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MONOGRAPH
Mathematics Education for changing

THE CHALLENGE OF MATHEMATICAL DISCUSSIONS IN TEACHERS' PROFESSIONAL PRACTICE

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Abstract: We seek to identify patterns of action in mathematics teachers' classroom practice regarding class discussions and to determine how these actions provide learning opportunities for students. The study is based on a framework that focuses on two key elements of teaching practice: 1) the tasks that teachers propose to their students and, 2) the way they handle classroom communication. Qualitative methodology is used with data collected from video recordings of a grade 6 class that is studying rational numbers. We conclude that challenging situations usually require teacher preparation and follow-up with supporting/guiding and informing/suggesting actions so that the students can learn what is involved in the teacher's motivations.

Keywords: teacher practice, classroom communication, teacher's action, challenge, rational numbers.

RETOS EN LOS DEBATES MATEMÁTICOS EN LA PRÁCTICA PROFESIONAL DE LOS PROFESORES

Resumen: *En este artículo nos proponemos identificar patrones de acciones en la práctica de los profesores de matemáticas en el aula durante debates en clase y averiguar cómo estas acciones proveen a los estudiantes de oportunidades de aprendizaje. El estudio se centra en dos elementos clave de la práctica docente: las tareas que proponen los profesores a los estudiantes y la manera en que se lleva a cabo la comunicación en el aula. La metodología es cualitativa con datos recogidos a través de la grabación de un grupo de 6º curso que estaba estudiando los números racionales. Nuestra conclusión es que las situaciones de aprendizaje desafiantes requieren normalmente de preparación y seguimiento con acciones de apoyo/guía e información/sugerencia para que los estudiantes puedan alcanzar los objetivos establecidos por el profesor.*

Palabras clave: *práctica docente, comunicación en el aula, acciones del profesor, retos, números racionales.*

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Introduction

In an exploratory approach to mathematics teaching, the students assume an active role in interpreting the tasks proposed by the teacher and in constructing their own solving strategies, using mathematical representations in a flexible way (Ponte, 2005; Ruthven, 1989). Without an immediately applicable solution method to deal with such tasks, the students are called to mobilize their knowledge, constructing and deepening their understanding of concepts, representations, procedures, and other mathematical ideas. They are also encouraged to present and justify their solutions and to question the solutions of their colleagues, justifying their reasoning, and thus developing their communication and argumentation capacity.

An exploratory lesson usually involves three phases (Christiansen & Walther, 1986; Ponte, 2005): (i) presentation and interpretation of the task, carried out in whole class; (ii) undertaking of the task by students individually, in pairs, or small groups; and (iii) presentation and discus-sion of the students' strategies and results and final synthesis, again in whole class. The teacher, rather than teaching directly procedures and algorithms, showing examples and giving exercises to practice, promotes a guided discovery work (Gravemeijer, 2015), at the same time that provides moments of negotiation of meanings and whole class discussions.

In this paper, we center our attention in the moment of the discussion. We do not intend to establish a normative framework, with «musts» and «don'ts», but to analyze the phenomena that take place in this classroom moment, striving to understand the situations that may arise and the possible alternatives that the teacher may follow. In such a lesson, a great variety of situations may occur as a consequence of factors such as the students' age level, their mathematical capacity, the classroom culture, the topics under study, as well as of factors such as the teacher's concerns with students' assessment, the school policies, the textbooks and other resources available, the physical conditions of the classroom, the views of the parents about mathematics education, and many others. Our purpose is essentially analytical, seeking to contribute towards the construction of a conceptual framework that may be useful to researchers and teachers in the analysis, unders-tanding and preparation of situations of whole class discussions conducted within an exploratory approach. In this paper, specifically, we seek to analyze the patterns of actions that the teacher is called to carry out during whole class discussions and to know how these actions provide learning opportunities for students.

