

Economic Model Predictive Control for Smart and Sustainable Agriculture*

G.B. Cáceres¹, P. Millán¹, M. Pereira¹ and D. Lozano²

Abstract—The joint effects of rise of global population, climate change and water scarcity makes the shift towards an efficient and sustainable agriculture more and more urgent. Fortunately, recent developments in low-cost, IoT-based sensors and actuators can help us to incorporate advanced control techniques for efficient irrigation system. This paper proposes the use of an economic model predictive control at a farm scale. The controller makes use of soil moisture data sent by the sensors, price signals, operative restrictions, and accurate dynamical models of water dynamics in the soil. Its performance is demonstrated through simulations based on a real case-study, showing that it is possible to obtain significant reductions in water and energy consumption and operation costs.

Index Terms—Sustainable Agriculture, Model Predictive Control, Economic Optimization.

I. INTRODUCTION

Global population has surpassed seven billion people, and almost one in every nine people in the world suffer from hunger. Considering that total world's population is expected to exceed 9.7 billion by 2050, it is projected that the demand for food may need to increase by between 25% and 70% to meet demand [1]. This figures, in conjunction with the effects of climate change, will increase the pressure over the agriculture sector, which will need to uptake a technological revolution to prevent food insecurity and to enhance both its efficiency and sustainability. Consequently, policy makers all around the world are creating strategies to increase the sustainability of food production along the whole value change, stressing the need of an efficient management of water resources (see, for instance, the Farm to Fork Strategy [2] of the European Green Deal [3]), and

It is important to consider that, in many countries, farming uses 60% of the total fresh water and up to 80% in some areas [4]. Therefore, the challenge is not only increasing food production, but also doing so with an sustainable use of water and other resources [5]. To assess water and food sustainability in agriculture, exists three indicators, one of them is the Water Footprint (WF) that is the total volumen of freshwater consumed to produce the product [6], [7], [8], and the color blue footprint indicates how much water is used

specifically in crops. For instance, part of the modernization process of irrigation systems to reduce water consumption in recent years has been based on updating the irrigation infrastructures to pressurize irrigation networks, and consequently, energy requirements for irrigation have significantly raised in many modern farms. Thus, deficient water management is a huge concern, not only for the depletion of this vital resource, but also because over-irrigation results on higher use of energy, lost of competitiveness, reduction on crop productivity and pollution of aquifers by fertilizers [9].

Recent advances in IoT-based sensors and actuators with fast-growing computation and communication capabilities makes it possible to incorporate advanced control techniques to minimize the use of water and energy at farm scale. In particular, Model Predictive Control (MPC) techniques, which have been successfully applied in highly technologically equipped greenhouses [10], are now being extended to other traditional farming methods. For instance, centralized and distributed MPC-based strategies based for the optimal management of irrigation canals are developed in [11] and [12], respectively. This framework, flow and water regulation of irrigation channels, have centered the attention of researchers in the last 5 years, and many predictive controllers of different configurations have been successfully developed: non-cooperative [13], distributed [14], adaptative [15], or hierarchical [16], [17].

However, there is little work in the literature regarding the predictive control and optimization of irrigation water and energy at a farm scale, and the strategies have been focused on the optimization of energy use in pressurized irrigation networks have been developed taking into account the minimization of both the investment and operational costs [18], [19]. At this scale, an adequate dynamic modelling of the water fluxes in the soil is key [5], as it makes possible to optimize irrigation using the soil as a water buffer, introducing at the same time energy-aware considerations.

This paper formulates an periodic economic MPC to reduce the water consumption and electricity costs at a farm scale and without compromising crop growth. The controller makes use of an extended version of a tested dynamical model [20] to predict water fluxed over the soil. Besides, it takes advantage of the quasi-periodic nature of important variables: radiation, transpiration, electricity prices, etc, to steer the irrigation system to a periodic optimal operation. The proposed controller is mainly composed of two layers. The first one has the function of calculating the best economic trajectory taking into account the periodic behaviour of several properties of the real system, a set of constraints related with the soil moisture, values and the

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cost of the electricity and water purchasing. The second layer adapts an MPC for tracking developed in [21], which guarantee convergence and recursive feasibility event when the parameters of the cost function change with time.

The paper is structured as follows: Section II introduces the nonlinear dynamical model that characterizes the dynamics of water in a cultivated soil. Section III formulates the proposed MPC controller and its associated variables, constraints and objectives. Section IV presents the simulation results over a real case study. Finally, Section V draws the main conclusions of this paper.

