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MAINTENANCE POLICY UNDER MULTIPLE UNREVEALED FAILURES

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The unrevealed failures of a system are detected only by inspection. In this work, an inspection policy along with a maintenance procedure for multi-unit systems with dependent times to failure is presented. The existence of an optimum policy is also discussed.

Keywords: Maintenance, reliability, replacement, optimum policy

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1. INTRODUCTION

The classical age and block replacement policies are only useful when the failures of a unit are detected as soon as they occur. However, the failures of spare units or in a system during the stand-by mode, may remain undiscovered until the next demand of activity, causing important availability losses. Periodic inspection or testing is the only way to fight against failures of this sort (see Lewis (1987)).

The works due to Nakagawa and Yasui (1991), Vaurio (1995, 97, 99) and Berrade (1999) deal with unrevealed failures; all of them consider maintenance policies for single-unit systems. On the contrary, Ebrahimi (1997) provides an extension of the age replacement policy for multi-unit systems under revealed failures.

In this work we provide a new inspection policy to detect the occurrence of failures in a system, otherwise being unrevealed, along with a maintenance procedure to improve its reliability. This policy can be carried out in multi-unit systems with an only type of failure, or in single-unit systems with multiple «hidden» causes of failure. Many systems of this sort can be found in practice: the fluid levels in a car are checked periodically so that an eventual leakage of oil, water, etc can be observed. The inspection policies are concerned not only with engineering systems: the periodic tests carried out to detect disorders of health in humans constitute the most common example of such procedure.

We assume the possibility of dependent times to failure between units or types of failure which is a realistic assumption when the occurrence of a type of failure is likely to have an effect on the probability of an event of another type. We characterize optimum policies so as to minimize the cost per unit of time for an infinite time span, thus, $\lim_{t\to\infty}\frac{C(t)}{t}$ with $C(t)$ the cost incurred in $[0,t]$.

Under this policy, the unit is periodically checked and replaced, hence, the maintenance process yields a renewal process. In fact, it corresponds to the particular class of the renewal-reward processes with the term cost being used instead of reward. The properties of the renewal reward processes ensure that the previous cost function converges, with probability 1, to the next one

$$Q(T)=\frac{E\left[C(\tau)\right]}{E(\tau)}$$

A cycle, denoted τ, is the time span between two consecutive renewals of the system (see Ross (1996)) with $C(\tau)$ denoting its associated (random) cost. From now on $Q(T)$, depending on the time between consecutive inspections, T, will denote the objective function.

In section 2, the maintenance policy is described along with the conditions that guarantee the existence of an optimum policy for a series system (Theorem 1) and a parallel system (Theorem 2). In addition, we consider multivariate distributions which are natural extensions of the exponential random variable: the bivariate Gumbel distribution, as well as the Marshall-Olkin model. These distributions allow the dependence between components and, provided they hold, we obtain particular conditions for the existence of an optimum policy. Finally, section 3 contains some examples involving the previous distributions which illustrate the theoretical results.

2. THE MODEL

The system consists of n units which are periodically checked at times kT, $k = 1,2,\ldots$. In addition each unit, had it failed or not, is replaced by a new one. Times of inspections and replacements are assumed to be negligible.

The following notation will be handled:

- X_i, is unit i's random time to failure $(i = 1,2,\ldots,n)$
- μ_i, F_i, f_i, R_i and r_i are, respectively, the mean value, cumulative distribution function, density function, reliability function and failure rate of X_i, $(i = 1,2,\ldots,n)$
- $T^\star$: optimum replacement time

Let c_1 be the cost of the replacement when no failure has occurred and c_{2i}, $(i = 1,2,\ldots,$ $n)$ the corresponding cost if the unit i has failed. In practice, it is usual to consider $c_1 < c_{2i}$. In addition, we will take into account the cost derived from the down-time which represents the losses incurred while the failure of the system remains undiscovered. It can be due to defective production, unavailability of the system when it is needed, etc.

