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Power Allocation in HARQ-based Predictor Antenna Systems

Hao Guo, *Student Member, IEEE*, Behrooz Makki, *Senior Member, IEEE*,
Mohamed-Slim Alouini, *Fellow, IEEE*, and Tommy Svensson, *Senior Member, IEEE*

Abstract—In this work, we study the performance of predictor antenna (PA) systems using hybrid automatic repeat request (HARQ). Here, the PA system is referred to as a system with two sets of antennas on the roof of a vehicle. In this setup, the PA positioned in the front of the vehicle can be used to predict the channel state information at the transmitter (CSIT) for the receive antenna (RA) that is aligned behind the PA. Considering spatial mismatch due to mobility, we derive closed-form expressions for the optimal power allocation and the minimum average power of PA systems under different outage probability constraints. The results are presented for different types of HARQ protocols and we study the effect of different parameters on the performance of PA systems. As we show, our proposed approximation scheme enables us to analyze PA systems with high accuracy. Moreover, for different vehicle speeds, we show that HARQ-based feedback can reduce the outage-limited power consumption of PA systems by orders of magnitude.

Index Terms—Channel state information, mobile relay, moving backhaul, outage probability, power allocation, predictor antenna.

I. INTRODUCTION

Vehicle communication is one of the important use cases in the fifth generation of wireless networks (5G) and beyond [1]. Here, the focus is to provide efficient and reliable connections to cars and public transports, e.g., busses and trains. Channel state information at the transmitter (CSIT), plays an important role in achieving these goals, as it enables advanced closed-loop transmission schemes such as link adaptation, multi-user scheduling, interference coordination and spatial multiplexing schemes. However, typical CSIT acquisition systems, which are mostly designed for (semi)static channels, may not work well as the speed of the vehicle increases. This is because, depending on the vehicle speed, the position of the antennas may change quickly and the CSIT becomes inaccurate.

To overcome this issue, [2] proposes the concept of predictor antenna (PA). Here, a PA system is referred to as a setup with two (sets of) antennas on the roof of a vehicle. The PA positioned in the front of the vehicle can be used to improve the CSIT for data transmission to the receive antenna (RA) that is aligned behind the PA. The potential of such setups have been previously shown through experimental tests [2]–[4], and its performance has been analyzed in, e.g., [5]–[7].

H. Guo and T. Svensson are with the Department of Electrical Engineering, Chalmers University of Technology, 41296 Gothenburg, Sweden (email: hao.guo@chalmers.se; tommy.svensson@chalmers.se).

B. Makki is with Ericsson Research, 41756 Gothenburg, Sweden (email: behrooz.makki@ericsson.com).

M.-S. Alouini is with the King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia (e-mail: slim.alouini@kaust.edu.sa).

One of the challenges of the PA setup is spatial mismatch that causes CSIT for the RA to be partially inaccurate. This occurs if the RA does not reach the same spatial point as the PA, due to, e.g., the delay for preparing the data is not equal to the time that is needed until the RA reaches the same point as the PA [3]. Moreover, the performance of typical PA setups could be further improved in case the PA could be used not only for channel prediction, but also for data transmission. We address these challenges by implementing hybrid automatic repeat request (HARQ) in PA systems as follows.

In this work, we analyze the outage-limited performance of PA systems using HARQ. With our proposed approach, the PA is used not only for improving the CSIT in the retransmissions to the RA, but also for data transmission in the initial round. In this way, as we show, the combination of PA and HARQ protocols makes it possible to improve the power-constrained outage probability, and adapt the transmission parameters to mitigate the effect of spatial mismatch.

The problem is cast in the form of minimizing the average transmission power subject to an outage probability constraint. Particularly, we develop approximation techniques to derive closed-form expressions for the instantaneous and average transmission power as well as the optimal power allocation minimizing the outage-limited power consumption. The results are presented for the cases with different repetition time diversity (RTD) and incremental redundancy (INR) HARQ protocols [8], [9]. Moreover, we study the effect of different parameters such as the antennas separation and the vehicle speed on the system performance.

