The Travels of Astronomical Tables within Medieval Islam: A Summary

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Abstract: This survey of the movements of astronomical tables within medieval Islam provides a backdrop for the remaining articles in this volume. We discuss the types of document that contained astronomical tables, and some of the analytical tools that have produced discoveries with respect to the transmission of tables from one author or subculture to another. We conclude with a summary of table transmission: from origins with the appropriation of traditions of astronomical tables taken from Greece and India, through the establishment of a Ptolemaic tradition in the 9th century and a uniquely Islamic expansion of methods and concepts starting in the 10th, to important sub-traditions in al-Andalus and the Maghrib.

Keywords: astronomy, numerical tables, Ptolemy, India, al-Andalus

1. Introduction

Much has been written about the transmission of astronomy between ancient Greece, early India, and medieval Islam. Links between the astronomical theories in the works of certain Muslim scientists and their later European counterparts, especially Copernicus, have also garnered much attention. However, not enough work has gone into transmission between various medieval cultures, so it is salutary that this issue of *Suhayl* is bringing this important topic to light. While the articles in this issue deal with different aspects of transmission from Islam to other medieval cultures, it is important to recognize that medieval Islam itself was far from a single entity in either time or space. Astronomical subcultures had their own theoretical preferences and predilections. These emerge from the study of numerical astronomical tables in distinctive ways: for an astronomical table theory takes second place to practice, changing the goals of the astronomical enterprise. Given the unique nature of tables as opposed to text and diagrams, a study of astronomical transmission within Islam, focusing on tables rather than texts, would be most welcome.

This short article is not that study. Rather, we provide here a summary of what is known today about the movements of tables within Islam, as well as a short discussion of the tools (both actualized and in potential) to study these transmissions. Our hope is to provide an appropriate background for the studies in this volume, and motivate interest in a more thorough account of table transmission within Islam.

2. The Documents

Astronomical tables are a significant focus of research in Islamic science; several dozen articles over the decades have given hints at the diversity and depth of knowledge represented in these texts[1]. Even so, the secondary literature has hardly scratched the surface; newly discovered tables continue to reflect novel astronomical insights and mathematical techniques. There is, therefore, still a long way to go before the table tradition is really understood. Tables do not reveal their secrets in the same transparent way that written texts do; the analytical tools are still being developed. In addition, the study of tables requires a different sort of historical sensibility: in particular, an awareness that table making was a different skill with different goals. Rather than produce new theoretical insights, tables had a more pragmatic purpose – to produce useable predictions of the motions of the heavenly bodies for their audience, regardless of the underlying geometric model. Thus, what makes a table special often differs from what makes a text special.

The largest part of the tabular literature is the *zīj*, a compendium of numerical tables (usually, between about 50 and 400) with astronomical significance, with instructions for their use and only occasionally with some accompanying theoretical discussion. The *zīj* was a universal guide to tracking the movements

[1] See King, David A; Samsó, Julio; and Goldstein, Bernard (2001). "Astronomical handbooks and tables from the Islamic world (750–1900): an interim report", *Suhayl* (2): 9-105. for an update of Kennedy's classic survey of Islamic astronomical tables Kennedy, Edward S (1956). "A survey of Islamic astronomical tables", *Transactions of the American Philosophical Society*, (46): 123-177.; see also Van Brummelen, Glen (to appear). "Tables transformed: Innovations in tabular theory and methods in medieval Islamic astronomy", in Dominique Tournes, (ed.), *Histoire des Tables Numériques*. for an account of how the table makers transformed the inherited tradition.

in the heavens, and all the resulting applications. It was a document to be used, not studied; thus it was often unimportant to the author or reader to explain the parameters and geometric models. The *zīj* was in some aspects specific to the author's locality in space and time, since the parameters included the author's geographic latitude and often an epoch close to the author's life. After building up the mathematical preliminaries (trigonometry and spherical astronomy), a typical *zīj* would include a set of tables to convert between several calendars. This set the basis for the heart of the work, the determination of the positions of the celestial bodies. This completed, it was now possible for the author to construct tables to predict various phenomena related to celestial positions, such as eclipses and planetary visibilities. Often, a *zīj* also contained a star catalogue, tables of geographical positions, and tables of astrological quantities.

