the Mamluks" (n. 11), pp. 547-548.

for horizontal, vertical, and special sundials, mostly for the latitude of Cairo, 30°. This work survives in numerous copies.[276]

5. Najm al-Dīn al-Miṣrī (see 4.7.3) compiled some unusual tables for sundial construction *ca.* 1325, extant in MS Dublin Chester Beatty Persian (*sic*) 102 and another manuscript in a private collection.[277]

6. Taqī al-Dīn (Istanbul, *ca.* 1580) (see 4.8.10) compiled a treatise on sundial construction with numerous tables for the latitude of Istanbul, extant in a unique copy, MS Istanbul Kandilli 208, unstudied.

A survey of these and numerous other Islamic tables for sundial construction would be worthwhile.

4.13 Astrological tables

There are all manner of astrological tables to be found in works other than *zīj*es. A few examples must suffice here:

1. Several medieval Egyptian manuscripts, as well as the Hyderabad copy of the *zīj* of Ibn Isḥāq (see 3.4.4) contain a table for computing the length of life of a new-born, in some sources stated to have been written in gold ink in the treatise *al-Kāmil fi 'l-nujūm* for the treasury of ʿAbdallāh ibn Ṭāhir, the governor of Khurasān (d. 844). This table was found originally in the *Anthologiæ* of the 2nd-century astrologer Vettius Valens, and in some of our sources is indeed attributed to 'Wālīs'.[278]

[276] Available in al-Marrākushī, *Mabādi' wa-ghāyāt*.

[277] F. Charette, *Mathematical Instrumentation in 14th-century Egypt and Syria*, doctoral thesis, Johann Wolfgang Goethe University, Frankfurt am Main, 2001. The second manuscript is featured in *Christie's London Islamic Art and Manuscripts Catalogue 11.4.2000*, lot 22.

[278] D. Pingree, ed., *Vettii Valentis Antiocheni Anthologiarum libri novem*, Leipzig 1986, pp. 308-315; *idem*, *The Thousands of Abū Ma'shar* (n. 53), pp. 21-24; and King, "Some Arabic Copies of Vettius Valens' Table for Calculating the Duration of Life", in *Symposium Græco-Arabicum* II, G. Endress ed., Amsterdam 1989, pp. 25-28.

2. The extensive treatise entitled *al-Jāmi' al-Shāhī* by 'Abd al-Jalīl al-Sijzī (*ca.* 975), extant in several copies, including MS Cairo Dār al-Kutub *mīqāt* 887 (150 fols.) and Dublin Chester Beatty 4079 (282 fols.).[279]

3. A treatise entitled *Iẓhār mā kāna mustakhfiyan fī aḥkām al-nujūm* by Najm al-Dīn Ayyūb ibn 'Ayn al-Dawla ibn Naṣr Allāh al-Ikhlāṭī (*ca.* 1300?), full of astrological tables of an unusual variety, extant in MS Cairo DM 4, as yet unstudied.[280]

4-6. Extensive astrological tables, several of them textual rather than mathematical, are found in the *Safīnat al-aḥkām* of Naṣīr al-Dīn al-Ṭūsī (see 3.3.9);[281] the two surviving Rasulid Yemeni ephemerides (see 4.3); and a Persian encyclopedia compiled in Isfahan *ca.* 1413 for Iskandar Sulṭān ibn 'Umar Shaykh and extant in MS Istanbul Topkapı B411.[282]

4.14 Miscellaneous tables

Muslim astronomers sometimes succumbed to an uncontrollable urge to tabulate any function that took their fancy. Thus, for example, when Ḥabash developed a device for time-keeping by the stars for all latitudes, he tabulated all of the stellar data in a form that would enable the user to mark it on the instrument with facility.[283] When Pseudo-Naṣīr al-Dīn al-Ṭūsī was thinking about the lengths of daylight that were to be marked on astrolabe plates, he tabulated the function for a series of localities.[284] And when some

279 Pingree, *The Thousands of Abū Ma'shar* (n. 53), pp. 70-127; also Sezgin, *GAS*, VII, pp. 178-181, and *Cairo ENL Survey*, no. B56.

