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SOME RECENT RESULTS ON HOCHSCHILD HOMOLOGY OF COMMUTATIVE ALGEBRAS(*)(**)

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Abstract

This article is a brief survey of the recent results obtained by several authors on Hochschild homology of commutative algebras arising from the second author's paper [**21**].

1. Introduction.

The Hochschild (co)homology theory is originally defined for associative algebras over commutative rings (see, e.g., [**5**, Chapter 9]). In this paper our interest is the homology of commutative algebras. After remembering the definition and some "clasic" properties of the theory, we briefly explain the results of some recent works, which have arisen from the point of view adopted by the second author in [**21**].

2. Preliminaries.

If $K \to A$ is a commutative ring homomorphism and M is an $A \otimes_K A$-module, the n-dimensional Hochschild (co)homology for A with coefficients in M is defined as

$$H_n(A, M) = \operatorname{Tor}_n^{A \otimes_K A}(A, M)$$
$$H^n(A, M) = \operatorname{Ext}^n_{A \otimes_K A}(A, M)$$

In both cases, A is considered as $A \otimes_K A$-module via the multiplication $A \otimes_K A \to A$, $a \otimes a' \rightsquigarrow aa'$.

This is the definition used in [**5**] and, in general, is different from Hochschild's definition. Both coincide if A is a flat K-module (homology)

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or if A is a projective K-module (cohomology). In particular, if A is a flat K-module, then $H_n(A,A)$ is the n-th homology of the complex of A-modules $(C_*(A,A),d_*)$, where

$$C_n(A,A) = A^{\otimes(n+1)} = A \otimes_K \overset{n+1}{\cdots} \otimes_K A$$

with the A-module structure induced by multiplication in the first factor, and

$$d_n(a_0 \otimes a_1 \otimes \cdots \otimes a_n) = \sum_{i=0}^{n-1} (-1)^i a_0 \otimes \cdots \otimes a_i a_{i+1} \otimes \cdots \otimes a_n + (-1)^n (a_n a_0) \otimes a_1 \otimes \cdots \otimes a_{n-1}.$$

$H_*(A,A) = \oplus_{n\geq 0} H_n(A,A)$ becomes a strictly anti-commutative graded algebra over A [**13**, Theorem 2.2, p. 225]. If A is flat K-module, the product is induced by the shuffle product:

$$C_p(A,A) \otimes_A C_q(A,A) \to C_{p+q}(A,A)$$

$$(a \otimes a_1 \otimes \cdots \otimes a_p) \otimes (a' \otimes a_{p+1} \otimes \cdots \otimes a_{p+q}) \to \sum \operatorname{sig}(\sigma) aa' \otimes a_{\sigma^{-1}(1)} \otimes \cdots \otimes a_{\sigma^{-1}(p+q)}$$

where the sum is taken over all permutations σ of $\{1,\dots,p+q\}$ such that $\sigma(1) < \cdots < \sigma(p)$ and $\sigma(p+1) < \cdots < \sigma(p+q)$.

Consider the Kähler differentials A-module $\Omega^1_{A|K} = I/I^2$, where I is the kernel of the ring homomorphism $A \otimes_K A \to A$, and the canonical derivation $d = d_{A|K} : A \to \Omega^1_{A|K}$, $a \to (a \otimes 1 - 1 \otimes a) + I^2$. There is a natural A-module isomorphism

$$\Omega^1_{A|K} \cong H_1(A,A)$$

determined by da$\rightsquigarrow$class of $(1 \otimes a)$. This isomorphism extends to a homomorphism of graded A-algebras

$$\gamma : \Omega^*_{A|K} \to H_*(A,A)$$

$\Omega^*_{A|K}$ being the exterior algebra of $\Omega^1_{A|K}$.

The study of the homomorphism γ begin in [**9**], where G. Hochschild, B. Kostant and A. Rosenberg prove that γ is isomorphism if K is a perfect field and A is the coordinate ring of an affine algebraic variety nonsingular over K.

