

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

# Strain rate dependent material model for polymer composites

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Gothenburg, Sweden 2020

Strain rate dependent material model for polymer composites  
VIVEKENDRA SINGH

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Cover: Rate dependent response in Abaqus/Explicit.

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## Abstract

We propose a micromechanical model that is able to predict the nonlinear behaviour and failure of unidirectional fibre reinforced polymer composites subjected to dynamic loading conditions. This novel material model is heterogeneous on the micro level and homogeneous on the ply level.

The fibres are assumed to be hyperelastic transversely isotropic and the matrix obeys a hypoelastic viscoelastic/plastic constitutive model enhanced by a continuum damage model. To model the matrix, a Zener rheological model for the viscoelastic behaviour combined with a Bingham model for the viscoplastic behaviour is assumed.

The proposed model is formulated in a framework that separates the fibre and the matrix contributions. Typical applications are unidirectional composites manufactured, for example, from unidirectional fibres embedded in a polymer matrix. Generally, the quasi-brittle compressive failure behavior of composites happens during fairly large strains in the matrix. Therefore, a geometrically nonlinear description has been developed.

Finally, using this model, we characterize the shear induced post-failure behaviour in compression of the composite material. Finite element simulations are conducted to predict the rate dependent properties of unidirectional polymer composites. The predictions of the finite element simulations are compared to published experimental results of an IM7/8552 material system under compression loading at different strain rates. The results are in a reasonably good agreement with the experiments.

**Keywords:** strain rate, unidirectional composites, continuum damage, finite element, viscoelasticity, viscoplasticity.



## Acknowledgements

This research would not have been feasible without the great support of many people.

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Vivekendra Singh, Gothenburg, December 2020



# List of publications

This thesis is based on the following appended papers:

**Paper A.** Larsson, R., Singh, V., Olsson, R., Marklund, E., 2020. A micromechanically based model for strain rate effects in unidirectional composites. *Mechanics of Materials* 148, 193–212.

**Paper B.** Larsson, R., Singh, V., Olsson, R., Marklund, E., 2020. A micromechanically based model for dynamic damage evolution in unidirectional composites. (Manuscript: To be submitted)

The two appended papers were prepared in collaboration with the co-authors.

In **Paper A** and **Paper B**: Larsson conceptualised and developed the methodology for the model. He contributed in software development and writing the original draft. Singh contributed in software development and data curation. He also validated the model, wrote the original draft and coordinated the contributions from the other authors. Olsson and Marklund gave new ideas and direction from the composites point of view throughout the model development, and assisted in the paper writing.





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## Contents

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**Part I**  
**Extended Summary**



# 1

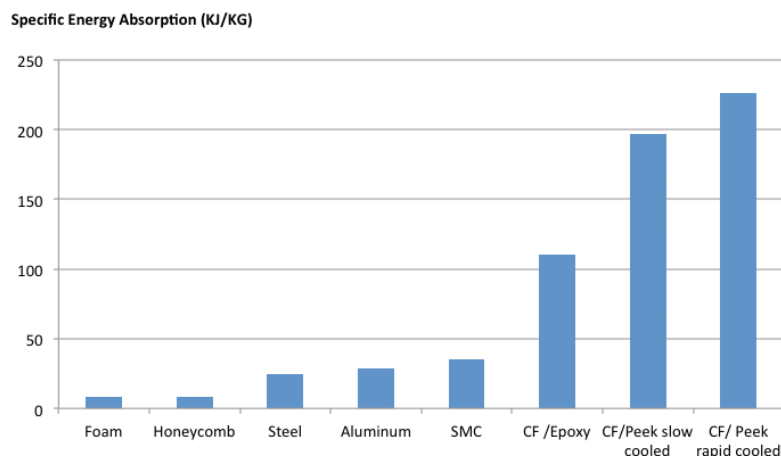
## Introduction

### 1.1 Motivation and Background

The growing demand for lighter structures has resulted in increasing replacement of metal parts by carbon fibre-reinforced polymers (CFRPs). They are increasingly considered for high technology applications in industries such as automotive, construction, marine, and aerospace.

They are capable of achieving high strength because the fibre reinforcement contributes with high strength and stiffness to the polymer composites while the polymer matrix is responsible for low weight and corrosion resistance. The matrix is an isotropic homogeneous material in which the fibre system of a composite is embedded.

