ALGORITHM OF GLOBAL EXTREMUM SEARCH AREA DEFINITION FOR SEVERAL VARIABLES FUNCTION

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Abstract: The article is devoted to the problem of global extremum search for several variables function. A modified algorithm is developed for the search of global extremum function, based on evolutionary calculations and differing by the approach of an area development to create an initial population of agents. They developed the algorithm for the function extremum search area definition, which ultimately performs the decomposition of the research area into subsets. It is suggested to take into account the knowledge of an expert, an agent and the available group agents. Based on the available knowledge, the region is divided into three subsets with different priorities. At that, the possibility of the function extremum drift is taken into account and a separate procedure of a search area definition is implemented, taking into account the retrospective information on the drift of parameters.
Keywords: multicriteria optimization, multiextremal function, global extremum, extremum drift, solution search area, decision support system.

Introduction. During the solution of applied problems, the problem of finding a solution arises often taking into account the multi-extremality of an objective function, which consists in a global extremum finding [1]. In some problems, the objective function can be non-differentiable, subject to distortions, and can also be presented in conjunction with the limitations in the form of a "black box". This problem is complicated considerably in the conditions of the objective function multidimensional dependence. At present, multiagent self-organizing systems are developing actively with the ability to function in the conditions of uncertainty and decision support systems, for which the search for an optimal solution is the basis for this goal achievement and the deviation from the global extremum can lead to significant costs. The modern means of function extremum search are focused on a solution search closest to an optimal one [2].

Some known algorithms of function extremum search solve two contradictory problems [3]:

- the task of use, consisting in the maximum application of already received information, that is, in the reduction of an extremum search area, in which the function values are sufficiently high;
- the research task, which consists in the viewing of as many points as possible on the entire search space.

Classical and evolutionary algorithms for function extremum finding in a multidimensional space do not provide a guarantee solution, except for a full-search algorithm, the use of which is not permissible for objective reasons. Thus, nowadays the task of tool development that increases the efficiency of a function global extremum search in a multidimensional space is an urgent one.

Methodology. They propose to use the expert's knowledge contained in the knowledge base and the retrospective information consisting in the system accumulated experience of solution finding in the process of search for the global extremum of the multi-extremal function of many variables. In a situation where a self-organizing multi-agent system is involved, agents that are available for information exchange can act as a source of knowledge. Evolutionary algorithms are considered as a basic search method. These algorithms have a number of advantages in comparison with classical methods under the conditions of a function uncertainty and non-differentiability [3, 4]. Taking into account the existing knowledge, it is proposed to realize the narrowing of extremum search area and to conduct the initial population development in an identified area.

The developed approach is oriented to the classification of a multidimensional search field into three subsets, which represent a promising, not known and a negative area of research. A negative area is developed on the basis of integrated knowledge of the surroundings, the study of which did not lead to an extremum obtaining. A prospective field of research is formed taking into account the knowledge about function extremum location. An unknown area is studied when the necessary solution is not obtained in the prospective area.

A special attention is paid to the development of the region with the knowledge of extremum drift [5]. It is necessary to develop the trajectory of extremum location coordinate change. Individual incremental values are calculated for each parameter, the consideration of which allowed the prediction of the extremum coordinates and the formation of a new study area based on the state of the region in which the extremum was found during the previous search step. Thus, the modified search algorithm adjusts the search mechanism of the global extremum depending on the operational information. The approach is proposed to develop an extremum search area based on the decomposition of a multidimensional domain into the subsets with different priorities.

Main part. During the solution of global optimization problems, we can distinguish static and dynamic problems. In static problems, the function is not subject to time change together with the range of permissible values. Consequently, the positions of the extrema in the search domain remain unchanged. In the case of a dynamic problem, the positions of the extrema vary with time, and the optimization problem is complicated by the need to track the trajectory of global extremum change [6]. Thus, if the classical methods are permissible for the static problems of multidimensional optimization, then dynamic methods require the use of heuristic methods based on stochastic search algorithms.