This result was generalized by M. André in [**2**], using simplicial methods through the homology theory of commutative algebras $H_n(K, A, -)$ (André-Quillen homology [**1**], [**19**]). André proves that for a flat homomorphism of noetherian rings $K \to A$ the following conditions are equivalent

i) $K \to A$ is regular (i.e., its fibres are geometrically regular)
ii) $\Omega^1_{A|K}$ is a flat A-module and γ is an isomorphism.

In positive characteristic very little is known on the homomorphism γ. Nevertheless, if A is of characteristic zero (i.e., if A contains the rational numbers), it is not difficult to prove, using the Hochschild complex $C_*(A, A)$, that γ has left inverse. More generally, D. Quillen obtain in [**19**] the following. If $K \to A$ is a flat homomorphism with A of characteristic zero, then there exists an A-module isomorphism

$$H_n(A, A) \cong H_n(A, A)^{(1)} \oplus \cdots \oplus H_n(A, A)^{(n)}, \quad n \geq 0$$

(Hodge decomposition of the Hochschild homology), where

$$H_n(A, A)^{(p)} = H_{n-p}(\wedge^p L_{A|K}),$$

$L_{A|K}$ being the cotangent complex of A over K. In particular

$$H_n(A, A)^{(1)} = H_{n-1}(K, A, A)$$

and $H_n(A, A)^{(n)}$ identifies to $\Omega^n_{A|K}$ by γ.

Question 2.1. *Let K be a field (of any characteristic) and A a K-algebra. Is $\gamma : \Omega^*_{A|K} \to H_*(A, A)$ injective?*

The answer is affirmative for complete intersections [**6**] (in fact in [**6**] an analogue to the Hodge decomposition is obtained for complete intersections in any characteristic). Nevertheless, in the general case is unknown.

There are analogous results on cohomology.

First there exists a graded homomorphism

$$\omega : H^*(A, A) \to \mathrm{Hom}_A(\Omega^*_{A|K}, A).$$

If $K \to A$ is a smooth homomorphism of noetherian rings, then ω is isomorphism.

On the other hand, if $K \to A$ is flat with A of characteristic zero, ω has right inverse. More generally, in this case

$$H^n(A, A) \cong H^n(A, A)_{(1)} \oplus \cdots \oplus H^n(A, A)_{(n)}$$

with $H^n(A, A)_{(p)} = H^{n-p}(\mathrm{Hom}_A(\wedge^p L_{A|K}, A))$.

3. Regular homomorphisms and vanishing of Hochschild homology.

First we shall refer to [**20**], where the following result is obtained:

Theorem 3.1. *If $K \to A$ is a homomorphism of noetherian rings such that A is of characteristic zero and $A \otimes_K A$ is a noetherian ring, then are equivalent:*

i) *$K \to A$ is a regular homomorphism*
ii) *$K \to A$ is flat and $fd_{A\otimes_K A}(A) < \infty$.*

The proof uses that $H_*(K, A, -)$ is a direct summand of $H_*(A, -)$ (in characteristic zero) so as some deep results of the homology of commutative algebras. (Let us say i) $\Rightarrow$ ii) is proved without hypotheses on the characteristic.)

In [**21**] it is managed to avoid the restriction on the characteristic, and in part of the ring $A \otimes_K A$, proving, in a relatively elementary way, the following:

Theorem 3.2. *If $K \to A$ is a flat homomorphism of noetherian rings and $fd_{A\otimes_K A}(A) < \infty$, then $K \to A$ is a regular homomorphism.*

Moreover it is conjectured:

Conjecture 3.3. *Let K be a field of characteristic zero and A a K-algebra of finite type. If $H_n(A, A) = 0$ for n sufficiently large, then A is a smooth K-algebra (which, in this case, is equivalent to the regularity of the ring A).*

The reciprocal of the theorem 3.2 is false, as it is seen by applying the result of André, above mentioned, to a separable field extension $K \to A$ with $\Omega^1_{A|K}$ not of finite type over A.