Other collections of tables usually dealt with certain special topics. Tables helping users to tell the time of day by the altitude of the sun appeared in a surprising variety of configurations, since there are a variety of quantities that may enter such calculations[2]. There were several types of tables designed for the purpose of Islamic ritual. These included: (i) determining the direction of Mecca (the *qibla*) from one's position on the Earth's surface; (ii) calculating the five daily prayer times (which is determined by the Sun's altitude in the sky); and (iii) predicting the beginning of the lunar month (especially Ramadan), which takes place when the lunar crescent becomes visible again after new moon. The latter may have been the most difficult of all functions to calculate. Tables of *auxiliary functions* were popular methods of solving via tables many astronomical problems at once: by tabulating cleverly-chosen simple combinations of trigonometric functions, these functions could be “snapped” together in various ways to attack virtually any sophisticated problem. Finally, ephemerides giving the positions of the celestial objects for a set of dates were usually computed from tables found in *zījes* or elsewhere.

3. Tools to Trace Transmission

Faced with a historical table, the scholar may access several different tools to detect the influence of some antecedent table. The most obvious method is simply to search for a direct reference to the older table in the text accompanying the newer one. However, this occurred rather infrequently, much less than one might expect even when tables were copied wholesale. In some

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[2] Surveys and studies of tables for celestial timekeeping are compiled the magisterial King, David A (2004), *In Synchrony with the Heavens: Studies in Astronomical Timekeeping and Instrumentation in Medieval Islamic Civilization*, vol. 1: *The Call of the Muezzin*, Leiden: Brill.

cases the older table is altered in minor ways, but is still at least partly recognizable in the newer document.

Generally, however, the transmission is more subtle, and such obvious signs are absent. In these cases, hope is not lost. Fairly commonly, the value of some numerical parameter (such as the Sun's eccentricity or the obliquity of the ecliptic) affects every value in the table. In this situation, a method designed by Benno van Dalen to reconstruct the parameter's value from the entries in the table[3] may be able to identify the particular value used by the astronomer, which in turn places the table within one tradition or another[4].

In other cases some older table (say, a trigonometric or auxiliary table) would be used by an astronomer to compute a new table for a more specialized purpose. In these cases, it might be possible to detect such a use by tracing the errors in the underlying table through to the errors in the dependent table. This is the idea behind Van Brummelen's table dependence test[5], which has been used both to identify connections between tables from one author to the next, and to link tables computed by the same author.

Finally, it is often possible to identify the method of computation used to compute a table, and a shared computational method between two tables might suggest (although not prove) that the authors were in contact, or at least working within the same tradition. A simple example: the "method of sines" and "method of declinations" were common techniques for computing the solar equation within the *Sindhind* tradition (as opposed to the Ptolemaic). The entries in a solar equation table are usually precise enough that one can tell whether or not

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[3] van Dalen, Benno (1989). "A statistical method for recovering unknown parameters from medieval astronomical tables", *Centaurus* (32): 85–145.

[4] Although van Dalen's method is the first systematic approach to parameter estimation, parameters have been studied for the purpose of understanding transmission for decades. For instance, for various studies of the transmission of parameters in Ibn al-Aʿlam's *zīj* see Mogenet, Joseph (1962). "Une scolie inédite du *Vat. gr.* 1594 sur les rapports entre l'astronomie arabe et Byzance", *Osiris* (14) : 198–221., Kennedy, Edward S (1977). "The astronomical tables of Ibn al-Aʿlam", *Journal for the History of Arabic Science (1)*, 13–23., Mercier, Raymond (1989). "The parameters of the *zīj* of Ibn al-Aʿlam", *Archives Internationales d'Histoire des Sciences* (39): 22–50., and Tihon, Anne (1989). "Sur l'identité de l'astronome Alim", *Archives Internationales d'Histoire des Sciences* (39): 3–21. See also Mielgo, Honorino (1996). "A method of analysis for mean motion astronomical tables", in Josep Casulleras and Julio Samsó, (eds.), *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Juan Vernet*, vol. 1, Barcelona: Instituto "Millás Vallicrosa", 159–179. for a method of analyzing parameters in mean motion tables.

[5] Van Brummelen, Glen and Butler, Kenneth (1997). "Determining the interdependence of historical astronomical tables", *Journal of the American Statistical Association* (92): 41–48.

these methods were used simply by recomputing the tables according to the various techniques and comparing the results with the table[6].