As we show through analysis and simulations, the implementation of HARQ as well as power allocation can improve the outage-limited performance of PA systems by orders of magnitude, compared to the cases with no retransmission. For example, consider an outage probability constraint of 10^{-4}, initial rate $R = 2$ nats-per-channel-use (npcu) and a maximum of two transmission rounds. Then, compared to the cases with no retransmission, our proposed scheme can reduce the required signal-to-noise ratio (SNR) by 18 dB and 20 dB for the RTD and the INR schemes, respectively.

II. PROBLEM FORMULATION

Here, we first introduce the basics of PA systems which is followed by our proposed HARQ-based PA setup.

A. Standard PA System

Figure 1 illustrates the standard PA system. Here, the PA first receives pilots from the base station (BS) in time t. Then, the BS gets the BS-PA channel h_1 from the PA and sends the
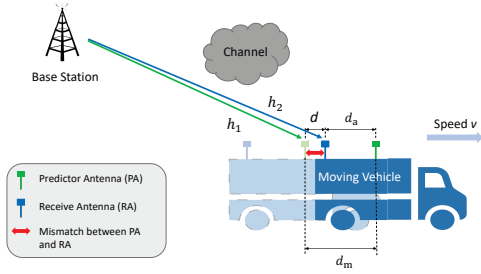
Fig. 1. Predictor antenna system with mismatch problem.

data in time $t+\delta$ to the RA where δ depends on the processing time at the BS. At the same time, the vehicle moves forward d_m while the antenna separation between the PA and the RA is d_a. Then, considering downlink transmission in the BS-RA link, the signal received by the RA is

$$y = \sqrt{P}h_2 x + z. \tag{1}$$

Here, P represents the transmit power, x is the input message with unit variance, and h_2 is the fading coefficient between the BS and the RA. Also, $z \sim \mathcal{CN}(0,1)$ denotes the independent and identically distributed (IID) complex Gaussian noise added at the receiver.

We represent the probability density function (PDF) and cumulative density function (CDF) of a random variable A by $f_A(\cdot)$ and $F_A(\cdot)$, respectively. Due to spatial mismatch between the PA and the RA, assuming a semi-static propagation environment, i.e., assuming that the coherence time of the propagation environment is much larger than δ [1], h_2 and h_1 are correlated according to [5, Eq. 5]

$$h_2 = \sqrt{1-\sigma^2}h_1 + \sigma q, \tag{2}$$

where $q \sim \mathcal{CN}(0,1)$ which is independent of the known channel value $h_1 \sim \mathcal{CN}(0,1)$, and σ is a function of the mismatching distance $d = |d_\mathrm{a}-d_\mathrm{m}|$ [5, Eq. 4]. Defining $g_1 = |h_1|^2$ and $g_2 = |h_2|^2$, the CDF $F_{g_2|g_1}$ is given by

$$F_{g_2|g_1}(x) = 1 - Q_1\left(\sqrt{\frac{2g_1(1-\sigma^2)}{\sigma^2}}, \sqrt{\frac{2x}{\sigma^2}}\right), \tag{3}$$

where $Q_1(s,\rho) = \int_\rho^\infty x e^{-\frac{x^2+s^2}{2}} I_0(xs)\mathrm{d}x, \ s,\rho \geq 0$, is the first-order Marcum Q-function. Also, $I_n(x) = (\frac{x}{2})^n \sum_{i=0}^\infty \frac{(\frac{x}{2})^{2i}}{i!\Gamma(n+i+1)}$ is the n-order modified Bessel function of the first kind, and $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}\mathrm{d}x$ represents the Gamma function. In this way, although parameter adaptation is performed based on perfect CSIT of h_1 at time t, the spatial mismatch may lead to unsuccessful decoding by the RA at $t + \delta$.

B. Proposed HARQ-based PA System

Along with the spatial mismatch problem, in the typical PA systems the PA is used only for channel estimation. On the other hand, because the PA system includes the PA-BS feedback link, in a frequency-division duplex (FDD) setup HARQ can be supported by the PA structure. For this reason, we propose a setup as follows.
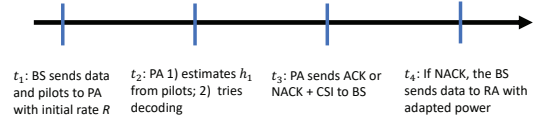
[1]This has been experimentally verified in, e.g., [4]

Fig. 2. Time structure for the proposed PA-HARQ scheme.