In [**17**], Majadas and Rodicio pose the problem to characterize the regular homomorphisms $K \to A$ for which $fd_{A\otimes_K A}(A) < \infty$. This seems a difficult problem and, in fact, they resolve it only for two particular cases:

a) K and A fields: $fd_{A\otimes_K A}(A) < \infty \Leftrightarrow A|K$ is a separable field extension and $\operatorname{tr.deg.}(A|K) < \infty$.
b) K perfect field and A Dedekind domain: $fd_{A\otimes_K A}(A) < \infty \Leftrightarrow \operatorname{tr.deg.}(A|K) < \infty$.

To give an idea of the difficulties of the proofs, we will say that the proof of b) needs the Néron desingularization theorem so as André theorem on the localization of formal smoothness.

In view of a) and b) we pose the following question.

Question 3.4. *Let K be a field and A a noetherian local K-algebra. If $K \to A$ is regular and* $\operatorname{tr.deg.}(A|K) < \infty$*, is $fd_{A\otimes_K A}(A) < \infty$?*

The reciprocal is true [**17**].

The partial results a) and b) allow to give examples which tend to delimit the position of the flat homomorphisms with $fd_{A\otimes_K A}(A) < \infty$ in commutative algebra. So, we have a diagram of implications and not implications relative to the flat homomorphisms of noetherian rings

$$\begin{array}{ccccc} \text{absolutely flat} & \Longrightarrow & fd_{A\otimes_K A} < \infty & \Longrightarrow & \text{regular} \\ & \searrow & \not\Uparrow \quad \not\Downarrow & \nearrow & \\ & & \text{smooth} & & \end{array}$$

Moreover, if $fd_{A\otimes_K A}(A) < \infty$, then: $K \to A$ is smooth $\Leftrightarrow$ the A-module $\Omega^1_{A|K}$ is of finite type.

On the other hand, if $fd_{A\otimes_K A}(A) < \infty$ and K is a quasi-excellent ring, then A is not necessarily quasi-excellent (unlike what happens with the absolutely flat homomorphisms).

Next we will explain the history of the Conjecture 3.3.

4. Case of complete intersections.

In [**16**] the Conjecture 3.3 has been proved in the case A is locally complete intersection (l.c.i.), i.e., $A = R/J$, where R is a polynomial K-algebra of finite type and J is locally generated by a regular sequence. It is achieved using a spectral sequence

$$E_{p,q} = H_p(A, A) \otimes_A \wedge^q J/J^2 \Rightarrow A \otimes_R \Omega^n_{R|K}.$$

In this proof it is essential the condition of K be of characteristic zero. Nevertheless, Rodicio manages to avoid in [**22**] this hypothesis proving previously the following result (which is a weak form of the Question 2.1).

Proposition 4.1. *Let K be a field and A a K-algebra of finite type. If $H_n(A, A) = 0$ for some natural number n, then $\Omega^n_{A|K} = 0$ (see Proposition 5.7, below).*

In [**18**] Majadas and Rodicio study more in detail the above spectral sequence, and they use it to calculate the homology of the coordinate rings of the hypersurfaces (in characteristic zero).

The authors discover in [**11**] a new method for the study of the Hochschild homology of l.c.i. algebras. Let us suppose, to fix ideas, that R is a localization of the polynomial ring $K[x_1, \dots, x_n]$, with K field of characteristic zero, J and ideal of R generated by a regular sequence $f_1, \dots, f_m$, and $A = R/J$. Then they prove that in the Hodge decomposition

$$H_n(A, A) \cong H_n(A, A)^{(1)} \oplus \cdots \oplus H_n(A, A)^{(n)}$$

it is verified

$$\begin{aligned} H_\alpha(A, A) &= H_{\alpha-p}(\wedge^p L_{A|K}) \\ &= H_{n-m+\alpha-2p+1}(\mathbf{K}(\mathbf{a}; n-m+\alpha-p+1)), \quad 1 \le p \le \alpha \end{aligned}$$

where $\mathbf{a}$ is the matrix of the canonical homomorphism

$$J/J^2 \to A \otimes_R \Omega_{R|K},$$

with respect to the basis induced by f_i and x_j.