A number of test multi-extremal functions have been developed that can reflect the dependence of a function on a set of arguments to analyze the optimization algorithms. Rastrigin function is an example [7], which clearly demonstrates the problem of a global extremum finding for a multiextremal function of numerous variables.

Multidimensional optimization problem statement. Let a multidimensional functional dependence is given in the following form $f(x_1, x_2,..x_n)$, where $x_1 \in X_1, x_2 \in X_2,,..x_n \in X_n$. It is necessary to find the best value of the objective function $f(x_1, x_2,..x_n) \rightarrow \min/\max$ at given constraints on the range of parameter variation $X_1, X_2,..X_n$. At that, the objective function can be non-differentiable, can have discontinuities, be multi-extremal one and subject to changes under the influence of external factors.

Optimization methods can be divided conditionally into classical and heuristic ones. The representatives of classical methods are a complete search, realizing the search of all possible variants, and a gradient descent method that realizes the optimization of steepest descent [8]. The restrictions of classical methods application for multidimensional optimization problem solution are stochasticity, large dimension, the choice of the initial study point, non-differentiability and function discontinuities. Separately, one can point out the problem when a saddle point is searched [9]. The evolutionary algorithms of global optimization are not subject to these limitations and can be used for optimization problems with nonlinear functions. Unlike classical methods that realize the search by one individual trajectory, several variants of the problem solution are processed in evolutionary algorithms simultaneously. The representatives of evolutionary algorithms are the genetic algorithm, ant algorithm, gravitational search, artificial immune systems, bee algorithm and so on [10]. The technology of evolutionary optimization algorithm operation includes the following stages [11]:

- Initial initialization of the population. A certain number of agents is formed, each of which represents some approximation to the solution of the problem in the field of search.

- A solution search. The value of the objective function is determined and the best solution is chosen for each population agent.

- The migration of population agents or population modification. The agents are moved in a search area by the given algorithms, which approximate the solution to the function extremum during migration. Population modification implies the development of a new population, taking into account the retrospective information about agents (crossing) and mutations.

- Search completion. If the search term is satisfied, then the calculations are completed and the selection of an agent from a population with the best approximate solution takes place. If the termination condition of the search is not fulfilled, then the transfer to the migration stage of population agents or population modification takes place.

Evolutionary algorithms can be used in aggregate, thus developing hybrid approaches to global optimization [12]. They form a swarm intellect, manifested in the complex behavior of the population (even during simple behavior of individual agents), as well as in the presence of self-organization mechanisms. Thus, the behavior of evolutionary algorithm agents has the following peculiarities: the algorithms of migration and agent population modification have a random component (the chance to search for the best solution in an unpromising area), the decentralized search for a solution by agents, the trajectory of the agents is formed autonomously or in part depending on other agents, the agents have retrospective information about the part of the search field they research, the agents can exchange information.

In the situation with no a priori information, the agents of the initial population are distributed evenly throughout a search area. The availability of expert knowledge and the mechanisms for retrospective information collection about the function will reduce the time for a solution finding significantly. Therefore, the modification of a search algorithm is necessary by the identification of the most promising area of research in a multidimensional parameter space. Fig. 1 shows a modified algorithm for the extremum function search with the preliminary development of a prospective search area. A target function and parameters appear at an algorithm input.

Then the function of a search area definition decomposes the multidimensional space into three areas of research: a prospective one, most likely containing a global extremum; a negative one, formed on the basis of unsuccessful experience concerning an extremum search; an unknown area in which no research was conducted. The knowledge of function extremum drift forms the region taking into account the drift of parameters.