280 Sezgin, *GAS*, VII, p. 22; and *Cairo ENL Survey*, no. C3.

281 Sezgin, *GAS*, VII, pp. 22-24.

282 King, *World-Maps* (n. 43), pp. 143-145.

283 *Ibid.*, pp. 354-358. See now F. Charette and P. Schmidl, "A Universal Plate for Time-keeping with the Stars by Ḥabash al-Ḥāsib: Text, Translation and Preliminary Commentary", *Suhayl* 2 (2001).

unidentified Egyptian astronomer was pondering the fact that the ventilators of medieval Cairo were aligned in the direction of the main axis of the city, he compiled a table showing the altitude of the Sun throughout the year when it was in the direction of the main axis of the rectangular base of the ventilator, that is, in the direction perpendicular to the city axis, which was, conveniently or by design, towards winter sunrise.[285] And when Najm al-Dīn al-Miṣrī wrote the introduction to his universal auxiliary table with its main table for each degree of each of the three arguments filling some 850 pages, he calculated how much paper would have been filled if he had made the table for each minute of each argument: the result was 5,300 volumes of the same size as the one he actually produced.[286] Here, admittedly, we see the passion of the Muslim astronomers for compiling tables getting a little out of control. Najm al-Dīn did not succumb to the temptation he contemplated, claiming as an excuse "the shortness of life and the lack of money", but others in medieval Cairo did, if only for much smaller tables: several Egyptian manuscripts are known containing 'stretched' tables of functions relating to timekeeping, such as the oblique ascensions of the ascendant at the time when the muezzin announces a blessing on the Prophet, computed for each three minutes of solar longitude.[287] But these were exceptions, for the majority of astronomers who compiled a *zīj* were content to restrict the tables and text to one or two hundred pages. Nevertheless, it will be no surprise if future researchers uncover other categories of tables as yet unknown. This is most likely in manuscripts labelled, or even catalogued, in one language or another as "Anonymous: astronomical tables".

[284] King, *World-Maps*, pp. 75-76.

[285] On this curious table for orienting ventilators see King, "Architecture and Astronomy: the Ventilators of Medieval Cairo and their Secrets", *JAOS* 104 (1984), pp. 97-133 (repr. as *SATMI*, IXb), esp. pp. 103-106.

[286] Charette, "Monumental Table" (n. 240), pp. 44-45.

[287] *Cairo ENL Survey*, nos. C122-123.

5 Concluding remarks

The scope of the *zīj* literature is a clear indication of the interest of Muslim scholars in astronomy for over a millennium. It is an accident of Islamic history that a few of these *zīj*es became known in medieval Europe and that this spawned an interest there too. Alone the computational accuracy of the vast majority of the tables in the *zīj*es is a clear sign of the competence of their compilers. Their errors, if they are original and not due to copyists, can provide useful clues to the way in which the tables were calculated. But the accompanying texts also merit our attention.

The study of Islamic mathematical astronomy is progressing slowly but constantly, and the reader should understand that this brief overview is intended to encourage the reader to consult Kennedy's *zīj* survey and the many writings it has inspired already and will continue to inspire. A new version of the survey is currently under preparation by B. van Dalen. A plea to have some of the most important manuscripts of *zīj*es published in facsimile may one day be taken seriously.[288] Given the small number of scholars engaged in deciphering these sources, we can assert that there is plenty of material available already. However, as new manuscript sources become accessible, researchers will continue to locate new *zīj*-related materials of historical consequence that are worthy of detailed study. Anyone who would doubt this would do well to consult A. Mestres' doctoral thesis on the Hyderabad manuscript of the anonymous recension of the *zīj* of Ibn Isḥāq. Ibn Khaldūn stated that this *zīj* was the basic reference work for contemporaneous Maghribī astronomers. Only in 1978 did the Hyderabad manuscript (400 pages of text and tables) "come to light" (even though it was correctly identified as "*Zīj Ibn Isḥāq*" in the published handlist of the library manuscript holdings from *ca.* 1930). In brief, the discovery of this manuscript and its subsequent evaluation have occasioned a revision of the history of astronomy in the Maghrib as well as in al-Andalus. Similar cases could be mentioned, and others have yet to happen.

