

APPLICATION OF ANALYTICAL AND CFD MODELS OF LIQUID FUELS COMBUSTION IN A FLUIDIZED BED

by

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Original scientific paper
<https://doi.org/10.2298/180226317M>

In Laboratory for Thermal Engineering and Energy of Institute Vinča, University of Belgrade, a 2-D CFD modeling procedure of numerical simulation of unconventional liquid fuel combustion in bubbling fluidized bed has been developed. This procedure is based on a two-fluid Euler-Euler approach modeling a fluidized bed with the determination of the velocities field of gas and particulates in two-phase, granular flows, analog to the kinetic theory of gases. This model describes in detail the unsteady motion of gas and solid phases, the formation and movement of bubbles with the combustion process in the fluidized bed, but temperature profiles calculated by the bed height differ to some extent from the experimentally obtained profiles. This discrepancy is probably due to the inability of a two-fluid CFD model to give a realistic simulation of the liquid fuel mixing in a fluidized bed. Therefore, an analytical model has been developed, where one of the basic assumptions is that the particles are mixed in the vertical direction of fluidized bed mainly by the bubble wakes. The proposed zonal type of calculating procedure is based on Davidson and Harrison two-phase model of the bubbling fluidized bed, where fluidized bed is divided into zones within which material and energy balances are set.

Key words: *fluidized bed, combustion, analytical model, CFD model*

Introduction

Modern society generates large amounts of waste and the problem of its removal is an increasingly challenging one. This work is indirectly concerned with the combustion of liquid waste, of high volatility and density, such as deposits from crude oil reservoirs. Such fuels are very difficult to combustion in conventional furnaces, and therefore the consideration of the mechanism of the whole combustion process is very important for optimizing the process of its thermal disintegration.

There are many benefits of the fluidized bed (FB) combustion of the unconventional fuels (with high amounts of water and other ballast matter). Principally, the high thermal capacity of the FB, the thermal conductivity, and the intensity of heat transfer between the bed inert material and the fuel facilitate a stable combustion process of a wide variety of unconventional fuels, accompanied by a low sensitivity to fuel quality. The zone of intensive combustion occupies a relatively small volume because most of the fuel is burning in the bed itself, and burning-off in the *splash* zone and above the bed. As FB furnaces operate at lower temperature (≈ 850 °C) this is also optimal from the aspect of the decreased NO_x emission. In addition, these

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furnaces are favorable for the efficient in-bed desulphurization, when it is necessary. Consequently, FB technology is recommended for the combustion of waste by EU [1].

Experimental methods and numerical simulations are equally employed in research and development in the fields of energy and process engineering. The numerical models provide great opportunities for saving resources and time in the development of facilities and technologies. However, it should be noted that the numerical tools for simulation of complex processes, such as FB combustion, are not completely developed due to difficulties in describing relatively complex two-phase flow and specificity of heat and mass transfer in bubbling fluidized bed (BFB). In addition, many of existing numerical methods do not comply with engineering needs, due to their complexity and necessity of expensive computer equipment, which makes them unsuitable for development and engineering application [2-4].

Many authors use Eulerian-Eulerian modeling approach for CFD modeling gas-solid flows in FB reactors [5-8]. This method, also called granular flow model, assumes that both phases can be considered as fluids and take into consideration the interpenetrating effect of each phase using drag models. Consequently, the proper selection of drag model in Eulerian-Eulerian modeling is crucial. The Eulerian two-fluid approach is an extension of the fluid dynamics formulation of the single phase to multiphase flow. Particles in gas-solid flow is treated as magnified molecules, and the analogy of their behavior to the gas molecules is the reason for the wide use of the kinetic theory of granular flow (KTGF) for modeling the particles motion. The KTGF is based on the concept of the granular temperature, as a measure of the random oscillations of the particles and defined as the average of the three variances of the particle's velocities [9]. In spite of detailed mathematical modeling of the complex processes in FB, the drag laws used in two-fluid models are semi-empirical in nature. Therefore, it is essential to use a drag law that correctly predicts the minimum fluidization conditions wherein the particles are in a state of suspension as the result of the balance between interfacial drag and body forces. The model inter-phase interaction drag force by Syamlal and O'Brien [10] is commonly used, wherein the coefficient between the fluid and solid (granular) phase depends only on the phase void fraction, but not of the fluidization conditions.

