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Efficiency of a Voluntary Speed Reduction Algorithm for a Ship's Great Circle Sailing

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ABSTRACT: The great-circle is the shortest distance between two points on the surface of the earth. When planning a ship's sailing route (waypoints and forward speeds) for a specific voyage, the great circle route is commonly considered as a reference route, especially for ocean-crossing seaborne transport. During the planning process, the upcoming sea weather condition is one of the most important factors affecting the ship's route optimization/planning results. To avoid encountering harsh conditions, conventional routing optimization algorithms, such as Isochrone method and Dynamic Programming method, have been developed/implemented to schedule a ship's optimal routes by selecting waypoints around the great circle reference route based on the ship's operational performances at sea. Due to large uncertainties in sea weather forecast that used as inputs of these optimization algorithms, the optimized routes may have worse performances than the traditional great circle sailing. In addition, some shipping companies are still sailing in or making charting contracts based on the great circle routes. Therefore, in this study, a new optimization algorithm is proposed to consider the voluntary speed reduction with optimal speed configuration along the great circle course. The efficiency of this method is investigated by comparing these two methods for optimal route planning with respect to ETA and minimum fuel consumption. A container ship sailing in the North Atlantic with full-scale performance measurements are employed as the case study vessels for the comparison.

1 INTRODUCTION

In the maritime industry, as the increasing regulatory pressure and the economic competition, ship energy efficiency becomes one of the most controversial topics in every shipping company's agenda, in addition to the traditional safety issues. To ensure better economic benefit as well as reduce negative environmental impact from shipping emissions, a voyage optimization system is recognized as an efficient measure by the shipping industry (DNV 2015). A voyage optimization system is used to plan a ship's courses/schedules in order to reach her destination with certain pre-defined objectives, e.g., the expected time of arrival with minimum fuel

consumption and air emissions, etc. (Bowditch, 2002). The core element of such a voyage optimization system is to implement a proper optimization algorithm, which generates optimal ship routing based on the sea weather conditions along the ship's sailing area, the ship's characteristics and operational capabilities, as well as some constraints for a particular voyage, etc. (Wang et al. 2018). Many route optimization algorithms have been implemented and used in the shipping market and maritime research community, e.g., the modified isochrone method (Hagiwara, 1989), the isopone method (Klompstra, 1992), dynamic programming method (De Wit, 1990), DIRECT algorithm (Simonsen et al. 2014), etc.

Most of the optimization algorithms are two-dimensional and optimization process is performed by changing a ship's course based on a reference route, e.g., the great circle course. The variables that are optimized are the ship's sailing waypoints/course (in a more specific way, ship heading). The optimal routes obtained from these algorithms can differ significantly from the great circle courses to avoid harsh storms, implying the sacrifice of the shortest travelling distance. Nevertheless, to avoid encountering harsh storms can be also achieved by proper management of the ship speed, but still keeping the shortest sailing distance along the great circle course.

In the voyage optimization process, sea weather forecast plays an extremely important role as it is the main dynamic input for the optimization algorithm. The reliability of the optimized route highly depends on the accuracy of the weather forecast. However, the accuracy of weather forecast often starts to be questioned after 3-5 days, while even large discrepancy happens after 5-7 days. An example of the comparison of significant wave height between forecast data and hindcast data is presented in Fig. 1. The observation point is selected in the North Atlantic Ocean. The figure indicates that the accuracy of forecast data starts to decrease after 5 days. As the consequence, even the route given by the optimization algorithms is optimal based on forecast data, the actual voyage may end up with encountering very harsh weather conditions.