Let also consider the ordered times to failure of the units, thus, $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$. In addition, $\mu_{(i)}$, $F_{(i)}$, $f_{(i)}$, $R_{(i)}$, and $r_{(i)}$ are the mean value, distribution function, density function, reliability function and failure rate corresponding to $X_{(i)}$. Finally, the multivariate distribution and reliability functions of the system are, respectively

$$\begin{aligned} F(x_1,x_2,\ldots,x_n) &= P(X_1 \le x_1, X_2 \le x_2, \ldots, X_n \le x_n) \\ R(x_1,x_2,\ldots,x_n) &= P(X_1 > x_1, X_2 > x_2, \ldots, X_n > x_n) \end{aligned}$$

In what follows, we will analyze the maintenance policy for series and parallel systems, and we will begin with the former.

CASE I. The system works if all the component work.

In this case, the time to failure of the system is given by

$$X_{(1)} = \min(X_1, X_2, \ldots, X_n)$$

being $R_{(1)}(t) = R(t,\ldots,t)$ its corresponding reliability function. Now, c_{di} is the cost derived from the down-time when component i fails.

The next result provides the expression of the cost function.

Proposition 1 *Consider a n-out-of-n system whose failures are only detected by inspection. Under the maintenance policy previously described, the following results hold*

(1) $$E[C(\tau)] = c_1 R_{(1)}(T) + \sum_{i=1}^{n} c_{2i} F_i(T) - \sum_{i=1}^{n} c_{di} \int_0^T R_i(u)du + \sum_{i=1}^{n} c_{di} T$$

(2) $$Q(T) = \sum_{i=1}^{n} c_{di} + \frac{c_1 R_{(1)}(T) + \sum_{i=1}^{n} c_{2i} F_i(T) - \sum_{i=1}^{n} c_{di} \int_0^T R_i(u)du}{T}$$

Proof

At times kT, $(k = 1,2,\ldots,)$ the system is renewed, hence, the mean length of a cycle is $E[\tau] = T$. The cost of a cycle under this maintenance procedure is

$$C(\tau) = c_1 I(X_1 > T, X_2 > T, \ldots, X_n > T) + \sum_{i=1}^{n} c_{2i} I(X_i \leq T) + \sum_{i=1}^{n} c_{di} \left[T - \min(X_i, T)\right]$$

where $I(.)$ represents the indicator function. The expectation of the expression above, leads to (1).

Both $E[\tau]$ and (1) provide $Q(T)$ given in (2). $\square$

The following theorem gives a sufficient condition which guarantees the existence of an optimum policy.

Theorem 2 *Consider the same conditions given in Proposition 1, under the next inequality*

(3) $$\sum_{i=1}^{n} c_{2i} < \sum_{i=1}^{n} c_{di} \mu_i, \qquad i = 1,2,\ldots,n$$

there exists $T^\star$ in $(0,\infty)$ minimizing $Q(T)$ in (2). Moreover, T^* is one of the roots of the following equation

$$T\left[-c_1 f_{(1)}(T) + \sum_{i=1}^{n} c_{2i} f_i(T) - \sum_{i=1}^{n} c_{di} R_i(T)\right] - \\ - \left[c_1 R_{(1)}(T) + \sum_{i=1}^{n} c_{2i} F_i(T) - \sum_{i=1}^{n} c_{di} \int_0^T R_i(u)du\right] = 0$$

(4)

Proof

$Q(T)$ can be expressed as

$$Q(T) = \sum_{i=1}^{n} c_{di} + \frac{a(T)}{T}$$

where

$$a(T) = c_1 R_{(1)}(T) + \sum_{i=1}^{n} c_{2i} F_i(T) - \sum_{i=1}^{n} c_{di} \int_0^T R_i(u)du$$

$a(T)$ is a continuous function in $[0,\infty]$ verifying $a(0) = c_1 > 0$. From (3), it is derived that $a(\infty) = \sum_{i=1}^{n} c_{2i} - \sum_{i=1}^{n} c_{di}\mu_i < 0$. Note that the two limiting cases, $T = 0$ and $T = \infty$, correspond, respectively, to a continuous inspection and no inspection at all. Therefore, there exists T_0 in $(0,\infty)$ such that $a(T_0) < 0$, hence, $Q(T_0) < \sum_{i=1}^{n} c_{di} = Q(\infty)$. In addition, $Q(0) = \infty$, so, there exists T_1, $0 < T_1 < T_0$, satisfying $Q(T) \leq \inf_{x\in[0,T_1]} Q(x)$ for all $T > T_1$.