The last-mentioned model has been used in the numerical simulation of combustion of liquid fuels in a FB [11]. Although this complex model simulates the processes of unsteady combustion in BFB well, the locations of intensive reactions differ somewhat from the experimentally obtained. Therefore, in this paper preparation of the analytical model has been started which, despite its simplicity, still offers the possibility of easier access to global processes of combustion in FB. The proposed analytical model is based on Davidson *et al.* [12] approach to the modeling of FB. Comparison of analytical and CFD KTGF model with experimental results has been presented in the paper.

Analytical model

The basic assumption of the proposed analytical model is to neglect the changes in the radial (lateral) direction of the FB. The model also does not take into account the velocity fields in a reactor. By modeling of the size, rate of bubbles, and also proportion of bubbles in FB (by Davidson, Harrison, Kunii and Levenspiel model), important data necessary for the calculation of the reaction rate along the height of the reactor were obtained. These FB aerodynamic data are also important for determining the energy balance along the reactor height. For calculation mass and energy balances along the height of the FB reactor, it is necessary to divide it into zones. For each zone, the energy and mass balances are calculated and then coupled with other zones. The combustion reaction of liquid in FB is considered in two steps: devolatilization process and the

combustion reaction of volatile fuel components. The model has assumed that the evaporation of the liquid takes place in a dense phase of the FB, and the combustion of the gaseous fuel components takes place in the bubble phase, according to FB scheme by Kunii and Levenspiel [13], fig. 1.

Basic concept of the analytical model

In accordance with the two-phase FB model, the following scheme may be adopted, fig. 2.

- Air-flow in the bubble phase = bubble velocity × (bubble fraction in FS × total air-flow)
- Air-flow in the dense phase = total air-flow - air-flow in the bubble phase
- Liquid fuel-flow with bubbles = bubble speed × bubble fraction in FS × fraction of trace in bubbles × total fuel-flow
- Liquid fuel-flow in dense phase = total fuel-flow - Liquid fuel-flow with bubbles
- Vapor fuel-flow in bubble phase = liquid fuel-flow with bubbles × evaporation rate
- Vapor fuel-flow in dense phase = liquid fuel-flow in dense phase × evaporation rate
- Width (height) of the control zone: Δx
- Bubble passing time through the control zone: $\Delta\tau = \Delta x/u_B$, where u_B is bubbles velocity
- The velocity of the bubble grows from the zone to the zone due to the increase in flue gases.

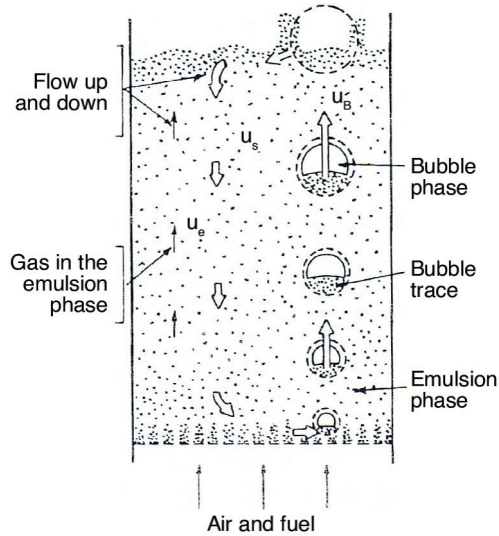


Figure 1. Bubble FB model

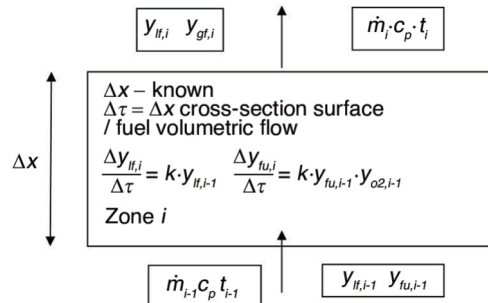


Figure 2. Zonal model scheme

The mass-flow of liquid fuel decreases in the zone for $\dot{m}_{f,i-1} \Delta y_{f,i} = \Delta \dot{m}_{gf,i}$ and this difference increases the mass-flow of fuel vapor in the zone i .