Here, for a matrix $\mathbf{b}$ and an integer t, $\mathbf{K}(\mathbf{b}; t)$ denote the generalized Koszul complex associated to $\mathbf{b}$ and t, introduced by D. Kirby in [**10**].

This result allows to use the grade-sensitivity of the complexes $\mathbf{K}(\mathbf{b}; t)$ to study the $H_\alpha(A, A)$. So the following results are obtained.

Theorem 4.2. *Assume that there exists a natural even number α and a natural odd number β such that*

$$H_\alpha(A, A) = 0 = H_\beta(A, A).$$

Then A is a smooth K-algebra.

Theorem 4.3. *Let r be a natural number. The following conditions are equivalent*

i) $\gamma_\alpha : \Omega^\alpha_{A|K} \to H_\alpha(A, A)$ *is an isomorphism for all* $\alpha \le r$.

ii) A_p *is a smooth K-algebra for all prime ideals P of A such that* $ht(P) < r - 1$.

In [**11**] it is also proved a result on cohomology:

Theorem 4.4. *If there exists a natural even number α such that $H^\alpha(A, A) = 0$. Then A is a smooth K-algebra.*

Furthermore for the coordinate rings of the hypersurfaces, is enough to require the vanishing in only one dimension (for homology and cohomology).

These results of the authors, have been generalized by J. A. Guccione and J. J. Guccione to arbitrary characteristic in [**7**].

5. General case.

The affirmative answer to the conjecture 3.3 has been obtained by A. Campillo, J. A. Guccione, J. J. Guccione, M. J. Redondo, A. Solotar and O. E. Villamayor (B.A.C.H.) in [**4**]. In fact they prove the more general result:

Theorem 5.1. *Let K be a field of characteristic zero, A a K-algebra of essentially finite type and P a prime ideal of A. If A_p is not regular, then $H_i(A,A) \neq 0$ for every i congruent to $q = \max\{j/\Omega^j_{A_p|A} \neq 0\} \bmod 2$.*

Therefore, if $H_\alpha(A,A) = 0 = H_\beta(A,A)$ for some natural number α even and some natural number β odd, then A is a smooth K-algebra.

The proof uses the explicit construction of a free resolution of A over $A \otimes_K A$, which is a differential graded algebra.

The result of B.A.C.H. was also obtained by L. Avramov and M. Vigué-Poirrier in [**3**] for a field of any characteristic:

Theorem 5.2. *Let A be an algebra of finite type over a field K. If $H_\alpha(A,A) = 0 = H_\beta(A,A)$ for some α even and some β odd, then A is a smooth K-algebra.*

The proof of Avramov and Vigué use arguments from differential graded homological algebra (in particular the Eilenberg-Moore Tor functor) so as intuition from Algebraic Topology. The proof is rather complicated from the technical point of view.

The definitive result (till the present) in this direction, has been obtained by Rodicio, who generalizes in [**24**] the Avramov and Vigué-Poirrier theorem simplifying its proof drastically.

The principal novelty is to deal with homology of augmented algebras instead of Hochschild homology. A R-algebra S is augmented if the canonical homomorphism $\phi : R \to S$ verify $\psi\phi = 1$, where $\psi : S \to R$ is a ring homomorphism. The result obtained is:

Theorem 5.3. *Let R be a ring and S a noetherian augmented R-algebra with augmentation ideal I. If $\operatorname{Tor}^S_\alpha(R,R) = 0 = \operatorname{Tor}^S_\beta(R,R)$ for some α even and some β odd, then I is locally generated by a regular sequence.*

