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Vehicle Routing Problem with multiple trips and time constraints (VRPMTTC): A Case Study

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Bibliographic Note

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Abstract

The purpose of this dissertation is to create a model capable of improving the distribution of blood and other biological products collected from several health centers and delivered to a central warehouse. A real case study is used to address a Vehicle Routing Problem with Multiple Trips and Time Constraints (VRPMTTC), which is a variant of the classical Vehicle Routing Problem (VRP). The blood has a special treatment because it has a due date associated: the blood has to arrive to the central laboratory at most two and a half hours after extraction at the health centers.

A Mixed Integer Linear Programming (MILP) model is proposed to solve this problem. The purpose is to minimize the total traveled distance while fulfilling a large set of constraints such as the time limit previously mentioned. The solution will provide the order in which the health centers should be visited and at which time, according to the opening and closing hours of the health centers, the service times, travelling times and the blood due date. The results provided by the model show that there are several solutions possible for the same total traveled distance. The solution obtained in this case study prevents negative impacts on the environment because no blood will be wasted since the time limit previously mentioned will be respected.

Keywords: Vehicle Routing Problem; Vehicle Routing Problem with multiple trips and time constraints; Optimization; Minimization

Resumo

O objetivo desta dissertação é criar um modelo capaz de melhorar a distribuição de sangue e outros produtos biológicos recolhidos em vários centros de saúde e transportados para um armazém central. É usado um caso de estudo real para resolver o problema de escalonamento e roteamento de veículos com múltiplas viagens e restrições temporais, que é uma variante do clássico problema de roteamento de veículos. O sangue tem um tratamento especial pois tem uma data de caducidade associada: o sangue tem de chegar ao laboratório central no máximo duas horas e meia após a sua extração nos centros de saúde.

É proposto um modelo de programação linear inteiro misto de forma a resolver este problema. O objetivo é minimizar a distância total percorrida e juntamente cumprir uma grande variedade de restrições tais como o limite temporal referido anteriormente. A solução indicará a ordem pela qual os centros de saúde devem ser visitados e a que horas, de acordo com as horas de abertura e de fecho dos centros de saúde, os tempos de serviço, os tempos de viagem e a caducidade do sangue. Os resultados fornecidos pelo modelo indicam que existem diversas soluções possíveis para a mesma distância total percorrida. A solução obtida neste caso de estudo permite eliminar o impacto negativo no ambiente já que o sangue não será desperdiçado, uma vez que o tempo limite previamente mencionado é respeitado.

Palavras-Chave: Problema de roteamento de veículos; Problema de roteamento de veículos com múltiplas viagens e restrições temporais; Otimização; Minimização

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Acronyms

CVRP Capacitated vehicle routing problem

ERDF European Regional Development Fund

FCT *Fundação para a Ciência e a Tecnologia*

FEP *Faculdade de Economia da Universidade do Porto*

HPH *Hospital Pedro Hispano*

ILS Iterated Local Search

LP Linear Programming

MILP Mixed Integer Linear Programming

MTVRP Multiple trips Vehicle routing problem

OPL Optimization Programming Language

PRA Path-relinking algorithm

VRP Vehicle routing problem

VRPB Vehicle routing problem with backhauls

VRPMT Vehicle routing problem with multiple trips

VRPMTTC Vehicle routing problem with multiple trips and time constraints

VRPMTTW Vehicle routing problem with multiple trips and time windows

VRPSD Vehicle routing problem with split deliveries

VRPSDP Vehicle routing problem with simultaneous deliveries and pickups

VRPTW Vehicle routing problem with time windows

Chapter 1 – Introduction

The dissertation emphasis is a specific case study, therefore the data being used is a practical application and was previously used in a master thesis (Carneiro, 2019).

This specific case study fits the human health sector, more specifically the clinical analysis. There was a big increase in the number of companies representing Portugal in this sector in the past two decades. This industry is very important because it allows to improve the quality and access of citizens concerning health care[1].

It is safe to say that every day, people all around the world in different organizations and countries are generating a lot of data anchored with the development of the digital world.

So, nowadays there exists data everywhere around us. Often these data need to be carefully analyzed in order to allow for decision making.

The Vehicle routing problem (VRP) is one of the most significant problems in distribution management. The main objective of this kind of problems is to find optimal routes for distribution of various shipments, such as goods, mail, raw materials among others (Bräysy, 2009).

Several optimization problems have been addressed. Generally speaking, the aim is to optimize a certain goal that can be described by a mathematical expression to be minimized or maximized subject to a set of constraints. These constraints limit the scope of decisions by imposing conditions that must be satisfied.

The goal of this specific case study is to improve the distribution of blood and other biological products collected from several health centers to a central warehouse. Many companies in different sectors face similar management problems while planning their daily work. In the present case study questions such as: "Which health center should be visited first?" need to be answered.

The problem in this dissertation will be formulated as a Mixed Integer Linear Programming (MILP) model that minimizes the total traveled distance. The model will

[1] Pordata:
https://www.pordata.pt/Portugal/Empresas+total+e+por+sector+de+actividade+econ%c3%b3mica-2856

be solved by using the software IBM ILOG CPLEX Studio IDE version 12.9.0[2]. By minimizing the total traveled distance, consequently the total traveled time will be minimized too. The time saved can be used in different ways, for example, the companies can acquire more clients and subsequently improve the incomes.

Motivation

This study represents the last step of the Master in Modelling, Data Analysis and Decision Support Systems and the choice of the theme came from the will of learning more about optimization problems. Every day each of us needs to make decisions that might be easy or very difficult. The same happens with the companies. Some decisions might be very hard to make because they have a lot of information to consider as well as some complex or incomplete information. Nowadays, companies have the support of many tools and computer programs in order to make hard decisions in an easier way. The tools and computer programs are used considering company's own characteristics. Despite of having the opportunity to use that kind of tools, wrong decisions may be made and good decisions can be improved.

The tools exist but they need to be managed correctly and the person that is using them needs to understand how the tool works. A lot of companies try to standardize processes for similar tasks in order to facilitate the decision making of the persons that might be in charge.

The choice of VRPMTTC was suggested by Professor Dalila Martins Fontes. This problem can be applied to real life cases, which constitutes a big incentive to proceed with this study.

This dissertation will allow to apply some of the knowledge acquired during the master.

Clinical analysis is of crucial significance around the world and belongs to the healthcare field that consequently impacts the public sector.

Once extracted, blood and other biological products, here and hereafter designated only as blood, need to be delivered to the laboratory where they are tested and analysed. There is a time limit to collect, transport, and deliver this type of products after extraction. The

[2] https://www.ibm.com/support/pages/detailed-system-requirements-ibm-ilog-cplex-optimization-studio

optimization of the delivery process leads to better and cheaper delivery. If delivered on time, no material is wasted which in turn leads to more suitable operations both financial and environmental.

More precisely, my motivation relies on the following reasons. The optimization of the delivery process may have big impact in several companies. Thus, trying to optimize the routes on a daily basis is an important work to do because it will help to minimize the total distance/time performed by the vehicles. In addition, the optimization of the routes is very important because of the possible waste associated that if present will have an impact environmentally and financially. For example, in a clinical laboratory where it exists a time limit associated to the blood delivery since it is extracted until it arrives to the place where it will be analysed, if this time limit is not met, another blood extraction will be necessary, and the first extraction will be wasted. For this reason, the environment is a strong motivation for studding this optimization problem.