1. Teacher's actions in an exploratory approach

The exploratory approach is framed by the nature of the proposed tasks, and by the kind of communication that takes place in the classroom, closely related to the ways of organizing the work of the students (Ponte, Branco, & Quaresma, 2014). Tasks are of paramount importance because of the students' activity that they may originate. In fact, as Christiansen and Walther (1986) indicate, what students learn in the mathematics classroom mostly results from the activity that they un-dertake and from the reflection that they carry out about that same activity. So, it is fundamental to choose appropriate tasks that may support a rich and multifaceted mathematical activity from the students. Ponte (2005) suggests that different kinds of tasks such as exercises, problems, in

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vestigations and explorations may be useful for different purposes. Structured tasks with a small degree of challenge (exercises) mostly aim the consolidation of knowledge and structured tasks with a high degree of challenge (problems) aim the creative application of the knowledge that the student already has; open tasks with a high challenge (investigations) aim the creative use of already known representations, concepts and procedures and open tasks with some level of challenge (explorations) mostly aim the construction of new knowledge. To select the tasks according to the objectives defined for each lesson, taking into account their fit the students which will carry them out, is an important aspect of the teachers' role.

Different ways of work may be used to undertake these tasks in the classroom. A possibility is the whole class, or collective, mode in which the teacher interacts with all students and these also interact among them. It may be also used individual work seeking to foster the student's concentration and reflection capacities. Other possibilities are group work or work in pairs, aiming to provide students the possibility to exchange impressions with their colleagues, mutually helping each other. In this way, the students may participate at two levels of classroom discourse – the collective that includes everybody in the classroom and the private that they carry out with just a few colleagues.

The kind of classroom communication frames in a decisive way students' learning oppor-tunities (Bishop & Goffree, 1986; Franke, Kazemi, & Battey, 2007). Such communication may be univocal, when it is dominated by the teacher, and dialogical, when students' contributions are highly valued (Ponte, 2005). It is the teacher that has the responsibility to define the communica-tion patterns, propose the tasks to be carried out, and define the ways of working in the classroom but needs to do a constant negotiation of the classroom norms (Yackel & Cobb, 1996) with the students – which often is very far from easy. An important aspect of the work of the teacher is the way how he/she seeks to support in a subtle way the students in mastering the mathematical language. An important tool to do this is through revoicing, that is, reformulating the students' statements in a progressively more correct language. In addition, the teacher may assume the sole role of mathematical authority or share it with the students, seeking to stimulate their reasoning and argumentation capacity.

A particular form of communication are mathematical discussions, with several participants assuming, all of them, a role of authority regarding their ideas. A classic example is the fictional lesson of Lakatos (1978), in which the students alpha, beta and gamma and the teacher, in a climate of heated discussion, present conjectures, arguments, counter examples, new conjectures and new counter examples, proving and refuting statements about polyhedra, and thus constructing new mathematical knowledge. Another example, but based in real lessons, is given by Lampert (1990), showing how grade 5 students present their solutions and justify their reasoning in questions involving powers of whole numbers. These are interesting examples, but they raise the question of knowing what is necessary so that they may be regular classroom practice.

A fundamental starting point is, of course, the tasks proposed by the teacher. In an exercise, what is at stake is the selection and application of a solution method that students already know, and very often this does not lead to a very interesting discussion. More fruitful situations may occur as a consequence of challenging tasks that tend to yield a diversity of students' strategies that

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may be compared and evaluated. In addition, it is necessary that the students have the opportunity to work in a productive way on the task, organizing their solutions to present to their colleagues. In such conditions, the teacher needs to prepare the discussion, taking into account the work carried out by the students and the available lesson time. To do this, Stein, Engle, Smith and Hughes (2008) propose what they call five practices that go beyond show and tell (i) anticipating possible students' difficulties; (ii) monitoring students' work, collecting the necessary information; (iii) selecting the aspects to highlight during the discussion; (iv) sequencing students' interventions; and, during the discussion, (v) establishing connections among students' different solutions. Such preparation is an important support for conducting a discussion. However, to lead a discussion involves many aspects besides the establishment of connections that is impossible to foresee before its beginning and that the teacher must be prepared to face. That is the reason why it is necessary to take a closer look at the dynamics of discussions.