The mass-flow of liquid fuel coming out of the zone I is:

$$\dot{m}_{f,i} = \dot{m}_{f,i-1} - \dot{m}_{f,i-1} \Delta y_{f,i} = \dot{m}_{f,i-1} (1 - \Delta y_{f,i}), \Delta y_{f,i} = k_{ev} \Delta y_{f,i-1} \Delta\tau$$

The mass-flow of fuel vapor coming out of the zone is:

$$\dot{m}_{gf,i} = (\dot{m}_{gf,i-1} + \Delta \dot{m}_{gf,i}), (1 - \Delta y_{gf,i}) \Delta y_{gf,i} = k y_{gf,i} \Delta\tau$$

Mass-flow of products coming out of the zone i : $\dot{m}_{pr,i} = \dot{m}_{pr,i-1} + s_{pr} (\dot{m}_{gf,i-1} + \Delta \dot{m}_{gf,i}) \Delta y_{gf,i}$ where s_{pr} is the stoichiometric ratio of products and fuel.

Mass-flow of air coming out of the zone i : $\dot{m}_{air,i} = \dot{m}_{air,i-1} - s_{air} (\dot{m}_{gf,i-1} + \Delta \dot{m}_{gf,i}) \Delta y_{gf,i}$ where s_{air} is the stoichiometric ratio of products and fuel.

Fuel conversion

The reaction rate of evaporations modeled according to the following expressions:

$$R_{dev} = \frac{dy_{fl}}{d\tau} = (1 - \delta_b) k_d y_{fl}, \quad k_d = k_{0,d} e^{-\frac{E_{ad}}{RT}} \quad (1)$$

where y_{fl} is the mass fraction of liquid fuel phase and δ_b is the proportion of the volume of bubbles in the total volume of the bed.

The reaction rate of volatiles combustion can be estimated using following expressions:

$$R_{fu} = \frac{dy_{fu}}{d\tau} = \delta_B k_c \gamma_{fu} \gamma_{O_2}, \quad k_c = k_{0c} e^{-\frac{E_{ac}}{RT}} \quad (2)$$

Applying the discretization of eqs. (1) and (2), it is possible to estimate the fuel mass fraction change, $\Delta y_{fu,i}$ in each zone (i) along the FB reactor:

$$\Delta y_{fu,i} / \Delta \tau_i = \delta_{B,i} k_c \gamma_{fu,i} \gamma_{O_2,i} \quad (3)$$

The residence time of fuel in a given zone $\Delta \tau_i$ is obtained by dividing the volume of the zone with the volumetric fuel-flow through a given zone. Since the fuel-flow changes along the height of the reactor then the residence time varies with height.

The proportion of the volume of bubbles in the total volume of the bed, δ_B , can be determined on the basis of the following expression:

$$\delta_B \approx \frac{u_f - u_{mf}}{u_B} \quad (4)$$

where u_B is velocity multitude of bubbles, and $u_f - u_{mf}$ is the difference between the velocity of the gas in the FB (on a cross-section of the reactor) and the minimum fluidization velocity.