Outline of Dissertation

The dissertation is organized as follows: In Chapter 2, Literature Review, the most important references related to our work as well as some concepts are discussed. Chapter 3, Problem definition and formulation, provides the description of the problem and of the specific case study. In Chapter 4, Methodology, the data regarding the case study is presented and the solution and the computational experiments are reported. Finally, in Chapter 5, Conclusions, some conclusions are pointed out.

Chapter 2 - Literature Review

The big increase in transportation allowed the mobility of goods and people making distances look shorter. The transportation has a giant impact in companies because it changed the way in which business strategies are done, generating a set of new opportunities and challenges that contribute to the current globalization and connection of economies. Despite of transportation having several advantages, there are also negative effects associated like environmental impacts.

Different companies make diverse decisions while planning their routes because they have varied objectives subject to certain constraints, that can be either endogenous or exogenous. Endogenous constraints can be, for example, the resources available (either people or vehicles, among others). The legislation is an example of exogenous constraints.

The main objectives regarding transportation are, usually, minimize the total distance travelled, minimize the total time travelled, minimize the environmental impact, maximize the number of customers served, or maximize customer satisfaction, among others.

2.1. Vehicle routing problem (VRP)

The classical VRP was introduced by Dantzig and Ramser (1959). Since then, the classical VRP and its many variants have been extensively researched.

The classical vehicle routing problem (VRP) requires finding a set of routes, one for each vehicle, beginning and ending at a central warehouse, usually designated as depot. Each customer is serviced exactly once by exactly one vehicle such that the total cost is minimized (Mardaneh et al., 2017). Other objective functions, such as minimizing the total distance or the number of vehicles used, have also been considered.

The VRP has drawn enormous interest from many researchers in many application areas. For example, Grasas et al. (2014) address problems with the healthcare, where the main challenges are to deliver high-quality services with limited resources; Mirmohammadi et al. (2017) consider environmental impacts since the major objective is to minimize the total amount of carbon dioxide emissions produced by the vehicles. The application in many different areas provides evidence of the importance of the VRP. Furthermore, its many uses in real cases have shown its potential both in cost savings and environmental benefits.

2.1.1. Variants of the vehicle routing problem

Over the years, several variations of the VRP have been studied, namely: the capacitated VRP (CVRP) that extends the VRP in the sense that the delivery of goods to the customers is done with vehicles that have a limited carrying capacity, typically, regarding weight or volume (Hajishafee et al., 2015); the VRP with time windows (VRPTW) in which a vehicle has to visit each customer within a pre-specified time frame (a vehicle arriving earlier must wait) (Kallehauge, 2006); the VRP with backhauls (VRPB) that includes both a set of customers to whom products are to be delivered, and a set of vendors whose goods need to be transported back to the distribution center and imposes that on each route all deliveries are made before any goods are picked up (Oesterle et al., 2016); the VRP with simultaneous deliveries and pickups (VRPSDP) is a case of combined demands where each vehicle in a single stop may both deliver and pick up goods (Rieck et al., 2013); the VRP with split deliveries (VRPSD) which allows for each customer demand to be satisfied by more than one vehicle (Bortfeldt et al., 2020); the multiple trips VRP (MTVRP), also designated by VRP with multiple trips (VRPMT)[3], that allows each vehicle to perform multiple trips. Many other variants can be found in the literature, see, e.g., Tirkolaee et al. (2019).

2.2. Vehicle routing problems with multiple trips (VRPMT)

The problem under study is closely related to the VRPMT; therefore, in here, we describe in greater detail the VRPMT and other routing problems involving multiple trips.

The VRPMT was first addressed by Fleischmann (1990) under the name Vehicle Routing Problem with Multiple Use of Vehicles, in the context of the solution of a series of distribution problems involving a heterogeneous fleet of vehicles and time windows.

[3] Here and hereafter used indistinctly.

Nevertheless, until quite recently research on this problem appeared sporadically in the literature as illustrated in Figure 1.

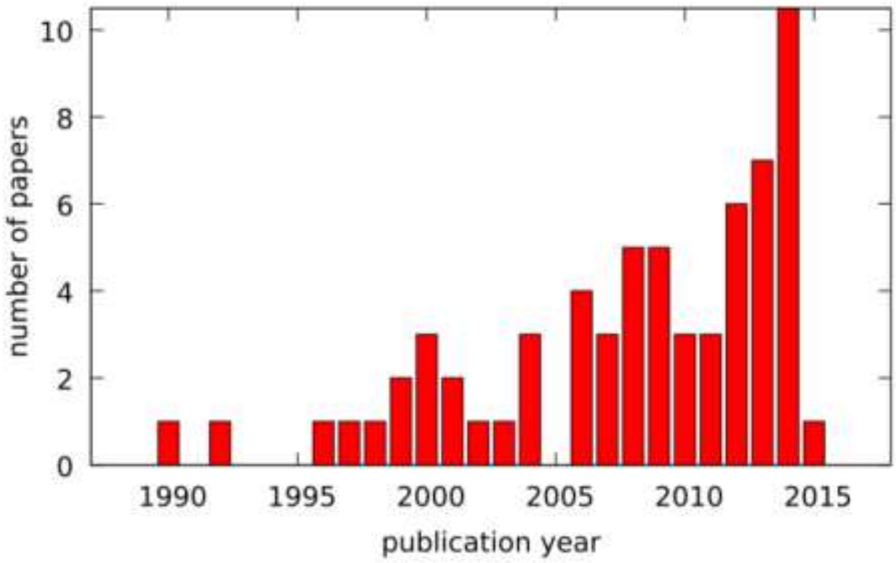


Figure 1: Publications per year addressing multi-trip transportation (Ph.D. theses excluded) (Cattaruzza et al., 2018).

The VRPMT is a proven NP-hard problem (Olivera et al., 2007) that started to receive significant attention only in the last two decades.

The recent interest in this problem is probably due to the new policies regarding final customer distribution, such as the use of electrical vehicles and of smaller vehicles, time restrictions to access city centers (Cattaruzza et al., 2014), and also to the growing importance of last mile delivery, which according to Xiao et al. (2017) accounts for nearly 53% of the total shipping costs.

This problem had several designations over the years as already reported in Cattaruzza et al., (2018): multitrip VRP (Prins, 2002), VRP with multiple routes (Azi et al., 2007), VRP with multiple trips (Olivera et al., 2007), VRP with multiple depot returns (Tsirimpas et al., 2008), multiple trip VRP (Battarra et al., 2009), vehicles multiple traverses (Taniguchi et al., 2000) and recycling of trucks (Van Buer et al., 1999). Recently, it has been designated by multi-trip VRP (Li et al., 2019). In this thesis, we use indistinctly multi trip VRP (MTVRP) and VRP with multiple trips (VRPMT).

The VRPMT consists of finding a set of trips to allocate to a set of vehicles that minimizes travelled time/distance while satisfying the following conditions (Olivera et al., 2007):

1. every customer is visited exactly once;

2. every trip starts and ends at the depot;

3. the total demand of each route does not exceed vehicle capacity; and

4. the total time of the trips assigned to each vehicle does not exceed the planning period (working day duration).

As reported in Cattaruzza et al. (2018) several extensions of the VRPMT have been reported. While some are academic extensions, others are motivated by practical applications. Although many interesting variants exist such as product late arrival at the depot (Mu et al., 2013), new requests arriving during the time horizon (Azi et al., 2012), and limited number of trips per vehicle (Seixas et al., 2013); in here, only VRPMT variants regarding time are considered.

Vehicle routing problem with multiple trips and time windows **(**VRPMTTW) – in this version a time interval $[e, l]$ is specified for each customer, during which the customer has to be visited. The vehicle may arrive at a customer earlier than e, waiting to perform the service, but cannot arrive later than l (or l minus service time). This problem version, typically, includes service time at customers and loading time at the depot (Hernandez et al., 2016).