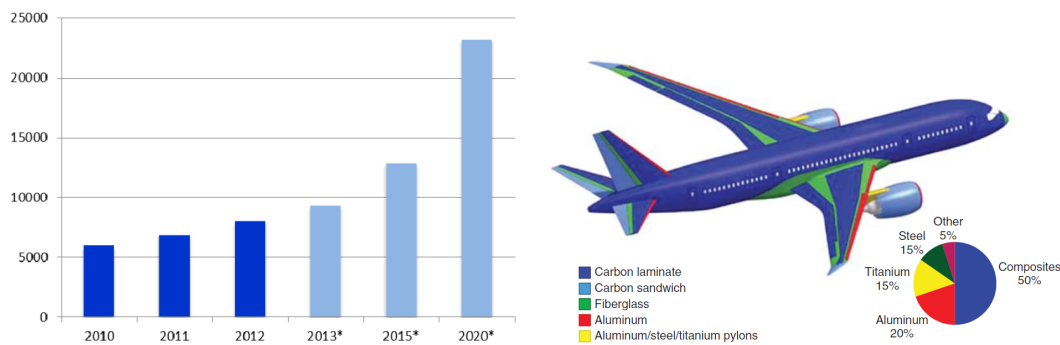
Fibre reinforced composites also have a high specific energy absorption compared to conventional metals such as steel and aluminum as shown in Figure 1.1. One of the critical requirements for the transport industry is the crashworthiness of a vehicle, which describes its ability to absorb kinetic energy in the event of a crash in a controlled manner such that the vehicle will decelerate at a rate that limits the load on the occupants [1]. In applications where weight is an important consideration, it is more appropriate to make comparisons on the basis of specific properties (i.e. per unit weight) of the materials rather than the absolute values. High specific stiffness and high specific strength of CFRPs has been an important consideration in aerospace applications [2].



**Figure 1.1:** Comparison of materials based on their specific energy absorption [1].

In the aeronautic industry, composite materials are expected to increase efficiency without compromising safety. For reasons of lack of confidence for safe design, limited technology readiness and the comparatively high cost for the material and manufacturing, in the past, the application of composites remained mainly limited to military applications where extensive testing programs could be undertaken.

Initially, composites were used in military applications in non-critical secondary structures such as access doors and flaps. With experience, composites were incorporated into primary (load-bearing and flight-critical) structures and complex-shaped components such as wing skins, stabilizer skins etc.



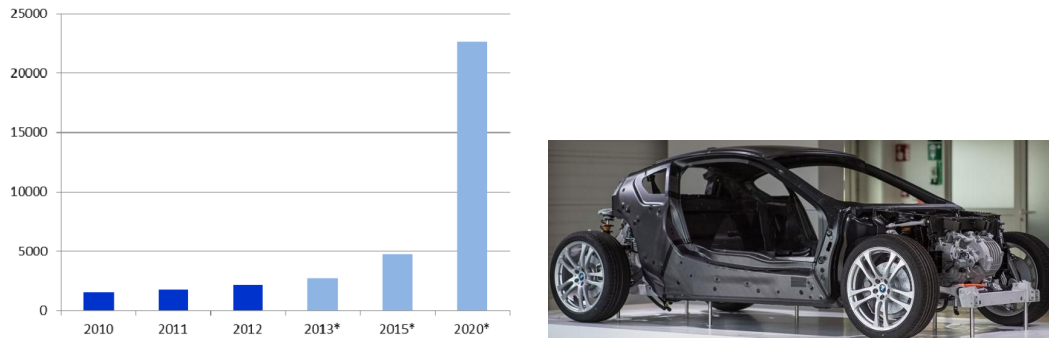
**Figure 1.2:** Carbon fibre consumption in tonnes in the market segment Aerospace Defense(\* estimated) [3] and Boeing 787 Dreamliner structural material distribution [2].

The reduction in material prices within the last decades together with the improved understanding of the material behaviour and rising awareness for environmentally friendly technology have allowed for composites to be employed in civil aerospace applications. This can be easily seen from Figure 1.2. Another motivation could be from the fact that composite materials are more durable than aluminum and, therefore, may require lower maintenance and fewer inspections. A good example of substitution of metallic components by composite or hybrid materials is the Boeing 787 Dreamliner jet [2].

Another cost driven sector is automotive and transportation. Historically, race cars and high-performance street vehicles took advantage of the light weight of composites through their use in components such as chassis, hoods, wheels, and roofs. They are used in high-end sports cars which can afford the high material and labor costs.

However, rising fuel prices, lower costs for the material and comparatively less time in manufacturing have triggered the request for fuel-efficient vehicles. The requirement of high-volume production for automotive applications is fulfilled by reasonably efficient resin transfer molding (RTM) and compression molding processes. This opened way for the composites to penetrate into the automotive industry to reduce vehicle weight.

Composites have been widely used to develop lightweight automotive components because they have many advantages, such as a high specific strength/stiffness and



**Figure 1.3:** Carbon fibre consumption in tonnes in the market segment Automotive (\* estimated) [3] and Life module passenger cell made from carbon-fibre-reinforced plastic [4].

resistance to fatigue and corrosion, when compared to conventional metallic materials. In addition to weight reduction, composites also offer other advantages such as the ability to make complex shapes with integrated parts, innovative styling, and lower corrosion. Composites are also being increasingly used in heavy trucks and mass transit buses.

Currently, it is expected that CFRPs will be increasingly applied in automobiles, because of their high specific energy absorption in compressive fracture as shown in Figure 1.1. The composites make transportation more energy efficient and, therefore, their use in the automotive and mass transportation industry has been steadily growing. One such example is the cockpit of BMW i8 made out of CFRPs to improve passenger safety in the high-end car [4].