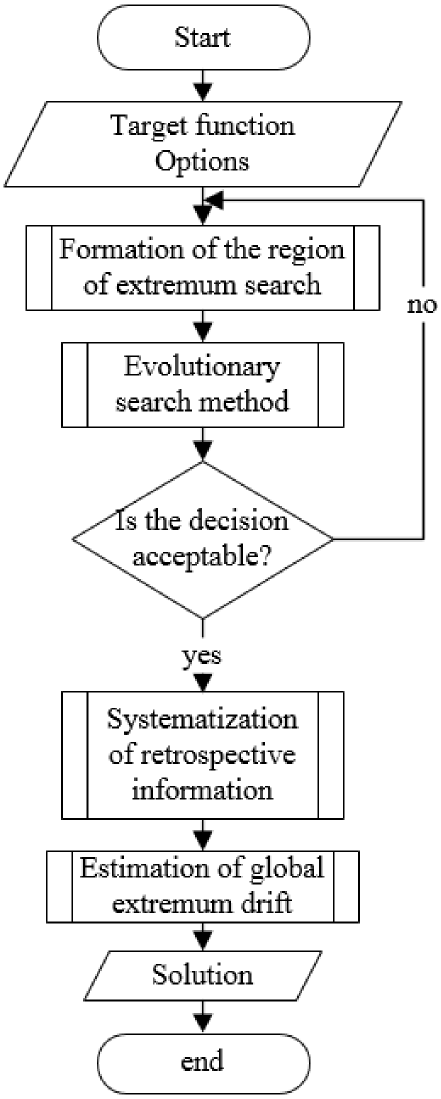

Fig. 1. Modified evolutionary algorithm for function extremum search

Initially, the input of the evolutionary search algorithm receives the parameters with the values from a prospective search area. If an obtained solution satisfies the specified requirements, the retrospective information is systematized, which consists of an agent's gained experience summarizing by the agent during a problem solution with the available knowledge. If the search in the prospective area has not provided an acceptable solution, then the search in an unknown area is performed and the retrospective information is systematized. Then, if there is knowledge about a function extremum drift, its estimate is realized, which consists in a drift trajectory or functional dependence clarification. The result is the provision of the best solution.

The quality and the speed of a function extremum search in evolutionary algorithms is influenced by the choice of a function research perspective area. In this paper, we propose the approach to the identification of a promising research area, based on a formal description of participant knowledge concerning a problem solution process in the form of subsets of values for each parameter of the function $f(x_1, x_2, ...x_n)$:

- $ExpP_1 \subseteq X_1, ExpP_2 \subseteq X_2, ...ExpP_n \subseteq X_n$ - Expert knowledge about the most promising area of extremum search.

- $ExpN_1 \subseteq X_1, ExpN_2 \subseteq X_2, ...ExpN_n \subseteq X_n$ - Expert knowledge about the negative search area (the search did not provide an acceptable solution).

- $AgP_1 \subseteq X_1, AgP_2 \subseteq X_2, ...AgP_n \subseteq X_n$ - individual agent's knowledge about the most promising area for an extremum search.

- $AgN_1 \subseteq X_1, AgN_2 \subseteq X_2, ...AgN_n \subseteq X_n$ - individual agent's knowledge about the negative area of extremum search, that is, the consideration of the studied area.

- $AgGrP_1 \subseteq X_1, AgGrP_2 \subseteq X_2, .. AgGrP_n \subseteq X_n$ - the knowledge of available agents about the most promising area of extremum search. At that, $AgGrP_i = \bigcup_{j=1}^{k} AgGrP_j$, where k is the number of available agents of the group and $i = \overline{1,n}$.

- $AgGrN_1 \subseteq X_1, AgGrN_2 \subseteq X_2, .. AgGrN_n \subseteq X_n$ - the knowledge of available agents about a negative search area. At that $AgGrN_i = \bigcup_{j=1}^{k} AgGrN_j$.

- The consideration of a drift possibility (the change of coordinates) of the extremum function in dynamic systems by the calculation of individual increments $V_1, V_2, ..., V_n$ for each function parameter $f(x_1, x_2, .. x_n)$.