These techniques described in this section have so far hardly scratched the surface of what might be possible to learn about table transmission in the medieval period. Their applications so far have proved their potential, but the table literature is so vast that the work left to be done is almost boundless.

4. Table Travels

Although we are not prepared to give a detailed picture of table transmission within Islam, the broad strokes of the story are at least traceable – especially the parts that share in the larger context of scientific transmission. The rest of this short article summarizes briefly what is known of the various traditions and how they interacted, in order to provide both background for the articles in this volume, and a list of sources for readers seeking more information on the topic.

Arrival from the East: The astronomical tradition that eventually gave rise to the tables in *zījes* and elsewhere likely came from both west and east. Indian astronomy bears certain characteristics that suggest to some researchers (but not others) an origin in pre-Ptolemaic Greek astronomy[7]. If this transmission did in fact occur, without doubt the theory was transformed substantially in Indian hands; different approaches to science and the role of geometric theory would transform it into a unique discipline. Indian astronomy in turn likely inspired Sasanian scientists in Persia, although Greek influences were also present in Persia. Now, the modern image of a table, as a grid of numbers corresponding to values of some mathematically-defined function, did not exist in early Indian texts; rather, the extant texts contain lists of numbers, embedded within the prose text, that might have been suitable for memorization. There is virtually no documentary evidence for what might have appeared in Sasanian texts, so it is

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[6] See Salam, Hala; and Kennedy, Edward S (1967). "Solar and lunar tables in early Islamic astronomy", *Journal of the American Oriental Society* (87): 492–497 for a survey of different methods of calculation of solar and lunar equation tables.

[7] The question of the extent of Greek influence in medieval India, and through India to Sasanian and Arabic texts, is both complicated and vexed. David Pingree was a leader in tracing evidence of pre-Ptolemaic Greek astronomy through the earliest Indian sources (Pingree, David (1976). "The recovery of early Greek astronomy from India", *Journal for the History of Astronomy* (7): 109–123.) and onward to early Islam (Pingree, David (1973). "The Greek influence on early Islamic mathematical astronomy", *Journal of the American Oriental Society* (93): 32–43.); some others have denied that there was a transmission, arguing for a native birth or rebirth of the subject in early India, or transmission in the reverse direction.

unclear at what point in the transmission that tables recognizable as such to a modern reader started to appear[8].

In the eighth century Indian astronomy spread even farther west, especially through the works of Brahmagupta[9]. Two of the earliest *zījes*, the *Zīj al-Arkand* (AD 735) and the *Zīj al-Shāh* (AD 790), were inspired by his *Khandakhādyaka.* However, the most famous transmission came about through a diplomatic voyage from India to Baghdad in the early 770s; some unnamed person took with him an astronomical work (*siddhanta*), probably based on Brahmagupta's *Brāhmasphutasiddhānta*, but not identical to it[10]. Versions of this work, composed in Arabic at the request of caliph al-Manṣūr, were made by al-Fazārī and Yaʿqūb ibn Ṭāriq[11], triggering the *zīj* tradition known as *Sindhind.* These works, as far as we know, resided mostly in the Indian genre (such as the mean motion parameters, geometric models, and computational techniques); a few Sasanian influences, such as the Persian calendar, may also be found[12].

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[8] The standard account of the term "*zīj*" is that it derives from the Pahlavi term "*zīk*", or a cord from a loom, a metaphor for the appearance of the table's grid (see Kennedy, Edward S (1956). "A survey of Islamic astronomical tables", *Transactions of the American Philosophical Society*, (46): 123—177: 123–124). However, Mercier, Raymond (2000). "From tantra to *zīj*", in Menso Folkerts and Richard Lorch, (eds)., *Sic Itur ad Astra: Studien zur Geschichte der Mathematik und Naturwissenschaften*, Wiesbaden: Harrassowitz, 451–460. casts doubt on a connection between the Pahlavi word and the table's gridlike appearance.

[9] See Pingree, David (1973). "Indian influence on Sasanian and early Islamic astronomy and astrology", *Journal of Oriental Research Madras* (34-35): 118–126. for a survey of Indian influence on Sasanian and early Arabic astronomy. The question of the rise of interest in science in early Islam more generally has been discussed in many places, recently in Saliba, George (2007). *Islamic Science and the Making of the European Renaissance*, Cambridge, MA: MIT Press.; major factors include the shared language of Arabic and a learned administrative class.