CFRPs used in the above applications are also frequently subjected to dynamic loads such as automotive crashes or bird strike on aeroplanes. Particularly for application of CFRPs to automobiles, it is imperative to predict the dynamic response under dynamic loading. Under such situations, one may expect the behaviour of CFRPs composites to be different from that of quasi-static loads.

Therefore, to increase the use of composites in aviation and automotive applications requires reliable and cost efficient design tools. Realistic rate dependent constitutive models for numerical analysis tools like Finite Elements (FE), for instance, can enhance performance and safety by more sophisticated structural design and at the same time decrease the development costs and time, as experimental design verification can be significantly reduced.





# 2

## Strain rate dependence

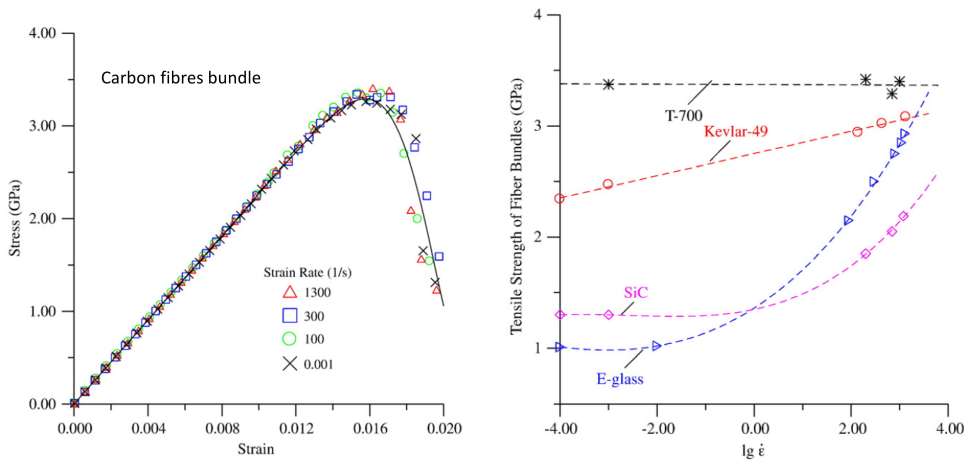
In many structural applications composite materials are exposed to high energy, high velocity dynamic loadings, particularly use of composites in high speed aircraft, missiles, automotive and other transportation industries. Therefore, it is important to understand the dynamic behaviour of CFRPs and to develop numerical models which can capture the dynamic response in high technology applications.

Micromechanics can be used to estimate the dynamic mechanical behaviour of a composite by considering the influence of each constituent. In order to do that, it is important to first understand the strain rate effects on the CFRPs and their individual constituents, i.e. the fibre and the matrix.

Strain rate studies found in the literature are based on different types of composite material systems such as unidirectional (UD), woven fabrics, metal-matrix composites and many more. In this thesis work we have considered only UD composites with an emphasis on carbon/epoxy or glass/epoxy systems.

### 2.1 Rate dependence of fibres

Only limited experimental data exist regarding the strain rate effect on the mechanical properties of fibres or fibre bundles. In general glass and polymer fibres are clearly rate sensitive, while carbon fibres are not.



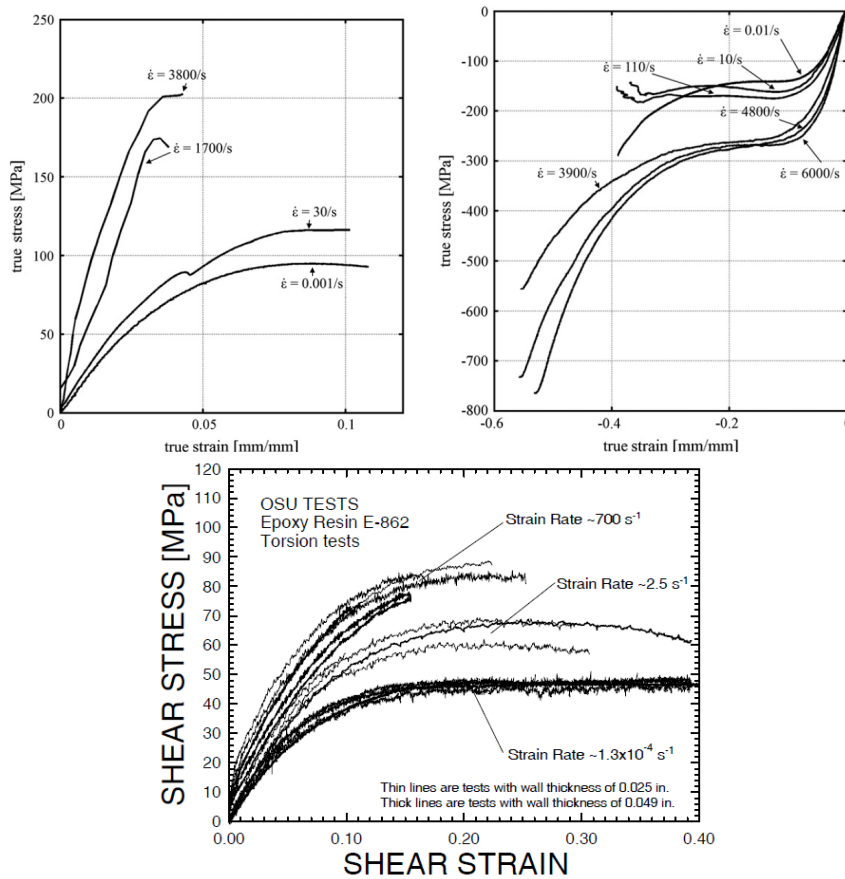
**Figure 2.1:** Comparison of mechanical properties of fibres at different strain rates [8]