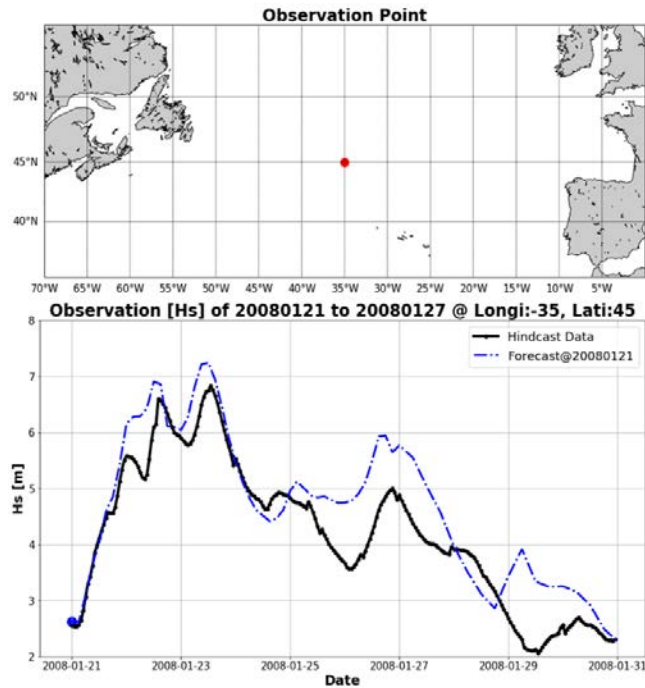


Figure 1. Comparison of significant wave heights obtained from forecast data and hindcast data

In this paper, a new voyage optimization method, which considers voluntary speed reduction in the great circle course is proposed. Then, a comprehensive comparison between the proposed method and conventional two-dimensional Dijkstra's algorithm is conducted to investigate the reliability and efficiency of the proposed optimization method. The investigation also includes the influence of the

uncertainty of weather forecast on the optimization results.

2 DESCRIPTION OF THE PROPOSED METHOD

The great circle course is divided by N waypoints and each waypoint is associated with a passing time. A ship's state variable represents the location and the associated passing time of the i -th waypoint as,

$$\mathbf{s}_i = [x_i, y_i, t_i]^T \quad (1)$$

where x_i, y_i, t_i represent longitude, latitude and time of the i -th waypoint. All the waypoints along the route is described by $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_N]$.

Normally, for a conventional voyage optimization algorithm, a ship's speed and heading at each waypoint are selected as control variables that needs to be optimized during the voyage,

$$\mathbf{U}(\mathbf{S}) = [\mathbf{u}_1(\mathbf{s}_1), \mathbf{u}_2(\mathbf{s}_2), \dots, \mathbf{u}_N(\mathbf{s}_N)] \quad (2)$$

$$\mathbf{u}_i(\mathbf{s}_i) = [v_i(\mathbf{s}_i), \theta_i(\mathbf{s}_i)]^T$$

where \mathbf{v} and $\boldsymbol{\theta}$ are vectors of ship speed and heading of all waypoints such as $\mathbf{v} = [v_1, v_2, \dots, v_N]$.

In this study, we fixed the heading to the great circle course while keep speed varying. The weather conditions encountered during a voyage depends on the state variable vector \mathbf{S} , denoted by $\mathbf{W}(\mathbf{S})$. The sea conditions contain information of wave height, wave direction, wave period, wind and current, etc. In addition, a ship's sailing is often limited under some operational constraints. The constraints function is a Boolean function that returns if the ship is feasible to sail under a state \mathbf{S} and a control variable \mathbf{U} of a waypoint. Here, the constraints include the land-crossing avoid, maximum speed/engine power constraints.

The objective of the voyage optimization is to minimize the fuel consumption during the voyage. The total fuel consumption C is estimated by,

$$C = \int_{t_0}^{t_N} f(\mathbf{U}(\mathbf{S}), \mathbf{W}(\mathbf{S})) dt \quad (3)$$

where $f(\mathbf{U}(\mathbf{S}), \mathbf{W}(\mathbf{S}))$ is the instantaneous cost function (fuel cost between two waypoints) for a series of ship state \mathbf{S} under control $\mathbf{U}(\mathbf{S})$; t_0, t_N are the departure time and arrival time, respectively.

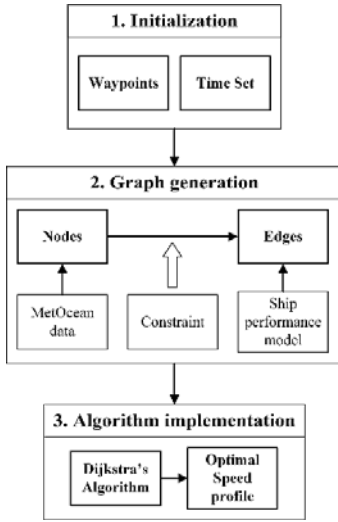


Figure 2. Scheme of the proposed method

The proposed method is to optimize the speed profile, i.e., the speed vector \mathbf{v} , along great circle waypoints \mathbf{S} . The workflow of the method is shown as Fig.2.