[10] See Pingree, David (1976). "The Indian and pseudo-Indian passages in Greek and Latin astronomical and astrological texts", *Viator* (7): 141–195.

[11] The surviving fragments of these works are collected and summarized in Pingree, David (1970). "The fragments of the works of al-Fazārī", *Journal of Near Eastern Studies* (29):103–123. and Pingree, David (1968). "The fragments of the works of Yaʿqūb ibn Ṭāriq", *Journal of Near Eastern Studies* (27): 97–125.

[12] A major primary source (although not always a reliable one) on *zījes* for this early period is al-Hāshimī's late ninth-century *Book of the Reasons Behind Astronomical Tables* (Haddad, Fuad, Kennedy, E. S.; and Pingree, David (1981). ʿAlī ibn Sulaymān al-Hāshimī, *The Book of the Reasons Behind Astronomical Tables*, Delmar, NY: Scholars' Facsimiles and Reprints). Back in 1956, Otto Neugebauer said: "In view of this rather unsatisfactory situation [the dearth of primary literature in the Sindhind tradition, and the difference in methods between it and Indian astronomy] it has become customary to assign to Indian influence all those parts of Muslim tables...which cannot be traced directly to Greek predecessors...But it is by no means impossible that occasionally a method is called Indian when it actually may come from a Greek source, lost or unpublished, or may be the contribution of a Muslim astronomer" (Neugebauer, Otto (1956), "The

By far the most important *zīj* in the *Sindhind* tradition was al-Khwārizmī's, from the early ninth century[13]. No longer available in the original Arabic, this *zīj* had a large following in an altered form in al-Andalus, and through al-Andalus later to Europe. Thus we have some idea of its contents, although it has been modified through translations and a number of intermediaries. Al-Khwārizmī worked in the court of caliph al-Ma'mūn, where documents from diverse sources and many scholars were in regular contact with each other[14]. His *zīj* was influenced by, or even said to derive from, the *Brāhmasphutasiddhānta*; it was once described as an abridgment of al-Fazārī's *zīj*. Our ability to judge the accuracy of these statements is clouded by overlays of new content that were added when the *zīj* arrived in al-Andalus. We even find within the existing *zīj* four tables taken from Ptolemy's *Almagest* and *Handy Tables*. However, the heart of the theory is Indian, taking its inspiration from the tradition of Brahmagupta.

The Ptolemaic incursion: Just how familiar al-Khwārizmī was with Ptolemy is unclear. Two translations of the *Almagest* were made under al-Ma'mūn, so he cannot have been completely ignorant of it. However, the Ptolemaic and *Sindhind* traditions are not easy bedfellows; astronomers trying to compose coherent works would need to choose one or the other as a starting point. Around the middle of the ninth century, Islamic astronomers chose Ptolemy. This required several changes to their astronomical practice. In particular, the geometric models for planetary motions became more axiomatic and less empirical; that is, the models were taken as the truth and therefore as the source of predictions, rather than as malleable tools needed to generate predictions. Secondly, systematic observational programmes were instituted to re-evaluate the values of various astronomical parameters, and to adjust for new

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transmission of planetary theories in ancient and medieval astronomy", *Scripta Mathematica* (22): 165—192 espec. 171-172). Although the manuscript tradition is better established than it was then, Neugebauer's caution is still warranted.

[13] There is a large literature on this *zīj* and its transmission; the foundational works are an edition by Suter, Heinrich (1914). *Die Astronomischen Tafeln des Muḥammed ibn Mūsā al-Khwārizmī, D. Kgl. Danske Vidensk. Selsk. Skrifter* (7), a translation and study in Neugebauer, Otto (1962). *The Astronomical Tables of al-Khwārizmī, Hist. Filos. Skr. Dan. Vid. Selsk.* (4), and Goldstein's edition of a commentary by Ibn al-Muthannā Goldstein, Bernard (1967). *Ibn al-Muthannā's Commentary on the Astronomical Tables of al-Khwārizmī*, New Haven: Yale University Press.

[14] The usual story, that scholarly activity took place at the House of Wisdom, has been questioned by Gutas in Gutas, Dimitri (1998). *Greek Thought, Arabic Culture*, New York: Routledge. Certainly at this time and roughly in this place, a great deal of knowledge was changing hands and form, if not actually in the House of Wisdom.

geographical locations. Seven hundred years after Ptolemy, this was sorely needed.