$Q(T)$ is a continuous function, therefore it has a minimum, $T^\star$, in $[T_1,\infty]$. Moreover, next inequality

$$Q(T_1) \geq Q(\infty) > Q(T_0)$$

leads to $T_1 < T^\star < \infty$. Therefore, $T^\star$ is also the minimum in $[0,\infty]$, and the result holds. Finally, by differentiation of $Q(T)$, (4) is obtained. □

The particular case of time to failure exponentially distributed is analyzed in the following result.

Proposition 3 *Consider the same conditions given in Proposition 1, and X_i, exponentially distributed with failure rate λ_i, $i = 1,2,\ldots,n$. If the next two conditions hold*

(5) $$\lambda_i < \frac{c_{di}}{c_{2i}}, \quad i = 1,2,\ldots,n$$

(6) $$r^2_{(1)}(T) > r'_{(1)}(T)$$

with $r'_{(1)}(T)$ being the derivative of $r_{(1)}(T)$. Then, T^* is the only root of (4).

Proof

Denote $A(T)$ the left-hand side in equation (4). In this case, the derivative of $A(T)$ is expressed as

$$\frac{dA(T)}{dT} = T\left[c_1\left(r^2_{(1)}(T) - r'_{(1)}(T)\right)R_{(1)}(T) - \sum_{i=i}^{n}\lambda_i^2 e^{-\lambda_i T}\left(c_{2i} - \frac{c_{di}}{\lambda_i}\right)\right]$$

The hypotheses (5) and (6) assure that the expression above is positive and, therefore, $A(T)$ has an only root. $\square$

The multivariate extensions of exponential distribution arise easily to model the time to failure when the maintenance of multi-unit systems is studied. We consider the following bivariate extensions

1. The bivariate Gumbel distribution (Gumbel (1960))

The corresponding bivariate reliability function is given by

$$R(x,y) = e^{-x(\lambda_1+\lambda_3 y)-\lambda_2 y}, \ \ 0 \leq \lambda_3 \leq \lambda_1\lambda_2, \ \ \lambda_1, \lambda_2 > 0, \ \ x,y \geq 0$$

This distribution has exponential marginal distributions, however, the failure rate defined as

$$r(x,y) = \frac{f(x,y)}{R(x,y)}$$

is not a constant, unlike the univariate exponential.

2. The Marshall-Olkin model (Marshall-Olkin (1966))

Marshall and Olkin derived the joint reliability distribution

$$R(x,y) = e^{-\lambda_1 x-\lambda_2 y-\lambda_3 \max(x,y)}, \quad \lambda_1, \lambda_2 > 0, \ \ \lambda_3 \geq 0, \ \ x,y \geq 0$$

so as to describe a two-component system subject to shocks following a Poisson process. An important property of this distribution is that $X = \min(T_1, T_2)$, $Y = \min(T_2, T_3)$ where T_1, T_2 and T_3 are independent exponential random variables. It follows that $\min(X,Y)$ is also exponentially distributed. In addition, (X,Y) is the only distribution to have the so-called no-aging property

$$P(X \geq x+t, Y \geq y+t \mid X \geq t, Y \geq t) = P(X \geq x, Y \geq y) \quad x,y,t \geq 0$$

which means that the remaining lifetime, given that both components have survived up age t, does not depend on t.

In the former models, both X and Y are exponentially distributed. Hence, by direct application of Proposition 2, the following corollaries are derived

Corollary 4 *Consider a two-out-of-two system whose time to failure is distributed as the bivariate Gumbel model. Under the following conditions*

$$\lambda_i < \frac{c_{di}}{c_{2i}}, \quad i = 1,2$$

$$(\lambda_1 + \lambda_2)^2 > 2\lambda_3$$

T^* is the only root in equation (4).