Xia et al. [6] reported a significant strain rate effect on the modulus, strength and failure strain of glass fibre bundles as shown in Figure 2.1. In a follow up study Wang and Xia [7] investigated rate dependent behaviour of the Kevlar® aramid fibre bundles and found that they are sensitive to strain rate, but the degree of rate sensitivity is not as pronounced as found for glass fibre bundles.

Experimental studies are also performed by Zhou et al. [8] on the tensile behaviour of T700 carbon fibre bundles and reported that strain rate has no effect on the mechanical properties of carbon fibres ( Figure 2.1).

## 2.2 Rate dependence of polymers

Polymers in tension show an increase in stiffness modulus and strength, while failure strain decreases with strain rate in the resin [9, 10, 11].



**Figure 2.2:** Strain rate dependence of RTM-6 epoxy resin in tension and compression [11]. Rate dependent shear behaviour of E-862 [10].

Gilat et al. [9] investigated the rate dependent tensile behaviour of an epoxy resin using a conventional machine for quasi-static loading and a tensile Split Hopkinson Bar (SHB) for a dynamic testing. A transition from ductile to brittle response was observed when progressing from low strain rate to high strain rate. In a recent study, Gerlach et al. [11] reported an increase in failure strength and stiffness modulus and

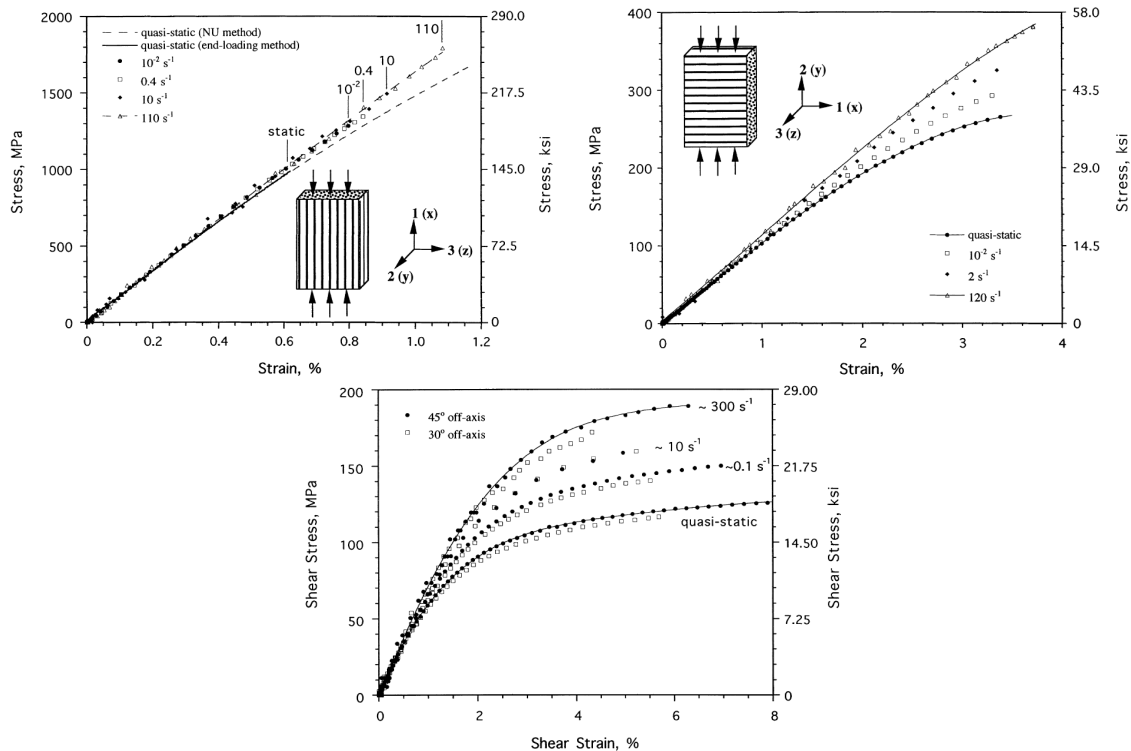
a decrease in failure strain at different strain rates. The experiments were performed at strain rates from  $10^{-3}$  to  $10^4$  /s using a tension SHB (see Figure 2.2).