In leading a class, the teacher draws on the established action plan and keeps making decisions, some relatively straightforward and other requiring complex deliberations (Schoenfeld, 2010). Potari and Jaworski (2002), to analyze the teaching and learning process in the classroom, created the model of the teaching triad, with three main elements: mathematical challenge, sensitivity to students and management of learning. They suggest that a suitable harmony between mathematical challenge and sensitivity is critical to support students' learning. Very strong challenge that does not take into account students' ability is likely to lead to their anxiety or lack of interest in responding. Very weak challenge, that oversights students' capability, is likely to drive them off task. Focusing her attention in the discussion moments, Wood (1999), in a study carried out in a primary school class, calls the attention for the potential of exploring disagreements among students as starting point for fruitful discussion moments. When two students are in disagreement, the teacher seeks that they justify their positions and encourages the remaining students to get involved in the discussion. The author shows how this strategy may yield excellent argumentation moments in the classroom. More recently, Cengiz, Kline and Grant (2011) developed a framework to analyze the actions of the teacher in conducting mathematical discussions. They propose a distinction between three fundamental kinds of actions, which aim is to lead students to present their methods (eliciting actions), to support their conceptual understanding (supporting actions), and to extend or deepen their thinking (extending actions). In their perspective, the more frequent and efficient are extending actions the successful will be the lesson. Drawing on elements from these different models, Ponte, Mata-Pereira and Quaresma (2013) developed a framework to analyze discussions that establishes a distinction between the actions of the teacher directly related to the mathematical topics and processes and actions that are related to classroom management (Figure 1). Centering their attention on the actions related to mathematical aspects, they distinguish four fundamental kinds:

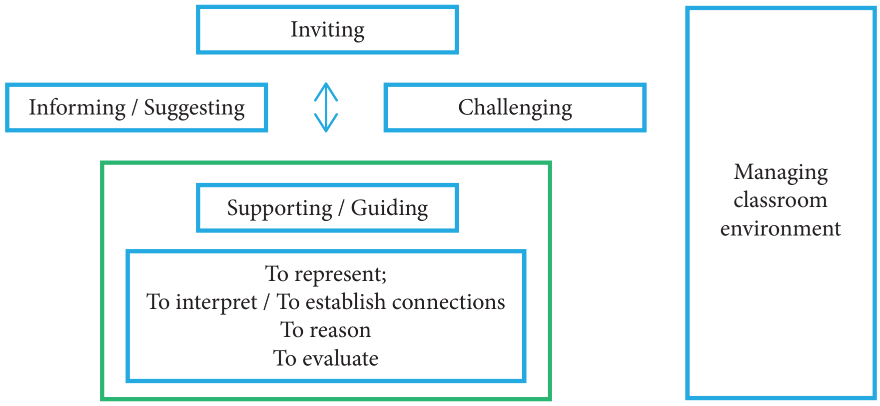

Figure 1: Framework to analyze the actions of the teacher.

- Inviting, actions aimed at initiate a discussion;
- Supporting/Guiding, actions intended to lead students in solving a task through questions or observations and that point, in an explicit or implicit way, the path that they may follow;
- Informing/Suggesting, actions in which the teacher introduces information, gives suggestions, present arguments or validates students' responses;
- And, finally, challenging actions in which the teacher seeks that students assume the role of producing new representations, interpret a statement, establish connections, or formulate a reasoning or an evaluation.

These four kinds of actions may be found in lessons with very different characteristics, with different frequencies and roles that is interesting to study. What differentiates informing/suggesting, supporting/guiding, and challenging actions is the amount of information provided by the teacher, which is explicit in the first kind of actions, implicit but clearly identifiable in the second, and not identifiable (by most students) in the third. Inviting actions usually mark the beginning of a discussion segment.

In all these actions one recognizes fundamental aspects of mathematical processes such as to represent (both in the same language as converting to a different representation), to interpret (revoicing a statement using different words and establishing connections with other concepts), to reason (making inferences, that is, arriving at new conclusions, with backing in the provided information) and to evaluate (making global judgments about the aspects related with solving the task). As Bruner (1999) indicates, representations may be active, iconic or symbolic, involving oral and written language, gestures, drawings, diagrams and mathematical symbols. Without representations it is impossible to work in mathematics and the success in solving a problem largely depends on an appropriate choice of the representations to use. The interpretation (sense making) is essential so that the mathematics work has meaning, what requires both an understanding of the nature of the task as of the details, regarding other concepts and experiences. Reasoning includes to develop problem solving strategies and to make conjectures and generalizations (inductive and abductive reasoning) and to present justifications (deductive reasoning), eventually organized in chains of inferences (proofs). Finally, evaluation includes all aspect that lead to appreciate the value of a concept, of a representation or a strategy of solving a task.

