Simultaneous scheduling of preventive system maintenance and of the maintenance workshop

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Abstract

While a system operates, its components deteriorate and in order for the system to remain operational, maintenance of its components is required. Preventive maintenance (PM) is performed so that component failure is avoided. This research aims at scheduling PM activities for a multi-component system within a finite horizon. The system to be maintained possesses positive economic dependencies, meaning that each time any component maintenance activity is performed, a common set-up cost is generated. Each component PM activity generates a cost, including replacement, service, and spare parts costs. We start from a 0-1 mixed integer linear optimization model of the PM scheduling problem with interval costs, which is to schedule PM of the components of a system over a finite and discretized time horizon, given common set-up costs and component costs, of which the latter vary with the maintenance interval. We extend the PMSPIC model to incorporate the flow of components through the maintenance/repair workshop, including stocks of spare components, both the components that require repair and the repaired ones. Our resulting model is a tight integration of the PM and the maintenance workshop scheduling. We investigate two different contract types between stakeholders, present and analyze preliminary numerical results obtained.

1. INTRODUCTION

When planning the maintenance for any system, the decisions to be made concern when each of its components should be maintained (i.e., replaced, repaired, or serviced) and what kind of maintenance should then be performed, with respect to the operational schedule of the system. Preventive maintenance (PM) can often be planned well in advance, while corrective maintenance (CM) is done after a failure has occurred, which may come on very short notice. On the other hand, an unexpected but necessary CM action may provide an opportunity for the PM at which the maintenance actions can be rescheduled, starting from the system's current state. While both PM and CM are aimed at restoring the components in order to put the system back in an operational state, CM is often much more costly than PM, due to a longer system down-time and also due to possible damages to other components caused by the failure.

We present an application from the aerospace industry. On one side, we consider a system of aircraft that has an operational demand to fulfill, and on the other, the maintenance workshop that repairs the components coming from the aircraft and makes them available for usage again. Hence, there are two stakeholders, an *aircraft operator* and a *maintenance workshop*, whose collaboration is normally predefined by a contract. We define and discuss a number of optimization objectives corresponding to two different contract types, so-called *availability* and *turn-around-time* contracts.

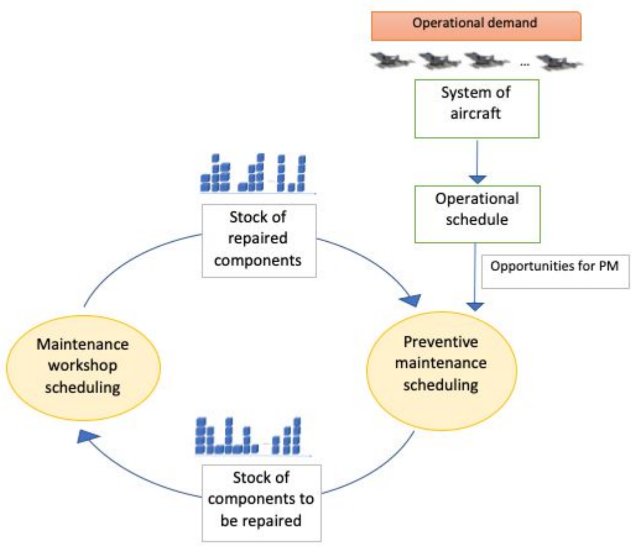


Figure 1: The operating systems and the maintenance workshop, with the operational demand as input and component maintenance schedules as output.

The model of the maintenance scheduling problem presented in this article is partly based on the *preventive maintenance scheduling problem with interval costs* (PMSPIC) model presented in Gustavsson et al. (2014). The PMSPIC considers one or several systems with multiple component types; we generalize these models in the sense that also the individual components are considered and can be placed in any of the systems. We also take into account the operational schedules for the systems which provide time windows in which the different maintenance activities may or must be performed.

The PMSPIC considered and generalized in this article is an extension of the opportunistic replacement problem (ORP) studied in Almgren et al. (2012). The PMSPIC takes into account the interval between two maintenance occasions for each component; the length of that interval determines the assigned cost.

An efficient way of generating the operational/flying schedules (e.g., timetables) is presented in Gavranis et al. (2015), in which the availability of a fleet of aircraft is maximized subject to requirements on the transport missions and maintenance of the vehicles' components.

1.1. Motivation

The aircraft maintenance-and-supply chain is often a major bottleneck for the performance of today's total aircraft systems. Hence, an even higher pressure to boost aircraft supply chain performance in tomorrow's aircraft systems will be crucial since

- the required turn-around-time—the time on ground due to maintenance—will decrease in future aircraft operations,
- demands on flexibility of resources on ground as well as airborne will increase, and
- cost efficiency over the life-cycle is increasingly important.

The motivation for considering the tight integration of the maintenance planning for the systems and the scheduling of the maintenance workshop is threefold. First, it provides a planning tool for systems in which the maintenance workshop is in reality integrated with the operating system. That would mean that the stakeholder operating the aircraft is also responsible and performs maintenance of its components. Secondly, when there is more than one stakeholder, a tightly integrated model will provide an optimistic estimate of the results—in terms of costs for maintenance, of costs for lateness (under a turn-around contract), or of the lower limit of items on the stock and/or the average availability (under an availability contract)—that could be obtained in reality and which can be used as a benchmark. Lastly, the integration enables an investigation of different types of contracts that can be set-up between the stakeholders.