2.1 Speed/passing time initiation along the route

The first step is to generate waypoints on the great circle course between the departure and the destination. The waypoints contain the information of longitude and latitude. Then, it will add a time set to each waypoint. A time set is an array, which contains a series of feasible arrival times to a waypoint. Figure 3 gives a simple example on how the time set is added on a waypoint. The procedure is done as follows:

- 1 Generate a set of discrete time with even space for each waypoint. The space can be chosen from several hours to days.
- 2 Add constraints to the time set. Here we consider both the sailing speed and ETA as constraints.
- 3 Select feasible times from each waypoint shown as round dots in the figure.

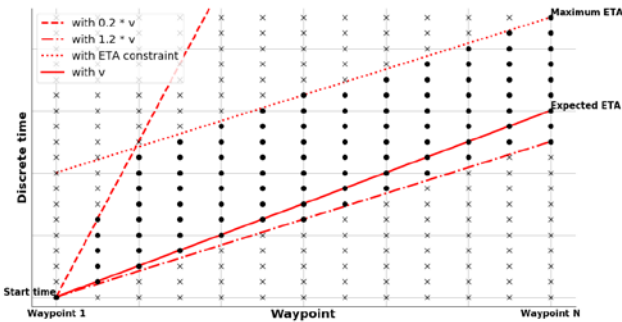


Figure 3. Illustration of a "time set" along the route

2.2 Cost function for edge generation

In this study, a ship energy performance model is used as the cost function through the optimization and the ship's speed is considered as the input for the optimization.

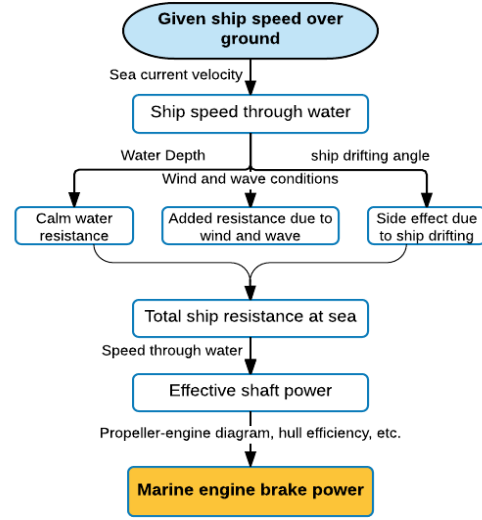


Figure 4. Ship energy performance models

For the optimization, the ship's engine power needed to push the ship forward at the input speed should be estimated as the cost based on encountered sea conditions, the ship's characteristics, and operational profiles etc. The workflow for the ship speed-power performance modelling is presented in Fig.4.

2.3 Graph generation

A great circle course is geometrically divided into a number of waypoints and each waypoint contains a set of feasible arrival time. With this information a graph $G = (\mathbf{S}, \mathbf{A})$ can be formed, where \mathbf{S} is a set of waypoints (also considered as ship state variables) and \mathbf{A} is a set of edges composed of pairs of nodes. A waypoint \mathbf{s}_i contains a number of ship states,

$$\mathbf{s}_i = \begin{bmatrix} \mathbf{s}_{i,1} \\ \vdots \\ \mathbf{s}_{i,m} \end{bmatrix} = \begin{bmatrix} x_i & y_i & t_{i,1} \\ \vdots & \vdots & \vdots \\ x_i & y_i & t_{i,m} \end{bmatrix} \quad (4)$$

where i is the i th waypoint, m is the number of feasible arrival time of the i th waypoint.