Here, as seen in Fig. 2, with no CSIT, at t_1 the BS sends pilots as well as the encoded data with certain initial rate R and power P_1 to the PA. At t_2, the PA estimates the channel h_1 from the received pilots. At the same time, the PA tries to decode the signal. If the message is correctly decoded, i.e., $R \leq \log(1 + g_1 P_1)$, an acknowledgment (ACK) is fed back to the BS at t_3, and the data transmission stops. Otherwise, the PA sends both a negative acknowledgment (NACK) and high accuracy quantized CSI feedback about h_1. The number of quantization bits are large enough such that we can assume the BS to have perfect CSIT of h_1 (see [6] for the effect of imperfect CSIT on the performance of PA systems). With NACK, in the second transmission round at time t_4, the BS transmits the message to the RA with power P_2 which is a function of the instantaneous channel quality g_1. The outage occurs if the RA cannot decode the message at the end of the second round.

III. ANALYTICAL RESULTS

Let ϵ be the outage probability constraint. Here, we present the results for the cases with RTD and INR HARQ protocols. With an RTD protocol, the same signal (with possibly different power) is sent in each retransmission round, and the receiver performs maximum ratio combining of all received copies of the signal. With INR, on the other hand, new redundancy bits are sent in the retransmissions, and the receiver decodes the message by combining all received signals [8], [9].

Considering Rayleigh fading conditions, as the worst-case scenario, we have $f_{g_1}(x) = e^{-x}$, and the outage probability at the end of Round 1 is given by

$$\Pr(\text{Outage, Round 1}) = \Pr\{R \leq \log(1 + g_1 P_1)\}$$
$$= \Pr\left\{g_1 \leq \frac{e^R - 1}{P_1}\right\} = 1 - e^{-\frac{\theta}{P_1}}, \tag{4}$$

where $\theta = e^R - 1$. Then, using the results of, e.g., [8, Eq. 7, 18] on the outage probability of the RTD- and INR-based HARQ protocols, the power allocation problem for the proposed HARQ-based PA system can be stated as

$$\min_{P_1, P_2} \quad \mathbb{E}_{g_1}[P_\mathrm{tot}|g_1]$$
$$\text{s.t.} \quad P_1, P_2 > 0,$$
$$P_\mathrm{tot}|g_1 = \left[P_1 + P_2(g_1) \times \mathcal{I}\left\{g_1 \leq \frac{\theta}{P_1}\right\}\right], \tag{5}$$

with

$$F_{g_2|g_1}\left\{\frac{\theta - g_1 P_1}{P_2(g_1)}\right\} = \epsilon, \quad \text{for RTD} \tag{6}$$

$$F_{g_2|g_1}\left\{\frac{e^{R-\log(1+g_1 P_1)} - 1}{P_2(g_1)}\right\} = \epsilon, \quad \text{for INR.} \tag{7}$$

Here, $P_\mathrm{tot}|g_1$ is the total instantaneous transmission power for two transmission rounds (i.e., one retransmission) with given

g_1, and we define $\bar{P} \doteq \mathbb{E}_{g_1}[P_{\text{tot}}|g_1]$ as the expected power, averaged over g_1. Moreover, $\mathcal{I}(x) = 1$ if $x > 0$ and $\mathcal{I}(x) = 0$ if $x \le 0$. Also, $\mathbb{E}_{g_1}[\cdot]$ represents the expectation operator over g_1. Here, we ignore the peak power constraint and assume that the BS is capable of transmitting sufficiently high power. Finally, (5)-(7) come from the fact that, with our proposed scheme, P_1 is fixed and optimized with no CSIT at the BS and based on average system performance. On the other hand, P_2 is adapted continuously based on the predicted CSIT.