The velocity of multitude of bubbles, u_B , by Davidson *et al.* [12] is defined as follows:

$$u_B = u_{B\infty} + (u_f - u_{mf}) \quad (5)$$

Numerous studies have confirmed [12, 14] that the rise velocity of a single bubble in the FB can be expressed by a formula derived for a gas bubble in liquid [15]: $u_{B\infty} = 0.711(gD_B)^{1/2}$. For FB of large particles used in furnaces with FB combustion, the following formula is recommended for the size and growth of slow bubbles:

$$D_B = 0.0326 (u_f - u_{mf})^{1.11} h^{0.81} \quad (6)$$

The minimum fluidization velocity can be evaluated using the expression:

$$\text{Re}_{mf} = \frac{d_p u_{mf} \rho_f}{\mu_f} = (A^2 + B \text{Ar})^{0.5} - A, \quad A = 17, \quad B = 0.037 \quad (7)$$

where Ar is Archimedes number: $\text{Ar} = \frac{g d_p^3 \rho_f (\rho_p - \rho_f)}{\mu_f^2}$.

Combining expressions (2)-(7) the change of fuel mass-flow in zone i can be determined:

$$\Delta \dot{m}_{fu,i} = \Delta y_{fu,i} \dot{m}_{fu,i} \quad (8)$$

Determination of total composition of combustion products

The total combustion process of the fuel converted in zone i ($\Delta m_{fu,i}$) can be analyzed by setting the stoichiometric expression of the generalized fuel molecule combustion:



where λ is the excess air coefficient, n_j – the number of atoms of chemical species (j) in the effective molecule of fuel with no moisture and ash.

A number of atoms or the effective molecules in fuel can be determined based on the data of ultimate and proximate analysis of the fuel. Fuel analysis data gives the mass fraction of the atomic species of C, H, O, N, and S (Y'_j), as well as the mass fraction of moisture (Y'_w) and ash

(Y'_a) in the fuel. Mass fraction of the atomic species C, H, O, N, and S (Y_j) in the fraction of fuel without moisture and ash can be calculated as $Y_j = Y'_j m' / m$, where m' and m are mass of fuel and mass without moisture and ash, respectively. The number of atoms of chemical species in the effective molecule of fuel without moisture and ash can be expressed as follows: $n_j = Y_j m / A_j$ where A_j is the atomic mass of the chemical species j . Number of moles of the moisture in 1 kg of fuel (x) is defined by the expression $x = Y'_w m' / M_{H_2O}$.

Based on mass balance, set in accordance with the expression (9), mole numbers, v_j , of all components in the stoichiometric eq. (9) for combustion 1 kg of fuel are obtained:

$$v_{CO_2} = n_c, \quad v_{H_2O} = x + \frac{n_h}{2}, \quad v_{SO_2} = n_s, \quad v_{n_2} = \frac{n_n}{2} + 3.76 \lambda c, \quad v_{O_2} = c \lambda \quad (10)$$

In expression (10) c represents the number of oxygen moles that participate in the combustion when fuel and oxidizer ratio is stoichiometric, *i.e.* there is no excess oxygen:

$$c = n_c + \frac{n_h}{4} + n_s - \frac{n_o}{2} \quad (11)$$

Since the expression (9) is defined per kilogram of fuel then the excess air coefficient (air-fuel equivalence ratio, λ) can be determined using the following expression:

$$\lambda_i = \frac{\dot{m}_{O_2,i}}{\dot{m}_{fu,i} c M_{O_2}} \quad (12)$$

On the basis of presented analysis it is possible to determine the change of the fuel and oxygen flow and also changes of product chemical species mass-flow that leaves zone i :

$$\dot{m}_{fu,i+1} = \dot{m}_{fu,i} - \Delta \dot{m}_{fu,i} \quad \dot{m}_{O_2,i+1} = \dot{m}_{O_2,i} - \lambda c \Delta \dot{m}_{fu,i} M_{O_2} \quad \dot{m}_{j,i+1} = \dot{m}_{j,i} + \Delta \dot{m}_{fu,i} v_j M_j \quad (13)$$

where v_j are mole numbers of gaseous components and M_j is a molar mass of components in stoichiometric eq. (9). The mass-flow rate of nitrogen is constant through all the zones.