Edges are also required to be added for proceeding the optimization. Adding edges is to pair two nodes/ship states from two adjacent waypoints as the following equation shows.

$$\mathbf{A}_{i,j,k} = \begin{bmatrix} \mathbf{s}_{i,j} \\ \mathbf{s}_{i+1,k} \end{bmatrix} = \begin{bmatrix} x_i & y_i & t_{i,j} \\ x_{i+1} & y_{i+1} & t_{i+1,k} \end{bmatrix} \quad (5)$$

where i is the i th waypoint and j, k are the j th and k th of the time corresponding to the i th and $i+1$ th waypoints. Constraints must be considered to eliminate the infeasible edges/connections between nodes/ship states when $\mathbf{A}_{i,j,k}$ is going to be added to \mathbf{A} :

- 1 if $t_{i+1,k} > t_{i,j}$;
- 2 the corresponding shaft power between $\mathbf{s}_{i,j}$ and $\mathbf{s}_{i+1,k}$ doesn't exceed the maximum engine power.

For the great circle course based optimization, each edge $\mathbf{A}_{i,j,k}$ is assigned with a cost/weight $C_{i,j,k}$ which is computed by the cost function, where

$$C_{i,j,k} = f\left(\mathbf{U}(s_{i,j}), \mathbf{W}(s_{i,j})\right)(t_{i+1,k} - t_{i,j}) \quad (6)$$

The generation of the graph \mathbf{G} is complete when all the feasible nodes \mathbf{S} are generated and all the edges \mathbf{A} are added to \mathbf{G} .

2.4 Implementation of Dijkstra's algorithm

Dijkstra's algorithm works in a graph by visiting edges starting from the source node to the target node. In this study, the source node and the target node refer to the starting ship state and the end ship states. It is an algorithm for finding the shortest path between two graph nodes in a graph. In this study, instead of using Dijkstra's algorithm to find the shortest path, it is used to find the path with minimum cost.

Before implementing Dijkstra's algorithm, a vector of total cost from the starting ship state $s_{0,0}$ to every node/ship state C should be constructed. This means $C(s_{i,j})$ should represent the cost from $s_{0,0}$ to any node/ship state $s_{i,j}$ in \mathbf{S} . Then a dictionary \mathbf{V} representing all unvisited nodes within \mathbf{G} is generated.

The implementation of Dijkstra's algorithm is performed as follows. For every $s_{i,j} \in \mathbf{S}$,

- 1 Initialize the cost set $C(s_{i,j})$ to infinity. An infinite cost in C for a given node/ship state means no path has been found from the start node to $s_{0,0}$ to $s_{i,j}$.
- 2 Add $s_{i,j}$ to \mathbf{V} , indicating $s_{i,j}$ is unvisited.
- 3 Set $C(s_{0,0})$ to 0.
- 4 If \mathbf{V} is not empty, select node $s_{i,j}$ with the smallest C -value from C .
- 5 Remove $s_{i,j}$ from \mathbf{V} . For every adjacent node \mathbf{u} of $s_{i,j}$, if $C(s_{i,j}) + \text{weight}(s_{i,j}, \mathbf{u}) < C(\mathbf{u})$, then $C(\mathbf{u}) = C(s_{i,j}) + \text{weight}(s_{i,j}, \mathbf{u})$. Go back to step 4, until \mathbf{V} gets empty.

In this study, it should be noted that the *weight* function above refers to Eq. (6). We have a number of end ship states at waypoint N as shown in Fig. 3. This means a number of optimal speed configuration along the route can be generated for different ETA.

3 CASE STUDY SHIP FOR INVESTIAGTION

In order to demonstrate the advantage and disadvantage of the proposed great circle based voyage optimization method, a case study ship equipped with full-scale measurement devices is chosen here. In addition, the influence of the uncertainty of the weather forecast data on the optimization results is also investigated. Thus, both the weather forecast data and hindcast data are used in this study.

Moreover, the interpolation method used in interpolating sea weather conditions for the ship's waypoints can also influence the optimization results.

In this study, trilinear interpolation (Weiser et al. 1988) is used. The influence of different interpolation methods is not considered here.

3.1 Details of the case study

In this case study, a 2800TEU container ship sailing in the North Atlantic is taken as a case study ship (Mao 2014). The main particulars of the ship are listed in Table 1. For this case study ship, the full-scale measurements of the ship's performance at sailing in the North Atlantic during the year 2008 are available in this study.