II. MODEL DESCRIPTION

The most usual way to measure the soil moisture in cultivated lands is through the volumetric water content (VWC), which is the ratio of water volume to soil volume. This variable plays a crucial role for irrigation control and, ideally, it should be above permanent wilting, the soil moisture level at which plants cannot absorb water, and below the field capacity [22], the soil moisture beyond which the excess water is rapidly drained away [23].

An crucial and yet overlooked aspect to design advanced controllers for irrigation systems is to count on an appropriate dynamical model to predict the water fluxes in the field. With this model, a predictive controller can optimize irrigation using the soil as a water buffer.

In this paper, we rely on an extended version of the dynamical model developed in [20]. This model contains a comprehensive set of variables and parameters to model and understand the water fluxes in a cultivated soil, which divided in three layers (1 - surface; 2 - root zone; and 3 - drainage zone). The governing equations are as follows:

$$\frac{d\theta_1}{dt} = \frac{1}{D_1} \left[I_{rr} + P_t - Q_{1,2} - \frac{1}{\rho_w} E_g \right] \quad (1a)$$

$$\frac{d\theta_2}{dt} = \frac{1}{D_2} \left[P_t - Q_{2,3} - \frac{1}{\rho_w} E_{tr} \right] \quad (1b)$$

$$\frac{d\theta_3}{dt} = \frac{1}{D_3} [Q_{2,3} - Q_3] \quad (1c)$$

where θ_i is the volumetric soil moisture content of each layer, D_i is the soil thickness of each layer, I_{rr} is the irrigation flow, P_t is the precipitation, $Q_{i,i+1}$ is the flux between layer i and layer $i+1$, Q_3 is the flux out of the bottom layer, E_g and E_{tr} are evaporation from the soil surface and transpiration from the vegetation canopy, respectively, and ρ_w is the water density.

To characterize the water fluxes between layer one can make use of equations (36) in [24] which, after a finite

difference discretization, yields to:

$$Q_{i,i+1} = \left(\frac{-\hat{\psi}}{0.5(\hat{D})} + 1 \right) \left(\frac{\hat{K}}{\hat{\psi}} \right) \quad (2a)$$

$$\hat{\psi} = \psi_{i+1} - \psi_i \quad (2b)$$

$$\hat{D} = D_i + D_{i+1} \quad (2c)$$

$$\hat{K} = K_i \psi_i - K_{i+1} \psi_{i+1} \quad (2d)$$

$$\psi_{i,i+1} = \psi_{sat} \left(\frac{\theta_i}{\theta_{sat}} \right)^{-B} \quad (2e)$$

$$K_i = K_{sat} \left(\frac{\theta_i}{\theta_{sat}} \right)^{2B+3} \quad (2f)$$

where K_i is the hydraulic conductivity of each layer, ψ_i is the matrix potential of each layer, θ_{sat} is the soil porosity, K_{sat} is the hydraulic conductivity at saturation, B is an empirical parameter related to soil texture, and the drainage out of the bottom layer is assumed to be K_3 .

The thicknesses of each layer depend on the cultivated crops. Some typical values are in the range of 3 to 10 cm for the first layer, while the layer 2 (root zone) can be more than 10 times thicker [25]. After a thorough simulation analysis of the model (1) with typical values of the agronomic parameters in (2), one easily concludes that the discretization of the water fluxes results in significant errors. However, it is straightforward to extend the previous model to work with sufficiently small layers. In this paper, we consider eight: surface layer, root zone (divided in six layers), and drainage zone (see Figure 1). This new model was checked, demonstrating a good accuracy (see Figure 4).

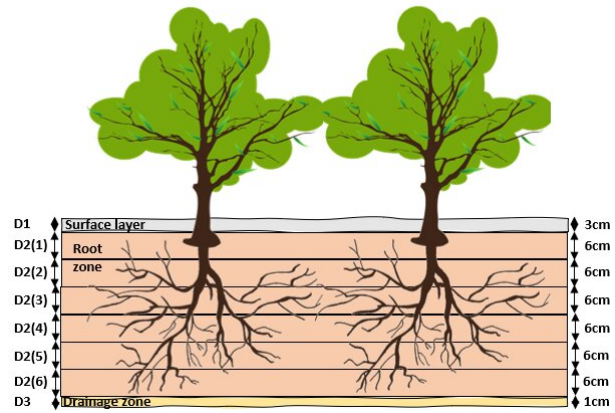


Fig. 1. Structure of the soil layer with the proposed division in eight layers.