Using (5), the required power in Round 2 is given by

$$P_2(g_1) = \frac{\theta - g_1 P_1}{F_{g_2|g_1}^{-1}(\epsilon)}, \tag{8}$$

for the RTD, and

$$P_2(g_1) = \frac{e^{R-\log(1+g_1 P_1)} - 1}{F_{g_2|g_1}^{-1}(\epsilon)}, \tag{9}$$

for the INR, where $F_{g_2|g_1}^{-1}(\cdot)$ is the inverse of the CDF given in (3). Note that, $F_{g_2|g_1}^{-1}(\cdot)$ is a complex function of g_1 and, consequently, it is not possible to express P_2 in closed-form. For this reason, one can use [10, Eq. 2, 7]

$$Q_1(s, \rho) \simeq e^{\left(-e^{\mathcal{I}(s)}\rho^{\mathcal{J}(s)}\right)},$$
$$\mathcal{I}(s) = -0.840 + 0.327s - 0.740s^2 + 0.083s^3 - 0.004s^4,$$
$$\mathcal{J}(s) = 2.174 - 0.592s + 0.593s^2 - 0.092s^3 + 0.005s^4, \tag{10}$$

to approximate $F_{g_2|g_1}$ and consequently $F_{g_2|g_1}^{-1}(\epsilon)$. In this way, (8) and (9) can be approximated as

$$P_2(g_1) = \Omega\left(\theta - g_1 P_1\right), \tag{11}$$

for the RTD, and

$$P_2(g_1) = \Omega\left(e^{R-\log(1+g_1 P_1)} - 1\right), \tag{12}$$

for the INR, where

$$\Omega(g_1) = \frac{2}{\sigma^2}\left(\frac{\log(1-\epsilon)}{-e^{\mathcal{I}\left(\sqrt{\frac{2g_1(1-\sigma^2)}{\sigma^2}}\right)}}\right)^{-\frac{2}{\mathcal{J}\left(\sqrt{\frac{2g_1(1-\sigma^2)}{\sigma^2}}\right)}}. \tag{13}$$

In this way, for different HARQ protocols, we can express the instantaneous transmission power of Round 2, for every given g_1 in closed-form. Then, the power allocation problem (5) can be solved numerically. However, (13) is still complicated and it is not possible to solve (5) in closed-form. For this reason, we propose an approximation scheme to solve (5).

Let us initially concentrate on the RTD protocol. Combining (5) and (8), the expected total transmission power is given by

$$\bar{P}_{\text{RTD}} = P_1 + \int_0^{\theta/P_1} e^{-x} P_2 \mathrm{d}x = P_1 + \int_0^{\theta/P_1} e^{-x}\frac{\theta - xP_1}{F_{g_2|x}^{-1}(\epsilon)}\mathrm{d}x. \tag{14}$$

Then, Theorem 1 derives the minimum required power in Round 1 and the average total power consumption as follows.

Theorem 1. *With RTD and outage constraint ϵ, the minimum required power in Round 1 and the average total power are, respectively, given by $\hat{P}_{1,RTD} = \frac{-m\theta}{\mathcal{W}_{-1}\left(\frac{m^2}{ce} - \frac{1}{e}\right)+1}$ and*

$$\hat{\bar{P}}_{RTD} = \hat{P}_{1,RTD} + \frac{c}{m^2}\left(\hat{P}_{1,RTD}e^{-\frac{m\theta}{\hat{P}_{1,RTD}}} - \hat{P}_{1,RTD} + m\theta\right), \tag{15}$$

where $m = 1 + \frac{1-\sigma^2}{\sigma^2}$ and $c = \frac{-1}{\sigma^2 \log(1-\epsilon)}$, and $\mathcal{W}_{-1}(\cdot)$ is the Lambert W function with the -1 branch [11, Eq. 16].

Proof. Plugging (3) into (6), we have

$$1 - Q_1\left(\sqrt{\frac{2g_1(1-\sigma^2)}{\sigma^2}}, \sqrt{\frac{2(\theta - g_1 P_1)}{\sigma^2 P_2}}\right) = \epsilon. \tag{16}$$

By using the approximation [12, Eq. 17] for moderate/large σ, i.e., if $1 - Q_1(s, \rho) = 1 - \epsilon$, then $\rho = Q_1^{-1}(s, 1 - \epsilon) \simeq \sqrt{-2\log(1-\epsilon)}e^{\frac{s^2}{4}}$, we can obtain

$$\sqrt{\frac{2(\theta - g_1 P_1)}{\sigma^2 P_2}} \simeq \sqrt{-2\log(1-\epsilon)}e^{\frac{g_1(1-\sigma^2)}{2\sigma^2}}. \tag{17}$$