Another incentive for our research is the fact that it is applicable not only for aircraft systems but for any kind of system possessing similar properties (such as heavy vehicles, trains, and trucks).

1.2. Background

Until now, advanced optimization models have been developed for each part of the supply chain of aircraft maintenance—from tactical scheduling of aircraft to missions or maintenance, to depot level maintenance planning and scheduling (see, e.g., Gavranis and Kozanidis (2015), Erkoc et al. (2016), De Bruecker et al. (2015), and Kurz (2016)). Further, contracts and other static prerequisites, like dimensioning and design issues, constrain the possibility to efficiently perform maintenance operations at a low cost, as described in Ekström et al. (2015) and Olde Keizer et al. (2016). However, in line with an increased interoperability via Internet of things and Interoperable service architectures—in the form of *system-of-systems*—new and promising possibilities to integrate data and intelligence within the whole supply chain have evolved. Separate planning problems can be integrated in a network and simultaneously optimized with respect to performance.

1.3. Outline of the article

The remainder of this article is organized as follows. In Section 2, we first define the aircraft maintenance scheduling problem as well as the maintenance workshop scheduling. Then, we introduce the stock dynamics in order to integrate these two problems. In addition, we include the operational demand as an input to our model. In Section 3, we present two types of contracting forms between the respective stakeholders and the corresponding optimization objectives. Test and preliminary numerical results are presented in Section 4. Lastly, conclusions and future research ideas are presented in Section 5. The full mathematical model is presented in the *Appendix.*

2. PROBLEM DEFINITION

2.1. Aircraft maintenance scheduling

We consider a fleet of K aircraft with I component types and J_i individual components of each type $i \in I$. Maintenance can be scheduled at any time step t within the finite and discrete planning horizon T. A maintenance occasion of an aircraft k at time step t generates a maintenance cost. The maintenance interval (i.e., the interval between two maintenance occasions) of a component generates an interval cost, which is increasing with the length of the interval. For each component type, by defining higher costs for scheduling maintenance after the end of—and close before—its life, unexpected failures are avoided; thereby our approach may stay within the scope of PM scheduling. We model this problem as a 0-1 mixed integer linear optimization problem (see Conforti et al. (2014)); the decision variables and constraints are described below.

Decision variables. To determine the maintenance intervals of the components, we let the decision variable take the value 1 if the individual component j of type i from aircraft k receives PM at time steps s and t, but not in-between. Otherwise, $x_{st}^{ijk} = 0$. We further let the decision variable z_t^k take the value 1 if aircraft k is scheduled for maintenance at time t, and 0 otherwise.

Constraints. For each aircraft and component type, a maintenance interval starts at time 0 (*App.*; (1b)), while at each time step t the same number (i.e., 0 or 1) of maintenance intervals must end and start (*App.*; (1a)). If a maintenance interval of component type i in an aircraft ends at time step t, then maintenance of that aircraft must occur at time step t (*App.*; (1c)). Lastly, we ensure that each component (i,j) is in at most one aircraft at each time step (*App.*; (1d)).

2.2. Maintenance Workshop scheduling

Components that should be maintained are sent to the *maintenance workshop*, which contains a number (L) of (identical) parallel repair lines for component repair, each of which has a repair capacity of one unit while each component repair requires one unit of this capacity per time step during a prespecified (component type-specific) and consecutive (i.e., preemption is not allowed) number of time steps. When a component arrives at the workshop it is available for repair and assigned a due date, at which the repair should be finished, and the component be returned back to the system operator. This problem is identified as an *identical parallel machines scheduling problem* (IPMSP; Brucker and Knust (2012)). A component that finishes repair prior to (after) its due date generates a non-positive (non-negative) penalty cost, which applies only in the case of a turn-around-time contract. A solution to the maintenance workshop scheduling problem specifies at which time each component arriving at the workshop should start maintenance.

Decision variables. For each individual component *j* of each type *i* and for each time step *t*, we define $u_t^{ij} \in \{0,1\}$ which takes value 1 if component (*i,j*) starts repair at time *t*, zero otherwise. The number of active parallel repair lines at each time step *t* is defined by the integer variable l_t.

Constraints. At each time step, the number of active parallel repair lines equals the number of active repair lines in the previous time step plus the difference of the numbers of components starting and finishing repair (*App.*; (2)); this number may never exceed the workshop capacity *L*.

To connect the mathematical models in Sections 2.1 and 2.2, we next introduce the *stock dynamics*, to model the interface between the variables defined for the two respective problems.

2.3. Stock dynamics

When an individual component is taken out of an aircraft it is sent—with no time delay—to the stock of damaged components, where it stays until it is scheduled for repair. The transport time between the stock of damaged components and the maintenance workshop is prespecified. Upon being repaired, it goes to the stock of repaired (i.e., as good as new) components, again with a prespecified transport time between the workshop and stock of repaired components, where it is kept until its scheduled time for placement into an(other) aircraft. We assume that all transport times are represented by non-negative integers.

Decision variables. To model the flow of components, we define the following binary variables: a_t^{ij} (b_t^{ij}) takes the value 1 if component (*i,j*) is on the stock of damaged (as good as new) components at time step *t;* otherwise, it takes the value 0. Furthermore, $_t^{ij}$ takes the value 1 if component (*i,j*) is taken out of some aircraft and placed on the stock of damaged components at time step *t*, and $_t^{ij}$ takes the value 1 if component (*i,j*) leaves the stock of repaired components and is placed in some aircraft at time step *t*.