In compression, an increasing strength and modulus have been reported by authors, e. g. in [11, 13], while no effect was reported in [12, 14]. A conventional testing machine for a quasi-static and a compressive SHB for a dynamic testing was used by Gerlach et al. [11] to perform a mechanical testing on a thermosetting resin. A significant increase of yield stress and compressive modulus was reported at high strain rates (Figure 2.2).

Gilat et al. [10] studied the shear response of an epoxy system at different strain rates and reported that shear modulus and strength increases with increasing strain rate. A ductile material behaviour in the stress-strain response was observed for all strain rates as shown in Figure 2.2.

### 2.3 Rate dependence of UD composite

For CFRPs, in tension along the fibre direction, no effect in stiffness modulus, strength and failure strain was reported by authors in [16, 17, 18]. However, in the transverse direction and other off-axis, an increase in modulus and strength and no effect on the failure strain was reported by authors [17, 18] using different test machines.



**Figure 2.3:** Measured quasi-static and dynamic response of unidirectional carbon/epoxy composite in longitudinal and transverse compression [19] and shear [23]

Under longitudinal compression, a slight increase in the modulus and a significant

increase in the strength was reported in [19, 20, 22]. An increase in failure strain was also reported in [19, 22]. Hsiao and Daniel [19] investigated the dynamic compressive behaviour of an UD carbon/epoxy composite (Figure 2.3). They performed by dynamic tests in a drop tower up to 110 /s.

In transverse compression, an increase in transverse modulus and strength was reported in [20, 23, 24]. A decreasing failure strain has been reported by authors in [20, 21]. Recently, Koerber et al. [24] performed a quasi-static and dynamic tests on an UD carbon/epoxy composite in a transverse compression. Quasi-static tests were carried out in a standard test machine and dynamic tests using a compressive SHB at strain rates between 90 and 350 /s.

Authors, e.g [17, 18, 24, 23], reported an increase in shear modulus and strength and a decreasing failure strain under in-plane shear loading. Hsiao et al. [23] tested unidirectional carbon/epoxy composite laminates under dynamic in-plane shear loading at an axial strain rate of 300 /s. Koerber et al. [24] tested quasi-static and dynamic loadings under in-plane shear loading and found an increase in the modulus and yield strength.

A more through review of rate dependent behaviour of UD carbon/epoxy and other material systems such as glass/epoxy is available in technical report by Singh [5].

# 3

## Rate dependent models for UD composites

A variety of methods have been applied to model the rate dependent response of polymer matrix composites. The models have been developed at both the macromechanical (ply) level and the micromechanical (constituent) level.

### 3.1 Macromechanical approaches

In the macromechanical approach at ply level, the composite material is modeled as an aniso-tropic, homogeneous material, without any attention being paid to the individual constituents. For example, Weeks and Sun [25] developed a macromechanical, rate dependent constitutive model based on a one-parameter plasticity model and a modified Johnson rate dependent model over a variety of strain rates.

The work was followed by Yoon and Sun [26] who proposed a one-parameter overstress viscoplastic model for unidirectional off-axis specimens subjected to different strain rates. Gates et al. [27] proposed a constitutive model to describe the elastic/viscoplastic behaviour of composites under plane stress conditions. This model was formulated for a quasistatic plasticity and time dependent viscoplasticity.

Thiruppukuzhi and Sun [28] later implemented the two parameter overstress viscoplasticity model into the constitutive model to simulate the nonlinear rate dependent behaviour of unidirectional and the woven composites at different strain rates.

### 3.2 Micromechanical approaches

Research has also been conducted in simulating the high strain rate deformation response of polymer matrix composites through micromechanics approaches. In micromechanics, the effective properties and response of the composite are computed based on the properties and response of the individual constituents. Several types of methodologies have been used in micromechanics analyses.

The simplest types of micromechanics techniques developed have been mechanics of materials based methods, in which various uniform stress and uniform strain assumptions were utilized within the composite unit cell to compute the effective properties and response of the material. Examples of this type of approach include

the traditional Voigt [30] and Reuss [31] based “rule of mixtures” equations.

These models incorporate limited use of microstructural information like fibre volume fraction and gives direct formulation for the effective mechanical properties which can later be used to form the stiffness matrix of the composite. While this approach involved a great deal of approximation and simplification, the resulting equations were very simple in form, very easy to implement within a computer code, and very computationally efficient.

A more sophisticated method to compute the effective properties of composite materials involved using continuum mechanics techniques. In this type of approach, the equations of continuum mechanics were solved in an average sense within the unit cell. Examples of this methodology include the Concentric Cylinders Model [32], the Self Consistent Method [32] and the Mori-Tanaka Method [33]. These scheme based models give direct estimation of the stiffness matrix of the composite by a continuum mechanics approach.

Continuum mechanics methods more completely satisfy the field equations of mechanics, resulting in a more accurate representation of the physics of the problem, in comparison to mechanics of materials techniques.