Below some VRPMT variants regarding time are described:

- Service-dependent loading time VRPMT – motivated by the fact that not all customers require the same products or amounts of products. Thus, the time required to load the vehicles at the depot for each customer depends on what has to be delivered, i.e., on the customers that are visited in each trip (Wang et al., 2014).

- VRPMT with limited trip duration - individual trips have a duration time limit, motivated by the existence of perishable goods or driving time constraints. The trip duration is, usually, measured as the time elapsed since loading until the start of the service of the last customer (Hernandez et al., 2014).

- VRPMT with double depot time window – the depot time window in addition to opening and closing times also has a time limit for the starting time of the vehicles last trip. Thus, there are three relevant time limits: opening time – earliest time at which vehicles can depart from the depot, closing time – latest time at which vehicles must finish their last trip, and last departure time – latest time at which vehicles may start a trip. This new time limit is motivated by the fact that facilities used for loading the vehicles can be closed earlier than the depot (Hajri-Gabouj et al., 2003).

- VRPMT with spread time – the motivation for a limit on the time a vehicle is used (spread time - S_T) comes from the eventual difference between depot opening hours (time horizon - T_H) and driver working hours (spread time - S_T). This constraint is redundant if the spread time and the time horizon are the same.

  However, if the spread time is smaller, as expected, then a vehicle (and driver) starting until $T_H - S_T$, the depot closing time (T_H) minus the spread time (assuming, without loss of generality, that the depot opens at time 0), must take no longer than S_T to be back to the depot; otherwise, a vehicle starting after $T_H - S_T$ must be back to the depot no later than time at (T_H) (Battarra et al., 2009).

- VRPMT with products release time – the motivation comes from the distribution from mutualized platforms, in which given a customer request the goods are first supplied by trucks in the platform (Cattaruzza et al., 2016) and from last-mile distribution of online orders, for which products are not always readily available at the warehouse (Li et al., 2019).

- VRPMT with product release time and due date – the release time is associated with the moment in time at which the product becomes available to be dispatched or picked up. The due date is associated to the deadline by which the products must be delivered or picked up (Reyes et al., 2018).

Regarding performance measures, the most commonly used ones are the minimization of the routing cost, time, or distance. Nevertheless, other objectives have been used. For example, Prins (2002), Battarra et al. (2009), and Cattaruzza et al. (2014) minimize the fleet size; while some works consider green objectives: Ayadi (2014) minimizes CO2 emissions and Cinar et al. (2015) minimizes fuel consumption.

Over the years, several solution approaches, both exact and heuristic, have been proposed for the VRPMT and the above-mentioned variants. The efficiency and effectiveness of these approaches have been tested on benchmark instances, i.e., academic sets of data. These instances were initially introduced in Taillard et al. (1996) and in Gehring et al. (1999). Other instances can be found in Solomon (1987), Mingozzi, et al. (2013), and Cattaruzza et al. (2018).

Exact methods have been proposed for the VRPMT and for some of its variants. However, due to the NP-hard nature of the problem only for some small sized instances an optimal solution has been found. Nevertheless, over the years larger instances and more instances have been being solved optimally.

One exact method seems to be the branch-and-cut algorithm proposed by Koc et al. (2011). The authors were able to solve optimally three of the eight instances attempted, all with 50 customers. However, Mingozzi et al. (2013) solved many more with a branch-and-price based method. A different branch-and-price method was proposed by Hernandez et al. (2013) – for the VRPMT with time windows (VRPMTTW) capable of solving most instances with 25 customers (25 out of 27) and a few instances with 50 customers (5 out of 27). Other authors solve the VRPMTTW with a limit on trip duration. These two constraints together force each trip to include just a small number of customers; therefore, complete enumeration is possible for small and medium size instances.

The branch-and-bound is a method used for enumerating the solutions of combinatorial optimization problems (Shelbourne, 2016). Branch-and-bound techniques are used limiting the gap between the dual bound computed at each node of the search tree and the integer optimal solution for solving problems. The two techniques more used in the last few decades and with a lot of success are branch-and-cut and branch-and-price (Feillet et al., 2010).

A branch-and-cut method combines a branch-and-bound method with a cutting algorithm. In each subproblem of the branch and bound tree, violated cuts are introduced and simultaneously branching, bounding and cutting will reduce the total number of subproblems that need to be solved (Shelbourne, 2016).

In a branch-and-price method a dual bound from the linear relaxation of a reformulation of the problem is computed. To compute this linear relaxation, column generation techniques are needed inferring multiple solutions of a generally computationally costly subproblem (Feillet et al., 2010).

The methods proposed by Azi (2010), Macedo et al. (2011), and Hernandez et al. (2014), all have two stages and in the first stage all employ the same method to identify all feasible trips. The second stage determines the set of trips for each vehicle and the methods used to do so are different: Macedo et al. (2011) introduces the time-indexed graph; Hernandez et al. (2014) uses column generation, where the columns represent trips, to choose the best set of trips so that all customers are visited and Azi (2010) uses column generation where each column represents a vehicle's workday. The computational experiments have shown that Macedo et al. (2011) and Hernandez et al. (2014) obtain comparable results and significantly improve upon Azi (2010).

It follows a more detailed review of the works addressing VRPMT with product release times and due dates since these are the ones that are closely related to the problem under study.

2.2.1. VRPMT with product release times and due dates

According to Azi et al. (2014), in a VRPMT with limited trip duration and time constraints, it might not be possible to serve all customers' requests. In these problems, there are two objectives: the first is to maximize the number of served customers and the second is to minimize the total distance travelled by the vehicles. An adaptive large neighborhood search, taking advantage of the ruin-and-recreate principle was proposed for solving the problem. This principle consists in searching for a better solution by destroying part of the current solution and reconstructing it in a different way.

Hernandez et al. (2014) designate VRPMT with time windows and limited duration. They consider that trips have a time limit so that a sequence of trips can be assigned to one vehicle. The limited duration of the trips can be motivated by management issues, for example, a limit on the driving time of the drivers or can depend on the nature of transported goods, that can be for example perishable goods. They proposed an exact two-phased algorithm to solve the VRPMT with limited trip duration. In the first phase, possible lists of clients are ordered that match the maximum trip duration criterion. In the second phase, a Branch and Price scheme is used in order to generate and choose the best set of trips so that all customers are visited.

Studies on VRPs with order releases are very recent.

A VRPMT with perishable goods is assumed as a VRPMT with product release times and due dates. A VRPMT with product release times and due dates has a limited trip duration and a release time. Thus, due dates can be considered as a special case of time windows. The availability of a product in order to be delivered or picked up can also be denominated as release time (Reyes et al. 2018).

In addition to a release date, a deadline may also be imposed as mentioned by Archetti et al. (2015). In such cases, the minimization of the total time or distance travelled has to be combined with the deadline of the goods that are delivered or picked up, which can be due to the existence of perishable goods.

A practical application in which steel coils are delivered from different production sites to customers with the explicit consideration of order release times can be found in Arda et al. (2014). In this case, a production plan and delivery schedule coexist. In the production plan, projected and confirmed release dates are announced by the plants. The delivery schedule is based on the delivery time windows requested by the customers. The vehicles are chartered according to confirmed coil released from production and according to customer's requirements. The authors also consider constraints regarding the capacity of the vehicles.

Archetti et al. (2015) address a routing problem where uncapacitated vehicles requested by customers are loaded with goods and arrive at the depot over time. The release date is considered as the product arrival time at the depot. Two problem variants are considered; one with a deadline to complete the distribution and another without any deadline. The variant with a deadline was combined with the objective of minimizing the total traveled distance. The variant without a deadline was considered together with the objective of minimizing the total time needed to complete the distribution.