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2. Research methodology

In this paper we present several segments of the activity that took place during the discussion of a task that the teacher selected aiming to lead grade 6 students (11 years old) to develop the notion of operator in a problem solving context. Our aim is to find patterns in the teacher's actions. During a part of the lesson the students worked in pairs and in the other part there was a whole class discussion. The teacher who conducted this lesson (the second author of this paper) has 6 years of experience and strives to put into practice in her lessons an exploratory approach. The students are from a grade 6 class of a public basic school of a rural socially deprived region, 50 Km away from Lisbon. The students' parents, in general, are lower class or middle/lower class whose schooling did not go beyond grade 6 or 9. The class has 19 students, from whom 4 were already retained at some point in previous academic years, whose ages vary among 12 and 17 years. The students have little involvement in the school activities and depict little working habits. Six lessons on rational numbers were video recorded and the whole class discussions were fully transcribed. In this paper we focus on several segments of the whole class discussion of a task in one of these lessons. Data analysis begun by identifying the segments in the discussion in solving this task, codifying the teacher's actions according to the categories presented in Figure 1. Then, we sought to establish relationships between these actions and key events concerning the representations, interpretations, and reasoning (strategies, generalizations and justifications) made by students.

The task asks students to use fractions as multiplicative operators to determine a part of a whole. It is a contextualized situation, involving a discrete magnitude, where the information is given under the form of whole numbers and fractions. It is sought that the result is a whole number and students are asked to justify their responses. It is an exploratory task since the students had not solved situations like this before in the mathematics class, so they had to give meaning to the question and seek ways of solving it.

> Task. For her birthday party, Rita brought 250 candies to give to her friends. She decided to give $\frac{1}{5}$ to the swimming friends, $\frac{3}{5}$ to her school colleagues, and she kept $\frac{2}{10}$ to give to the guests of her birthday party. How many candies did Rita gave to her swimming friends? Justify your answer.

Segment 1: A wrong answer as a starting point

After the students solved this task in pairs, the teacher begins a whole class discussion inviting some students that had wrong answers to present their solution. She seeks, in this way, to create opportunities to disagreements and for explanations and justifications to emerge. Daniel accepts the invitation and goes to the board to present the solution that he had got jointly with Marco, which shows several operations carried out with whole numbers (Figure 2).

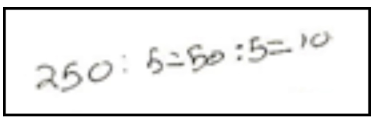

Figure 2: First solution of Daniel at the board.

These computations are obviously wrong since 250 : 5 is not equal to 50 : 5, and the final value obtained is not the answer to the task. In their strategy to solve the task, the students seemed

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to make use of the correct idea that to get ⅕ of an amount one may divide that amount by 5, but then went on dividing again the result by 5.

To focus the attention of all students, part of whom seemed to be absent minded, the teacher recalls again the statement of the task and asks Daniel to explain his reasoning. In his response, the student just uses symbolic representations and says that «everything is already explained» in the computations recorded at the board. He shows difficulty in providing further oral justifications for his strategy, seeming to consider that the computations speak for themselves and have all the necessary justifications. However, these symbolic expressions need to be interpreted with reference to the situation and the teacher reinforces the invitation, insisting with the student (through supporting/guiding actions) to explain how he solved the question. She encourages Daniel to speak, at the same time that she seeks that the other students do not intervene in the situation:

> Teacher: Not everything is explained there, Daniel... I do not understand, I look at there... And I need your help... Let Daniel explain... The way he thought.
>
> Daniel: In swimming, 250 is the number of candies to be divided by 5 which is the denominator, to see how much this would value, altogether...