The concentrations of gaseous components, *i.e.* their mass fractions, are determined using the following expression:

$$Y_{j,i} = \frac{\dot{m}_{j,i}}{\sum_j \dot{m}_{j,i}} \quad (14)$$

Heat balance in a zone

Combustion temperature is the maximum temperature reached by the products of combustion in the zone of intense reaction with ideal combustion conditions. Calculation of adiabatic combustion temperature is based on the heat balance *i.e.* on the equality of the total enthalpy of the fuel mixture and combustion products. Heat balance of the adiabatic combustion process in zone i of the fluidized reactor can be expressed:

$$c_{p,i} t_i \sum_j \dot{m}_{j,i} + \Delta \dot{m}_{fu,i} H_{fu} + \dot{m}_{p,i} c_{p,i} (t_{i+1} - t_i) = c_{p,i+1} t_{i+1} \sum_j \dot{m}_{j,i+1} \quad (15)$$

Lower heating value of the fuel, H_{fu} , if the fuel ultimate and proximate analysis are known, can be determined using the following expression:

$$H_{fu} = 3.40E7 \cdot Y'_c + 1.030E8 \cdot Y'_h - 1.09E7 (Y'_o - Y'_s) - 2.5E6 \cdot Y'_w \quad (16)$$

The $\dot{m}_{p,i}$ in eq. (15) represents the mass-flow rate of particles that are exchanged between the two neighboring zones. The proposed model assumes that the flow of particles in a BFB is primarily provided through the bubble wakes movement. Taking into consideration the aforementioned assumption, the flow of particles between the two-zones of FB can be defined using the following expression:

$$m_{p,i} = \alpha_w \delta_B S_R u_B \rho_w \quad (17)$$

where α_w is the ratio of the volume of particles dragged in the wake and the bubble volume (V_w/V_B). For most bubbles in a fluidized bed $Re_B < 20$ is valid, and it is assumed that the bubble *wake* in a FB can be determined from expression as follows:

$$\alpha_w = 0.037 Re_B^{1.4} \quad Re_B = \frac{D_B u_B}{\nu_f} \quad (18)$$

Other variables in equation (17) are defined by expressions (4)-(6).

The temperature of gases in the zone i is determined by iterative calculation of the expression (15) as the specific heat of gaseous components depends is a function of temperature. For solving this equation, the code in programming language FORTRAN 77 was used.

Results and discussion

Proposed analytical model of liquid fuels combustion in a FB has been compared with the Eulerian-Eulerian CFD model for gas-solid flows in FB reactors [11] and results of the experimental investigation of non-conventional liquid fuels combustion in FB [16].

The CFD model for combustion in FB

Euler-Euler FB modeling approach considers the gas and FB dense phase (gas-particle system under conditions of the minimum fluidization) as two fluids with different characteristics. In the transport equations for transfer of momentum of the effective fluid (the FB dense phase), fluid-particle interactions in conditions of the minimum fluidization velocity are modeled, as well as the interaction between the particles themselves. In the Eulerian-Eulerian approach, all phases have the same pressure, which is the pressure of the continuous-primary phase. This model solves the continuity and momentum equations for each phase and tracks the volume fraction. Further, the additional transport equation for the granular temperature (which represents the solids fluctuating energy) is solved, also the solids bulk and shear viscosity are determined using the KTGF.

It is also necessary to define the coefficients for calculating the inter-phase interaction term. For modeling the interactions between gas and particle phases, within the suggested Euler-Euler granular approach for FB modeling, the routines incorporated in the modules of the commercial CFD software package FLUENT 6.3.26 were used. This code allows the presence of several phases within one control volume of the numerical grid, by introducing the volume fraction of each phase. The solid phase represents a granular layer made of spherical particles, with uniform diameters. The mass and momentum conservation equations are solved for each phase separately.