Proof

The reliability function and the failure rate corresponding to $X_{(1)} = \min(X,Y)$ are, respectively

$$\begin{aligned} R_{(1)}(T) &= e^{-(\lambda_1+\lambda_2)T-\lambda_3 T^2} \\ r_{(1)}(T) &= (\lambda_1+\lambda_2)+2\lambda_3 T \end{aligned}$$

The hypotheses of Proposition 2 are satisfied and the result follows. □

Corollary 5 *Consider a two-out-of-two system whose time to failure is distributed as the Marshall-Olkin model. T^*, is the only root of equation (4), provided*

$$\lambda_i + \lambda_3 < \frac{c_{di}}{c_{2i}}, \quad i = 1,2$$

Proof

X and Y are exponentially distributed with parameters $\lambda_1+\lambda_3$ and $\lambda_2+\lambda_3$, respectively. Besides, the reliability function of $X_{(1)} = \min(X,Y)$ and its failure rate are respectively given by

$$\begin{aligned} R_{(1)}(T) &= e^{-(\lambda_1+\lambda_2+\lambda_3)T} \\ r_{(1)}(T) &= \lambda_1+\lambda_2+\lambda_3 \end{aligned}$$

again, Proposition 2 leads to the result. □

Next, the particular case of independence between times to failure of the components is considered.

Proposition 6 *Let $X_1, X_2, \ldots, X_n$ be independent random variables, and such that*

$$\alpha_i \leq \frac{c_{di}}{c_{2i}}, \quad i = 1,2,\ldots,n \tag{7}$$

with $\alpha_i = \sup_{0\leq t<\infty} r_i(t)$. Then, $T^\star$ is the only root of (4).

Proof

Denoting $A(T)$ the left-hand side of (4), its derivative can also be expressed

$$\begin{aligned} \frac{dA(T)}{dT} &= T\left[c_1 r_{(1)}^2(T)R_{(1)}(T) + \sum_{i=1}^{n} c_{2i} r_i'(T) R_i(T)\right] - \\ &- T\left[c_1 r_{(1)}'(T)R_{(1)}(T) + \sum_{i=1}^{n} r_i(T) f_i(T)\left(c_{2i} - \frac{c_{di}}{r_i(T)}\right)\right] \end{aligned}$$

The independence of $(X_1, X_2, \ldots, X_n)$ leads to

$R_{(1)}(T) = \prod_{i=1}^n R_i(T), \quad r_{(1)}(T) = \sum_{i=1}^n r_i(T), \quad r'_{(1)}(T) = \sum_{i=1}^n r'_i(T)$

hence

$$\begin{aligned} \frac{dA(T)}{dT} &= T\left[c_1\left(\sum_{i=0}^n r_i(T)\right)^2 \prod_{i=1}^n R_i(T) + \sum_{i=1}^n r'_i(T) R_i(T)\left(c_{2i} - c_1 \prod_{j\neq i} R_j(T)\right)\right] - \\ &- T\left[r_i(T) f_i(T)\left(c_{2i} - \frac{c_{di}}{r_i(T)}\right)\right] \end{aligned}$$

Usually, it is assumed $c_1 < c_{2i}$, $i = 1, 2, \ldots, n$. This fact along with the hypotheses given in (7), prove that the derivative of $A(T)$ is positive and the result holds. □

CASE II. The system works if at least one component works.

The time to failure in a parallel system is given by the longer lifetime, thus

$$X_{(n)} = \max(X_1, X_2, \ldots, X_n)$$

with $\mu_{(n)} = E\left[X_{(n)}\right]$. Its reliability function is

$$R_{(n)}(x) = 1 - P(X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x) = 1 - F(x, x, \ldots, x)$$

Now, we consider a unique c_d representing the cost derived while the failure of the whole components is not detected. First, the cost function is obtained.

Proposition 7 *Consider a 1-out-of-n system whose failures are only detected by inspection. Under the maintenance policy previously described, the following results hold*

$$E[C(\tau)] = c_1 R_{(1)}(T) + \sum_{i=1}^n c_{2i} F_i(T) + c_d T - c_d \int_0^T R_{(n)}(u)du \tag{8}$$

$$Q(T) = c_d + \frac{c_1 R_{(1)}(T) + \sum_{i=1}^n c_{2i} F_i(T) - c_d \int_0^T R_{(n)}(u)du}{T} \tag{9}$$

Proof

In Proposition 1, it was proved that $E[\tau] = T$. The cost of a cycle is

$$\begin{aligned} C(\tau) &= c_1 I(X_1 > T, X_2 > T, \ldots, X_n > T) + \sum_{i=1}^n c_{2i} I(X_i \leq T) + \\ &+ c_d\left[T - \min\left(X_{(n)}, T\right)\right] \end{aligned}$$

The expected value of the expression above is given by (8), therefore, (9) holds. □

The following result provides a sufficient condition for the existence of an optimum policy.