Daniel says that to know how many candies the swimming friends get, he divided 250 by 5. The process used by the student (dividing the given amount by 5) is correct, but since he had more things written at the board the teacher considers that the explanation is not sufficient. So, she asks him to explain where the fraction indicated in the statement operates, completing the solution with that information. She helps the student in completing his representation with verbal labels (as shown in Figure 3), and then she guides him in continuing his explanation by posing questions that support the interpretation of the situation:

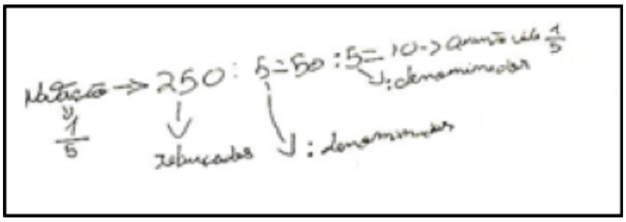

Figure 3: Solution of Daniel, completed by the student after the suggestions of the teacher.

> Teacher: So, put below swimming, put there fraction, please, so that we understand what you are saying... Below the word swimming put the fraction of the can-dies that she gave to the swimming friends. What was the fraction?
>
> Students: ⅕.
>
> Teacher: All right, OK, just for us to understand what you are speaking of... So, when you say that you divided by 5 it is because 5 is the denominator of that frac-tion ⅕... Go on...
>
> Daniel: And it gave 50.
>
> Teacher: And what does that 50 represents?
>
> Daniel: That 50 means how much values this 5 (points towards the denominator of the fraction ⅕)... And I did 50 divided by 5 again because of the denominator to see how much that one would value. It gave 10…

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In their solution, Daniel and Marco had used twice the operator ⅕, getting 10 as their response, but they have trouble in explaining why. Seeking to understand their reasoning, the teacher asks Daniel what is the meaning of the value 50 that he got in the first place. This is a critical question, which has a challenging nature because it seems that the student did not interpret 50 as the number of candies to give to the swimming friends and computed again ⅕ of this value. However Daniel seems to be «blind» by the symbolic representation and does not interpret his response in the context of the task, only indicating the computations that he did to get first 50 and then 10. His explanation is purely «computational» and he is unable to provide a clearer account of his thinking. At this moment, the teacher makes an important decision – instead of correcting the mistake of Daniel, she challenges other students in the class to assume a stand, seeking to promote the emergence of disagreements. That originates a new segment.

Segment 2 – Analysis of two correct responses

In response to the challenge of the teacher, several students voice immediately their disagreement with the answer of Daniel and Marco, showing their perplexity or pointing out the correct response:

> Students: And it gave 10 candies?
>
> I do not think so…
>
> I think it is just 50…
>
> Me also, I think it just gives 50…
>
> I also think that it gave 50…

One of these students, Jaime, asks to show his reasoning and the teacher gives him the opportunity to do so. Instead of presenting arguments to contradict the reasoning of his colleague, Jaime presents his own reasoning, based on the decimal representation. He transformed ⅕ in a decimal number, getting 0.2, and used this value as a multiplicative operator (Figure 4). His strategy was to make a change of representation, from a fraction to a decimal number, and to use his knowledge of multiplication of a decimal number by a natural number to solve the task. Jaime understands that the result of Daniel and Marco is incorrect because it is different from his, but did not identify (and much less did explain) the mistake of his colleagues.

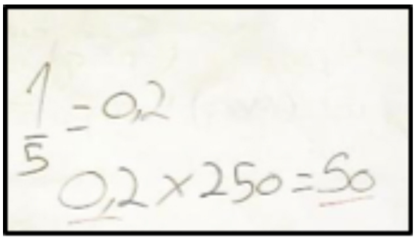

Figure 4: Answer of Jaime.

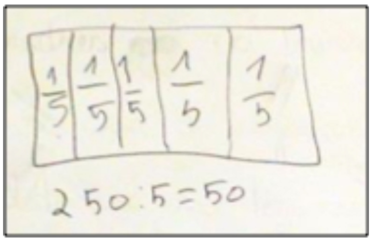

Figure 5: Answer of Vasco.

There are no voices against the solution of Jaime that seems to be accepted all other students. Vasco, who is already used to the norm that different solutions may be presented in the mathematics class, states that there is yet another way of solving the question. The teacher welcomes this offer and the student presents a solution based on a diagram representation (a rectangle divided in «slices») with symbolic elements (⅕ assigned to each slice) (Figure 5). This strategy is also based in a change of representation, and from that moment on it becomes the main reference in the discussion.

Vasco explains that each «slice» of the rectangle represents ⅕ and that to know the value of each «slice» he divided 250 by 5, getting 50 candies, which is the amount Rita must give to her swimming friends.