Therefore, the equations equations to describe the extended model are as follows:

$$\frac{d\theta_1}{dt} = \frac{1}{D_1} \left[I_{rr} + P_t - Q_{1,i} - \frac{1}{\rho_w} E_g \right] \quad (3a)$$

$$\frac{d\theta_i}{dt} = \frac{1}{D_i} \left[\hat{Q}_i - \frac{1}{\rho_w} E_{tr} \right], \forall i = 2 \dots 7 \quad (3b)$$

$$\frac{d\theta_8}{dt} = \frac{1}{D_8} [Q_{7,8} - Q_8] \quad (3c)$$

where $\hat{Q}_i = Q_{i-1,i} - Q_{i,i+1}$.

III. MODEL PREDICTIVE CONTROL ALGORITHM

A. Model Predictive Control structure.

The structure of the proposed economic and periodic model predictive controller is shown in Figure 2. The input for the MPC for tracking is an optimized reference trajectory, which is obtained from a real time optimizer layer (RTO). This RTO take into account a complex economic function providing the best periodic trajectory that must be tracked to obtain the best results controlling a linear or nonlinear model.

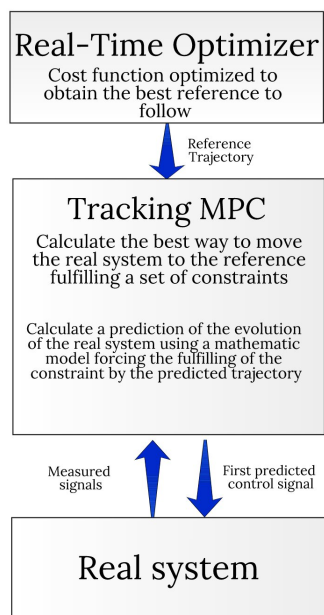


Fig. 2. Model predictive Control scheme

This MPC makes use of a linearized version of model (3) to obtain the optimal irrigation (control actions), the predicted evolution of the soil moistures, and the water consumption during a time window equal to the system period (1 day), fulfilling a set of constraints. These obtained predictions of the control actions are the best trajectories that can be applied in order to reach the best predicted soil moisture values along the system period. However, only the first control action is applied, and after that, the real system output(s) are measured again and delivered to the MPC for tracking (second layer). Then the optimization problem is recursively solved, following the classic receding horizon paradigm.

The second layer follows the paper [21] which guarantees the recursive feasibility and stability even when changes in certain parameters of the cost function happen. It is an interesting controller which increase the reachability region respect other classic tracking controllers.

The control objective in the second layer is usually to derive a control law $u(k) = \kappa(x(k), w(k))$ such that the

evolution of the closed-loop system fulfils the constraints (usually, the maximum and minimum in the soil moisture and irrigation flow) and the periodic tracking converge asymptotically to that computed by the RTO.

B. Model Linearization

The following linear model is proposed to be integrated into the model predictive controller scheme. This Linear Time Invariant (LTI) model is obtained from linearization of the non-linear model (3) described in previous section.

$$x(k+1) = Ax(k) + Bu(k) + B_d w(k) \quad (4a)$$

$$y(k) = Cx(k) \quad (4b)$$

where $x(k) \in \mathcal{R}^8$ represents the states of the model, $u(k) \in \mathcal{R}^1$ represents the control action and $w(k) \in \mathcal{R}^2$ represents the disturbances associated to this model. In this case, the states of the model are the soil moisture in every layer, the control action is the irrigation flow and the disturbances are transpiration E_{tr} and evaporation E_g .

The linearization was carried out using the System Identification in MATLAB, employing an algorithm called Prediction Error Minimization (PEM) and simulated input-output data. The comparison between some of the outputs (soil moistures) of the nonlinear model (3) and the linearized model are shown in Figure 3.

C. Economic and periodic model predictive control

In our economic and periodic MPC formulation for farm irrigation systems, both the soil moisture ($x(k)$) and irrigation flow ($u(k)$) are restricted within limits related to the crop needs and the irrigation system, respectively. Moreover, the system performance a weighted combination of the soil humidity, the water consumption and electricity costs. These terms are captured by a quadratic economic cost function $V_p(k, x, u)$, which depends on both the system state (soil moisture) and control inputs (irrigation flow).

In this paper we focus on the periodic operation of a closed-loop system with a fixed period T of 24 hours. The quasi-periodic behaviour of the main dynamic variables involved at a farm scale (radiation, crops transpiration, electricity prices), enables us to take advantage of a periodic, Real Time Optimizer and tracking layer, to achieve the better performance taking into account that we are using a linear model instead of a nonlinear model.

The main goal of this control structure consists of managing the irrigation to achieve an optimal economic performance, which optimizes a cost function reducing the irrigation flow and the costs associated to the water purchasing and energy consumption by pumps. This performance cost function V_p^* is used by a Real-Time Optimization (RTO) layer to provide an optimal trajectory. The optimal trajectory to operate the system is derived from the solution of the following optimization problem (5), where the initial state is a free variable.