In the practical voyage planning, the voyage optimization process is conducted before the voyage starts and the weather forecast data is usually used as the preliminary data source for optimizing the voyage. In this study, the optimization is conducted using the weather forecast information that can be accessed before the voyage. While the hindcast data is used to estimate the weather and sea conditions which the ship actually encountered. The forecast data is obtained from ECMWF Mars operational archive server (<https://apps.ecmwf.int/mars-catalogue/?class=od>) and the hindcast data is extracted from ECMWF ERA5 hindcast dataset (<https://cds.climate.copernicus.eu>). It should be noted that the forecast data is updated every 12 hours and the time resolution of the forecast data is 3 hours. As the ship's cost variation is mainly caused by encountered significant wave height (H_s), H_s is chosen to represent sea conditions.

Table 1. Main particulars of the case study ship

Length L_{oa}	235.0 m
Length L_{pp}	230.4 m
Beam B	32.2 m
Prop. Diam.	6.9 m
Displacement	55566 m ³
Wetted surface	10396 m ²
Length waterline	230.4 m
Draft T	10.78 m

In this study, the actual voyages of the case study ship sailing along the North Atlantic during 2008 are used as the reference for implementing the proposed method and comparing the method with other methods. As the storms in North Atlantic always moves from the east to the west, two westbound voyages are selected because it is more important for captains to plan their voyages for westbound voyages to ensure cargo safety and save fuel, in particular during the winter season. Furthermore, to demonstrate the capability of the proposed method, all optimizations are conducted in the open sea area in North Atlantic. The analysis of the strategy of slow steaming is conducted in the last case study.

3.2 Ship routing algorithms for the case study

This investigation compares three algorithms, i.e., 1) 2D Dijkstra's algorithm with fixed ship speed, 2) Great circle course with speed optimized (proposed method), and 3) Great circle course with fixed ship speed often for chartering contract.

The 2D Dijkstra's algorithm (2DDA) is one of the conventional voyage optimization algorithms derived from 2D Dynamic programming method (De Wit, 1990). It uses the Dijkstra's algorithm instead of the traditional dynamic programming approach to conduct the optimization. The grid system is generated based on the great circle reference course, which covers the accessible sailing area of the voyage shown in Fig. 5. The 2DDA is a two-dimensional optimization method. It is used to find optimal waypoints for a certain voyage with certain demands such as minimum fuel consumption, maximum ship safety.

The proposed method is used to optimize the ship speed profile during the voyage instead of the location of the waypoints for a voyage. The third method is to estimate the speed and passing time along the great circle course. It should be noted that constraints such as maximum-engine-rate constraint are applied to all of these methods in the optimization process. Thus, there is involuntary speed reduction occurring when encountering harsh sea conditions.

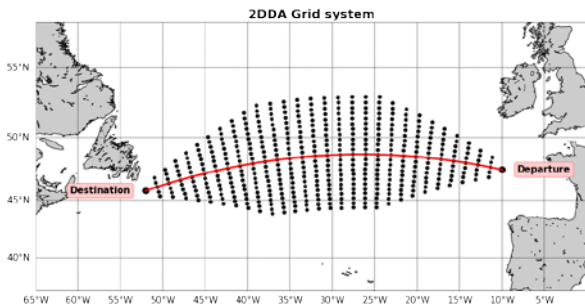


Figure 5. Example of the grid system for the 2DDA

4 RESULTS OF INVESTIGATING THE PROPOSED VOYAGE OPTIMIZATION

An actual voyage during Feb. 2008 is selected to demonstrate the capability of the proposed method. Two operation scenarios are investigated. One is for the ship sailing with her service speed using 85% MCR of the ship engine. Considering the slow steaming strategy is quite well adopted in today's shipping market, the other scenario is to investigate the capability of the proposed method for the ship to sail with a 25% speed reduction as her service speed.

4.1 A westbound voyage sailing in service speed

The trajectory of the optimized routes by the 2DDA method is shown in Fig.6, as well as the great circle course. The trajectory got from the 2DDA method implies that the encountered sea weather conditions were more severe in high latitudes. Therefore, the 2DDA method could try to find alternative waypoints (rather than the shortest distance) to avoid these harsh sea conditions.