The first major work to implement a rigorous series of observations was the *Mumtaḥan* ("Verified") *Zīj* by Yaḥyā ibn Abī Manṣūr, commissioned by caliph al-Ma'mūn in 828/829. This joint effort was the first of several large observational projects – in a sense, the beginning of big science[15]. While there is still a substantial presence of Indian and Iranian astronomy in the *Mumtaḥan Zīj*, the theories of planetary motion and spherical astronomy – the heart of the *zīj* – had become Ptolemaic. In the middle of the century Ḥabash al-Ḥāsib wrote three different *zījes*, shifting gradually but dramatically toward the Ptolemaic tradition. Ḥabash's works also benefited from improved observations and parameters, but Ḥabash also preserved the superior Indian-inspired methods of computation, which were to spark *zīj* authors at least through the 15th century. The century ended with al-Battānī's *al-Zīj al-Ṣābi'*, a tightly structured and almost purely Ptolemaic work; some of its tables are taken directly from the *Handy Tables*[16]. All three of these astronomers spent at least part of their time in Baghdad, the center of the scholarly world. However, movement through eastern Islam was quite fluid, and it is clear that scholars carried their ideas and influences with them as they traveled. For example, al-Battānī was born in Harran, Turkey; spent a large part of his working life in Raqqa, Syria; and finished his career in Baghdad. The Indian tradition had essentially run its course in the east; it would, however, continue through transmission to al-Andalus in the west.

The Islamic Enlightenment: Around the turn of our millennium, eastern Islam saw a flowering of intellectual activity, especially in the sciences, that took Islamic science in several new directions. With the support of Sharaf al-Dawla of the Buyid dynasty, a new observatory was built in Baghdad; one of its contributors was Abū l-Wafā'[17] His *zīj*, named the *Almagest* after Ptolemy's work, survives only without the tables[18]. Nevertheless the text reveals great

[15] Goldstein describes some of the observations made under al-Ma'mūn's supervision in Goldstein, Bernard (1986). "The making of astronomy in early Islam", *Nuncius* (1): 79–92.

[16] Al-Battānī's *zīj* was edited and translated to Latin in Nallino, Carlo Alfonso (1899-1907). *Al-Battānī sive Albatenii Opus astronomicum*, 3 vols., Milan: Pubblicazioni del reale Osservatorio di Brera in Milano.

[17] See Sayili, Aydin (1960). *The Observatory in Islam*, Ankara: Türk Tarih Kurumu Basimevi espec. 110–117.

[18] See Carra de Vaux, Bernard (1892). "L'Almageste d'Abû'l-Wéfa Albûzdjâni", *Journal Asiatique*, (19) : 408–471.

mathematical creativity: it incorporates the tangent function fluidly for the first time, and employs a newly conceived and improved spherical trigonometry. The great al-Bīrūnī corresponded with Abū l-Wafā' and performed some joint observations with him (although, after Abū l-Wafā''s death, they were on opposite sides of a priority dispute with respect to the new spherical methods[19]). Al-Bīrūnī's own *zīj*, the *al-Qānūn al-Masʿūdī*[20], contained a number of theoretical innovations and influenced several eastern *zījes*, but it was one of the earliest major works not to have an impact in al-Andalus.

Several communities developed from this fertile ground. The central focus of table-making activity shifted from Baghdad toward Iran[21], and subtraditions developed for instance in Egypt, Syria[22], and Yemen[23]. These regions developed new interests, especially in the area of solar timekeeping[24]. But it was the founding of observatories that most directly caused ideas to collide and new paths to be forged. Two were especially significant. The first was the 13th-century observatory at Maragha (now in northwestern Iran), led by Naṣīr al-Dīn al-Ṭūsī[25]. Al-Ṭūsī's most famous astronomical achievements were related to finding new geometric models of the motions of the planets differing from

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[19]One side of this dispute is documented in al-Bīrūnī's *Kitāb Maqālīd ʿIlm al-Hay'a*; see Debarnot, Marie-Thérèse, tr. (1985). *Al-Bīrūnī,* Kitāb Maqālīd ʿIlm al-Hay'a*: La trigonométrie sphérique chez les Arabes de l'Est à la fin du X^ème^ siècle*. Damascus: Institut Français de Damas.