Liu et al. (2017) developed a Tabu search and Lagrangian relaxation procedure to deal with the capacitated VRP with order available time. The challenges faced by one of the biggest Chinese e-commerce companies was the motivation for them to proceed with their work. At the beginning of their planning period the orders are not yet available for delivery and become available only when the order picking and packing stage is competed at the warehouse of the online grocer.

According to Shelbourne (2016) each customer has a release date associated, defining the earliest time that an order is available to leave the depot for delivery, and a due date, indicating the time by which an order should preferably be delivered to the customer. He proposed a path-relinking algorithm (PRA) for the VRP with release and due dates. The efficiency of the PRA is validated by comparing it with an ILS algorithm.

In our specific case study, we have a VRPMT with product release times and due dates. The product release time matches the date/time at which the product, in our case, blood, is extracted, being only available for transportation after this. The due date is associated to the time limit imposed on the product delivery, in our case the blood arrival at the laboratory.

Thus, our specific problem differs, for example from that of Archetti et al. (2015), on the definition of release date.

Chapter 3 - Problem definition and formulation

3.1. Problem Description

This dissertation addresses the Vehicle Routing Problem with multiple trips and time constraints (VRPMTTC). The warehouse is the central location where the vehicles start and end every trip in order to visit several customers with the aim of delivering or picking up a given product. Figure 2 depicts three trips. One (in grey) that starts at the warehouse – 0 and visits customer 1, 2 and 3, in that order, and returns to the warehouse – 0. Another trip (in black) starts at the warehouse – 0, visits customer 4 and returns to the warehouse – 0. A third trip (in dashed) begins at the warehouse – 0 visits customer 5, 6, 7, and 8 and returns to the warehouse – 0.

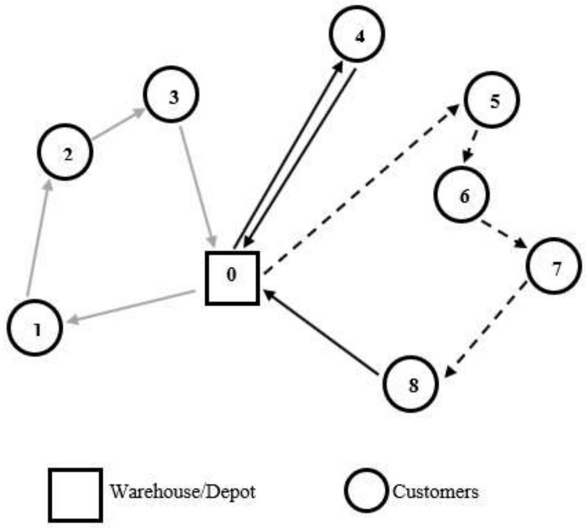


Figure 2: Vehicle routing problem with multiple trips.

As previously mentioned, the analysis of the VRPMTTC will be applied to an existing case study that is described next.

3.2. Case Study Description

In this section, we provide the detailed description of the specific problem addressed in this work. The detailed description is also used to introduce the notation used in the Mixed Integer Linear Programming (MILP) model, which is presented in Section 3.3.

In this case study, we have a central laboratory and a set of geographically disperse health centres. The central laboratory is the warehouse (also designated as depot) and the health centres are the customers. In the health centres, blood and other biological products are extracted and then they must be collected and delivered to the central laboratory to be analysed. The time elapsed from extraction to laboratory delivery is limited and thus, several trips to each health centre are needed. Finally, the delivery is done by a single vehicle.

This problem is a variant of the classical VRP that we designate by Vehicle Routing Problem with Multiple Trips and Time Constraints (VRPMTTC). Vehicle routing problem with multiple trips because a set of trips is allocated to a set of vehicles (in this case we have only one vehicle) and with time constraints because of the special treatment of the blood described below. The time constraints that are closely related to the problem under study are product release times and due dates (that is a special case of time windows, as previously mentioned). The product release time matches the date/time at which the blood is extracted, being only available for transportation after this and the due date, considered a special case of time windows, is associated to the time limit imposed on blood delivery since extraction. In this case study, we assume blood extraction to occur continuously and that the time limit imposed refers to the time since extraction to delivery at the depot. The due date is given by the released date plus the blood duration. The time limit to collect, transport, and deliver the blood after extraction is of two and a half hours. This means that the blood has to be delivered to the central laboratory in at most two and a half hours after extraction at the health centres.

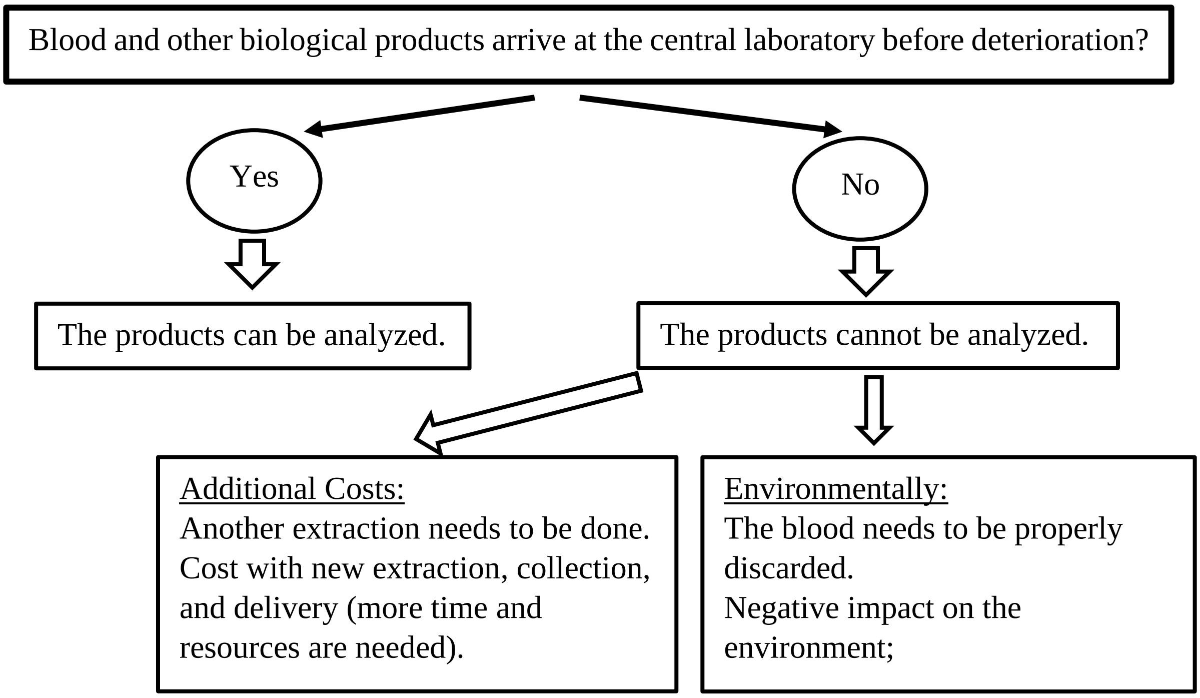
If the blood does not arrive on time at the central laboratory, it cannot be properly tested and analysed and thus, it has to be discarded and another extraction needs to be done. If this time limit to collect, transport, and deliver is not met, there will be a waste associated with the discarding of blood. This waste has negative impacts both on the company and on the environment. The negative impacts for the company arise from discarding of the blood and from additional extractions that also imply additional deliveries.