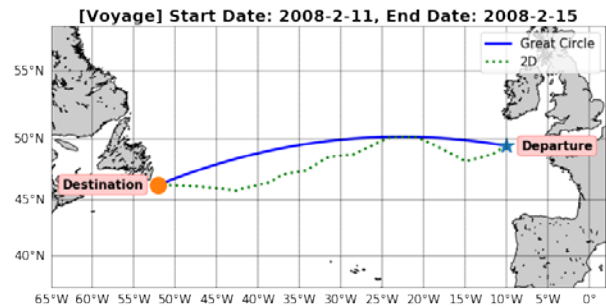


Figure 6. Route trajectories by 2DDA and the shortest distance (the great circle course).

Furthermore, the ETA, fuel consumption and sailing distance along the routes optimized by the three methods based on forecast sea weather conditions are listed in Table 1. It also gives the ship's overall operation performance from the post-voyage analysis. The post voyage analysis is performed by extracting the "true" weather information from hindcast data for the same waypoints (i.e., location and passing time) as the optimized routes. Then the ship's new fuel consumption and ETA are re-estimated based on the hindcast weather information. If the ship state is invalid in the actual sea condition (for example, the shaft power estimated by the actual sea condition exceed the maximum power rate), the current ship state as well as the following ship state will be recalculated.

For the voyage optimization based on the weather forecast data, Table 1 shows that the proposed method can save approximate 1% of fuel consumption in comparison with the conventional fixed speed great circle sailing method (as reference here). The optimized route from the proposed method can also keep the same ETA as the reference method. While, the ship routing generated from the 2DDA method gives the largest fuel saving, because this method can choose routes shifting away from the shortest distance to encounter calm sea environments.

Table 2. Optimization results for service speed sailing based on forecast and hindcast weather data

Method	Fuel cost [ton]		ETA [hour]		Distance [km]
	Forecast	Hindcast	Forecast	Hindcast	
(Method 1) 2DDA	228.0	234.8	100.4	100.8	3324
(Method 2) GC optimal speed	242.4	244.3	102.6	103.3	3121
(Method 3) GC fixed speed	244.4	247.9	102.7	103.3	3121

Table 3. Optimization results for slow steaming sailing based on forecast and hindcast weather data

Method	Fuel cost [ton]		ETA [hour]		Distance [km]
	Forecast	Hindcast	Forecast	Hindcast	
(Method 1) 2DDA	170.3	160.0	122.8	122.8	3380
(Method 2) GC optimal speed	195.4	188.4	117.6	117.6	3121
(Method 3) GC fixed speed	204.7	190.9	117.7	117.9	3121

Furthermore, Fig.7 presents the comparison of significant wave height (H_s) between obtained from weather forecast in the optimization and extracted from hindcast data after the voyage, as well as the comparison between optimized ship speeds (based on weather forecast) and actual ship speeds (estimated from hindcast data for post voyage analysis). It shows that for such a short passage, the weather forecast data is very reliable at the beginning of the voyage. However, after approximately 2-3 days sailing, the forecast data begins to deviate from hindcast data (but not very much). The post voyage analysis of speeds in the right column shows the same trend, i.e., ship speeds optimized from various method begin to differ in the late stage of the voyage.

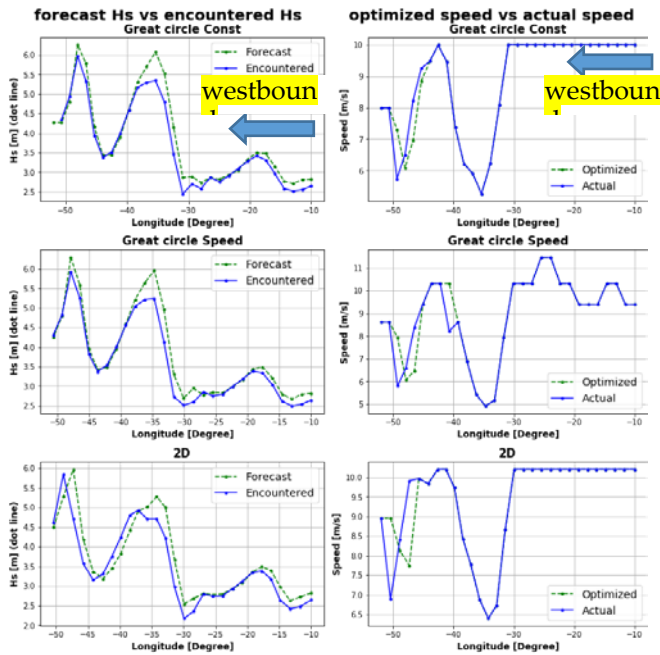


Figure 7. Comparison of H_s and sailing speeds from weather forecast for route optimization and hindcast data for post voyage analysis.