[20] See a detailed table of contents of this *zīj* in Kennedy, Edward S (1971). "Al-Bīrūnī's *Masudic Canon*", *Al-Abhath* (24): 59–81., and an edition in Krause, M., (ed) (1954-56). *Al-Bīrūnī, Al-Qānūn 'l-Masʿūdī (Canon Masudicus)*, 3 vols., Hyderabad.

[21] Kennedy, Edward S (1956). "A survey of Islamic astronomical tables", *Transactions of the American Philosophical Society*, (46): 123–177. espec. 168.

[22] For a survey of astronomy in Mamluk Egypt and Syria, see King, David A (1983). "The astronomy of the Mamluks", *Isis* (74): 531–555; on *zījes* see especially pp. 535–537.

[23] For a short summary of mathematical astronomy in Yemen, see King, David A (1979). "Mathematical astronomy in medieval Yemen", *Arabian Studies* (5): 61–65., and for a longer one, see King, David A (1983). *Mathematical Astronomy in Medieval Yemen – A Bio-Bibliographical Survey*, Malibu, CA: Undena.

[24] See King, David A (1976). "Astronomical timekeeping in fourteenth-century Syria", in Ahmad Y. Al-Hassan, Ghada Karmi, and Nizar Namnum, (eds.), *Proceedings of the First International Symposium for the History of Arabic Science*, vol. 2, University of Aleppo: Institute for the History of Arabic Science: 75–84 for a survey of astronomical timekeeping in Syria. But King, David A., Samso, Julio; and Goldstein, Bernard (2001). "Astronomical handbooks and tables from the Islamic world (750–1900): an interim report", *Suhayl* (2): 9–105. and King, David A (2005). *In Synchrony with the Heavens: Studies in Astronomical Timekeeping and Instrumentation in Medieval Islamic Civilization*}, vol. 2: *Instruments of Mass Calculation*, Leiden: Brill.

[25] See Sayili, Aydin (1960). *The Observatory in Islam*, Ankara: Türk Tarih Kurumu Basimevi: 187–223.

Ptolemy's (the *hay'a* tradition, exemplified in his *Tadhkira*). However, *hay'a* models seldom found their way into numerical tables; al-Ṭūsī's own *Zīj-i Īlkhānī*, which had an impact on a number of later authors, was itself a solidly Ptolemaic work. A significant counter-example to this was the collection of tables by Ibn al-Shāṭir, built on one of the alternate geometric models and strongly influenced by the Maragha tradition[26]. The Maragha observatory lasted only about fifty years, but its influence was felt for centuries. In fact, our second observatory was founded by Ulugh Beg, who early in life had visited the remnants of the Maragha observatory[27]. Its leading scientist, Jamshīd al-Kāshī, wrote his *Khāqānī Zīj* explicitly as a series of corrections to the *Īlkhānī Zīj*[28]. Ulugh Beg's own *Sulṭānī Zīj*, perhaps the most successful set of tables in the entire medieval period, seems to have been a collaborative effort, and includes (without mention) at least a few of the innovations presented in al-Kāshī's earlier work[29].

Al-Andalus and the Maghrib: The introduction of *zījes* into western Islam helped to promote interest in astronomy[30]. Two in particular, those by al-Khwārizmī and al-Battānī, enjoyed great success. Through al-Andalus, these two *zījes* were the conduits to the introductions of the *Sindhind* and the Ptolemaic traditions respectively into Europe. However, later eastern *zījes* did not generate the same

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26 See Roberts, Victor (1957). "The solar and lunar theory of Ibn al-Shāṭir", *Isis* (48): 428–432., Kennedy, Edward S.; and Roberts, Victor (1959). "The planetary theory of Ibn al-Shāṭir", *Isis* (50): 227–235., Roberts, Victor (1966). "The planetary theory of Ibn al- Shāṭir: Latitudes of the planets", *Isis* (57):208–219.; and for his tables, especially Abbud, Fuad (1962). "The planetary theory of Ibn al-Shāṭir: Reduction of the geometric models to numerical tables", *Isis*, (53): 492-499.

27 For information on Ulugh Beg's observatory, see Sayili, Aydin (1960). *The Observatory in Islam*, Ankara: Türk Tarih Kurumu Basimevi, 260–289 and Bruin, Frans (1967). "The astronomical observatory of Ulugh Beg in Samarkand", *al-Bīrūnī Newsletter*, (9).