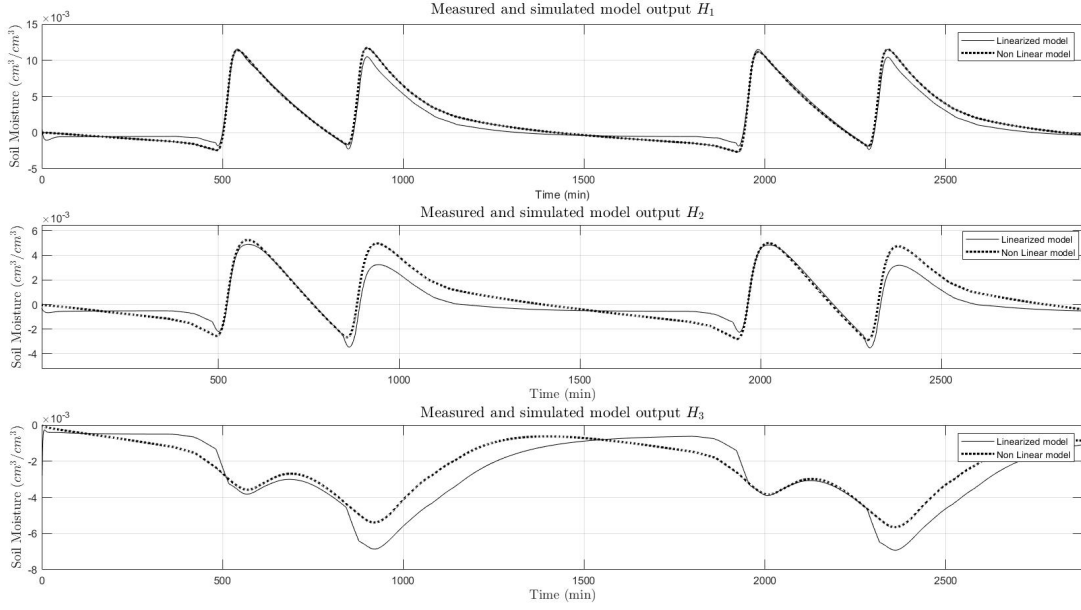


Fig. 3. Identification curves for the soil layers

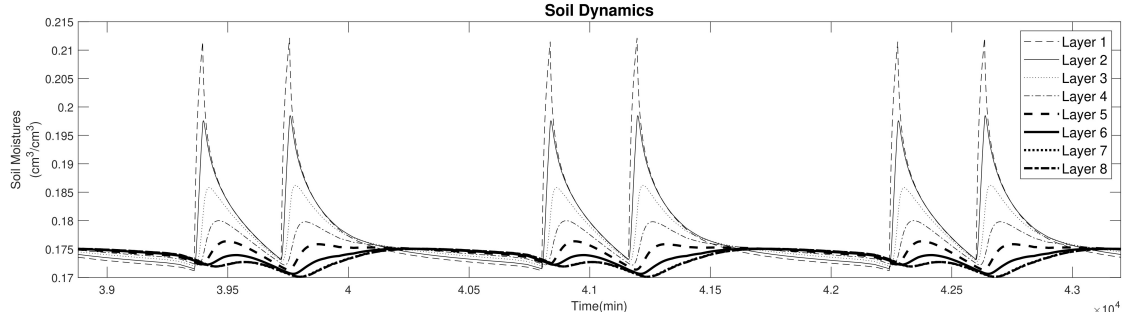


Fig. 4. Soil moistures for 8 layers

$$\min_{x(0), \mathbf{u}_\infty} V_p^*(x(0), \mathbf{u}_\infty) \quad (5a) \quad \text{which is denote as } \mathcal{P}_P(x, w).$$

$$s.t. \quad x(j+1) = f(j, x(j), u(j)), \quad (5b)$$

$$(x(j), u(j)) \in Z_r, \quad \forall j \geq 0, \quad (5c)$$

where the set Z_r is a closed polyhedron that encloses the above mentioned restrictions that affect the soil moisture and irrigation flows. The optimal state and input trajectories¹ are \mathbf{x}_∞^* and \mathbf{u}_∞^* respectively. In general, problem (5) has an infinite number of decision variables, however given the periodic nature of the dynamics, the constraints and the cost function, the optimal solution can be obtained from the solution of the following optimization problem at any given time instant k ,

$$\min_{x(0), \mathbf{u}_T} \sum_{k=0}^{T-1} V_p^*(x(0), \mathbf{u}_T) \quad (6a)$$

$$s.t. \quad x(k+1) = Ax(k) + Bu(k) + B_d w(k), \quad (6b)$$

$$(x(k), u(k)) \in Z_r, \quad \forall k \geq 0, \quad (6c)$$

$$x(0) = x(T) \quad (6d)$$

The optimal solution $(\mathbf{x}_T^*, \mathbf{u}_T^*)$ of the problem 6 ($\mathcal{P}_P(x, w)$) is used by the tracking optimization problem which is denote as $\mathcal{P}_N(x, w)$. The objective of this problem is to move the real system to the nearest position to the optimal trajectory