In comparison with the post voyage analysis and the voyage optimization based on forecast data, there is very little difference of fuel cost and ETA for the two great circle based routes, the different between forecast and hindcast based. This is because 1) the total sailing time is about 4 days and the weather forecast is quite reliable, 2) there is not so big margin to change too much of the ship's speed in the great circle course to avoid harsh storm and keep ETA at the same time. If there are too large and long voluntary speed reduction, the ship will not be able to increase her speed too much than the service speed to catch up with the same ETA.

However, on the other hand, even though the weather forecast is quite reliable for the 4 days' route, there are still large variation (3% in fuel cost) between the forecast and hindcast based optimization by the 2DDA method. It means that the 2DDA method is the most affected by the uncertainty of the weather forecast. For even long sailing voyages, this weather uncertainty may lead to that the 2DDA method will generate worse route than the simple shortest route.

4.2 A westbound voyage with slow steaming

The slow steaming strategy for the same voyage is used here to study the capability of the proposed method. The trajectory of the generated courses by different method is presented in Fig.8. It shows that the 2DDA method for the slow steaming have even large freedom to choose the waypoints different significantly from the great circle course.

The corresponding ship performance (fuel cost, ETA and distance) planned by these methods are listed in Table 3. For the slow steaming, the fuel consumption for all routes is reduced significantly, but the reduction is simple proportional (not exponential) to the speed reduction. The consequence is obvious that it takes more sailing time. It gives large margin for the proposed method to optimize the ship speed configuration along the great circle course. The result shows that it increases the fuel saving up to around 5% instead of 1% in the service speed sailing. For the comparison of fuel consumption between optimized routes by the forecast data and post voyage analysis by the hindcast data, the difference can be more than 7% due to long sailing time, which increased weather uncertainties. Furthermore, the proposed method performs much better than the simple fixed speed great circle route.

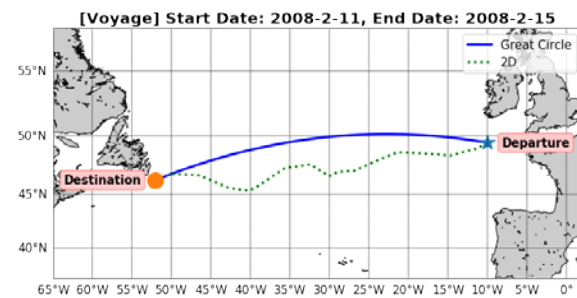


Figure 8. Trajectories generated by different methods

4.3 Cons and pros of the proposed method

For the proposed method to consider the voluntary speed reduction to avoid harsh environment along the great circle course, the fuel consumption of the generated route is a bit higher than the 2DDA method. One possible reason might be that the storms often appear at least for 12 hours, but the ship speed cannot be voluntarily reduced too much and last for too long time in order to keep the ETA.

Figure 9 shows the waypoint number and its corresponding weight graph with the square of significant wave height. The weight graph is generated based on the hindcast data. The solid line is the optimized result given by the proposed method. The dash lines can be considered as the boundary which assume that the ship sails with certain speed. It can be implied that without delaying the ETA, it is hard to avoid the bad weather conditions. It is also shown that the ship can slow down at the beginning and catch up later to avoid the heavy shadow area shown like the arrow line. However, to catch up the ETA, the ship has to accelerate in the shadow area which would cost more fuel than the optimized one. Thus, it is possible to avoid the storm by adjusting the ship's speed. However, when considering the item of

fuel saving, the speed acceleration to a very high level may lead to much higher fuel cost, in particular, sometimes, the maximum speed is limited by the ship's engine size.