Constraints. Whether a component is placed in an aircraft (i.e. taken from the stock of repaired components) or removed from one (i.e. placed on the stock of damaged components), we model by *App.*; (3a), (4a), respectively. Further, we formulate a set of constraints describing the state of a component (*i,j*) at time step *t* on both stocks: a component is either on one of the stocks or not (*App.*; (3b), (4b)). The state of a component at a stock at time *t* is affected by its state in the previous time step *t-1*, possible arrival to (*App.*; (3a)) and departure from (*App.*; (4a)) the stock. The modeling of the two stocks is analogous. Aside from having a physical lower limit of zero, we can constrain the level of *as good as new* components more (*App.*; (4c)).

2.4. Integration with the operational demand

The system of aircraft considered possesses an operational demand, represented by a flying/operational schedule that should be fulfilled.. The schedules define time intervals when the aircraft is either operating or grounded, i.e., accessible for maintenance. Therefore, the starting point for our modeling is precisely the operational demand.

Constraints To include the opportunities for scheduling maintenance occasions, we constrain z_t^k with the binary input parameter (*App.*; (5)), coming from the operational schedule. If the aircraft is in the air, maintenance is not allowed and z_t^k is forced to 0.

2.5. Optimization objectives

As mentioned above, we consider two stakeholders, the aircraft operator and the maintenance workshop. We study two contract types (availability and turn-around time) by defining two bi-objective optimization problems (see Ehrgott (2005)). The first objective is composed by the minimization of the maintenance cost (*App.*; (10)) and the maximization of the availability of components on the stock of repaired components (*App.*; (11)). The second objective is composed by the minimization of the maintenance costs and the minimization (maximization) of the penalties for lateness and ealiness. (*App.*; (12)). The minimization of the maintenance cost is of interest for the aircraft operator while the other two objectives are relevant for the maintenance workshop and represent the risk for lack of spare components.

When solving a multi-objective optimization problem, we are usually interested in finding Pareto efficient solutions (Ehrgott (2005)). A solution is called *Pareto efficient* (also called Pareto optimal or non-dominated) if none of the objective functions can be improved in value without degrading some of the other objective values. Without additional subjective preference from the decision maker, all Pareto optimal solutions are considered equally good. The *Pareto frontier* is the set of all Pareto optimal points. To find points on the Pareto front, we use the *ε-constraint method* (see Mavrotas (2009)) which, in the bi-objective case, optimizes one objective function while the other one is being constrained. In the next section, we investigate and analyze our two bi-objective problems and their Pareto frontiers.

3. TEST AND RESULTS

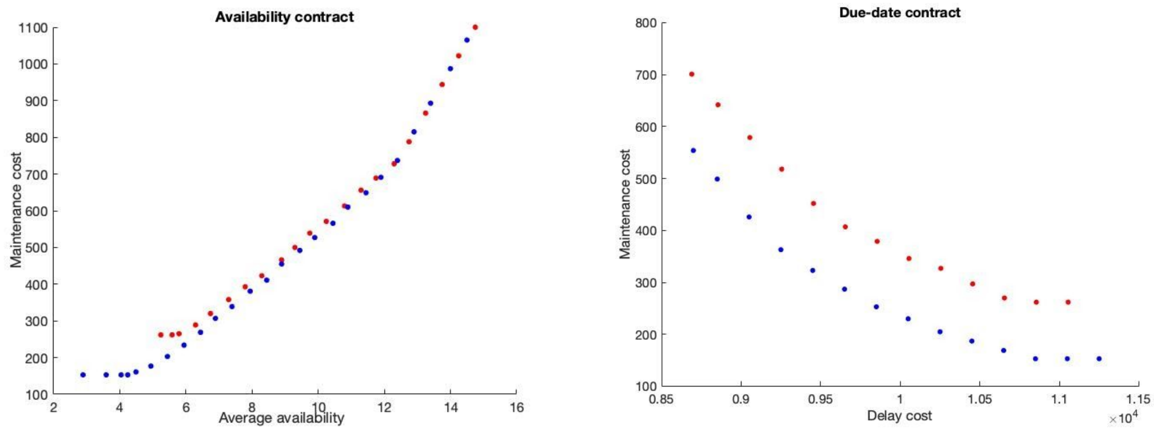
The implementation of our modeling was done using Julia (2012) and JuMP (2017) and the computations are performed by Gurobi (2020) on a laptop computer with a 2.4 GHz Intel Core i5 processor and 8 GB of RAM memory. The figures are produced in Matlab (2018).

As a test case, we use randomized data. We consider a set of five aircraft, each with (the same) three component types. There are ten individual components of each type. At each time step, each aircraft carries exactly one individual component of each component type. The differences of the component types are reflected in their repair times (in the maintenance workshop) and due dates (for delivery back to the system operator), which are chosen randomly and are of the same order of magnitude. For the turn-around-time objective, the penalty for late delivery differs between the component types, where lateness comes with twice higher penalties than earliness, for each component type. The planning horizon is 20 time steps and the workshop consists of ten parallel repair lines. The due-dates and processing times are component type-specific, randomized, and such that the due-dates are larger than the shortest possible turn-around-times. The (positive) transport times between the stocks and the maintenance workshop are fixed, while a component taken out of an aircraft is immediately (i.e., zero transport times) put on the stock of damaged components. Similarly, once a component is on the stock of repaired components (i.e., after it has been repaired in the workshop and transported to the stock), it is available for usage without any transport time. In our tests, we consider lower limits on the stock of repaired components of zero, one, and two.