¹Bold letters denote trajectories of signals over the prediction horizon/period.

$(\mathbf{x}_T^*, \mathbf{u}_T^*)$.

$$\begin{aligned} \min_{x_0^r, \mathbf{u}^r, \mathbf{u}} \quad & V_N(x, \mathbf{x}, \mathbf{u}, \mathbf{w}; x_0^r, \mathbf{u}^r, \mathbf{x}^r, \mathbf{w}) \\ \text{s.t.} \quad & x(0) = x \quad (7a) \\ & x^+ = Ax(i) + B_u u(i) + B_d w(i) \quad (7b) \\ & y(i) = Cx(i) + Cu(i) \quad i \in \mathbb{Z}_N \quad (7c) \\ & (x, u) \in Z_r \quad (7d) \\ & x(N) = x^r(N) \quad (7e) \\ & x^{r+} = Ax^r(i) + B_u u^r(i) + B_d w(i) \quad (7f) \\ & (x^r, u^r) \in Z_r \quad (7g) \\ & y^r(i) = Cx^r(i) + Cu^r(i) \quad i \in \mathbb{Z}_T \quad (7h) \\ & x^r(0) = x^r(T) \quad i = 1..T \quad (7i) \end{aligned}$$

where \mathbf{x}^r and \mathbf{u}^r are reachable trajectories by the linear model of the controller used to avoid problematic situation (loss of recursive feasibility, etc.) for the MPC controller. For more details, see [26].

The cost function of this controller is defined as follows:

$$V_N(x, \mathbf{w}; x_0^r, \mathbf{u}^r, \mathbf{u}) = V_S(x, \mathbf{w}; x_0^r, \mathbf{x}^r, \mathbf{x}, \mathbf{u}^r, \mathbf{u}) + V_T(x_0^r, \mathbf{u}^r)$$

and

$$\begin{aligned} V_S &= \sum_{i=0}^{N-1} \|x(i) - x^r(i)\|_Q^2 \\ &+ \|u(i) - u^r(i)\|_R^2 \quad (8) \\ V_T(x_0^r, \mathbf{u}^r) &= \sum_{i=0}^{T-1} \|x^r(i) - x_T^*(i)\|_W^2 \\ &+ \|u^r(i) - u_T^*(i)\|_S^2 \quad (9) \end{aligned}$$

In general, the initial soil moisture is an argument of the tracking optimization problem and taking into account that this controller presents a big reachability region and the size of the admissibility for the soil moisture, the possibility that the optimization problem become unfeasible is very reduced.

The constraints of the optimization variables are divided in four groups: constraints (7b)-(7c) provides the predicted state and input trajectories; constraint (7a) imposes that the initial state of the predicted trajectory is equal to the state of the system at time step k ; constraint (7f) states that the predicted state must reach the artificial reference in T steps; and constraints (7g)-(7i) provides the artificial references state and input trajectories. The artificial inputs and states are variable of decision of the optimization problem. This trajectories are reachable trajectories by the model and must be near to the reference, or if possible, must converge to the reference if the reference provided by the RTO is reachable by the model.

Must be remark that we are proposing the use of a nominal controller not a robust controller. We are avoiding unfeasibilities using a soft constraint in lower constraints of the soil moisture.

D. Economic cost function for agriculture

The economic function is composed by three main terms. The first term weights deviations of the soil moistures from optimal values for the crops. The second term is use a time-varying weight to minimize the electric cost related to irrigation water. Finally, the last term focuses on minimizing the use of water.

$$\begin{aligned} V_p^*(\mathbf{x}, \mathbf{u}) &= wp_1 * f^1(x^{op}; \mathbf{x}) + wp_2 * f^2(\mathbf{u}) + wp_3 * f^3(\mathbf{u}) \\ f^1(\mathbf{x}) &= \sum_{i=0}^{T-1} \|x(i) - x^{op}\|_Q \\ f^2(\mathbf{x}) &= \sum_{i=0}^{T-1} C_{elec}(i)u(i) \\ f^3(\mathbf{x}) &= \sum_{i=0}^{T-1} C_{water}u(i) \end{aligned}$$

where C_{elec} is a time-varying electric cost, C_{water} is a fixed cost associated to the water per m^3 , and x^{op} are the operational point values of the soil moisture. There is a set of weights related with every part of the cost function (wp_i).