The most accurate micromechanics techniques have been the numerically based methods. In this approach, the fibre and matrix are explicitly modeled using either finite elements or boundary elements. The effective response of the unit cell is then computed by conducting a finite element or boundary element analysis. Examples of this approach can be found in [34, 35]. This type of analysis yields the greatest accuracy, but the execution times required to conduct the analysis on a computer is often quite substantial.

More recently, continuum mechanics methods have been used to simulate the dynamic response of polymer matrix composites. For example, Wang et al. [37] developed a finite strain elastic-viscoplastic self-consistent model for polycrystalline materials. Schapery [38] proposed a nonlinear viscoelastic/plastic model and considered a Concentric Cylinder Assembly model to predict the viscoplastic behaviour of a glass fibre composite. The matrix was assumed as a viscoelastic material.

Goldberg et al. [39] developed a nonlinear, strain rate dependent deformation and strength model for the analysis of polymer matrix composites. The constitutive equations were then implemented into a mechanics of materials based micromechanics technique to predict the dynamic response of the composite. More recently, a constitutive model which takes into account the viscous effects in the mechanical behaviour of a unidirectional carbon-epoxy system using a fully 3D viscoelastic-viscoplastic material model at the ply scale was developed by Gerbaud et al. [40].

It is only very recent that physically based three dimensional failure theories started to emerge. These novel theories aim at representation of the failure mechanisms and enable a more realistic prediction of the various composite failure modes.

Gutkin et al. [41] presented a model accounting for damage growth during fibre kinking in the UD composites. Camanho et al. [42] proposed a smeared crack model to predict the ply failure. Costa et al. [43, 44] developed a physically based

model and Larsson et al. [46] proposed a set of CDM models for fibre kinking under compression in UD composites.

Modelling composites for dynamic loading requires constitutive models which incorporate the strain rate dependent behaviour for a realistic prediction of the stresses induced due to dynamic loading and coupled with a damage model for prediction of strength and subsequent damage evolution.

Tabiei et al. [47] developed a micromechanical model where nonlinear strain rate and pressure dependency is accounted by the matrix model and a CDM based failure model to incorporate the progressive post-failure behaviour. Another micromechanical model was proposed by Nguyen et al. [48] to predict nonlinear behaviour of UD composites. The fibres are assumed to be transversely hyperelastic isotropic and the matrix obeys a hyperelastic viscoelastic/plastic constitutive model enhanced by a multi-mechanism nonlocal damage model.

Eskandari et al. [51] developed a mesoscale model by considering a viscoelastic/plastic model coupled with CDM for composites at high strain rates. Recently, Tan and Liu [53] developed a micromechanical model to capture the matrix shearing and fibre rotation of CFRPs at different strain rates. The carbon fibre composite is homogenised, based on various inelastic slip systems identified from the fibre architecture.





# 4

## A material model for a UD composite

### 4.1 Objectives

The research presented in this thesis was mainly inspired by the desire to introduce composite materials in aerospace and automotive structures exposed to dynamic loading, e.g. impact and crash. The objective of the research is to enhance the available model capabilities for unidirectional CFRPs subjected to dynamic loading.

The proposed constitutive model is developed for explicit FE analysis, which has been demonstrated to be a powerful tool for the simulation of dynamic events. The key for successfully modelling dynamic events is a strain rate dependent constitutive model which considers three-dimensional stress states for strength prediction and subsequent damage evolution. Reaching this objective requires the following tasks to be addressed within the thesis:

- *incorporation of strain rate dependent material behaviour into the constitutive equations.*
- *development of damage evolution algorithms which predict the propagation of damage and the resulting energy dissipation.*
- *calibration of the proposed model with conducted experiments for model parameters.*
- *implementation/validation of the proposed constitutive model into an explicit FE environment.*

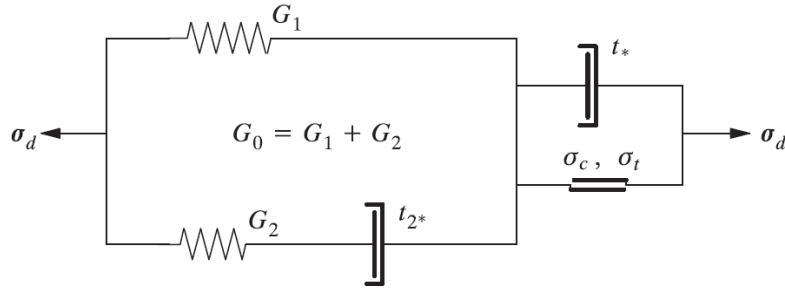
The current state of research requires a novel contribution to each of the tasks stated above. The focus of the constitutive model development lies in the implementation of a rate dependent model and to couple it with continuum damage for the prediction of the initiation of damage and the subsequent damage evolution. A macro and micro scales are considered to develop the formulation. The tasks stated above will be explained in more detail in the coming sections.

### 4.2 Rate dependent constitutive modelling

Composite materials are heterogenous materials. The constitutive model therefore depends on the scale on which modelling is performed. In general, three main scales are commonly identified for the composites used here.