In this way, P_2 in (14) is approximated by

$$P_2 \simeq (\theta - g_1 P_1)\frac{e^{-\frac{g_1(1-\sigma^2)}{\sigma^2}}}{-\sigma^2 \log(1-\epsilon)}, \tag{18}$$

and considering RTD, (14) can be rewritten as

$$\bar{P} = P_1 + \int_0^{\theta/P_1} e^{-x}(\theta - xP_1)\frac{e^{-\frac{x(1-\sigma^2)}{\sigma^2}}}{-\sigma^2 \log(1-\epsilon)}\mathrm{d}x$$
$$\overset{(a)}{=} P_1 + \frac{c}{m^2}\left(P_1 e^{-\frac{m\theta}{P_1}} - P_1 + m\theta\right), \tag{19}$$

where, in (a) we set $m = 1 + \frac{1-\sigma^2}{\sigma^2}$ and $c = \frac{-1}{\sigma^2 \log(1-\epsilon)}$ for simplicity. Then, setting the derivative of (19) with respect to P_1 equal to zero, the minimum P_1 for the minimum total power can be found as

$$\hat{P}_{1,\text{RTD}} = \underset{P_1>0}{\arg}\left\{1 + \frac{c}{m^2}e^{-\frac{\theta m}{P_1}}\left(\frac{m\theta}{P_1} + 1\right) - \frac{c}{m^2} = 0\right\}$$
$$= \underset{P_1>0}{\arg}\left\{e^{-\frac{\theta m}{P_1}}\left(\frac{m\theta}{P_1} + 1\right) = 1 - \frac{m^2}{c}\right\}$$
$$\overset{(b)}{=} \frac{-m\theta}{\mathcal{W}_{-1}\left(\frac{m^2}{ce} - \frac{1}{e}\right)+1}. \tag{20}$$

Here, (b) is obtained by the definition of the Lambert W function [11, Eq. 16]. Then, plugging (20) into (19), we obtain the minimum total transmission power as given in (15). ∎

A. On the Effect of CSIT Feedback/Power Allocation

In this part, we consider the case without exploiting CSIT, i.e., we consider the typical HARQ schemes where CSIT feedback is not sent along with NACK, and we do not perform power adaptation. Here, the outage probability, for the RTD and the INR are given by

$$\zeta_{\text{RTD}} = \Pr\left\{\log\left(1 + (g_1 + g_2)P\right) < R \ \& \ g_1 < \frac{\theta}{P}\right\}, \tag{21}$$

$$\zeta_{\mathrm{INR}} = \Pr\left\{\log\left(1+g_1 P\right) + \log\left(1+g_2 P\right) < R \ \& \ g_1 < \frac{\theta}{P}\right\}. \tag{22}$$

Note that as opposed to (5)-(7), where the transmit power in Round 2 is set instantaneously such that, for every given g_1, the instantaneous outage probability constraint is satisfied, (21) and (22) are based on the average outage probability, i.e., a less constrained, condition. Also, in both protocols, the total average power is given by

$$\bar{P} = P + P \cdot \Pr\{\log\left(1+g_1 P\right) < R\} = P(2 - e^{-\frac{\theta}{P}}). \tag{23}$$

Theorem 2. *Without CSIT feedback/power allocation, the outage probability of the RTD-based PA-HARQ scheme is given by (26).*

Proof. Using (21), we have

$$\zeta_{\mathrm{RTD}} = \Pr\left\{g_2 < \frac{\theta}{P} - g_1 \ \& \ g_1 < \frac{\theta}{P}\right\}$$

$$= \int_0^{\frac{\theta}{P}} e^{-x} F_{g_2|x}\left(\frac{\theta}{P} - x\right) \mathrm{d}x. \tag{24}$$

Considering (3), there is no closed-form solution for (24). For this reason, we use the approximation [13, Eq. 14]

$$Q_1\left(s, \rho\right) \simeq 1 - \left(1 + \frac{s^2}{2}\right) e^{-\frac{s^2}{2}} + \left(1 + \frac{s^2}{2} + \frac{s^2 \rho^2}{4}\right) e^{-\frac{s^2 + \rho^2}{2}}, \tag{25}$$

which simplifies (24) to

$$\zeta_{\mathrm{RTD}} \simeq -\frac{e^{-\frac{\theta}{P\sigma^2}}}{6\sigma^4}\left(\left(6\sigma^8 - 12\sigma^6\right) e^{\frac{\theta}{P\sigma^2}} + 12\sigma^6 + \right.$$

$$\left. \left(3\sigma^2 - 3\sigma^4\right)\frac{\theta^2}{P^2} + \left(12\sigma^4 - 6\sigma^6\right)\frac{\theta}{P} + \left(1-\sigma^2\right)\frac{\theta^3}{P^3} - 6\sigma^8\right) \tag{26}$$