Experimental investigation of non-conventional liquid fuels combustion in FB

Within the scope of the research activities of the Laboratory for Thermal Engineering and Energy of the Vinča Institute, the experimental installation for tests with nonconventional fuel combustion in FB has been developed [16]. In order to confirm the numerical models, experiments on the combustion of liquid fuels in FS were performed with sunflower oil and glycerine. Both liquids were selected for the so-called model fuels because they are characterized by an approximate thermal effect as well as oil sludge, but in contrast to it, do not contain mechanical and other impurities and is less viscous than oil sludge (liquid waste), and therefore it is more easily feeded. In addition, it is cheap and readily available.

The experimental installation fig. 3 has been dimensioned, designed and built in a way that the results obtained during investigations on it can be used as design parameters for the construction of real-scale facilities for combustion of solid or liquid fuel.

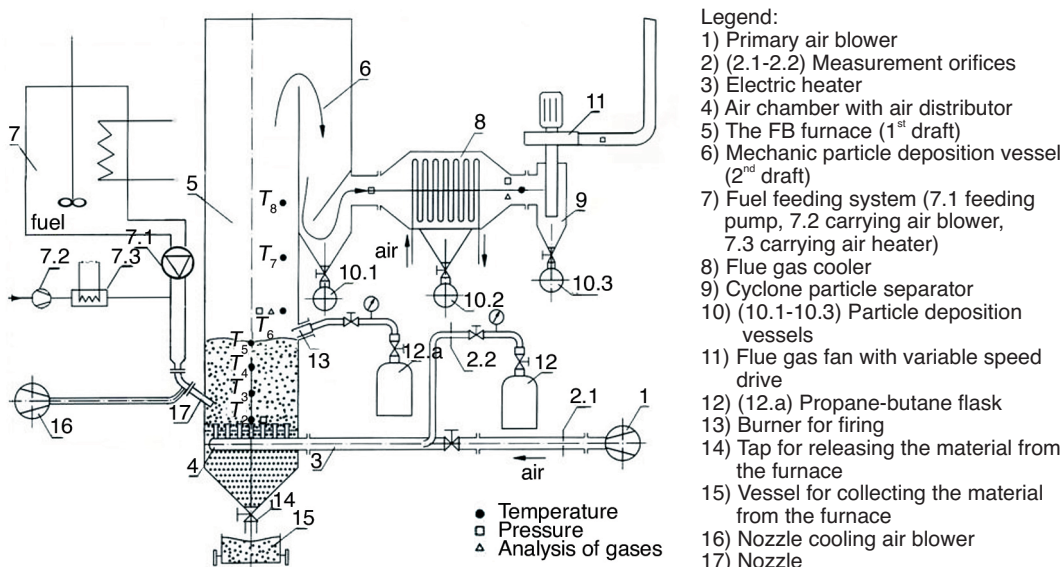


Figure 3. Scheme of the experimental facility with the fuel feeding system

The fuel is introduced into the experimental facility with the fuel feeding system 7 through the tubular nozzle 17. The fuel is fed into the FB. The air is supplied to the FB through the distributor 4. The bed material is quartz sand with a medium diameter of 0.8 mm, and a density of 1310-1585 kg/m³. The bed has a rectangular cross-section (295 × 290 mm) and height of bulk particles is 323-400 mm.

In tabs 1. and 2. data of the ultimate and proximate analysis of sunflower oil, glycerol and modeled fuel tab. 1 are given, as well as the operating parameters of experimental and modeled FB furnace tab. 2.

Comparison of experimental results and data obtained by models

Figure 4 shows a comparison of temperature profiles along the height of FB reactor during combustion of sunflower oil and glycerin with temperature profiles obtained by CFD KTGF calculations and using proposed analytical model. The *y*-co-ordinate in fig. 4 is the normalized temperature because the energy balances of the comparing cases do not match. In addition, on the *x*-co-ordinate is the dimensionless height of the FB reactor, normalized to the height occupied by the bed at the minimum fluidization velocity (H/H_{mf}). As it can be seen in fig. 4, the temperature profile obtained by the analytical model has a better fit with the experimental results than those obtained by CFD KTGF procedure. The analytical model and experiments show that the intensive combustion already starts deeper in the bed, while by CFD model significant part of the combustion occurs above the bed.