Theorem 8 *Consider the same conditions of Proposition 4, if*

$$\mu_{(n)} > \frac{\sum_{i=1}^{n} c_{2i}}{c_d} \tag{10}$$

then, there exists T^*, $0 < T^* < \infty$ *minimizing* $Q(T)$ *in (9). Moreover,* T^* *is one of the roots of the following equation*

$$\begin{aligned} & T\left[-c_1 f_{(1)}(T) + \sum_{i=1}^{n} c_{2i} f_i(T) - c_d R_{(n)}(T)\right] - \\ - & \left[c_1 R_{(1)}(T) + \sum_{i=1}^{n} c_{2i} F_i(T) - c_d \int_0^T R_{(n)}(u)du\right] = 0 \end{aligned} \tag{11}$$

Proof

$Q(T)$ can be also expressed as

$$Q(T) = c_d + \frac{a(T)}{T}$$

where

$$a(T) = c_1 R_{(1)}(T) + \sum_{i=1}^{n} c_{2i} F_i(T) - c_d \int_0^T R_{(n)}(u)du$$

Clearly, $a(0) = c_1 > 0$ and, from hypothesis (10), it is obtained $a(\infty) = \sum_{i=1}^{n} c_{2i} - c_d \mu_{(n)} < 0$. Hence, there exists T_0, $0 < T_0 < \infty$ such that $Q(T_0) < c_d = Q(\infty)$. In addition $Q(0) = \infty$, and the same strategy used in Theorem 1 shows the existence of T^*, finite, minimizing $Q(T)$.

On the other hand, the roots of the derivative of $Q(T)$ should verify (11). □

Remark 1 *Conditions (3) and (10) mean that an optimum policy exists whenever the mean cost incurred for the down-time is greater than the cost derived from the preventive maintenance.* □

3. EXAMPLES

We present two examples that aim at illustrating the results presented in this work. The first of them refers to a series system following a bivariate Gumbel distribution and the second deals with a parallel system whose time to failure is distributed as the Marshall-Olkin model.

The periodic tests which are carried out so as to detect disorders in the blood pressure and the cholesterol level, correspond to the case of a series system. No sooner does one of the causes appear than the health disorder arises. In general if the cholesterol level is high so does the blood pressure, that is to say, both turn out to be related and may indicate the occurrence of a more serious illness. The disorders in the blood pressure and the cholesterol level are unrevealed failures as, apart from the corresponding tests, there is no other way to detect them.

Consider now an engineering system with two spare units being available. Both can be modeled by a parallel system and should be tested periodically so as to detect its failures which may occur even while they are not in use. The failures may be dependent if the units are stored under the same conditions, hence a failed unit makes more likely the failure of the second spare component.

Table 1

$E[X_{(1)}]$	λ_1	λ_2	λ_3	T_1^*	$Q(T_1^*)$	T_2^*	$Q(T_2^*)$
2849.98	10^{-4}	2×10^{-4}	10^{-8}	12.92	1.56	15.82	1.28
2263.39	10^{-4}	3×10^{-4}	10^{-8}	11.19	1.80	14.16	1.46
1868	10^{-4}	4×10^{-4}	10^{-8}	10.01	2.02	12.93	1.56
1586.3	10^{-4}	5×10^{-4}	10^{-8}	9.14	2.21	11.97	1.57
1376	10^{-4}	6×10^{-4}	10^{-8}	8.46	2.39	11.20	1.81
1332.84	10^{-4}	6×10^{-4}	2×10^{-8}	8.46	2.39	11.20	1.81
1295.53	10^{-4}	6×10^{-4}	3×10^{-8}	8.46	2.39	11.20	1.81
787.44	5×10^{-4}	5×10^{-4}	2×10^{-7}	7.08	2.87	8.18	2.49
285	10^{-3}	2×10^{-3}	10^{-6}	4.09	5.01	5.01	4.11
158.63	10^{-3}	5×10^{-3}	10^{-6}	2.90	7.10	3.80	5.47
152.30	10^{-3}	5×10^{-3}	2×10^{-6}	2.90	7.10	3.80	5.47
147.02	10^{-3}	5×10^{-3}	3×10^{-6}	2.90	7.10	3.80	5.47
140.71	1.1×10^{-3}	5×10^{-3}	4×10^{-6}	2.88	7.17	3.75	5.55
47.81	0.01	0.01	10^{-5}	1.59	13.51	1.84	11.81
46.04	0.01	0.01	2×10^{-5}	1.59	13.51	1.84	11.81
42.14	0.01	0.01	5×10^{-5}	1.59	13.51	1.84	11.81
28.50	0.01	0.02	10^{-4}	1.30	16.58	1.60	13.74
22.63	0.02	0.02	10^{-4}	1.13	19.60	1.30	17.21