First, the microscale, where fibre, matrix and the interface between them are considered separately. Second, the mesoscale where the lamina is considered as a homogeneous material but the interface between laminas is still considered separate. Third, the macroscale, where several laminas of different orientation, including the interfaces, are considered as one homogeneous material. However, in this thesis work we have chosen a multiscale approach where the microscale models for the fibre and the matrix are combined using a homogenization technique to develop a mesoscale model for the ply.

The experimental evidence in Chapter 2 demonstrates that strain rate dependent material behaviour is very pronounced in composite materials. Therefore, the application of the proposed constitutive model to dynamic related problems requires that strain rate effects on the material properties are considered. The following section introduces how strain rate dependent material behaviour can be introduced into the constitutive equations.



**Figure 4.1:** Adapted rheological model for the viscoelastic-viscoplastic response of the polymer matrix of the composite.

The rate dependent behaviour in the composite material is caused by the viscoelastic/plastic behaviour of the matrix polymer. As stated in chapter 3, several models have already been proposed to derive the change of properties directly from the behaviour of the polymer resin in which the reinforcing fibres are embedded.

The focus of this study lies on unidirectional CFRPs. The stiffness of carbon fibres is assumed to be rate insensitive. Therefore, fibres are assumed as an elastic transversely isotropic material. The Young's and shear moduli transverse to the fibres show rate dependent behaviour as the matrix material significantly contributes to the response. The strain rate dependent stiffness behaviour of CFRPs is known to be dominated by the viscoelastic/plastic behaviour of the matrix material.

Viscoelastic material behaviour can be described by a spring damper system. Commonly used models are the Kelvin-Voigt (spring and damper in parallel), the Maxwell model (series of spring and damper) and the Zener model (Kelvin-Voigt with spring in series or Maxwell with spring in parallel). For the viscoplastic response, a Bingham and a Norton model are available in the literature. The viscoelastic/plastic effects are reflected in the constitutive model by a damper which controls the rate dependent stiffness response.

In order to enable a numerically efficient implementation, a Zener model for the

viscoelastic model and a Bingham model for the viscoplastic model have been chosen here as shown in the figure 4.1. The parameters involved are the quasi-static and dynamic shear moduli,  $G_1$  and  $G_2$ , the relaxation times  $t_{2*}$  and  $t_*$  associated with the viscoelastic and viscoplastic response and  $\sigma_t$ ,  $\sigma_c$  defining the quasi-static yield stress of the polymer matrix in tension and compression.

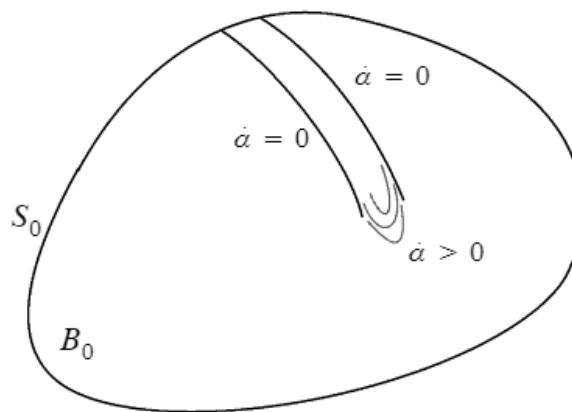
The framework, as given here, will be used in the following chapter to build a model for onset and evolution of damage.

### 4.3 Modelling damage initiation and propagation

Two main strategies are used to represent damage in composite materials, micromechanical modelling and Continuum Damage Mechanics (CDM).

Micromechanical models focus on explicit modelling of damage entities using fracture mechanics, while CDM use a homogenised approach where the local effect of damage is smeared out over the RVE. The focus of this work lies in ply modelling of composites as a continuum for which reason the micromechanical modelling is not considered.

This chapter proposes a model for damage evolution in composite materials for general three dimensional stress states in unidirectional CFRP. The heterogenous nature of composites results in a complex failure behaviour. UD composites fail due to several failure modes, some of which occur at relatively low load levels long before the maximal load is reached. The subsequent damage evolution within the composite results in complex stress redistributions which affect the global behaviour of the whole laminate.



**Figure 4.2:** A damage degrading solid in reference configuration  $B_0$  with a diffusive localized distribution of the damage field representing the separation of crack surfaces.

Predicting the dynamic response of composite materials therefore requires a realistic and a physically sound model for the damage representing all relevant failure modes at high strain rates and the subsequent damage evolution.