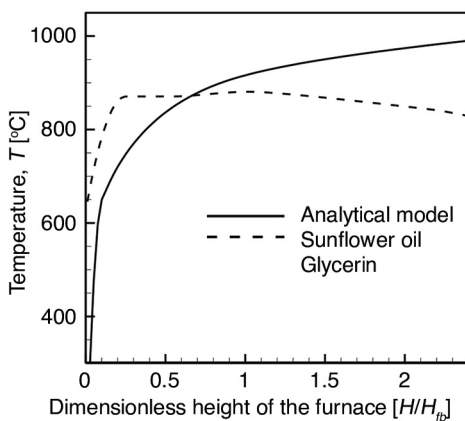
The reason for this probably lies in the fact that the analytical model takes into account the more intensive vertical mixing of particles, which is the movement of particles modeled by

Table 1. Proximate and ultimate analysis of fuels

Fuel	Sunflower oil	Glycerol	Model fuel
Moisture [%]	0.1	-	8
Ash [%]	0	-	2.4
Volatile[%]	99.17	99	-
Char [%]	0.73	1	-
C [%]	77.6	39.1	41.66
H [%]	11.5	8.7	5.32
O[%]	10.9	52.17	41.93
N [%]	0	0	0.66
S [%]	0	0	0
LHV [MJkg ⁻¹]	37.1	17	15

way of the bubble wakes. In addition, the analytical model assumption of the the ideal mixing of components in the radial direction of FB, is, better fit to the real combustion conditions in the FB than in the case with the CFD model.

Figure 5. shows the comparison of non-normalized temperature profile obtained by the analytical model with the measured temperature profiles. Calculated temperature profiles are in better agreement with the results of tests with glycerin, as is to be expected, bearing in mind the fuel characteristics have similar values in these two cases tab. 1.

**Figure 5. Temperature profiles along height of the FB combustor - comparison of analytical model and experiments****Table 2. Operating parameters of experimental FB furnace**

Fuel	Fuel mass-flow rate [kg h ⁻¹]	λ	Primary air-flow rate [kg h ⁻¹]
S.oil	4.12	2.79	145.68
Glycerin	7.92	2.98	122.62
Model fuel	4.65	3.0	67.0

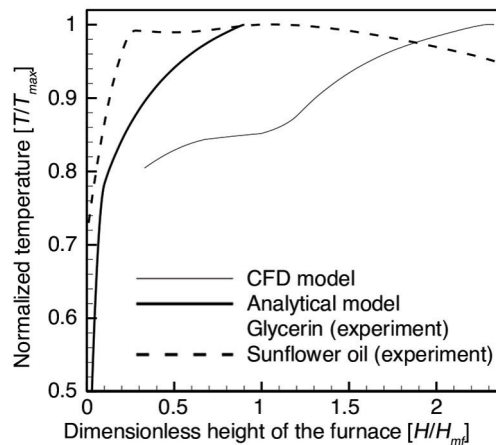
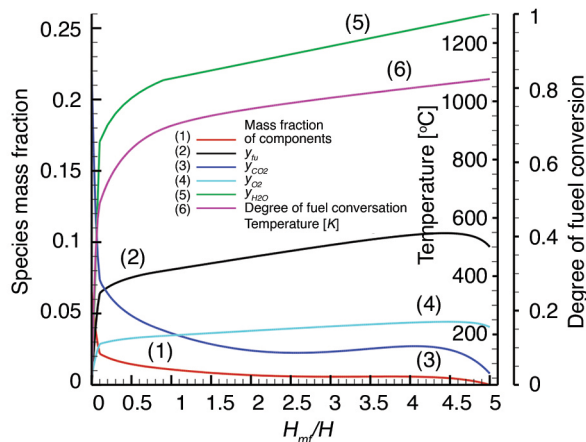
**Figure 4. Normalized temperature profiles along height of the FB combustor - comparison of models and experiments****Figure 6. Results of the analytical model**

Figure 6 shows, in addition to profiles of temperature also the calculated distribution of the components concentrations along the height of the fluidized reactor, as well as the degree of

fuel conversion along the height of the combustion chamber. As the fig. 6 shows, by the proposed model, at the exit of bed with fluidized particles about 80% of the fuel is burnt. A similar conclusion is obtained also by an analysis of the experimental results of liquid fuels combustion in FB. Thereby, on the level of about 5 heights of the bulk layer of particles (bed height at minimum fluidization -Hmf), all fuel is reacted.