Usually central laboratories need to decide how to collect the blood and other biological products on a daily bases because every day they might have extractions at different places and at different times that obviously will influence the order in which they should do the daily collection.

Our main objective is to minimize the total travelled distance. There are several constraints associated to this objective such as: the collection starts and ends at the central laboratory; each health centre is visited at most once in each trip; the health centres may only be visited after the production starting time; the last visit to each health centre needs to be done after normal working hours to ensure that every and all extractions are collected.

In our problem there are no limits regarding the capacity of the vehicles and therefore it is assumed that there is always enough capacity to deliver the goods. It is also assumed that all products that arrive at the central laboratory/warehouse before deterioration will be analysed. In addition, the service time at the central laboratory/warehouse when starting a trip is zero. There is only service time at the central laboratory/warehouse when the vehicle arrives at the central laboratory/warehouse and unloads the products collected at the health centres.

Without loss of generality, we assume that the distance travelled between the health centres and central laboratory are the same as the time required to travel between them.



From the previous scheme we can see that there are a lot of disadvantages if the blood and other biological products are not collected and carried from the health centers to the central laboratory before deterioration, in this case within two and a half hours of extraction. If delivered on time, the corresponding samples can be analyzed; otherwise they must be properly discarded, and another extraction needs to be done. Discarding of biological products is costly and impacts the environment negatively. A new extraction requires additional material, transportation, and resources such as vehicles and people to pick up the new extraction and all of this has costs associated. In our model the time limit of the two and a half hours is guaranteed and therefore no additional extraction needs to be done. No additional extraction means no waste of blood, no additional costs and no negative impact on the environment which constitutes a big advantage of our model. Since the time limit of the two and a half hours is guaranteed in our model, costs are not included in our decision-making process.

Therefore, we wish to determine a set of trips such that all blood collected at all health centres is delivered to the central laboratory within the two and a half hours limit, while minimizing the total distance travelled.

3.3. Problem formulation

In this section, the mathematical programming model developed for solving the vehicle routing problem of this specific case study is described. The problem is formulated as a MILP and the notation can be found in the tables below, which summarize the information regarding sets, indices, parameters and decision and auxiliary variables.

Sets and indices	Description
$V = \{1, \dots, n\}$	Set of n customer nodes, indexed by i, j and l;
$V' = V \cup \{0\}$	Set of customer nodes and departure depot node, indexed by i, j and l;
$V'' = V \cup \{n + 1\}$	Set of customer nodes and arrival depot node, indexed by i, j and l;

$W = V \cup \{0, n+1\}$	Set of all nodes (recall that the departure and arrival depots are the same, here duplicated for convenience of exposition), indexed by i, j and l;
$A = \{(i,j) : i \in V', j \in V'', i \neq j\}$	Set of arcs;
T	Set of trips $\{1, \dots, U\}$ ($T' = T \cup \{0\}$), indexed by k, s and t. (U is defined in equation (1)).

Table 1: Mathematical notation of sets and indices.

Parameters	**Description**
H_i	Allowed time between product production and product delivery to the depot for node $i \in V$;
st_i	Service time at node $i \in W$ ($st_0 = 0$);
e_i	Production starting time at node $i \in V$;
l_i	Production ending time at node $i \in V$;
t_{ij}	Travel time of arc $(i,j) \in A$;
d_{ij}	Travel distance of arc $(i,j) \in A$;
M	A large integer.

Table 2: Mathematical notation of parameters.

Decision Variables	**Description**
s_i^t	Pickup time at node $i \in W$ in trip $t \in T'$, 0 if node i is not visited in trip t;
sp_i^t	Pickup time at node $i \in V$ in trip $t \in T$ if trip t is the last trip performed for node i, 0 otherwise;
x_{ij}^t	Binary variable set to 1 if node j $\in V''$ is visited immediately after node $i \in V', i \neq j$ in trip $t \in T$ and to 0 otherwise;
v_i^{st}	Binary variable set to 1 if trips $s \in T'$ and $t \in T, s < t$ are consecutive visits of node i and set to 0 otherwise.

Table 3: Mathematical notation of decision variables.

Auxiliary Variables	Description
y^t	Binary variable set to 1 if trip t is used and 0 otherwise;
w_i^t	Binary variable set to 1 if node i is visited in trip t and 0 otherwise;
z_i^t	Binary variable set to 1 if the last visit to node i is done in trip t and 0 otherwise.

Table 4: Mathematical notation of auxiliary variables.

Lower and upper bounds on the number of trips required can be obtained as follows. The maximum time δ_i allowed between two consecutive collections to node $i \in V$ can be computed as

$$\delta_i = H_i - \left(st_i + t_{i(n+1)} + st_{n+1}\right), \qquad \forall i \in V;$$

Assuming the fastest possible delivery, i.e., direct return to the depot, the least number visits to node $i \in V$ is given by

$$m_i = \left\lceil \frac{l_i - e_i}{\delta_i} \right\rceil, \qquad \forall i \in V;$$

where $\lceil a \rceil$ denotes the smallest integer greater than or equal to a. Hence, lower L and upper U bounds on the total number of trips required are given by

$$L = max_{i \in V}\{m_i\} \;\; and \;\; U = \sum_{i \in V} m_i \tag{1}$$

$$Minimize \sum_{t \in T} \sum_{i \in V\prime} \sum_{j \in V\prime\prime} x_{ij}^t \times d_{ij} \tag{2}$$

Subject to

$$\sum_{i \in V} x_{0i}^t \leq 1, \qquad \forall t \in T \tag{3}$$

$$\sum_{i \in V} x_{0i}^t = \sum_{j \in V} x_{j,n+1}^t, \qquad \forall t \in T \tag{4}$$

$$\sum_{j \in V'':j \neq i} x_{ij}^t \leq 1, \qquad \forall i \in V', t \in T \qquad (5)$$

$$\sum_{j \in V'':j \neq i} x_{ij}^t = \sum_{j \in V':j \neq i} x_{ji}^t, \qquad \forall i \in V, t \in T \qquad (6)$$

$$y^t \leq y^{t-1}, \qquad \forall t \in T: t \geq 2 \qquad (7)$$

$$y^1 = 1, \qquad (8)$$

$$y^t = \sum_{i \in V} x_{0i}^t, \qquad \forall t \in T, \qquad (9)$$

$$s_0^1 \geq 0, \qquad (10)$$

$$s_0^t \leq M y^t, \qquad \forall t \in T: t \geq 2, \qquad (11)$$

$$s_0^t \geq s_{n+1}^{t-1} + st_{n+1} - M(2 - y^{t-1} - y^t), \qquad \forall t \in T: t \geq 2, \qquad (12)$$

$$s_i^t \geq e_i - M(1 - w_i^t), \qquad \forall i \in V, t \in T \qquad (13)$$

$$s_j^t \geq s_i^t + st_i + t_{ij} - M(1 - x_{ij}^t), \qquad \forall i \in V', j \in V'', i \neq j, t \in T, \qquad (14)$$

$$s_i^t \leq M\, w_i^t, \qquad \forall i \in V, t \in T \qquad (15)$$

$$\sum_{t \in T} z_i^t = 1 \qquad \forall i \in V \qquad (16)$$

$$sp_i^t \leq M * z_i^t \qquad \forall i \in V, t \in T \qquad (17)$$

$$sp_i^t \leq s_i^t - M * (z_i^t - 1) \qquad \forall i \in V, t \in T \qquad (18)$$

$$\sum_{t \in T} sp_i^t \geq s_i^t, \qquad \forall i \in V, t \in T, \qquad (19)$$

$$\sum_{t \in T} sp_i^t \geq l_i, \qquad \forall i \in V, \qquad (20)$$

$$s_{n+1}^t + st_{n+1} - s_i^s \leq H - M(v_i^{st} - 1), \qquad \forall i \in V, s \in T', t \in T, s < t, \qquad (21)$$

$$s_i^0 = e_i, \qquad \forall i \in V, \qquad (22)$$

$$w_i^t = \sum_{j \in V'': j \neq i} x_{ij}^t, \qquad \forall i \in V, t \in T, \qquad (23)$$

$$w_i^0 = 1, \qquad \forall i \in V, \qquad (24)$$

$$v_i^{st} + \sum_{k=s+1}^{t} w_i^k \geq w_i^s + w_i^t, \qquad \forall i \in V, s \in T', t \in T: s < t \qquad (25)$$

$$s_i^t, \qquad \forall i \in W, t \in T \qquad (26)$$

$$x_{ij}^t, v_l^{st}, w_l^s, y^t \in \{0,1\}, \qquad \forall i \in V', j \in V'', l \in V, s \in T', t \in T \quad (27)$$