IV. SIMULATIONS RESULTS

A. Case study

This section compares, fundamentally in terms of water usage and electricity costs, the performance of a classic irrigation strategy used by farmers to that of the MPC-based irrigation system proposed in this paper. To carry out this comparison, we use a case-study corresponding to a strawberry farm located in Huelva (Spain), with approximately 100 hectares of crops and average size of greenhouses tunnels of 6.6x50m.

In particular, we consider the typical irrigation patters of local farmers during the month of June, when the crops need more water. A significant number of farmers in Huelva apply water in pulses of 30–40 min [27], and in during this month the total duration of irrigation is between 60 to 90 minutes a day. In this case, we apply water in pulses of 35 min twice a day.

In all the simulations analysis, we use the nonlinear model described in (3) and simulated the system in Matlab/Simulink. The term P_i in (1) and (3) is assumed to be zero, because in Huelva strawberries are cultivated in under plastic, so rainfall does not affect the water balance. In case of open-field crops, P_i must be considered and must enter as disturbance, which prediction can be approximately forecasting using local and remote weather stations.

The evolution of the soil moistures with the described classic irrigation strategy is shown in Figure 4.

Furthermore, note that the evapotranspiration E_{ir} includes evaporation and transpiration. This is an important concept which is the common concern of hydrology, ecology and meteorology [?]. According to [28], the transpiration E_{ir} is the result of evapotranspiration that multiplies the crop coefficient K_c . Because of the K_c is higher than 0.85, obtained in [29] from the city of Huelva condition, the evaporation E_g from the soil surface is practically zero in comparison of the

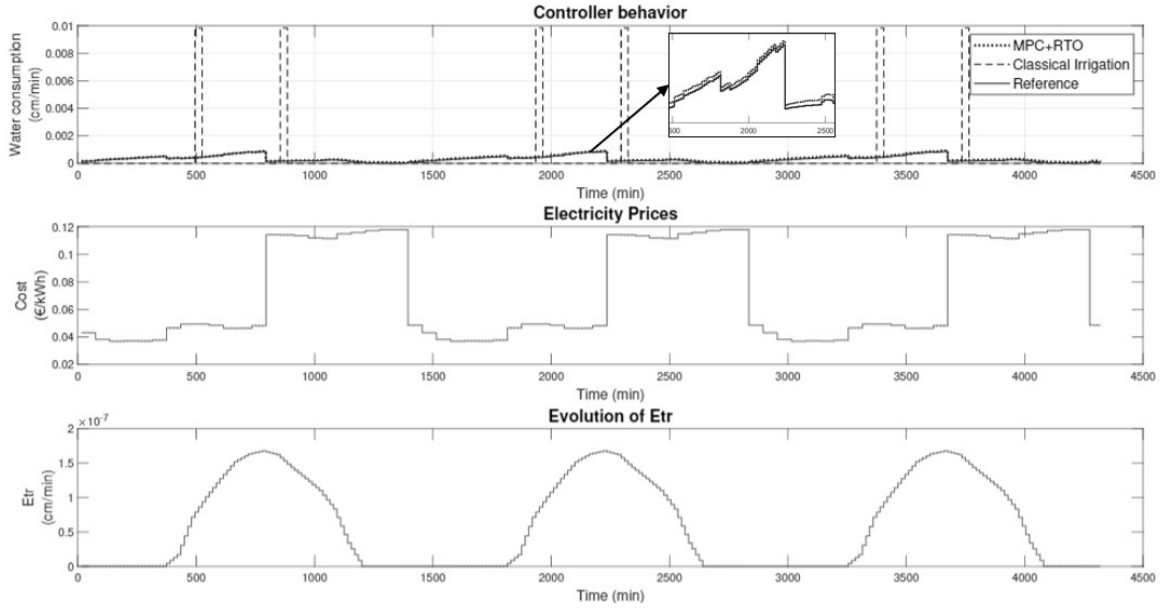


Fig. 5. (a) Irrigation flow by the Economic MPC Controller, optimal consumption by planning and classic irrigation methods during 3 days (b) Electricity prices (c) Evolution of the Etr

transpiration from the vegetation E_{tr} , so E_g is neglected for this application. Moreover, the simulations use real values for strawberries E_{tr} corresponding to a cloudless day on the month of June. These values are shown in Fig. 5(c).