288 King, "Some Remarks on Islamic Scientific Manuscripts and Instruments, and Past, Present, and Future Research", in *The Significance of Islamic Manuscripts*, J. Cooper ed., London 1992, pp. 115-143, esp. pp. 128-130 and 134.

Bibliographical abbreviations

We list here journals and frequently-cited works, including all those which are basic to the history of Islamic astronomy as well as the model editions and translations of, and commentaries on, the *zīj*es of al-Khwārizmī and al-Battānī. Studies of specific topics are listed in the footnotes.

AHES = *Archive for History of Exact Science.*

AIHS = *Archives internationales d'Histoire des Sciences.*

Aleppo 1976 Symposium Proceedings = A. Y. al-Hassan *et al.*, eds., *Proceedings of the First International Symposium for the History of Arabic Science*, 2 vols., Aleppo 1978.

ASP = *Arabic Science and Philosophy.*

Al-Battānī, *Zīj* = C. A. Nallino, ed., transl. and comm., *al-Battānī sive Albatenii opus astronomicum*, 3 pts., Milan, 1899-1907, pts. I-II repr. Frankfurt am Main 1969, pt. III (Arabic text) repr. Baghdad n.d. *ca.* 1960, pts. I-III, repr. 1 vol., Hildesheim & New York, 1977 [edited with a Latin translation and a commentary, also in Latin].

Astronomy across Cultures = *Astronomy across Cultures – The* [!] *History of Non-Western Astronomy*, H. Selin ed., Dordrecht, *etc.* 2000.

Al-Bīrūnī, *Maqālīd* = M.-Th. Debarnot, ed. and transl., *al-Bīrūnī – Kitāb Maqālīd ʿilm al-hayʾa – La trigonométrie sphérique chez les Arabes de l'Est à la fin du Xe siècle*, Damascus 1985.

— , *On Shadows* = E. S. Kennedy, *The Exhaustive Treatise on Shadows by Abū al-Rayḥān … al-Bīrūnī – Translation and Commentary*, Aleppo 1976.

— , *Tafhīm* = R. R. Wright, *The Book of Instruction in the Elements of the Art of Astrology by Abu'l-Rayḥān … al-Bīrūnī …* , London 1934, repr. Baghdad n.d.

Cairo ENL Survey = D. A. King, *A Survey of the Scientific Manuscripts in the Egyptian National Library*, (Publications of the American Research Center in Egypt, Catalogs, vol. 5), Winona Lake, Ind. 1986.

Cambridge Dibner Institute 1998 Conference Proceedings = selected papers given at a conference "New Perspectives on Science in Medieval Islam" held at the Dibner Institute, Cambridge, Ma., during Nov. 6-8, 1998, to appear.

Centaurus = *Centaurus – International Magazine of the History of Mathematics, Science, and Technology.*

DSB = *Dictionary of Scientific Biography*, 14 vols. and 2 supp. vols., New York 1970-80.

EHAS = *Encyclopedia of the History of Arabic Science*, R. Rashed with R. Morelon eds., 3 vols., London 1996; with French translation *Histoire de la science arabe*, Paris 1997.

EI[1] = *The Encyclopædia of Islam*, 1st edn., 4 vols., Leiden 1913-1934, repr. in 9 vols., Leiden 1987, paper-back edn., 1993.

EI[2] = *The Encyclopædia of Islam*, new edn., 10 vols. to date, Leiden 1960 to present.

Frankfurt IGN Festband = *Ad radices – Festband zum fünfzigjährigen Bestehen des Instituts für Geschichte der Naturwissenschaften, Frankfurt am Main*, A. von Gotstedter ed., Stuttgart 1994.