Therefore, a connection was established between the symbolic representations 250 : 5 (an already known operation involving natural numbers and expressed in symbolic language), 0.2×250 (another known operation involving natural and decimal numbers, also expressed sym-bolically), and a diagram depicting the process of dividing a rectangular whole in five equal parts. For the teacher, this created and agenda of exploring the connection among these representations and 250 × ⅕ to establish the meaning of n×a ⁄ b (a new operation for the students, involving a natural number n and a non-unit fraction a ⁄ b).

The way the teacher encouraged Daniel to present his reasoning in segment 1 created conditions for the emergence of expressions of disagreement from several students in segment 2 and the spontaneous offer first of Jaime and then of Vasco to present their solutions.

Jaime solved to problem by a change of representation from fractions to decimal numbers and Vasco by another change of representation, a diagram that helps to see that ⅕ is one fifth of the unit, that is, one of the parts that we get when we divide the whole in five equal parts. During this segment the teacher's actions are all supporting/guiding the students in explaining their responses. The explanation of Vasco seems quite clear, but the teacher is not sure that it was understood by all students and decides to lead a further discussion around it, which brings us to segment 3.

Segment 3 – Comparing a correct and an incorrect answer

The teacher recalls the disagreement between the solution of Daniel and Marco and the solutions of the other students:

> Teacher: We still have our problem… How many candies do we give to the swimming friends? 10 or 50?
>
> Guilherme: 50.

Guilherme immediately says 50, but the teacher does not know what the remaining students think. So, she decides to make a quick survey to know who the students that support one or another solution are:

> Teacher: 50, why? So you think... Who thinks it is 50, raise their hand. Ah, very well, so... You think it is 50. And you do not think it is 10... So, why it is not 10?

As the teacher verifies that the number of students who support the solution of Vasco is very high, she challenges them to present further arguments against the solution of Daniel and Marco. A student says «because that one is wrong», an argument that the teacher does not accept as valid, asking for a more complete justification:

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> Teacher: Take it easy... Because yes, it is no answer. Let us try... Jaime why do you think that is not 10? And I do not accept as an answer, «because I think it is 50». OK, that part I already understood. Now I want you to try to explain why... That solution is not correct.

Given this challenge, Jaime provides an explanation based on the diagram presented by Vasco:

> Jaime: I do not know about mine, but I know about Vasco's [response]. All that is...
>
> Guilherme: 250.
>
> Jaime: [All that is] 250 candies and each bit of that [is] ⅕... That is only asking ⅕ and we, let us see, that is, in decimal numbers how much values each bit and she only asks ⅕ for each... So, all that [is] 50.
>
> Guilherme: It is just a part [parcela].

In this way, Guilherme helps Jaime in his explanation. He seems to be thinking in terms of the diagram representation of Jaime (Figure 5) as meaning ⅕+⅕+⅕+⅕+⅕, so each ⅕ is an «addend» which sum provided the unit 1. This activity is very rich in terms of the students' interpretation of the situation, relating the fraction representation to the situation described in the task. An interesting aspect is that we see one student (Guilherme) providing an explanation based on a representation provided by another student (Vasco).

The teacher puts then another challenge, asking the students to indicate the differences between the two solutions:

> Teacher: So why is it?... What happened here? What happened here Daniel (points to his solution)? What is the similarity and what is the difference [between both solutions]... There is one difference...
>
> Daniel: It is in dividing the 50 by 5.
>
> Daniel recognizes that the difference is in the division of 50 by 5. That leads the teacher to seek to know why he had made that operation:
>
> Teacher: Why? Why did you divided 50 by 5?
>
> Daniel: I thought that... To see how much was worth the 5.
>
> Teacher: But here, when you got here what does it mean to divide by 5, what does it mean this division by 5? When you divided this by 5 what were you seeking to find?
>
> Daniel: The 50.
>
> Guilherme: ⅕.

The teacher challenges Daniel to justify why he divided 50 by 5. However, once more, the student answers only with the result of the operation and does not explain his solution strategy, and therefore, the meaning of the result 10 in the context of the problem. Finally, Guilherme intervenes again, indicating that the aim was to determine ⅕, what is the same as dividing by 5.