B. On the Effect of Introducing INR

For the INR scheme, by using Jensen's inequality and the concavity of the logarithm function [14, Eq. 30]

$$\frac{1}{n}\sum_{i=1}^{n}\log(1+x_i) \le \log\left(1 + \frac{1}{n}\sum_{i=1}^{n}x_i\right), \tag{27}$$

the closed-form expressions for the minimum required power, the average total power, as well as outage probability without power allocation are given by the following Corollary.

Corollary 1. *With INR, the minimum required power in Round 1 and the average total power are given by (28) and (29), respectively. Also, without power allocation, the outage probability of the INR-based scheme is given by (31).*

Proof. Using (7), the Jensen's inequality (27) and defining $\theta_1 = 2\left(e^{\frac{R}{2}} - 1\right)$, (9) can be approximated by $P_2(g_1) \simeq \frac{\theta_1 - g_1 P_1}{F_{g_2|g_1}^{-1}(\epsilon)}$, and, following the same steps as in Theorem 1, we obtain the minimum required P_1 for the minimum total power in the INR scheme as

$$\hat{P}_{1,\mathrm{INR}} = \frac{-m\theta_1}{\mathcal{W}_{-1}\left(\frac{m^2}{ce} - \frac{1}{e}\right) + 1}. \tag{28}$$
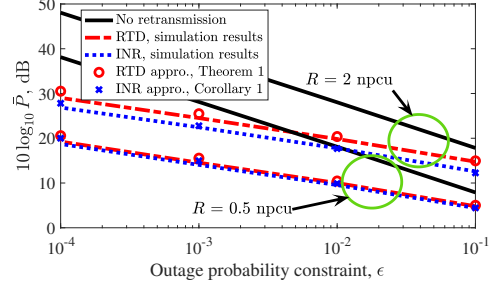

Fig. 3. Minimum required power under various outage constraints for the RTD and the INR, $\sigma = 0.8$, $R = 0.5, 2$ npcu.

Also, the minimum total power can be calculated by

$$\hat{\bar{P}}_{\mathrm{INR}} = \hat{P}_{1,\mathrm{INR}} + \frac{c}{m^2}\left(\hat{P}_{1,\mathrm{INR}} e^{-\frac{m\theta_1}{\hat{P}_{1,\mathrm{INR}}}} - \hat{P}_{1,\mathrm{INR}} + m\theta_1\right). \tag{29}$$

Finally, (22) can be further derived by

$$\zeta_{\mathrm{INR}} = \Pr\left\{g_2 < \frac{e^{R - \log(1+g_1)} - 1}{P} \ \& \ g_1 < \frac{\theta}{P}\right\}$$

$$\overset{(c)}{\simeq} \int_0^{\frac{\theta}{P}} e^{-x} F_{g_2|x}\left(\frac{\theta_1}{P} - x\right) \mathrm{d}x, \tag{30}$$

with (c) using Jensen's inequality. Following the same steps as in Theorem 2, the outage probability is found as

$$\zeta_{\mathrm{INR}} \simeq \zeta_{\mathrm{RTD}}(\theta = \theta_1). \tag{31}$$

∎

Then, as a benchmark, we consider the case with no retransmission where the outage probability is given by $1 - e^{-\frac{\theta}{P}}$. In this case, the required outage-constrained power without retransmission is given by $P \ge \frac{1-e^R}{\log(1-\epsilon)}$.

Finally, note that, for simplicity of analysis, we have presented the results for the cases with the perfect CSIT of the BS-PA link. This is motivated by the fact that, compared to direct transmission, relatively short feedback signals are required to provide the transmitter with fairly accurate quantized CSIT. Then, as we have shown in [6, Section III.C] the effect of imperfect CSIT can be well modeled as an extra additive Gaussian noise in (2), and we can follow the same procedure to analyze the system performance with imperfect CSIT.