Goldstein, *Studies* = B. R. Goldstein, *Theory and Observation in Ancient and Medieval Astronomy*, London (Variorum) 1985.

Goldstein: see also Ibn al-Muthannā, *Commentary on al-Khwārizmī*.

Hartner, *Studies*, A-B = W. Hartner, *Oriens-Occidens – Ausgewählte Schriften zur Wissenschafts- und Kulturgeschichte – Festschrift zum 60. Geburtstag*, Hildesheim, 1968 (A); and *Oriens-Occidens – Ausgewählte Schriften zur Wissenschafts- und Kulturgeschichte – Band II*, Y. Maeyama ed., Hildesheim, *etc.* 1984 (B).

HistMath = *Historia Mathematica.*

HistSci = *Historia Scientiarum* (Brussels).

Ibn al-Muthannā, *Commentary on al-Khwārizmī* = B. R. Goldstein, *Ibn al-Muthannā's Commentary on the Astronomical Tables of al-Khwārizmī*, New Haven, Ct. 1967.

İhsanoğlu, ed., *Ottoman Astronomy* and *Ottoman Mathematics* = E. İhsanoğlu, ed., *Osmanlı astronomi literatürü tarihi*, 2 vols., and *Osmanlı matematik literatürü tarihi*, 2 vols., Istanbul (IRCICA Studies and Sources on the History of Science, nos. 7-8) 1996 and 1999.

IJHS = *Indian Journal of History of Science.*

Isis = *Isis – An International Review devoted to the History of Science and its Cultural Influences.*

Islamic Mathematics and Astronomy = *Islamic Mathematics and Astronomy*, F. Sezgin *et al.* eds., 106 vols., Frankfurt am Main (Institut für Geschichte der Arabisch-Islamischen Wissenschaften), 1997-2000. [Reprints of texts, studies and articles from the 19th and early 20th centuries: see n. 5.]

Istanbul 1977 Symposium Proceedings = *International Symposium on the Observatories in Islam, Istanbul, 19-23 September, 1977*, M. Dizer ed., Istanbul 1980.

Istanbul 1981 Congress Proceedings = *I. International Congress on the History of Turkish and Islamic Science and Technology, 14-18 September, 1981*, 5 vols., Istanbul 1981.

Istanbul 1986 Congress Proceedings = *II. International Congress on the History of Turkish and Islamic Science and Technology, 28 April - 2 May, 1986*, 3 vols., Istanbul 1986.

Istanbul 1987 Symposium Proceedings = *Transfer of Modern Science Technology to the Muslim World*, E. İhsanoğlu ed., Istanbul (IRCICA) 1982.

Istanbul 1991 and 1994 Symposia Proceedings = *Science in Islamic Civilisation*, E. İhsanoğlu and F. Günergun eds., Istanbul (IRCICA) 2000.

JAOS = *Journal of the American Oriental Society.*

JHA = *Journal for the History of Astronomy.*

JHAS = *Journal for the History of Arabic Science* (Aleppo).

JNES = *Journal of Near Eastern Studies.*

Kennedy, *Studies* = E. S. Kennedy, *Astronomy and Astrology in the Medieval Islamic World*, Aldershot & Brookfield, Vt. (Ashgate-Variorum) 1998.

— , "Zīj Survey" = *idem*, "A Survey of Islamic Astronomical Tables", in *Transactions of the American Philosophical Society*, N. S., 42:2 (1956), pp. 123-177, repr. *ibid.* n. d. [*ca.* 1990], with separate pagination.

Kennedy & Kennedy, *Islamic Geographical Coordinates* = E. S. and M. H. Kennedy, *Geographical Coordinates of Localities from Islamic Sources*, Frankfurt am Main 1987.

Kennedy *et al.*, *Studies* = E. S. Kennedy, Colleagues and Former Students, *Studies in the Islamic Exact Sciences*, M. H. Kennedy and D. A. King eds., Beirut 1983.

Kennedy Festschrift = *From Deferent to Equant – A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E. S. Kennedy*, D. A. King and G. Saliba eds., Annals of the New York Academy of Sciences, d [=500] (1987).