For the *availability contract*, we observe in Figure 2(a) that the optimal maintenance costs (for each value of the limit ε, on the average availability) increase slightly with the lower limit (when increased from 0 to 1) on the stock of repaired components. However, as the average availability, i.e., the average number of repaired components over the planning period, increases (in this case, when it goes above twelve components), the two Pareto curves converge since the lower limit being 0 or 1 loses impact with an increasing mean availability. The maximum value of the average availability for the lower limit of 0 (1) is in the interval 2.9–14.5 (5.25–14.75).

For the *turn-around-time contract* (see Figure 2(b)), the differences between maintenance costs (for the lower limit on the stock of repaired components of 0 and 1) are larger than when having an availability contract and these differences seem approximately constant when the value of ε is varied (see Mavrotas (2009)).

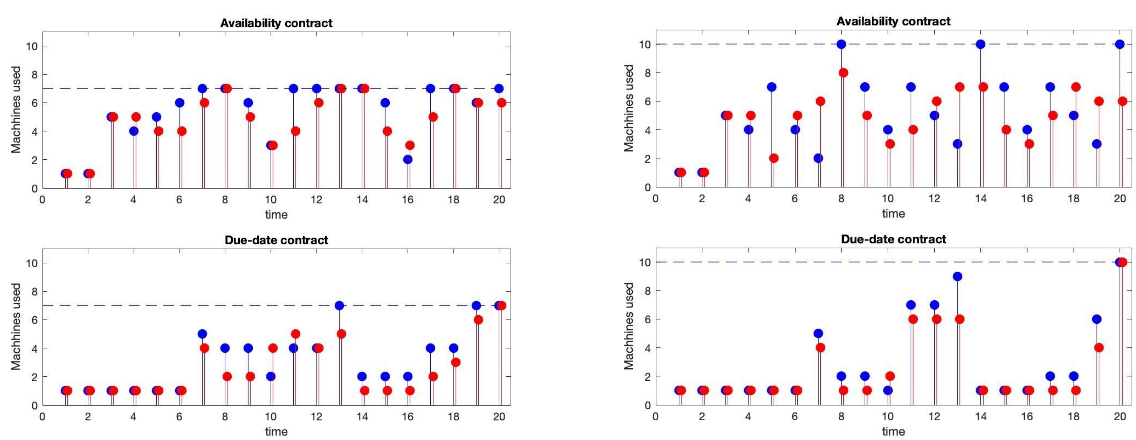
We observe that the minimum possible maintenance cost for both contract types is 153 (lower limit on the stock of repaired components of 0) and 262 (lower limit on the stock of repaired components of 1), which represents an instance with the lower limit on the availability of components being 0 (1), no delay/earliness penalties, and no reward for a high average availability. Further, the maximum maintenance cost in the case of an availability contract has a much higher value 1065 (lower limit on the stock of repaired components of 0) and 1100 (lower limit on the stock of repaired components of 1) as compared to 554 (lower limit on the stock of repaired components of 0) and 701 (lower limit on the stock of repaired components of 1), which is the value in the case of a turn-around-time contract. Observe, though, that the very high maintenance costs in the availability contract come with very high requirements on the average availability, where the definition of *a very high cost* is in the end a subjective preference of the decision maker.



(a) Average availability vs maintenance cost (b) Delay cost vs. maintenance cost

Figure 2: The computed points on the respective Pareto fronts for L=10 and c^i_{early}=10, 15, 20 for i=1, 2, 3. The blue (red) dots correspond to zero (one) as a lower bound on the stock of available components.

Figure 3 illustrates the resulting number of active parallel repair lines with availability and due-date contracts over time, for the workshop capacity being seven and ten parallel repair lines, respectively, and with the lower limit on the stock of repaired components being 0 and 1, respectively. When reducing the workshop capacity below L=7, for the instance considered, finding optimal schedules becomes computationally too expensive when considering a due-date contract. When considering an availability contract, the same effect occurs when the workshop capacity goes below L=6. We can see that availability contracts with a higher lower limit on the stock of available components impose higher workload on the workshop since a higher number of repaired components must be available in the stock.

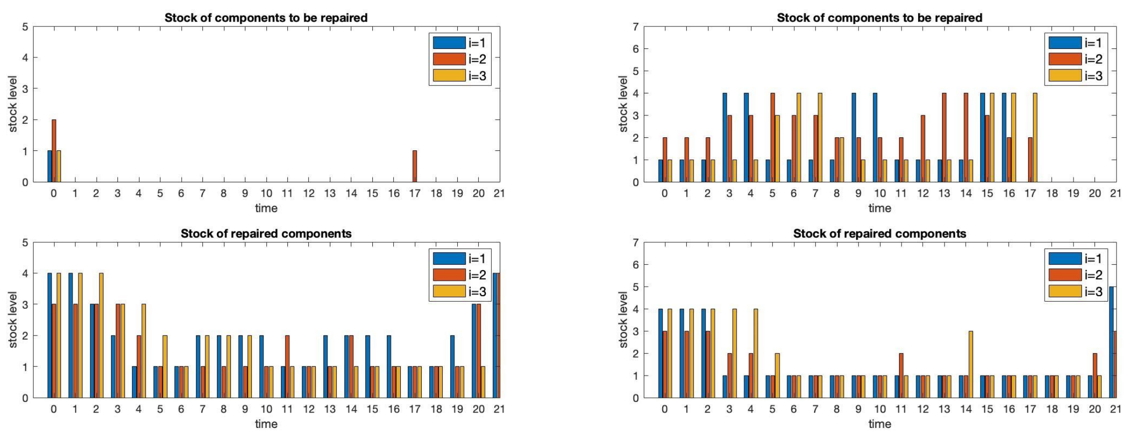


(a) L=7 (b) L=10

Figure 3: Number of active parallel repair lines for different types of contracts over time, where blue (red) points correspond to a lower limit of 0 (1) on the stock of repaired components. Pareto points for (b): availability contract: epsilon = 0.5, maintenance cost = 153 (262), availability =