To end up this discussion, using guiding questions, the teacher promotes a negotiation of the meaning of the expression ⅕ relating it to the «fifth part». This exchange is carried out with reference to the diagram representation provided by Jaime and ends up with an informing action that summarizes the meaning of fifth part:

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Teacher: One fifth. Tell me another way of saying one fifth... How did you in the 1st cycle[1] say one fifth... You used not to call this one fifth... How used you to say the half [a metade]? Oh! One of two equal parts [um meio]... I already said... How did you say one of two equal parts? You called it a half. How did you say one third at the 1st cycle?

Guilherme: The third part.

Teacher: The third part, OK. So when you here divide by 5 what are you doing?

Edgar: The fifth part...

Teacher: They are figuring out how much is the fifth part of...

Student: 250.

Teacher: 250, and, did you understand?... That this whole are the 5/5 and if we divide the 5/5, that is, the whole, in five equal parts we find... The...

Juliana: The fifth part.

Teacher: Ah! I discover its fifth part! Very well...

In this segment we note the exploration of the disagreement related to different solutions made by the teacher. She invites the students to consider the two responses presented (10 and 50) and challenges students to indicate what they regard as the mistake made by Daniel and Marco. Several students provide contributions for an explanation, based on the diagram representation. The teacher still poses a new challenge, asking the students to indicate the differences between both solutions, and Daniel recognizes that he made two divisions and not just one. The teacher further challenges Daniel to justify why he made a second division. Finally, the teacher uses the opportunity to promote a negotiation of meanings of the «fifth part», using guiding questions and ending with an informing action.

Segment 4 – Generalization: How to multiply a fraction by a natural number

In the previous discussion it was established that to find the fifth part corresponds to divide a quantity by 5. The teacher challenges the students to make a generalization:

Teacher: So we will try to get to a more general conclusion, you tell me...

Rui: Whenever we want to make a computation like that ($\frac{1}{5} \times 250$)...

Teacher: Yes.

Rui: It is just to divide the denominator by that thing that is before... That in this case are the candies.

Teacher: How is it? Explain that... Give me more examples...

Rui: For example, ¼, if it is another example, how many candies? 150 for example... Whenever there is a computation like that, I can do the 4 or the denominator to be divided by the number...

And it yields the result. As his colleagues, Rui also presents difficulties in mathematical

1 These students are in the 2nd cycle (grades 5 and 6). By saying «1st cycle», the teacher refers to the previous school years students had attended (grades 1 to 4).

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communication interchanging the names of the terms of the fraction and of the division. However, one understands that he intends to say that in a situation in which a unit fraction has the operator meaning, it is just necessary to divide the cardinal of the given set by the denominator of the fraction. Given the generalization made by Rui, albeit using a rather confusing mathematical language, the teacher decides to challenge him to apply this generalization to non-unit fractions:

> Teacher: So, now I am going to ask you a question... Does that apply if I have 2/4×250?

Rui strives to find a relationship but he is not able to. Guilherme asks to speak and extends Rui's generalization to the multiplication of a natural number by a non-unit fraction:

> Guilherme: May I teacher?
>
> Teacher: You may. Guilherme tell me... You think it is all right, or you think it is wrong and why... What is the result that you think that...
>
> Guilherme: I think that... It may be done in the same way but it is necessary to add something...
>
> Teacher: We need to follow... Oh Rui, that is yours... He is saying that is not true, you need to defend yourself... Look Guilherme, so, continue your explanation.

The teacher begins by supporting the explanation of Guilherme asking him to say what he thinks about what his colleague already said. This student seems to understand the situation, but also shows difficulty in using mathematical language, and therefore the teacher decides to support his explanation revoicing his statements in a mathematically more correct way:

> Guilherme: We may do 150 divided by 4... (...) It gives 37.50.
>
> Teacher: So 150 divided by 4, so, pay attention…
>
> Guilherme: 150 divided by 4, then we make the result times the denominator.
>
> Teacher: The one on the top or at the bottom?
>
> Guilherme: The one at the top...
>
> Teacher: Ah, the numerator.
>
> Guilherme: Numerator...
>
> Teacher: OK, so, let us go, let us see, let us move on... So we would do... What does it mean to do... How much is 150 divided by 4. First, I have to know what it means 150 divided by 4... What is this 37.5?
>
> Guilherme: It is ¼ of 150.
>
> Teacher: It is ¼ of 150, OK. So... Here I want... How many fourths?
>
> Guilherme: 2 fourths! That is why we are going to make times 2.