The damage generated due to impact loading is represented using the framework of CDM. The model describes the quasi-brittle failure process of the matrix material in compression under dynamic loading. The coupling between damage and viscoplastic formulation is obtained by reformulating the dissipation rate  $\mathcal{D} \geq 0$ . The model predicts the onset and evolution of damage ( $\dot{\alpha}$ ) using the Bingham type damage evolution law as shown in the following equation

$$l_c \dot{\alpha} = v^* \langle \alpha^s[\alpha] - \alpha \rangle \text{ with } \alpha^s = \frac{\mathcal{A}_T[\alpha] l_c}{\mathcal{G}_c} \quad (4.1)$$

where  $\alpha^s$  represents the source of damage energy, the internal length parameter  $l_c$  describes the diffusive character of the fracture area,  $v^*$  is the fracture area progression speed parameter that controls the damage evolution  $\dot{\alpha}$ ,  $\mathcal{A}_T$  is the damage driving energy and  $\mathcal{G}_c$  is the fracture energy [46].

The concept of damage degradation is similar to the 'radial return' method of plasticity. Loading the material beyond the elastic limit results in inadmissible stress states. As a consequence, the material degrades (e.g. lower stiffness and reduced strength) which is represented by increasing damage. The current inadmissible stress state in the model therefore needs to be returned to the damaged (shrunk) failure surface to represent material degradation.

## 4.4 Model calibration

The calibration of composite damage models has been widely discussed in the composite research community. Ideally, the rate dependent stiffness and damage parameters should be directly measured from experiments. The brittle nature of most composites, however, makes this a difficult task.

A recent approach for parameter identification is the use of numerical optimisation algorithms such as least square method. This approach relies on numerical optimisation techniques to obtain best guesses for parameters for a given set of experimental results and a certain constitutive model.

This approach has been used in this thesis work for predicting the viscoelastic/plastic and damage parameters for the matrix model using the experimental response when fibres are at an angle of  $45^\circ$  from the longitudinal direction and under quasi-static and dynamic loading. Explicit procedures are in Paper A-B.

Currently, the calibration is implemented within MATLAB, which then provides the required parameters. For future applications, it would be possible to implement the calibration algorithm as a preprocessing routine, thus avoiding the external calibration procedure.

## 4.5 Numerical implementation and validation

This section explains the implementation of the proposed constitutive model in the explicit finite element solver Abaqus/Explicit. The model is implemented into Abaqus/Explicit as a user defined material model (VUMAT).

For dynamic analyses, explicit dynamic FE has already proven to be a powerful tool. The non-linear problem is solved incrementally with the calculation increment (time step) being defined by the material properties and the spatial discretization. For each time step, an increment of stress is predicted for a given increment of strain. The assessment of the current state of stress (at time  $t$ ) is usually performed by an incremental stress predictor or trial stress. The type of element used is the 8-node brick, C3D8R, with reduced integration and enhanced hourglass control.

The constitutive model is validated by comparing the finite element predictions with the published experimentally measured stress-strain responses of an UD polymer composite in compression and subjected to quasi-static and dynamic loading [52].



# 5

## Conclusions

An extensive literature review of available data for rate dependent UD composite material behaviour has been carried out. The review identified the need to include strain rate effects in material models as well as the need of further characterisation of rate dependent material behaviour in other carbon/epoxy architectures, such as non-crimp fabric (NCF).

The present work has presented a novel computationally efficient model for fibre reinforced materials influenced by strain rate. A viscoelastic-viscoplastic-damage model accounting for the pressure dependency and strain rate effects is used to model the matrix behaviour. This approach covers material with fibrous microstructures such as unidirectional fibre matrix combinations.

Fibres are idealized as elastic transversely isotropic. This concept can be extended to fully orthotropic description, if necessary. Viscoelasticity and viscoplasticity are considered as two main aspects of the matrix material characteristics. Depending on application and composite components, it has to be decided whether infinitesimal or finite strains apply.

For small and finite strains, efficient homogenization procedure based on constant strain assumption along the fibre direction and constant stress transverse to the fibres has been derived and verified by finite element simulations on the ply scale.

This model is enhanced by a continuum damage formulation to model post-peak softening and failure. Under monotonic loading applied to the composite, the homogenized stress-strain behaviour starts with a linear elastic stage followed by a pre-peak nonlinear stage and after that the post-peak stage lasts until ultimate failure.

The present focus is on the homogenized macroscopic material level in order to provide a formulation suitable for structural finite element simulations. The numerical simulations are performed in Abaqus/Explicit using a user defined material model (VUMAT).

Model parameters are identified via a calibration method. This approach should yield considerably more accuracy for the modelling of dynamic events. The simulation of a UD composite under quasi-static and dynamic experiments showed promising results for the prediction of rate dependent stiffness and damage evolution in compression.





# 6

## Future work and outlook

The homogenization technique based on the assumption of constant stress and strain is quite simplified and requires further development. A more advanced viscoelastic/-plastic model is needed to simulate a wider range of strain rates but is currently not included in the constitutive model. The need is especially evident for the quasi-static loading where the current model response is softer than the experiments.

Quasi-brittle continuum damage is implemented to simulate rate dependent response in compression. A different failure criterion is required in tension and should be addressed in the future. Furthermore, there is an ambition to extend the material model to orthotropic plies and textile reinforced composites.

At the macroscale the model applies to the ply level. In order to analyse laminates, the ply response need to be combined with cross plies using cohesive zone modeling for the delamination behaviour.



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