4,249 (5,59) for the lower limit on the stock of repaired components being 0 (1); due-date contract: epsilon = 200, maintenance cost = 554 (701), delay cost = 8700 (8690) for the lower limit on the stock of repaired components being 0 (1).

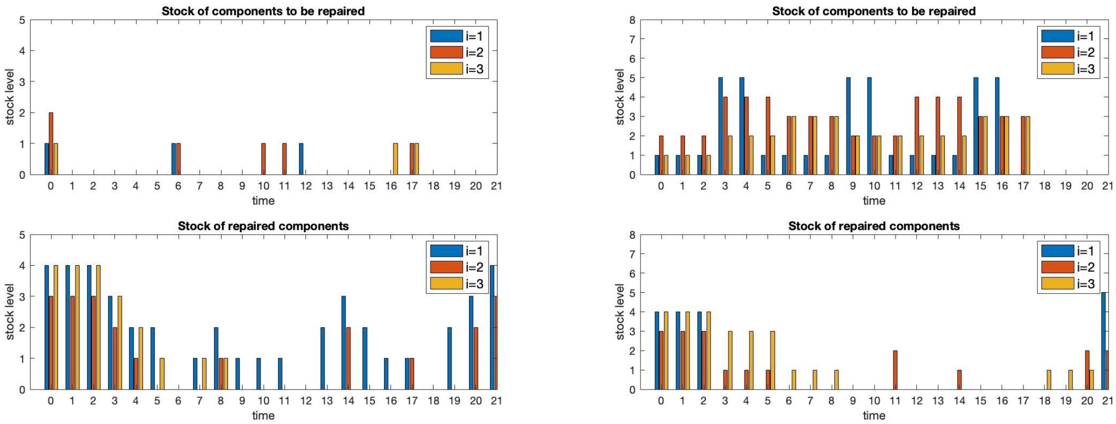
Figure 4 illustrates resulting stock levels with a demand on the stock of repaired components of one component of each type at every time step and the workshop capacity L=7. We can observe from Figure 4(a) that an availability type of contract, in combination with a lower limit of one on the stock of repaired components, forces the workshop to immediately repair components that are taken out of an aircraft in order to fulfil the demand for components. It is visible also in Figure 3(b) that the workshop operates at a high loading in this case. One can observe that having a due-date type of contract, for this instance size, allows much higher levels of damaged components waiting to go to the workshop for repair. By relaxing the requirement on the stock of repaired components from one to zero while having the same workshop capacity, overall fewer components occur in the stocks (see Figure 5). Since the processing times in the workshop, as well as the transport times to and from the workshop, are deterministic, we can conclude that the components spend more time being used by aircraft than in one of the stocks or the workshop. On the other hand, by not employing the positive lower limit on the availability in the stock of repaired components, there is a risk of not being able to fulfil the operational demand and the workshop could be forced to work at a higher capacity. That would require more parallel lines in the workshop, longer shifts and/or more personnel, which would come with certain costs. Investigations regarding these relations are left as further research.



(a) Availability contract

(b) Due-date contract

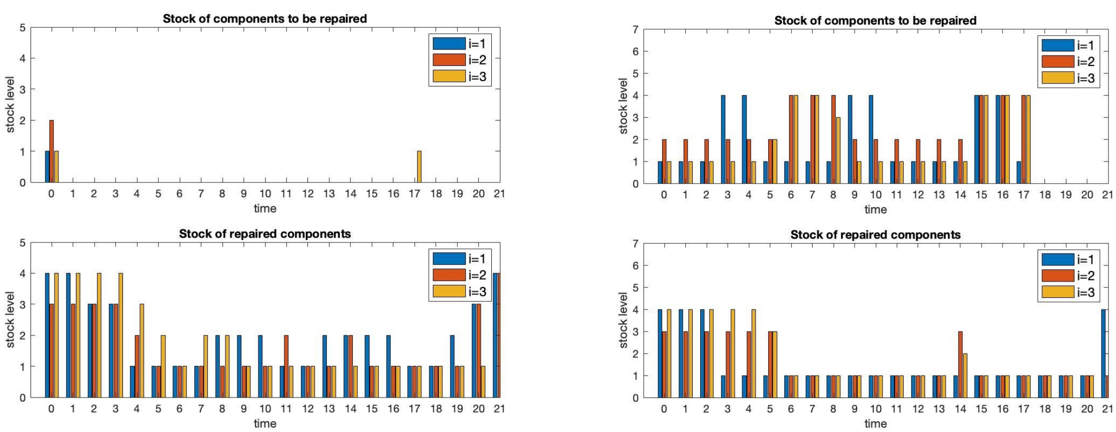
Figure 4: Resulting stock levels with a demand on the stock of repaired components of one component of each type at every time step and the workshop capacity L=7.