With the help of the teacher, Guilherme is able to reformulate correctly Rui's generalization and answer the challenge put by the teacher.

In this segment, the teacher challenges the students to make a generalization about what may be a multiplication of any fraction by a natural number. In this class, it is uncommon that the students assume the initiative of making generalizations, but with suitable challenging and

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guiding questions they achieve it. We highlight the successive interventions from the teacher that lead the students to improve their explanations as well as her care in revoicing the statements of the students to help them to improve their mathematical language.

Conclusion

In this study we see the mathematics teacher leading a whole class discussion on an exploratory task. The teacher initiates the discussion with the analysis of a wrong response of a student that she had previously identified as a promising starting point. Then, she accepts the offer of other students to present their answers and promotes the analysis of two responses based on different representations (decimal number and diagram). In a third moment, the teacher promotes the detailed comparison between the (correct) solution based on the diagram and the initial (incorrect) solution based on a symbolic representation. Finally, she seeks to lead the students to verbally formulate the generalization concerning the multiplication of a fraction by a natural number that constitutes the aim of the lesson.

In the first segment, the teacher seeks to explore a wrong response to make incorrect ideas visible regarding how a fraction may be used as an operator. Most of her actions go in the direction of supporting/guiding a student to present his solution. However, in this segment, the teacher has two fundamental challenging actions, the first when she asks the student what is the meaning of the 50 that he got in his computation and the second when she challenges the other students to take a stand regarding the presented solution. The key point lies in the interpretation of the meaning of the computation and of the obtained value. In the second segment, the teacher seeks to explore disagreements among students (Wood, 1999), inviting several students to present their solutions. Her interventions are subtle, mostly supporting/guiding. She uses the introduction of a representation as a diagram from another student to support the discussion from that moment on, and seeks to establish connections among different representations. In this way, the aspects related to representations acquire great salience. In the third segment, to lead students to justify their responses, the teacher poses several challenges, based on this diagram, so that they compare the different solutions and find the mistake in a solution. In the final part, using guiding/suppor-ting questions, the teacher promotes the negotiation of the meaning of «fifth part», reinforcing the interpretation of this notion that underlies all the work on this task but that not all students seem to understand very well. Finally, in the fourth segment, to lead students in formulating a generalization concerning the multiplication of a fraction by a natural number, the teacher poses two challenges. She achieves this based on supporting/guiding actions, often revoicing students' contributions. The sought generalization is established, first in a simple way (for unitary fractions) and after in a more general form (for any fractions). In these segments the teacher makes challen-ging actions at critical moments, paving the way mostly with supporting/guiding actions (Ponte, Mata-Pereira, & Quaresma, 2013; Potari & Jaworski, 202).

The segments that we analyze show productive working moments that result from the exploratory approach (Ponte, Branco, & Quaresma, 2014; Ruthven, 1989) followed in this class. In fact, the students could not solve in an immediate way the task involving the multiplication of a fraction by a natural number and they had to draw on their previous knowledge. Besides the task, we also note the communication established by the teacher, framed by an incentive to students' participation, in a dialogic register, asking them questions, valuing their contributions, revoicing their interventions to support them to improve their mathematical language, and leading, when necessary, negotiations of meaning. We also note the attention of the teacher to the mathematical processes, especially representing, interpreting and reasoning, notably seeking the clarification of solving strategies and the production of justifications and the establishment of generalizations. The central attention to these processes, more than the simple emphasis on following procedures is a central feature of the exploratory approach, creating a variety of learning opportunities for students. Throughout these different segments there were many opportunities for students' participation, and high attention was given to important mathematical issues, showing how a balance was achieved in the establishment of a discourse community in the classroom (Sherin, 2002). As we saw in this paper and also in previous studies (Mata-Pereira, Ponte, & Quaresma, 2015; Ponte & Quaresma, 2015), intertwining challenging and guiding actions seems to be a key feature of the work of the teacher in exploratory environments. Figuring out how to scale up this perspective about classroom work will be another step in our research program.

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