The objective function for this problem is to minimize the total traveled distance as stated in expression (2). Constraints (3) to (5) ensure that if a trip is performed it must start and end at the depot and visits each node at most once, while constraints (6) are the usual balance equations and therefore ensure that a trip entering a node must also leave the node. Constraints (7) and (8) ensure that trip t can only be performed if trip $t-1$ has been performed and that at least one trip is performed, respectively. (Note that constraints (9) ensure the correctness of variables y^t, which take the value 1 if trip t is performed and zero otherwise.)

The visiting times are established through constraints (10) to (15). The first trip can start at any time, constraints (10), while subsequent trips start only if performed, constraints (11), and after the previous trip has been concluded and the vehicle unloaded, constraint (12). Nodes may only be visited after production starting time, constraint (13) and after the previous node in the same trip has been visited and serviced and the vehicle moved, constraint (14). If a node is not visited in a trip, then the corresponding visiting time is set to zero, constraint (15).

Constraints (16) to (20) ensure that the last visit to each node occurs after closing time, i.e., after the last blood extraction has been performed. More specifically, constraint (16) assures exactly one last visit to each node, while constraints (17) to (19) ensure the correctness of the visiting time value. Finally, constraint (20) force the last visit to happen after health center closing hours (after normal working hours).

Constraints (21) to (25) ensure that all goods are delivered to the depot before deterioration. More specifically, constraint (21) ensure that for any two consecutive visits to a node the time elapsed between the last node visit and current vehicle unloading at the depot is no more than the deterioration time. For the first visit, the starting of the working hours is considered as the previous visiting time (see constraint (22)). The correctness of the node visit variables and consecutive visits variables is ensured by constraints (23) and (24) and constraints (25), respectively.

Lastly, constraints (26) and (27) specify the nature of the variables.

Chapter 4 - Methodology

4.1 Data

As already mentioned, the MILP model proposed will be used to solve a real case study. The problem instance being solved refers to a collection of community healthcare centers and a hospital in the region. The data was retrieved from the master thesis by Carneiro (2019).

The warehouse/depot, in our case also denominated as central laboratory, is called *HPH – Hospital Pedro Hispano*. There are four customers, also designated as health centers, as follows:

1) *CS São Mamede de Infesta*
2) *USF Porta do Sol*
3) *CS Matosinhos*
4) *CS Sra. Hora*

The health centers have different time schedules regarding their working hours. The opening and closing times are summarized in the tables below in two formats, namely: hh:mm and minutes. The model implementation uses time in minutes and at the end converts them into the hh:mm format.

Health centers	Opening times (hh:mm)	Opening times (minutes)
CS São Mamede de Infesta	08:00	480
USF Porta do Sol	08:00	480
CS Matosinhos	07:30	450
CS Sra. Hora	08:00	480

Table 5: Opening times of health centers in hh:mm and minutes format.

Health centers	Closing times (hh:mm)	Closing times (minutes)
CS São Mamede de Infesta	10:30	630
USF Porta do Sol	11:00	660
CS Matosinhos	11:00	660
CS Sra. Hora	11:00	660

Table 6: Closing times of health centers in hh:mm and minutes format.

It is assumed that *HPH – Hospital Pedro Hispano* is always open, therefore there is no time schedule for the Central laboratory/warehouse.

The time window [opening time; closing time] defines, in this case, the period between which people can go to the health centers and have their blood extracted. In our problem, the last visit to each health center has to be done after their closing hours to ensure that every and all extractions are collected.

There is a time associated when travelling between all health centres and the central laboratory. Without loss of generality, we assume that the travel time is equal to the distance travelled. Below we can see a matrix with the travel times between all possible locations. This matrix is not symmetric as $d_{ij} \neq d_{ji}$ at least for some pairs i and j (d_{ij} is the traveling distance in km between the locations of health centers - i and j).

	CS São Mamede de Infesta	**USF Porta do Sol**	**CS Matosinhos**	**CS Sra. Hora**	**HPH**
CS São Mamede de Infesta	0	5	11	10	13
USF Porta do Sol	5	0	8	5	11
CS Matosinhos	12	10	0	5	5
CS Sra. Hora	10	6	2	0	4
HPH	12	10	4	5	0

Table 7: Matrix with travel times in minutes (equal to Matrix with travel distance in km).

The service time – time that the vehicle needs to pick up the blood and other biological products at each health centre and to unload the vehicle at the central laboratory is summarized in the table below:

Health centers and warehouse	Service time (minutes)
CS São Mamede de Infesta	5
USF Porta do Sol	10
CS Matosinhos	6
CS Sra. Hora	10
HPH - Hospital Pedro Hispano	15

Table 8: Service time in minutes of the health centers and central laboratory.

From the previous table we can conclude that the service time at the central laboratory, reported as 15 minutes in Table 8, refers to vehicle unloading at *HPH*. No service time is considered at *HPH* when a trip is started.

It is considered that there is no limit regarding the capacity of the Vehicle, which means that the vehicle has always enough capacity.

As mentioned previously there is a time limit to collect, transport, and deliver blood after extraction. This time limit in the implementation of the model is considered as the allowed time between product production and product delivery from the health centers to the central laboratory and is two and a half hours, that is, 150 minutes, and applies to all health centers.

The data described above was collected from an already existing master thesis and was used as input data. We decided to use this data because it concerns to real data and because the same data was available for all the health centers. This data constitutes all the input data that we need in order to proceed with our analysis.

Regarding data storage, Microsoft Excel was used with the data inputs concerning the service times, opening times, closing times, travel times, distance times, and time limit between product production and product delivery. Since the data is small, we were able to use only one Excel file with several sheets.

In the table below we can find the notation used to represent the health centers and central laboratory in the implementation. The names were substituted by numbers to facilitate the implementation in CPLEX. Recall that the departure and arrival depots (central laboratory) are the same but are here duplicated for convenience of exposition as mentioned in Chapter 3.

Health centers and warehouse	Number
HPH - Departure Depot	0
CS São Mamede de Infesta	1
USF Porta do Sol	2
CS Matosinhos	3
CS Sra. Hora	4
HPH - Arrival depot	5

Table 9: Number associated to Health centers and warehouse.

4.2. Solution Approach

The commercial software IBM ILOG CPLEX Studio IDE version 12.9.0 was used to solve the MILP model proposed in Chapter 3. This software is usually used to solve models such as ours in an efficient and robust way. Optimization Programming Language (OPL) is a type of language that can be used in CPLEX. The OPL supports Linear Programming (LP), MILP, and constraint programming. We have used the OPL to implement our MILP model. Using mathematical and constraint programming, based in OPL, enables rapid development and deployment of decision models (IBM ILOG, 2017).