If $K \to A$ is a ring homomorphism, then the homomorphism $A \to A \otimes_K A$, $a \rightsquigarrow a \otimes 1$, converts to $A \otimes_K A$ in an augmented A-algebra with $\psi : A \otimes_K A \to A$, $a \otimes a' \rightsquigarrow aa'$. Particularizing to this case, the theorem 5.3 affirms:

Corollary 5.4. *Let $K \to A$ be a flat homomorphism of commutative rings such that $A \otimes_K A$ is noetherian. If $H_\alpha(A,A) = 0 = H_\beta(A,A)$ for some α even and some β odd, then A is a smooth K-algebra.*

As a consequence it is obtained the reciprocal of the previously mentioned result of Hochschild, Kostant and Rosenberg.

Corollary 5.5. *Let $K \to A$ be a flat homomorphism of commutative rings such that $A \otimes_K A$ is noetherian. The following conditions are equivalent:*

i) *A is a smooth K-algebra*
ii) *The homomorphism $\gamma : \Omega^*_{A|K} \to H_*(A,A)$ is an isomorphism*
iii) *The algebra $H_*(A,A)$ is generated by elements of degree 1.*

The proof of the theorem 5.3 is reduced to the case where R and S are local rings. It is considered then an "acyclic clousure" (see [**8**]) (X,d) of the S-algebra R, that is, a free *DG*-*S*-algebra resolution of R with a certain property of minimality. Now the condition of S be augmented over R, imply that X is a minimal resolution in the usual sense, i.e. $NX \supseteq dX$, N being the maximal ideal of S. This observation is what allows us to simplify the proof of Avramov and Vigué, and is a consequence of a result of Avramov and Rahbar-Rochandel proved many years ago (see [**12**, Theorem 2.5]).

It would be interesting to know if there exists some generalization of the theorem 5.3 to the case in which S fails to be noetherian. For example:

Question 5.6. *Let R be a noetherian ring, M a R-module and $S = S_R(M)$ the symmetric algebra of M. Let us suppose $fd_S(R) < \infty$. Is M a flat R-module?*

If the answer to this question is affirmative, then it is not difficult to deduce the following consequence: $fd_S(R) \leq n \Leftrightarrow M$ is R-flat and $\wedge^{n+1} M = 0$.

To show other example in which the point of view of the augmented algebras provide very much simplifications in the proofs, we will focus our attention on the homomorphism γ and we shall obtain a generalization of the Proposition 4.1.

Proposition 5.7. *Let S be a local noetherian ring with residue field L and I an ideal of S such that S is an augmented $R = S/I$-algebra. Then the canonical homomorphism*

$$\gamma \otimes L : (\wedge^* I/I^2) \otimes_R L \to \mathrm{Tor}^S_*(R,R) \otimes_R L$$

is injective. In particular, if $\mathrm{Tor}_n^S(R,R) = 0$ *for some* n*, then* $\wedge^n I/I^2 = 0$.

Proof: Let X be an acyclic closure of $S \to R$ which, as we have said, is also a minimal resolution. Then $\mathrm{Tor}^S(R,L) = H_*(X \otimes_S L) = X_* \otimes_S L$. In dimension p, $(\wedge^p I/I^2) \otimes_R L = (\wedge^p I) \otimes_S L$ has as basis the images of the elements of the form $dT_{i_1} \wedge \cdots \wedge dT_{i_p}$, $1 \le i_1 < \cdots < i_p \le n$, where $T_1, \dots, T_n$ are the variables introduced of degree 1, so that $\{dT_1, \dots, dT_n\}$ is a minimal set of generators for the ideal I. It follows that the canonical homomorphism

$$(\wedge^* I/I^2) \otimes_R L \to \mathrm{Tor}^S(R,L)$$

is injective. The commutativity of the following diagram shows that $\gamma \otimes L$ is injective:

$$\begin{array}{ccc} \wedge^* I/I^2 \otimes_R L & \xrightarrow{\gamma \otimes L} & \mathrm{Tor}^S(R,R) \otimes_R L \\ & \searrow \quad \swarrow & \\ & \mathrm{Tor}^S(R,L) & \end{array}$$