28 Several aspects of the *Khāqāni Zīj* have been studied, but alas, E. S. Kennedy's translation was never completed. See Kennedy, Edward S (1998). *On the Contents and Significance of the Khāqānī Zīj by Jamshīd Ghiyāth al-Dīn al-Kāshī*, Frankfurt: Institut für Geschichte der Arabisch-Islamischen Wissenschaften for a detailed table of contents.

29 Again, several individual studies of Ulugh Beg's *zīj* have appeared; but a thorough treatment of the entire work remains to be completed. Sédillot, L. P. E. A (1847-1853). *Prolégomènes des Tables Astronomiques d'Oloug Beg*, 2 vols., Paris: Firmin Didot Frères contains a translation to French of the introductory material.

30 For a survey of Andalusian *zījes*, see Samso, Julio (2011). *Las ciencias de los antiguos en al-Andalus*, Almería: Fundación Ibn Tufayl, or for a shorter summary, see Samso, Julio (1992). "The exact sciences in al-Andalus", in Salma Khadra Jayyusi, (ed.). *The Legacy of Muslim Spain*, Leiden/New York/Köln, Brill: 952—973 espec: 960–966.

level of interest. Rather, an internal tradition became established especially through the late 11th-century *Toledan Tables*, with contributions by Ibn al-Zarqālluh and Ṣāʿid al-Andalusi.[31] Several unique astronomical features became part of the Andalusian corpus[32]: in particular, a motion of the solar apogee, and (perhaps due to the presence of Indian astronomy in al-Andalus) theories of "trepidation" – an oscillation in the precession of the equinoxes. Although eastern authors were aware of these western innovations, they failed to gain much traction. In al-Andalus itself, they continued through the works of twelfth-century authors such as Ibn al-Kammād[33] and Ibn al-Hā'im. Beginning in the early 13th century, several Maghribī authors, including Ibn Isḥāq[34] and Ibn al-Bannā'[35], composed astronomical tables in the spirit of al-Andalus. By the early 15th century, observations of precession brought trepidation into question, and this led to a return of eastern influence in the Maghrib[36].

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[31] The Toledan Tables has attracted much scholarly interest. See in particular the edition by Pedersen, Fritz S (2002). *The Toledan Tables. A Review of the Manuscripts and the Textual Versions with an Edition*, 4 vols., Copenhagen: Det Kongelige Danske Videnskabernes Selskab. and the survey by Toomer, Gerald (1968). "A survey of the Toledan Tables", *Osiris* (15): 5–174.

[32] For a survey of some of the unique features of astrology in the Islamic west, see Casulleras, Josep (2008–09). "Mathematical astrology in the medieval Islamic west", *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, (18): 241–268.

[33] On Ibn al-Kammād's *zīj* see Chabás, José; and Goldstein, Bernard. (1994), "Andalusian astronomy: *al-Zîj al-Muqtabis* of Ibn al-Kammâd", *Archive for History of Exact Sciences,* (48): 1–41.

[34] On Ibn Isḥāq's *zīj* see King, David A (1990). "An overview of the sources for the history of astronomy in the medieval Maghrib", *Deuxième colloque maghrébin sur l'histoire des mathématiques arabes: Tunis, les 1-2-3 Décembre 1988, Actes du colloque*, Tunis: University of Tunis: 125–157. and Mestres, A (1996). "Maghribī astronomy in the 13th century: A description of manuscript Hyderabad Andra Pradesh State Library 298", in Josep Casulleras and Julio Samsó, (eds.), *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Juan Vernet*, vol. 1, Barcelona: Instituto "Millás Vallicrosa": 383–443.

[35] Vernet, Juan (1951). *Contribución al Estudio de la Labor Astronómica de Ibn al-Bannā',* Tetuán: Editora Marroquí.

[36] For a survey of the *zīj* tradition in the Maghrib, including the entrance of the eastern tradition in the fourteenth century, see Samsó, Julio (1998). "An outline of the history of Maghribī *zijes* from the end of the thirteenth century", *Journal for the History of Astronomy* (29): 93–102. For a study of texts relating to the reception of Andalusi theories of trepidation in the Maghrib, see Comes, Mercè. (2002), "Some new Maghribī sources dealing with trepidation", in S. R. Ansari, (ed.), *Science and Technology in the Islamic World*, Turnhout: Brepols: 121–141.