Conclusions

The experimental investigations of an unconventional liquid combustion in a BFB, and numerical procedure, based on the two-fluid Euler-Euler approach modeling FB with an analogy with the KTGF, as well as analytical model based on a two-phase model of the BFB, have been developed.

The proposed zonal type of calculating procedure is based on a two-phase model of the BFB by Davidson and Harrison, where FB is divided into zones within material and energy balances are set. For each zone, the energy and mass balances are calculated, which are coupled with other zones. For the reaction rate modeling, the basic assumption is that the devolatilization of the liquid takes place in a dense phase of the FB, and the combustion of the gaseous fuel components takes place in the bubble phase. For heat and mass balance calculation, the proposed model assumes that the vertical flow of particles in a BFB primarily takes place through the bubble wakes movement.

The comparison of experimental results and two models show the better agreement of experimental results with the analytical model than with two-fluid CFD numerical procedure. Namely, the analytical model and experiments show that the intensive combustion starts already in the depth of the bed of particles, while by CFD model significant part of the combustion occurs above the bed. The reason for this probably lies in the fact that the analytical model takes into account the more intensive vertical mixing of particles, which is modeled by the movement of particles by way of the bubble wakes. Furthermore, the CFD model probably simulates insufficient intensive mixing of the reacting components at lower heights of the FB.

The presented results might be of practical importance for optimization of different liquid fuels combustion in the FB furnace, especially non-conventional fuels such as oil sludge and waste liquid materials, especially in order of the simplest numerical model selection that most accurately describes the processes in the FB.

Acknowledgment

The authors would like to thank the Serbian Ministry of Education, Science and Technological Development for financing the project "Improvement of the industrial fluidized bed facility, in the scope of technology for energy efficient and environmentally feasible combustion of various waste materials in fluidized bed", (Project Tr33042).

Nomenclature

A – empirical constant	k_c – combustion reaction rate constant
A_j – atomic mass of the chemical species j	k_d – devolatilization reaction rate constant
B – empirical constant	m – mass
c_p – specific heat	\dot{m} – mass-flow rate
D_B – bubble diameter	n_j – number of chemical species atoms
d_p – particle diameter	R – universal gas constant
E_a – activation energy	R_{dev} – the reaction rate of devolatilization
g – acceleration due to gravity	Re_B – bubble Reynolds number
H_{fu} – fuel heating value	Re_{mf} – particle Reynolds number based on minimum fluidization velocity
h – height in FB	R_{fu} – reaction rate of fuel combustion
k_0 – pre-exponential factor	

S_R – fluidization reactor cross-sectional area	<i>Subscripts</i>
u_B – velocity multitude of bubbles	' – fuel with moisture and ash
$u_{B_{sc}}$ – velocity of a single bubble in a fluidized bed	a – ash
u_f – velocity of the gas in the FB	c – combustion, carbon
u_{mf} – minimum fluidization velocity	d – devolatilization
y – chemical components mass fraction	f – fluid
Y – mass fraction of the atomic species	fl – liquid fuel
<i>Greek symbols</i>	fu – fuel (volatiles)
α_w – ratio of the volume of particles dragged in the wake and the bubble volume	h – hydrogen
δ_B – proportion of the volume of bubbles in the total volume of the bed	i – zone number
λ – excess air coefficient	j – chemical species
μ_f – dynamic viscosity of the fluid	n – nitrogen
v_j – mole number	o – oxygen
ρ – density	p – particle
ρ_w – bubble wake density	s – sulfur
τ – time	w – moisture

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