Regarding our case study, the first step of the implementation was to define the variables of the model and import the data in Microsoft Excel format as already mentioned. We had to tell CPLEX exactly which excel file it should read and which sheets and ranges it should read. The decision and auxiliar variables were defined as positive integers or as booleans according to their nature. This was the first big step done. Before proceeding with the implementation of the objective function and the constraints, the model has some initial calculations in order to know the number of some variables that were used on the indexes of the coding of the objective function and the constraints.

The CPLEX approach to solve MILP models uses methods like branch and cut or dynamic search.

In the Branch and cut method CPLEX solves a series of continuous subproblems, building a tree, in which each subproblem is a node. With this approach, the software is able to manage those subproblems efficiently. The continuous relaxations of the original MILP are the root of the tree. CPLEX, automatically, tries to search cuts if the solution to the relaxation has one or more fractional variables. Each cut is a constraint that cuts away areas of the feasible region of the relaxation that contains fractional solutions. If the solution to the relaxation still has fractional-valued integers variables after the software tries to add cuts, then the branches on a fractional variable will generate two new subproblems, each with more restrictive bounds on the branching variable (IBM ILOG, 2007).

The dynamic search method consists on the same concepts as branch and cut, LP relaxation, branching, cuts, and heuristics. IBM considers that for many models, the dynamic search method is able to find feasible and optimal solutions more quickly than the conventional branch and cut method IBM ILOG (2007).

After all the adjustments that we have made throughout the implementation, we were able to solve the problem in CPLEX. Although our model has a large number of constraints and variables, since the case study only involves four customers/health centers, the program managed to provide a solution quickly (about 20 seconds).

4.3. Computational experiments

The solution obtained in CPLEX for our problem considers that the vehicle must perform three trips to respect all the constraints mentioned in Chapter 3. Figures 3 to 5 depict the solution found by CPLEX. We provide the departure time, travel time, arrival time, service time and waiting time. The pickup time mentioned in Chapter 3 corresponds to the time at which the vehicle starts to do the service. The pickup time corresponds to the arrival time at the location plus the waiting time when it exists. If there is no waiting time, the pickup time is equal to the arrival time.

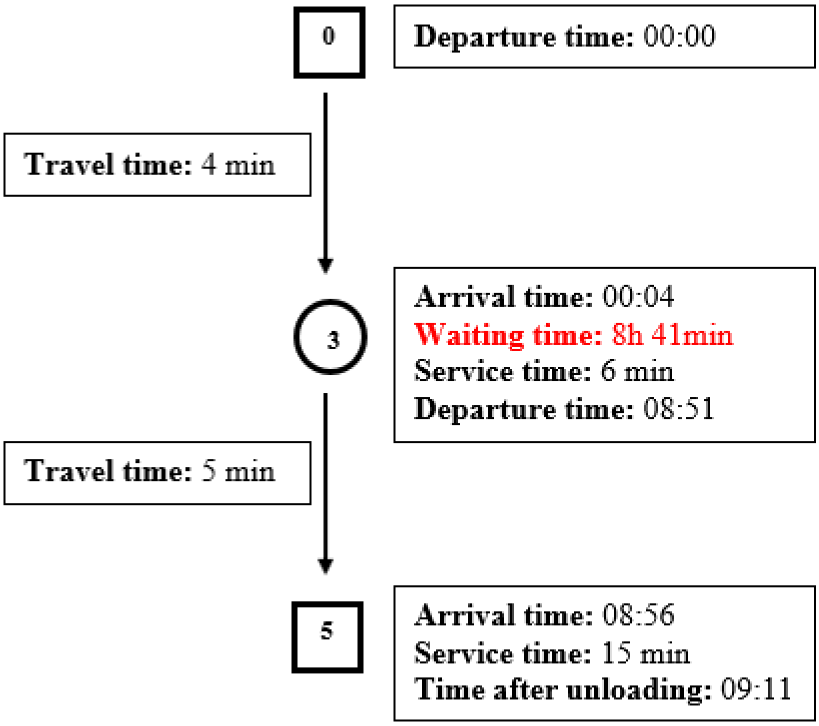


Figure 3: Scheme of the solution for trip 1.

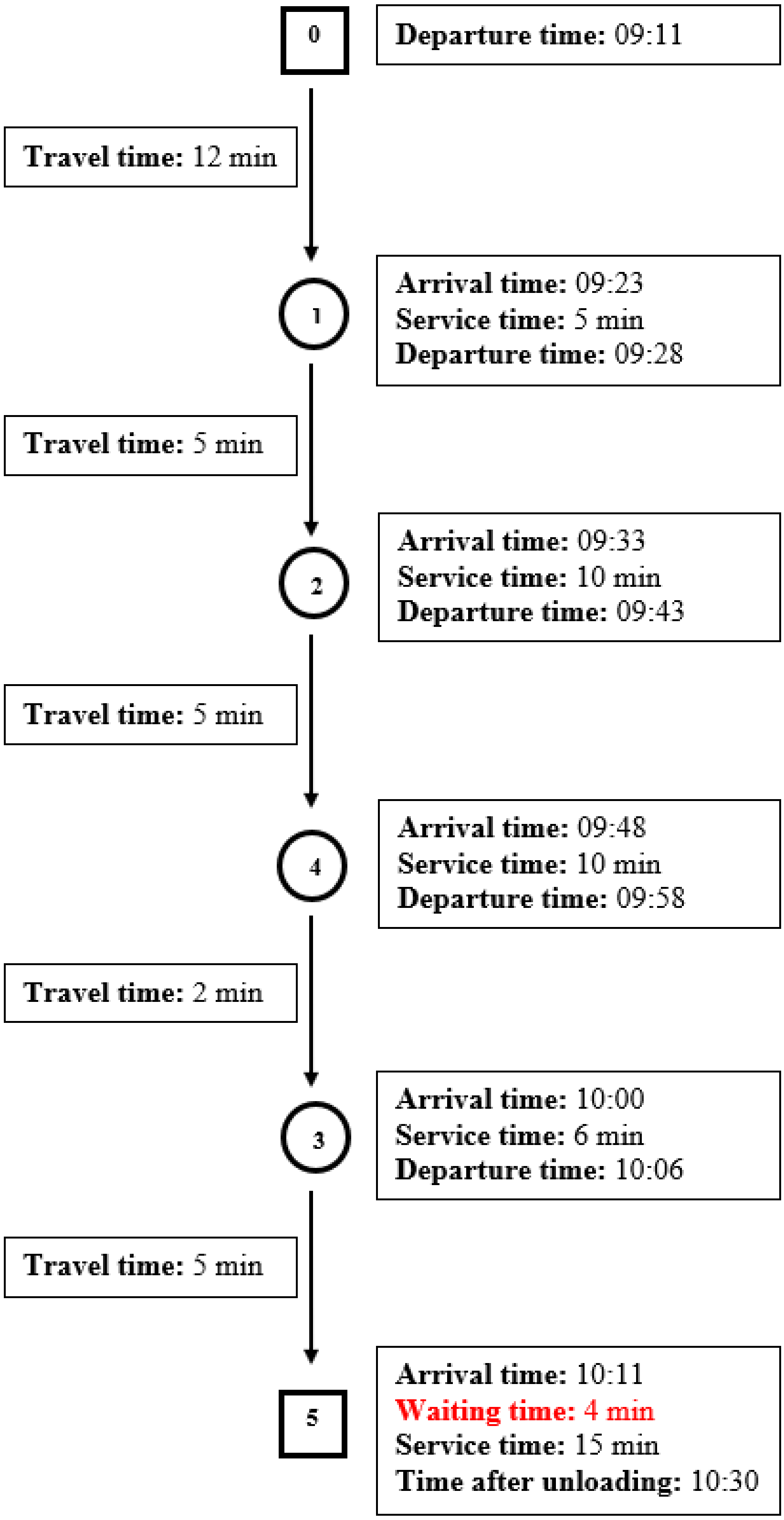


Figure 4: Scheme of the solution for trip 2.