What really demonstrates the proof given by B.A.C.H. of the Theorem 5.1, is the following. Let K be a field of characteristic zero, A a K-algebra of essentially finite type, P a prime ideal of A such that A_p is not regular and $q = \max\{j/\Omega^j_{A_p|K} \neq 0\}$. Then $H_{q+2i}(A,A)^{(q+i)} \neq 0$ for $i \ge 0$, i.e. $H_j(\wedge^{q+i} L_{A|K}) \neq 0$. Majadas study in [**15**] an analogue question in any characteristic and prove:

Theorem 5.8. *Let* $K \to A$ *be a flat homomorphism such that* $A \otimes_K A$ *is a noetherian ring and let* $q = \max\{j/\Omega^j_{A|K} \neq 0\}$*. If* $K \to A$ *is not smooth, then for every* $i \ge 1$*, there exists* r *such that* $i+1 \le r \le i+q$ *and* $H_i(\wedge^r L_{A|K}) \neq 0$.

(In fact, the paper of Majadas is written in the context of augmented algebras).

6. On the vanishing of the homology on only one dimension.

There are examples which show that the conclusion of the Theorem 5.3 cannot be deduced from the hypothesis $\mathrm{Tor}^S_\alpha(R,R) = 0$ for only one α. The authors don't know analogous examples for Hochschild homology. On the contrary, there are some results that seem to indicate that such examples don't exist.

Majadas proves in [**14**]:

Theorem 6.1. *Let K be a field of characteristic zero, I an ideal of $K[x_1, \dots, x_n]$ generated by a regular sequence of homogeneous polynommials $f_1, \dots, f_m$ and $A = K[x_1, \dots, x_n]/I$. If $H_\alpha(A, A) = 0$ for some α, then A is a smooth K-algebra.*

More generally, Rodicio study in [**23**] the rigidity of the generalized Koszul complexes and prove

Theorem 6.2. *Let K be a field and A a l.c.i. essentially of finite type K-algebra. If $H_\alpha(A, A) = 0$ for some α, then A is a smooth K-algebra.*

M. Vigué-Poirrier obtain in [**26**] an analogous result for any graded algebra over a field of characteristic zero.

That induce us to conjecture the following:

Conjecture 6.3. *Let K be a field and A an essentially of finite type K-algebra. If $H_\alpha(A, A) = 0$ for some α, then A is a smooth K-algebra.*

7. Characterization of complete intersections.

Let K be a field of characteristic zero and A an essentially of finite type K-algebra. If A is l.c.i., then in the Hodge decomposition

$$H_n(A, A) \cong H_n(A, A)^{(1)} \oplus \cdots \oplus H_n(A, A)^{(n)}$$

we have (as a consequence, for example, of the calculations in [**11**]) that $H_n(A, A)^{(p)} = 0$ for $p < n/2$. Vigué obtain in [**27**] a reciprocal:

Theorem 7.1. *If a natural number n_0 exists which verify $H_p(A, A) = 0$ for every p such that $0 < p < n/2$ and every $n > n_0$, then A is l.c.i.*

Let us say, finally, some words on the vanishing of Hochschild cohomology of non-complete intersection algebras.

Let K be a field of characteristic zero and A a reduced K-algebra of finite type. For $n = 2$, the Hodge decomposition is

$$H^2(A, A) \cong H^1(K, A, A) \oplus \mathrm{Hom}_A(\Omega^2_{A|K}, A).$$

It is not difficult to prove that $\mathrm{Hom}_A(\Omega^2_{A|K}, A) = 0$ is equivalent to $\dim(A) \leq 1$. Therefore: $H^2(A, A) = 0$ if and only if A is a rigid K-algebra and $\dim(A) \leq 1$. Nevertheless, the problem to determine if a rigid curve is nonsingular, is not solved (at least as far as the authors know).

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