Finally, regarding the soil characterization, its hydraulic parameters are chosen according to surveyed values of sandy soils during the month of June [30]. These values are shown in Table I.

Model parameters				
	θ_{sat}	K_{sat}	ψ_{sat}	B
Distribution	uniform	uniform	uniform	uniform
Units	cm^3/cm^3	cm/min	cm	-
Value	0.395	1.056	12	4.05

TABLE I

TABLE OF VALUES USED IN THE CASE STUDY

B. Linear model used in the proposed periodic predictive control

The Field Capacity (FC) plays a key role when using the soil as a water buffer or reservoir, because above it the excess water is rapidly drained away. A thorough simulation analysis of the nonlinear model (3) makes it possible to estimate the field capacity (FC). To this end, simulations with wet layers free of crops were conducted, and the points at which free drainage becomes negligible were determined. These values were used as the equilibrium point for the model identification.

The model linearization around the FC with a sampling time of 15 minutes result in the following system matrices:

$$A = [A1, A2] \quad (10a)$$

$$B = \begin{bmatrix} -5.2168 & 6.7321^5 & -16.0095 \\ -9.2607 & 7.3253^6 & -4.7959 \\ 0.7090 & -2.4302^6 & 3.1486 \\ 13.8103 & -1.3994^7 & 9.6757 \\ -414.1807 & -6.7976^7 & 20.0377 \\ -53.2899 & -1.1372^7 & 6.2628 \\ 194.2087 & 3.9110^7 & -20.0880 \\ -12.9238 & -6.4736^6 & 2.6168 \end{bmatrix} \quad (10b)$$

$$C = I_8 \quad (10c)$$

Where:

$$A1 = \begin{bmatrix} -0.1562 & -0.0140 & 0.0191 & -0.0330 \\ -0.3319 & -0.2622 & -0.0619 & -0.2079 \\ 0.04941 & -0.3586 & 0.1988 & -0.3130 \\ 1.3669 & -0.4635 & 0.8717 & -0.7543 \\ 3.4116 & -3.7417 & 3.4257 & -1.4387 \\ 1.0003 & -0.9481 & 0.6053 & -0.5312 \\ -3.1976 & 2.8731 & -2.6608 & 1.9631 \\ 0.4078 & -0.3419 & 0.1689 & -0.1799 \end{bmatrix}$$

$$A2 = \begin{bmatrix} -0.0094 & -0.0460 & -0.0311 & 0.0359 \\ -0.0302 & -0.3898 & -0.2077 & 0.1586 \\ -0.0983 & -0.0268 & -0.2064 & -0.0156 \\ -0.3102 & 0.3033 & -0.3756 & 0.4990 \\ -1.2169 & 0.8161 & -1.4873 & -1.1391 \\ -0.2530 & 0.0805 & -0.4299 & -0.4036 \\ 1.0304 & -0.1457 & 1.5170 & -0.3597 \\ -0.0774 & 0.1966 & -0.1246 & -0.4036 \end{bmatrix}$$

C. Description of the simulations.

The proposed scenario takes into account the evolution of the electricity price depicted in Figure 5(b). The water price used in this study case is constant and equal to 0.35 €/m³ [31].

The simulation has a duration of 3 days, with restrictions very near to the operational point x^{op} in order to check