(a) Availability contract (b) Due-date contract

Figure 5: Resulting stock levels with a zero lower limit on the stock of repaired components for each type at every time step and the workshop capacity L=7.

Similar conclusions can be drawn when increasing the workshop capacity from L=7 to L=10 parallel lines at a time (see Figure 6). The stock levels are then slightly different but the workshop has more freedom to fulfil the demand, while the costs for lateness get—as expected—lower.

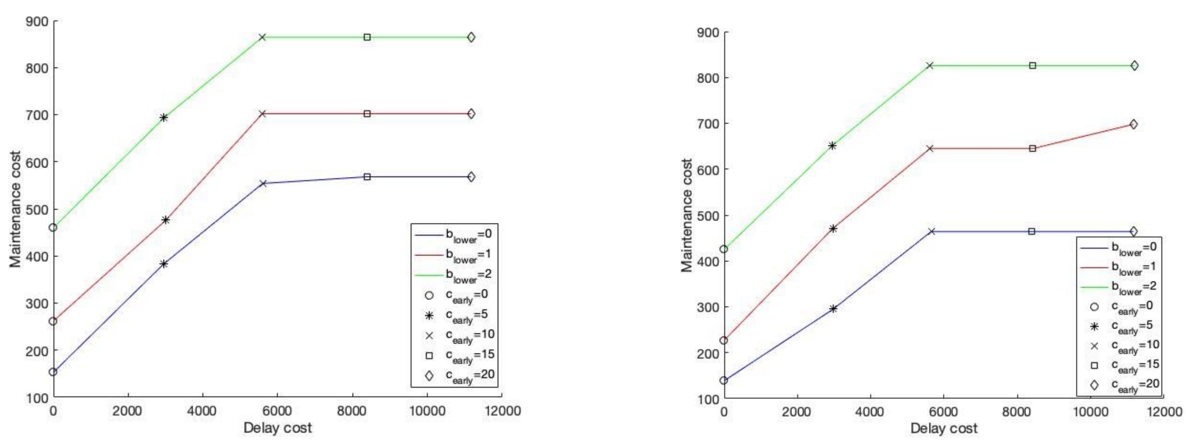


(a) Availability contract (b) Due-date contract

Figure 6: Resulting stock levels with a demand on the stock of repaired components of one component of each type at every time step and the workshop capacity L=10.

In order to investigate the interplay between the costs for aircraft maintenance (denoted by c) and the costs (i.e., fines) for late/early delivery of repaired components (denoted by c_{delay} and c_{early}, respectively), we performed a test where the sum of these costs are minimized. In this test, the level of the costs (fines) for late/early delivery is varied while the aircraft maintenance costs are kept constant. Figure 7(a) illustrates optimal values of the total costs (over the planning period) for aircraft maintenance and for delivery delay/earliness when the corresponding cost (fines) level is varied, and for different lower limits on the stock of repaired components (denoted by $b_{\underline{lower}}$), while the number of repair lines in the workshop is kept constant, i.e., L=10. The variation of the fines parameter c_{early} will then assign different importance to the two stakeholder's objectives. When c_{early}= 0, the turn-around-time contract assigns no fines for late/early delivery, meaning that only the aircraft maintenance costs and the lower limit on the stock of repaired components are taken into account in their combined production schedule. In fact, this corresponds to the availability contracting form of regulating the lower limit of repaired items. Thus, for the cases with the lower limit on the stock of repaired components being 0, 1, and 2, the aircraft operator can plan its maintenance optimally at a maintenance cost of 153, 262, and 460, respectively. These numbers correspond (approximately) to the costs of those solutions illustrated in Figures 2(a) and 2(b) having the lowest value of the aircraft maintenance cost (for lower limits 0 and 1, respectively). When c_{early} increases from 0, lateness/earliness is being penalized linearly and an optimal aircraft maintenance schedule is obtained at a higher maintenance cost. This appearance of the graph indicates that the increased penalty cost factor c_{early} invokes a shifted focus towards delivery on time, rather than the original aircraft operator objective—to perform aircraft maintenance at the lowest possible cost. An increased lower limit on the stock of repaired components (from 0 to 1 and 2), seems to increase the maintenance costs linearly (for most levels of the cost, c_{early}). In this problem set-up an availability contract with a lower limit of repaired components on the stock is the most favourable contracting form for both stakeholders.

In Figure 7(b), the cost structure in the aircraft maintenance scheduling model differs between the component types, while in Figure 7(a) it is equivalent for all three component types. The results are, however, similar for the two cases.



(a) Equal maintenance intervals (b) Unequal maintenance intervals

Figure 7: Optimal values of the aircraft maintenance and component delivery delay/earliness costs. The cost level for early delivery (denoted by c^i_{early}) is varied between 0, 5, 10, 15, and 20, for all values of *i* (cost for late delivery $c^i_{\text{delay}}= 2 \cdot c^i_{\text{early}}$), for the lower limits (denoted by b_{lower}; $\underline{b}$ in the *App*; (4c)) on the stock of repaired components being 0, 1, and 2, respectively. The number of repair lines in the workshop L=10. The maintenance intervals defining the cost structure in the

aircraft maintenance scheduling model are equal (a) and unequal (b), respectively, between the component types.