IV. SIMULATION RESULTS

In the simulations, we set $\delta = 5$ ms, $f_c = 2.68$ GHz, and $d_a = 1.5\lambda$. For simplicity, we have ignored the path loss. However, it is straightforward to extend the results to the cases with path loss which will affect the power terms in the figures correspondingly. Each point in the figures is obtained by averaging the system performance over 10^5 channel realizations.

Figure 3 shows the results of the power allocation problem (5) for both the RTD and the INR with different initial rates $R = 0.5, 2$ npcu, $\sigma = 0.8$, and different outage probability constraints ϵ. Here, the simulation results are obtained by solving the optimization problem (5) numerically, while the approximation results for the RTD and the INR are obtained from Theorem 1 and Corollary 1, respectively. In Fig. 4, we plot the outage probability using the RTD and the INR without power optimization. Here, we set $\sigma = 0.8$, and the initial rate
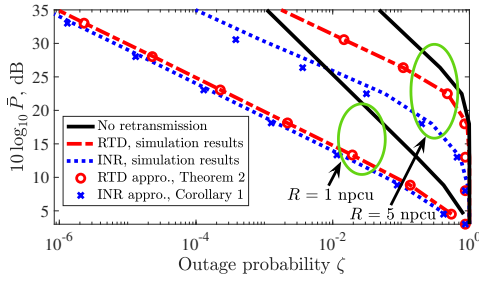
Fig. 4. Minimum required power without optimized power allocation (23) under various outage probabilities, $\sigma = 0.8$, $R = 1, 5$ npcu.
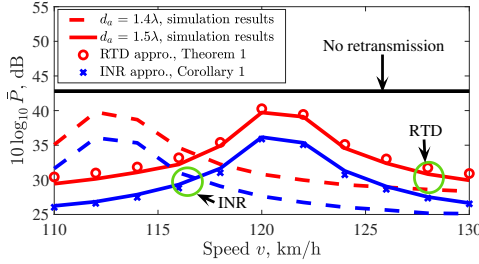

Fig. 5. Minimum required power for the RTD and the INR with given outage constraint, for different speed v. $R = 3$ npcu, $\epsilon = 10^{-3}$.

$R = 1, 5$ npcu. The simulation results are obtained by (21) and (22), while the analytical approximations are obtained from Theorem 2 for the RTD and Corollary 1 for the INR. Finally, in Fig. 5, we study the minimum required transmission power for different speeds v. Here, manipulating v is equivalent to changing the level of spatial correlation for given values of δ, f_c and d_a (see [6] for the detailed effect of the vehicle speed on the spatial correlation). Also, we study the effect of different values of d_a on the system performance. According to the figures, we can conclude the following:

- The approximation schemes of Theorem 1 and Corollary 1 are tight for a broad range of values of initial rate R, speed v, as well as ϵ (Figs. 3 and 5). Thus, for different parameter settings, the outage-constrained power allocation for the RTD and the INR protocols can be well determined by Theorem 1 and Corollary 1.
- Also, the approximation scheme of Theorem 2 is tight for a broad range of values of average power (23) as well as initial rate R (Fig. 4). Thus, for different parameter settings, the outage probability for the proposed PA-HARQ scheme without power allocation, can be well determined by Theorem 2 and Corollary 1.
- With the deployment of the PA and power allocation, remarkable power gain is achieved especially at low outage probabilities (Figs. 3 and 5). Moreover, as also indicated in, e.g., [8], INR outperforms RTD, in terms of outage-limited average power. However, the difference between the performance of these protocols decreases as the data rate or σ decreases (Figs. 3 and 5).
- Figures 3-5 emphasize the efficiency of HARQ as well as adaptive power allocation in the PA system. From Figs. 3-4, we can see that using the PA-HARQ setup, with different instantaneous and average outage-constrained power allocation cases, we can obtain considerable performance improvements compared to the case with no

retransmission. For instance, with $\epsilon = 10^{-4}$ and $R = 2$ npcu, the RTD- and INR-based PA-HARQ can reduce the required power, compared to no retransmission case, by 18 and 20 dB, respectively. Then, as can be seen from Fig. 5, the effect of power-optimized PA-HARQ increases with σ. This is because the larger σ provides better spatial diversity of the channel, which can improve the performance with retransmissions. For the peaks in Fig. 5, the channel for the retransmission has the largest correlation with the one in the first round, which leads to the smallest power gain. Moreover, when the antenna separation d_a decreases, the speed where the power gain is minimum, also decreases, due to the reduction of the mismatch distance d.