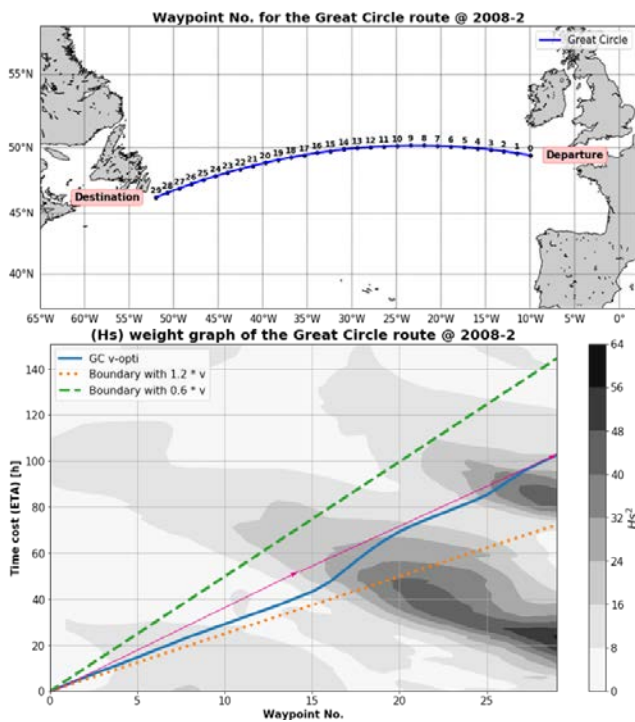


Figure 9. Waypoints and the weight graph

In addition to the voluntary speed reduction, the proposed method can deliver a Pareto front for end-users to select the best combination of ETA and fuel consumption to meet their interests. Figure 10 shows the Pareto front obtained from the proposed method based on the weather forecast data. The dots are a small part of the results extracted from the proposed method. The feasible ETA can be selected from approximately 80h to 140h with 0.3h interval.

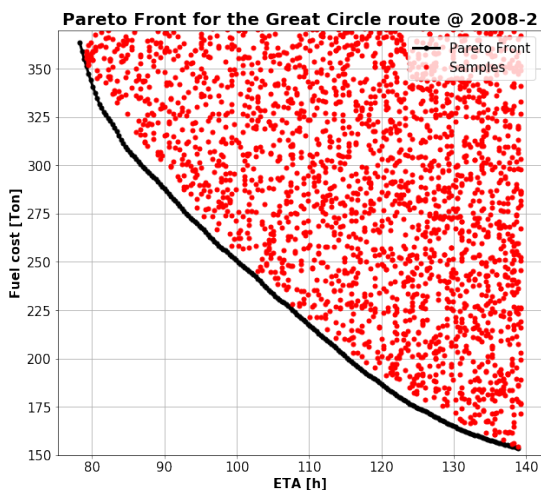


Figure 10. Pareto front for the Great Circle route given by the proposed method

5 CONCLUSION

Large uncertainty of weather forecast will affect a ship's actual performance for sailing along the routes

from voyage optimizations. In this study, the fixed great circle course considering voluntary speed reduction is compared with the conventional fixed speed shortest route planning and fixed speed varying course planned by the 2-dimensional Dijkstra algorithm (2DDA). Therefore, a new method is proposed for optimizing a ship's voluntary speed reduction along the great circle course. Through the comparison between the three methods based on an actual voyage collected by a containership, the proposed method shows better optimization results with respect to fuel saving than the simple fixed speed shortest route. Even higher energy saving can be expected if the slow steaming strategy is used. For the current case study, the 2DDA method can generate ship routes with lowest fuel consumption.

However, the performance of the 2DDA method can be easily affected by the uncertainty from the weather forecast. For the case study voyage lasting for about 4 days, the forecast data can be quite reliable. But still the variation of weather uncertainty can lead to more than 7% difference of fuel saving between optimization based on weather forecast and the post voyage analysis from hindcast data. It can be expected for even long-time-range voyages such as across Pacific Ocean, optimization methods such as 2DDA may give the bad optimization results in comparison with the proposed method, due to the deficiency of weather prediction by forecast. Furthermore, the proposed method can also generate Pareto Fronts for end-users to select the best combination with the respect of ETA and fuel consumption. It also has more potential for fuel saving in slow steaming scenario and long-distance sailing as it has more space to voluntary speed reduction during the voyage.

ACKNOWLEDGEMENTS

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