1. Series system with Gumbel distribution

Suppose that $c_1 = 10$, $c_{21} = 75$, $c_{22} = 35$, and $c_{d1} = 400$. We assume two different costs for the down-time due to failure in the second component: $c_{d2} = 400$ or $c_{d2} = 200$, being $T_1^\star$ and $T_2^\star$ their corresponding optimum inspection times.

Table 1 contains the optimum inspection times as well as the corresponding optimum cost for different mean times to failure of the system. The smaller $E[X_{(1)}]$ is, the smaller $T^\star$; on the contrary, $Q(T^\star)$ increases. In addition $T_1^\star < T_2^\star$ and $Q(T_1^\star) > Q(T_2^\star)$, which means that the higher the down-time cost is, the more frequent the inspection and, therefore, the more expensive the total cost.

2. Parallel System with the Marshall-Olkin model

Costs: $c_1 = 10$, $c_{21} = 75$, $c_{22} = 35$ and $c_d = 400$ ($T_3^\star$) or 200 ($T_4^\star$).

Table 2 shows that the optimum inspection time, T^*, is non-monotonic when the mean time to failure decreases. Moreover, $T_3^\star < T_4^\star$ and $Q(T_3^\star) > Q(T_4^\star)$; therefore, the conclusions derived are similar to those in the previous example.

Table 2

$E[X_{(n)}]$	λ_1	λ_2	λ_3	T_3^*	$Q(T_3^*)$	T_4^*	$Q(T_4^*)$
12150	10^{-4}	10^{-5}	7446×10^{-8}	25.92	0.79	36.67	0.56
11070	10^{-4}	10^{-4}	2967×10^{-8}	40.71	0.50	57.37	0.36
10632	10^{-4}	10^{-4}	3422×10^{-8}	37.97	0.54	53.55	0.38
10414	10^{-4}	2×10^{-4}	1229×10^{-8}	60.05	0.34	83.16	0.24
10371	10^{-4}	2×10^{-4}	1276×10^{-8}	59.11	0.34	81.93	0.25
10334	10^{-4}	2×10^{-4}	1317×10^{-8}	58.32	0.35	80.90	0.25
10290	10^{-4}	2×10^{-4}	1366×10^{-8}	57.42	0.35	79.71	0.26
3213	5×10^{-4}	2×10^{-4}	1507×10^{-7}	18.09	1.15	25.52	0.83
1215	10^{-3}	10^{-3}	1975×10^{-7}	15.19	1.40	21.14	1.03
1058	10^{-3}	2×10^{-3}	1055×10^{-7}	18.15	1.14	24.47	0.87
1053	10^{-3}	2×10^{-3}	1103×10^{-7}	17.90	1.16	24.18	0.88
1048	10^{-3}	2×10^{-3}	1160×10^{-7}	17.62	1.18	23.84	0.89
1041	10^{-3}	2×10^{-3}	123×10^{-6}	17.30	1.21	23.45	0.91
968	1.1×10^{-3}	2×10^{-3}	1358×10^{-7}	16.56	1.27	22.48	0.96
158	0.01	5×10^{-3}	2421×10^{-6}	4.34	5.51	6.06	4.21
154	0.01	5×10^{-3}	2628×10^{-6}	4.19	5.71	5.86	4.35
152	0.01	6×10^{-3}	2030×10^{-6}	4.62	5.18	6.40	3.98
122	0.01	0.01	1976×10^{-6}	4.47	5.33	6.13	4.14
77	0.02	0.01	5142×10^{-6}	2.96	8.63	4.13	6.72

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