Thus, taking into account the formal representation of knowledge, their aggregation is carried out in all parameters for $f(x_1, x_2, .. x_n)$ and the following research areas are developed:

- $PsaX_i = ExpP_i \cup AgP_i \cup AgGrP_i$ - a perspective one for the parameter $x_i \in X_i$.
- $Psa = \{PsaX_1, PsaX_2, .. PsaX_n\}$ - a perspective search area in n-dimensional space.
- $NsaX_i = ExpN_i \cup AgN_i \cup AgGrN_i$ - a negative one for the parameter $x_i \in X_i$.
- $Nsa = \{NsaX_1, NsaX_2, .. NsaX_n\}$ - a negative one in n-dimensional space.
- $UsaX_i = X_i \setminus (PsaX_i \cup NsaX_i)$ - unknown for the parameter $x_i \in X_i$.
- $Usa = \{UsaX_1, UsaX_2, .. UsaX_n\}$ - unknown in n-dimensional space.
- $PsaD = \{Psa_{-1}X_1 \pm V_1, Psa_{-1}X_2 \pm V_2, .. Psa_{-1}X_n \pm V_n\}$ - a perspective one in n-dimensional space taking into account the drift, obtained as the change of a perspective area for the previous solution $Psa_{-1}X_i$ y the calculated individual increment V_i for each parameter $f(x_1, x_2, .. x_n)$.

Taking into account the formal description of knowledge, Fig. 2, presents the algorithm to develop a region for a function extremum obtaining. If there is some knowledge about function extremum drift, then the function parameters are loaded corresponding to the extremum search area, in which the last optimal solution is obtained. The forecasting of the extremum is implemented taking into account the information about the trajectory of its coordinates change. This forecast is taken into account in the algorithm (Figure 1) to estimate the drift parameters and clarify the drift trajectory or functional dependence. Then, the function n parameters are adjusted to the individual increment value V_i for each parameter, since the drift value of the parameters can be different. The region $PsaD$ is developed with the increments taken into account.