4. CONCLUSIONS AND FUTURE RESEARCH

We present a mathematical model for the integration of preventive maintenance scheduling for the components in an aircraft system with the scheduling of the maintenance workshop, through the modeling of stock dynamics. We consider two stakeholders and two types of contracts between them, which lead to two bi-objective optimization problems. Our preliminary results indicate that different types of contracts will—through the optimization of the corresponding objectives—advocate different planning patterns. A related and important aspect is the resulting implication on the collaboration between different stakeholders, which is highly applicable to general maintenance and supply chain problems and well suited for our modelling framework.

We intend to further investigate the type of availability contract which is defined by the lower limits on the number of items on the stock of repaired components, and such that there is a fine charged every time step when these requirements are not fulfilled; this type of contract can then be complemented with a reward assigned to the average number of items on the stock over time.

Since we are dealing with a high computational complexity of the integrated problem, one of our future research questions is to investigate how to reduce computing times. For this purpose, we intend to investigate a controlled pre-optimal termination of the solver, polyhedral properties of the combinatorial optimization problems that our model consist of, and mathematical decomposition approaches. A possible extension, which will further increase the computational complexity, is to include the scheduling of the aircraft operations' systems, which is currently considered as input data to our model.

When an unexpected failure occurs, our current model can be used to reschedule the maintenance and repair workshop from that point in time, however, including a modelling of the mechanisms that drive the need for corrective maintenance (CM) would improve the utility that can be gained from our modelling.

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REFERENCES

Almgren, T., Andréasson, N., Patriksson, M., Strömberg, A.-B., Wojciechowski, A., and Önnheim, M. (2012) The opportunistic replacement problem: Theoretical analyses and numerical tests. *Mathematical Methods of Operations Research* 76(3):289–319

Arkin, E., Joneja, D., and Roundy, R. (1989) Computational complexity of uncapacitated multi-echelon production planning problems. *Operations Research Letters* 8(2):61–66

Boctor, F., Laporte, G., and Renaud, J. (2004) Models and algorithms for the dynamic demand joint replenishment problem. *International Journal of Production Research* 42(13):2667–2678

Brucker, P. and Knust, S. (2012) *Complex Scheduling*, 2nd edn. GOR-Publications, Springer-Verlag, Berlin

Conforti M., Cornuéjols G., Zambelli G. (2014) *Integer programming*, Springer, Switzerland

De Bruecker, P., Van den Bergh, J., Beliën, J. and Demeulemeester, E. (2015): A model enhancement heuristic for building robust aircraft maintenance personnel rosters with stochastic constraints, *European Journal of Operational Research* 246:661–673

Dunning, I., Huchette, J. and Lubin, M. (2017) JuMP: A Modeling Language for Mathematical Optimization. *SIAM Review* 52: 295–320

Ehrgott M (2005) *Multicriteria Optimization*, 2nd edn. Springer, Berlin

Ekström, T., Dorn, M. and Skoglund, P. (2015): Swedish defence acquisition transformation—A research agenda, *Proc. of the 12th Annual Acquisition Research Symposium*, Graduate School of Business & Public Policy, Naval Postgraduate School, Washington, USA

Erkoc, M. and Ertogral, K. (2016): Overhaul planning and exchange scheduling for maintenance services with rotable inventory and limited processing capacity, *Computers & Industrial Engineering* 98:30–39

Gavranis, A. and Kozanidis, G. (2015): An exact solution algorithm for maximizing the fleet availability of a unit of aircraft subject to flight and maintenance requirements, *European Journal of Operational Research* 242:631–643

Gurobi Optimization, LLC (2020) Gurobi Optimizer Reference Manual. http://www.gurobi.com

Gustavsson, E., Patriksson, M., Strömberg, A.-B., Wojciechowski, A., and Önnheim, M. (2014) The preventive maintenance scheduling problem with interval costs. *Computers & Industrial Engineering* 76:390–400

Julia: A fast dynamic language for technical computing (2012) version 1.5, https://docs.julialang.org/en/v1/

Kurz, J. (2016) Capacity planning for a maintenance service provider with advanced information, *European Journal of Operational Research* 251:466–477

MATLAB (2018) version 9.5. MathWorks https://se.mathworks.com/products/matlab.html

Mavrotas, G. (2009) Effective implementation of the ε-constraint method in multi-objective mathematical programming problems. *Applied Mathematics and Computation* 213:455–465

Olde Keizer, M.C.A., Teunter, R.H. and Veldman, J. (2016) Clustering condition-based maintenance for systems with redundancy and economic dependencies. *European Journal of Operational Research* 251:531–540

APPENDIX: The full mathematical model[1,2]

The feasible set for our integrated model is defined by the following sets of constraints.

Constraints modelling the aircraft maintenance scheduling:

$$\sum_{j\in\mathcal{J}_i}\sum_{s=0}^{t-1} x_{st}^{ijk} = \sum_{j\in\mathcal{J}_i}\sum_{r=t+1}^{T+1} x_{tr}^{ijk}, \qquad i\in\mathcal{I},\, t\in\mathcal{T},\, k\in\mathcal{K}, \tag{1a}$$

$$\sum_{j\in\mathcal{J}_i}\sum_{r=1}^{T+1} x_{0r}^{ijk} = 1, \qquad i\in\mathcal{I},\, k\in\mathcal{K}, \tag{1b}$$

$$\sum_{j\in\mathcal{J}_i}\sum_{s=0}^{t-1} x_{st}^{ijk} \le z_t^k, \qquad i\in\mathcal{I},\, t\in\mathcal{T},\, k\in\mathcal{K}, \tag{1c}$$

$$\sum_{k\in\mathcal{K}}\sum_{s=0}^{t-1} x_{st}^{ijk} \le 1, \qquad j\in\mathcal{J}_i,\, i\in\mathcal{I},\, t\in\mathcal{T}, \tag{1d}$$