V. CONCLUSION

We studied PA-HARQ systems with the spatial mismatch problem, in the context of outage-constrained power allocation. We derived closed-form expressions for the minimum instantaneous and total transmit power. The approximations are tight for a broad range of system configurations. Also, the results show that, while PA-assisted adaptive power adaptation leads to considerable performance improvements, the total transmission power and the outage probability are remarkably affected by the spatial mismatch.

REFERENCES

[1] S. Dang, O. Amin, B. Shihada, and M.-S. Alouini, "What should 6G be?" *Nat. Electron.*, vol. 3, no. 1, pp. 20–29, Jan. 2020. [Online]. Available: https://doi.org/10.1038/s41928-019-0355-6

[2] M. Sternad, M. Grieger, R. Apelfröjd, T. Svensson, D. Aronsson, and A. B. Martinez, "Using predictor antennas for long-range prediction of fast fading for moving relays," in *Proc. IEEE WCNCW*, Paris, France, Apr. 2012, pp. 253–257.

[3] D. T. Phan-Huy, M. Sternad, and T. Svensson, "Adaptive large MISO downlink with predictor antenna array for very fast moving vehicles," in *Proc. IEEE ICCVE*, Las Vegas, Nevada, USA, Dec. 2013, pp. 331–336.

[4] N. Jamaly, R. Apelfröjd, A. Belen Martinez, M. Grieger, T. Svensson, M. Sternad, and G. Fettweis, "Analysis and measurement of multiple antenna systems for fading channel prediction in moving relays," in *Proc. IEEE EuCAP*, Hague, Netherlands, Apr. 2014, pp. 2015–2019.

[5] H. Guo, B. Makki, and T. Svensson, "Rate adaptation in predictor antenna systems," *IEEE Wireless Commun. Lett.*, vol. 9, no. 4, pp. 448–451, Apr. 2020.

[6] H. Guo, B. Makki, M.-S. Alouini, and T. Svensson, "A semi-linear approximation of the first-order Marcum Q-function with application to predictor antenna systems," Jan. 2020, available at http://arxiv.org/abs/2001.09264.

[7] ——, "On delay-limited average rate of HARQ-based predictor antenna systems," Apr. 2020, available at https://arxiv.org/abs/2004.01423.

[8] B. Makki, A. Graell i Amat, and T. Eriksson, "Green communication via power-optimized HARQ protocols," *IEEE Trans. Veh. Technol.*, vol. 63, no. 1, pp. 161–177, Jan. 2014.

[9] T. V. K. Chaitanya and E. G. Larsson, "Outage-optimal power allocation for hybrid ARQ with incremental redundancy," *IEEE Trans. Wireless Commun.*, vol. 10, no. 7, pp. 2069–2074, Jul. 2011.

[10] M. Z. Bocus, C. P. Dettmann, and J. P. Coon, "An approximation of the first order Marcum Q-function with application to network connectivity analysis," *IEEE Commun. Lett.*, vol. 17, no. 3, pp. 499–502, Mar. 2013.

[11] D. Veberic, "Having fun with Lambert W(x) function," *arXiv preprint arXiv:1003.1628*, Jan. 2018.

[12] M. M. Azari, F. Rosas, K. Chen, and S. Pollin, "Ultra reliable UAV communication using altitude and cooperation diversity," *IEEE Trans. Commun.*, vol. 66, no. 1, pp. 330–344, Jan. 2018.

[13] S. András, A. Baricz, and Y. Sun, "The generalized Marcum Q-function: an orthogonal polynomial approach," *Acta Universitatis Sapientiae Mathematica*, vol. 3, no. 1, pp. 60–76, Oct. 2011.

[14] B. Makki, T. Svensson, T. Eriksson, and M. Alouini, "On the performance of RF-FSO links with and without hybrid ARQ," *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4928–4943, Jul. 2016.