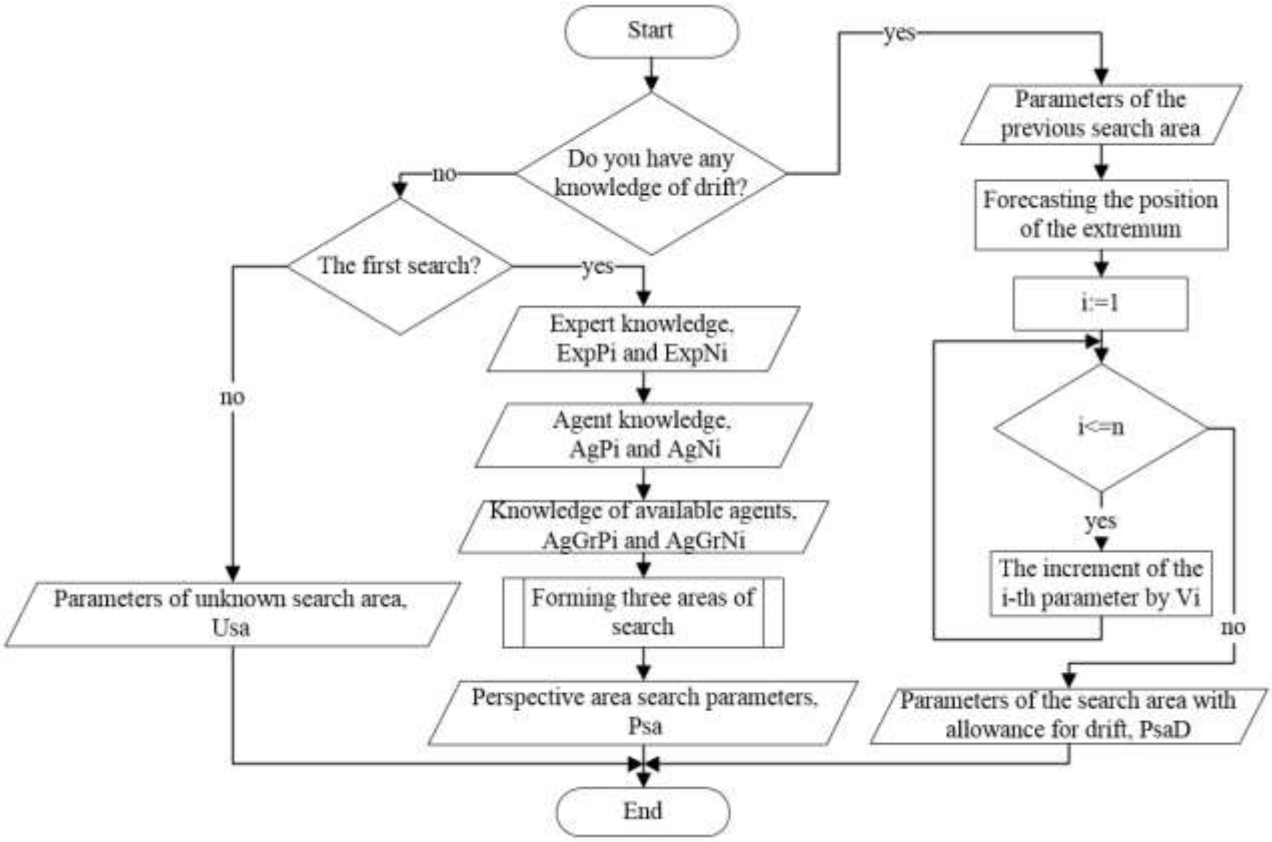

Fig. 2. The algorithm for search area definition of the function global extremum

When information about a drift is absent and the search is realized for the first time, a prospective area of research is developed Psa, based on available expert, own and available agent knowledge. Taking into account the received knowledge, the n-dimensional domain is divided into three subsets (perspective, negative and unknown one) and the derivation of study perspective area parameters. When the algorithm (Figure 2) is re-addressed to the sub program of a search area definition, that is, when no solution is found in a perspective area, the parameters of an unknown area are developed. The parameters of a negative area are necessary only to narrow the range of acceptable value search.

Summary. Thus, the developed algorithm for an extremum search area detection allows to organize a more accurate distribution of the initial population of agents in evolutionary algorithms through the knowledge base and the decomposition of the multidimensional area of research into the subsets with different priorities, which ultimately leads to the search speed increase of the global extremum. At that, the consideration of the extremum possible drift allows us to predict a search area for the next instant of time on the basis of increment individual value calculated for each parameter.

Conclusions. Thus, the tools were designed to improve the performance of evolutionary algorithms to obtain an extremum function of many variables. A developed modified evolutionary algorithm allows to form the initial distribution area of population agents taking into account the knowledge base containing the expert knowledge and the retrospective information accumulated by an agent. The algorithm has the systematization procedure for retrospective information, which allows to process the information received in the process of a set task solution in order to update the knowledge. The algorithm allows us to search for the function global extremum in which the drift of the extremum coordinates is possible. The subroutine evaluates the drift parameters of the extremum, which ultimately allows you to predict the value of the extremum of the function at the next instant of time, and to form a modified search area based on the increment of the parameters of the region containing the extremum at the previous iteration of the search.

An algorithm was developed to form a function extremum search area. The formalization of expert's, agent's and available group agents' knowledge was performed. During formalization the knowledge is divided into two categories (positive and negative study experience) which allow to decompose the entire field of research into three subsets: a perspective, a negative and an unexplored area of search. This approach allows us to narrow the scope of the initial study and to increase the solution search speed. If the prospective area study does not provide a necessary solution, then the algorithm starts the search in an unknown area, after which the agent's knowledge is adjusted. The algorithm of a search area definition takes into account the possibility of the function extremum drift and a separate procedure for a search area definition is implemented taking into account the retrospective information about parameter drift. The modifications introduced into the evolutionary algorithm make it possible to narrow the search area for a global extremum by the processing of knowledge about the subject area in question.

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