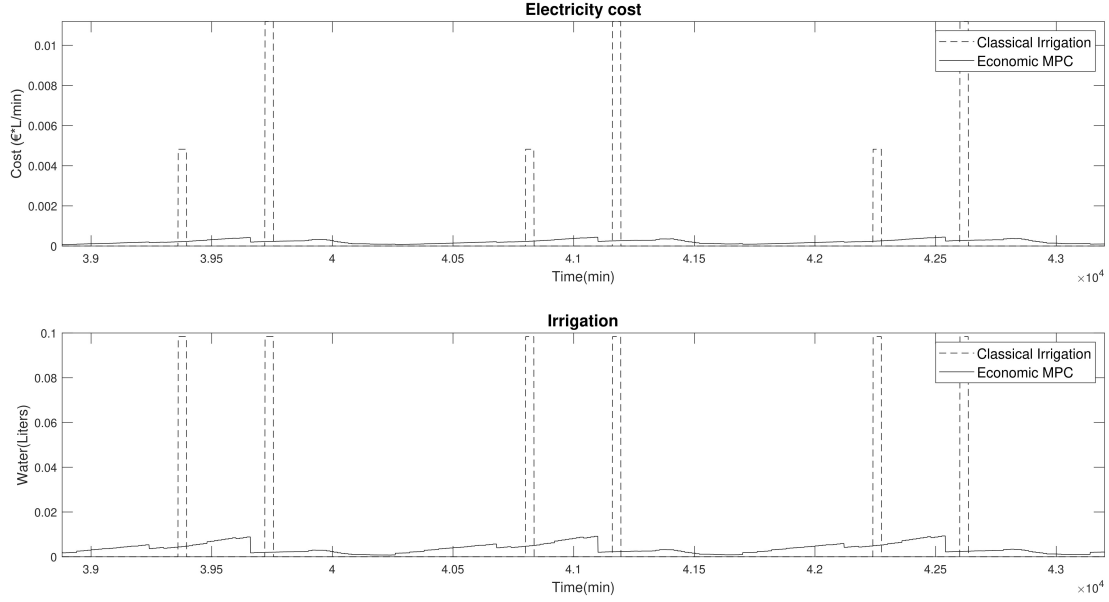


Fig. 6. Comparison between MPC controller and classical irrigation of (a) Electricity and (b) Water costs in euros and (c) Amount of water in liters used in the case study.

whether the controller performance. The constraints in Table II are assumed.

Constraints and MPC weights		
Variables	Range/values	Units
(x_{max}, x_{min})	[0.29 0.16]	%
(u_{min}, u_{max})	$\begin{bmatrix} 0 & - \end{bmatrix}$	cm/min
(wp_1, wp_2, wp_3)	$\begin{bmatrix} 0 & 10^{12} & 1 \end{bmatrix}$	-
E_g	0	cm/min

TABLE II

TABLE OF CONSTRAINTS AND WEIGHTS

The prediction horizon is chosen equal to the period, that is $N = T = 96$ min (24 hours). The cost matrices Q, R, S, W are chosen as follows

$$Q = 1 \cdot \mathcal{I}_8 \quad (11a)$$

$$R = 5000 \cdot \mathcal{I}_1 \quad (11b)$$

$$S = 1 \cdot \mathcal{I}_8 \quad (11c)$$

$$W = 1e12 \cdot \mathcal{I}_1 \quad (11d)$$

where \mathcal{I}_n is the identity matrix of dimension n .

To illustrate the comparison between a classic strategy and the proposed MPC, Figure 5(a) shows the applied water of both irrigation systems together with the references provided by RTO. Looking at figures 5(a) and 5(b) it can be check how the predictive controller tries to pump water when electricity prices are lower.

Furthermore, a simulation of 30 days during the whole month of June was also conducted. In order to simplify

the simulation, we assumed that the bomb consume 1 kWh/m^3 . The evolution of water and electricity consumption are shown in Figure 6. A summary of the obtained results is presented in Table III.

Simulation Results			
Studied terms	Classical Irrigation	MPC Irrigation	Units
Water usage	206.6	159.4	l
Electricity cost	16.81	9.492	€
Water cost	0.07231	0.0558	€

TABLE III

TABLE OF COMPARISON BETWEEN CLASSICAL IRRIGATION AND THE PROPOSED MPC

The crops total water needs during the whole month is about $157,5 \text{ l/m}^2$. As can be checked in Table III, a classical two-pulse irrigation strategy waste a 30% of water. However, the proposed MPC-based irrigation system waste only 1,2% of the water.

Simulation Results			
Study terms	Classical Irrigation	MPC Irrigation	Units
Water usage	68178	52602	l
Electricity cost	5547,3	3132,36	€
Water cost	23.86	18.414	€

TABLE IV

TABLE OF CASE STUDY COMPARISON BETWEEN CLASSICAL IRRIGATION AND THE PROPOSED MPC

Of course, these results are only approximation of what can be really obtained in farms, as some modeling simplification would increase consumption in real implementations.

Two of the most important simplifications are: i) homogeneous soil and crops, and ii) ideal and uniform irrigation network and filling/emptying dynamics. Nonetheless, these effect would also augment the water consumption of the traditional irrigation strategy, so total saving could be still similar.

Considering these simplifications, in an greenhouse of the study case with $330m^2$, the results are shown in Table IV, showing that the total water saving and costs will be considerable in large scale.

V. CONCLUSIONS

In this work, an economic, periodic MPC controller is successfully developed for irrigation management at farm scale. The MPC-based irrigation system is compared in simulation with a traditional water management in a real case scenario, showing significant reduction in percentage in water consumption and hence costs.

The proposed controller show a good performance, in which applies practically the water that the plant needs, saving 22.8% of water consumption and cost, and an 43.5% in electricity cost in comparison of the classical irrigation.

We assume that the performance of the controller on a large scale can give considerable returns, taking into account the aspects already considered above.

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