Al-Khwārizmī, *Zīj* = H. Suter *et al.*, eds., *Die astronomischen Tafeln des Muhammed ibn Musa al-Khwārizmī ...* , in *Kgl. Danske Vidensk. Skrifter*, 7. R., *Hist. og filos. Afd.*, 3:1 (1914) [providing an edition of the Latin text]; and O. Neugebauer, *The Astronomical Tables of al-Khwārizmī*, in *Kgl. Danske Vidensk. hist.-fil. Skrifter*, 4:2 (1962) [with an English translation of the text and a commentary].

King, "Islamic Astronomical Tables" = D. A. King, "On the Astronomical Tables of the Islamic Middle Ages", in *Studia Copernicana* 13 (1975), pp. 37-56, repr. in *idem*, *Studies*, A-I (with addenda).

—, *SATMI* = *idem*, *Studies in Astronomical Timekeeping in Medieval Islam*, 10 pts., including I: A Survey of Tables for Reckoning Time by the Sun and Stars, II: A Survey of Tables for Regulating the Times of Prayer; VI: Universal Solutions in Islamic Astronomy; VIII: Astronomical Instrumentation in the Islamic World, Leiden (E. J. Brill), to appear.

—, *Studies*, A-C = *idem*, *Islamic Mathematical Astronomy*, London (Variorum) 1986, 2nd rev. edn., Aldershot (Variorum) 1993 (A); *Islamic Astronomical Instruments*, London (Variorum) 1987, repr. Aldershot (Variorum) 1995 (B); and *Astronomy in the Service of Islam*, Aldershot (Variorum) 1993 (C).

King: see also *Cairo ENL Survey*.

Krause, "Stambuler Handschriften" = M. Krause, "Stambuler Handschriften islamischer Mathematiker", *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B: Studien, Band 3, Heft 4 (1936).

Kunitzsch, *Studies* = P. Kunitzsch, *The Arabs and the Stars*, Northampton (Variorum) 1989.

Kunitzsch Festschrift = *Sic itur ad astra – Studien zur Geschichte der Mathematik und Naturwissenschaften – Festschrift für den Arabisten Paul Kunitzsch zum 70. Geburtstag*, M. Folkerts and R. P. Lorch eds., Wiesbaden 2000.

Kyoto 1997 Conference Proceedings = *History of Oriental Astronomy, Pro-*

ceedings of Joint Discussion 17, 23rd IAU General Assembly, Kyoto, 25-26 August, 1997, S. M. R. Ansari ed., Dordrecht 2000.

Langermann, *Studies* = Y. T. Langermann, *The Jews and the Sciences in the Middle Ages*, Aldershot (Variorum) 1999.

Lorch, *Studies* = R. P. Lorch, *Arabic Mathematical Sciences – Instruments, Texts, Transmission*, Aldershot (Variorum) 1995.

Al-Marrākushī, *Mabādi' wa-ghāyāt*, A-B = J.-J. Sédillot, *Traité des instruments astronomiques des Arabes composé au treizième siècle par Aboul Hhassan Ali de Maroc ...* , 2 vols., Paris, 1834-1835, repr. in 1 vol., Frankfurt am Main 1985 (A); and L.-A. Sédillot, *Mémoire sur les instruments astronomiques des Arabes*, in *Mémoires de l'Académie Royale des Inscriptions et Belles-Lettres de l'Institut de France* 1 (1844), pp. 1-229, repr. Frankfurt am Main 1989 (B).

Matvievskaya & Rosenfeld, *MAMS* = G. P. Matvievskaya and B. A. Rosenfeld, *Matematiki i astronomi musulmanskogo srednevekovya i ikh trudi*, 3 vols., Moscow 1983.

Millás Vallicrosa, *Estudios*, A-B = J.-M. Millás Vallicrosa, *Estudios ...* and *Nuevos estudios sobre historia de la ciencia española*, (first published Barcelona 1949 and 1960, respectively), repr. together in 2 vols., Madrid 1987.

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