Constraints modelling the capacity of the repair lines in the maintenance workshop:

$$\ell_t = \ell_{t-1} + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}_i}\left(u_t^{ij} - u_{t-p_i}^{ij}\right) \le L, \qquad t\in\{2,\dots,T\}, \tag{2}$$

Constraints modelling the dynamics of the stocks of damaged (3) and repaired (4) components:

$$\alpha_t^{ij} = \sum_{k\in\mathcal{K}}\sum_{s=0}^{t-1} x_{st}^{ijk}, \qquad j\in\mathcal{J}_i,\, i\in\mathcal{I},\, t\in\mathcal{T}, \tag{3a}$$

$$a_t^{ij} = a_{t-1}^{ij} + \alpha_t^{ij} - u_{t+\delta_a^i}^{ij} \ge 0, \quad t\in\{1-\delta_a^i,\dots,T+1\},\, j\in\mathcal{J}_i,\, i\in\mathcal{I}, \tag{3b}$$

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[1] The constraints (7a)–(7c) are only relevant when minimizing the sum of the penalty costs for late (and early) deliveries of the repaired components.

[2] If all variables z and u possess binary values, the binary requirements on the variables x, a, b, α, and β can be relaxed to values in the interval [0,1], since any corresponding mixed-integer linear optimization problem will possess binary optimal solutions.

$$\beta_t^{ij} = \sum_{k\in\mathcal{K}} \sum_{r=t+1}^{T+1} x_{tr}^{ijk}, \qquad j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T} \qquad (4a)$$

$$b_t^{ij} = b_{t-1}^{ij} - \beta_t^{ij} + u_{t-\delta_b^i-p^i}^{ij}, \qquad j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T} \qquad (4b)$$

$$\sum_{j\in\mathcal{J}_i} b_t^{ij} \geq \underline{b}^i, \qquad i \in \mathcal{I}, t \in \mathcal{T}, \qquad (4c)$$

Constraints modelling the restrictions on maintenance from the operational schedule (5) and initializations for the component individuals (6):

$$z_t^k \leq \overline{z}_t^k, \qquad i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}. \qquad (5)$$

$$\sum_{k\in\mathcal{K}} \sum_{r=1}^{T+1} x_{0r}^{ijk} + a_0^{ij} + b_0^{ij} + \sum_{t=1-p^i-\delta_b^i}^{0} u_t^{ij} = 1, \qquad j \in \mathcal{J}_i, i \in \mathcal{I}, \qquad (6)$$

Constraints for modelling the penalty costs for late and early delivery of the repaired components (7):

$$v_{\text{tat}}^{ij} = (p^i + \delta_b^i)\, u_0^{ij} + \sum_{t=1}^{T+1} \left(\left(t + p^i + \delta_b^i\right) u_t^{ij} - t\alpha_t^{ij} \right), \quad j \in \mathcal{J}_i, i \in \mathcal{I}, \quad (7a)$$

$$v_{\text{early}}^{ij} \leq v_{\text{tat}}^{ij} - d_{\text{due}}^{ij} \left(a_0^{ij} + \sum_{t=1}^{T+1} \alpha_t^{ij} \right) \leq v_{\text{delay}}^{ij}, \qquad j \in \mathcal{J}_i, i \in \mathcal{I}, \quad (7b)$$

$$v_{\text{early}}^{ij} \leq 0 \leq v_{\text{delay}}^{ij}, \qquad j \in \mathcal{J}_i, i \in \mathcal{I}, \quad (7c)$$

Constraints on the variables required to take binary values (8) and non-negative values[3] (9):

$$z_t^k, u_t^{ij} \in \{0,1\}, \quad j \in \mathcal{J}_i, i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T} \qquad (8)$$

$$x_{st}^{ijk}, a_t^{ij}, b_t^{ij}, b_t^i, \ell_t, \beta_t^{ij}, \alpha_t^{ij} \geq 0, \qquad j \in \mathcal{J}_i, i \in \mathcal{I}, k \in \mathcal{K}, 0 \leq s < t+1 \quad (9)$$

The optimization objectives considered are defined as follows, where (10) models the minimization of aircraft maintenance costs, (11) models the maximization of the average component availability, and (12) models the minimization of the penalty costs for early and late delivery of repaired components:

[3] If all of the variables z and u possess binary values, the binary requirements on the variables x, a, b, α, and β can be relaxed to values in the interval [0,1], since any corresponding mixed-integer linear optimization problem will possess binary optimal solutions (Conforti et al., 2014, Ch. 4.1).

$$\text{minimize} \quad \sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} d_t z_t^k + \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}_i}\sum_{t=1}^{T+1}\sum_{s=0}^{t-1} c_{st}^{ik} x_{st}^{ijk}, \tag{10}$$

$$\text{maximize} \quad \frac{1}{T}\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}_i} b_t^{ij}, \tag{11}$$

$$\text{minimize} \quad \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}_i} \left(c_{\text{delay}}^{ij} v_{\text{delay}}^{ij} - c_{\text{early}}^{ij} v_{\text{early}}^{ij}\right). \tag{12}$$