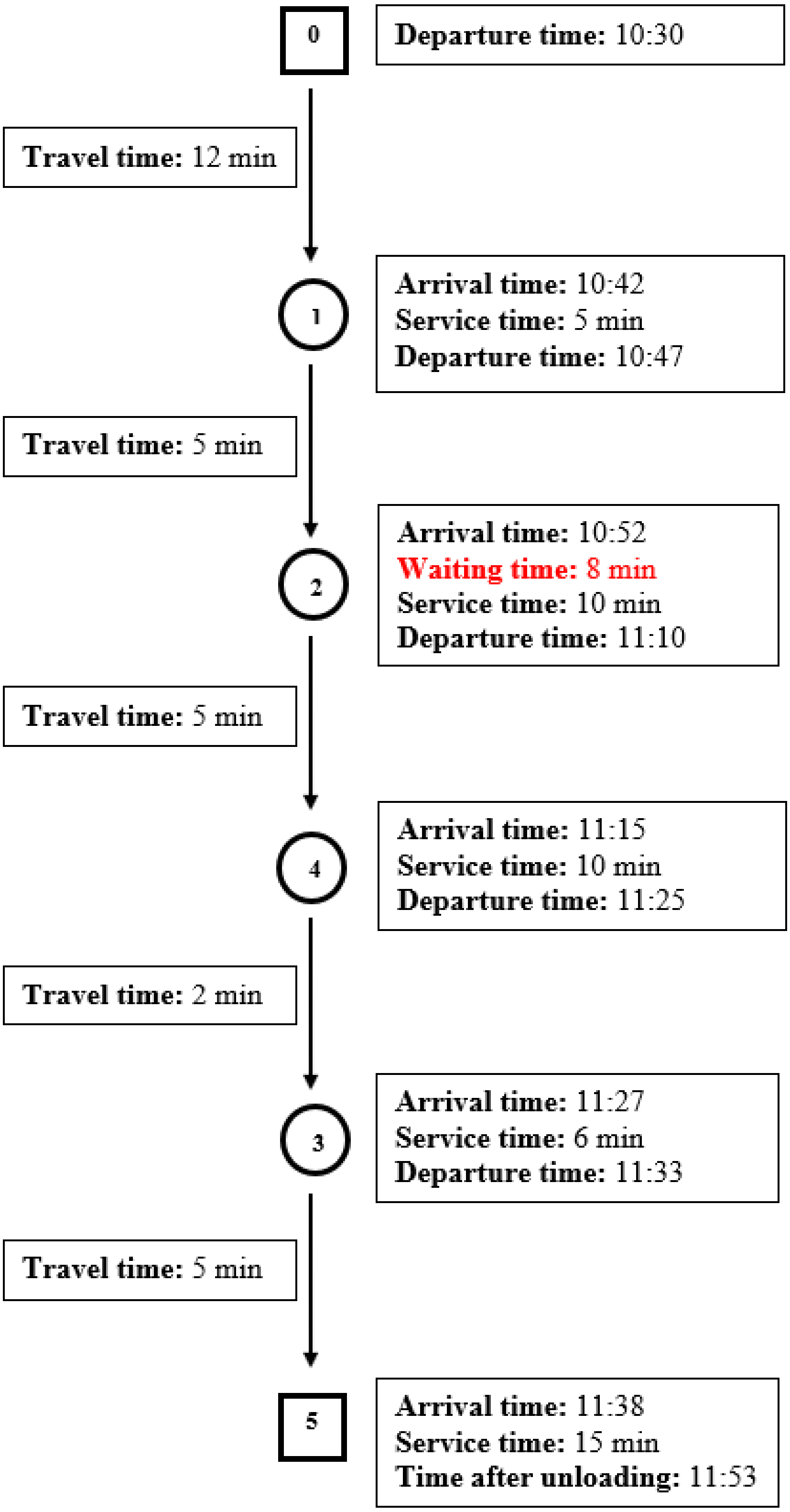


Figure 5: Scheme of the solution for trip 3.

With the solution obtained we can conclude that in the first trip only health center 3 - *CS Matosinhos* is visited. Here and hereafter the health centers will be designated as nodes according to the notation previously described (see Table 9).

In Figure 3 we can see that the first trip starts at the node 0 at 00:00 and the vehicle needs four minutes to travel from node 0 to node 3. The vehicle arrives at node 3 at 00:04 but has a waiting time of 8 hours and 41 minutes which means that he begins to do the service only at 08:45. Obviously, it makes no sense that the vehicle driver starts the trip at node 0 at 00:00 in order to wait 8 hours and 41 minutes at node 3. Since we don't have any time schedule for the central laboratory and according to the constraint (10) $s_0^1 \geq 0$, the first trip in node 0 starts whenever possible and in this case at 00:00 but the vehicle driver will not sleep 8 hours and 41 minutes at node 3 waiting to do the service since this makes no sense. Therefore, instead of begging the trip in node 0 at 00:00 it makes sense that the trip starts at node 0 at 08:41 since the vehicle needs four minutes to travel from node 0 to node 3 and so it will arrive at node 3 at 08:45 and begin immediately the service. Once arrived at node 3 the vehicle has a service time of six minutes and thus leaves node 3 at 08:51. The travel time from node 3 to node 5 is five min, arriving at node 5 at 08:56. When arriving at the central laboratory (node 5) the vehicle has to unload the samples collected in this case in node 3 and fifteen minutes are needed to do the service. The vehicle can start the second trip at or after 09:11. We can conclude that the solution provided by CPLEX is not the only solution possible for our problem. In the first trip, if the vehicle starts the trip at node 0 at 08:41 there will not be any waiting time in this trip and all the constraints of our problem are still be respected.

In Figure 4 the second trip starts at node 0 (depot), passes through nodes 1, 2, 4 and 3, in that order, and returns to the depot (node 5). The second trip begins at node 0 at 09:11. From the analysis of the first trip we concluded that the vehicle is ready to begin the second trip at or after 09:11 hours. This means that after unloading the samples collected at the first trip the vehicle begins immediately the second trip. The vehicle takes twelve minutes to travel from node 0 to node 1, arriving at node 1 at 09:23. The service time at node 1 is five minutes and therefore the vehicle is ready to leave node 1 at 09:28. The vehicle takes five minutes to travel from node 1 to node 2 arriving at node 2 at 09:33. Immediately after arriving at node 2, the blood is picked up and ten minutes are needed to do that, being ready to leave node 2 at 09:43. It follows the next visit that is done to node 4. Five minutes is the time needed to travel from node 2 to node 4, arriving at node 4 at 09:48. Ten minutes are needed to pick up the samples, thus leaving node 4 at 09:58. It follows the visit to the last customer. The last customer that is visited in the second trip is node 3. It takes two minutes to travel from node 4 to node 3. The vehicle arrives at node 3 at 10:00, performs the service for six minutes and leaves this node at 10:06 in order to go back to the warehouse (central laboratory/node 5). After traveling for five minutes the vehicle finally arrives at node 5 at 10:11. There is a waiting time of 4 minutes, meaning that the vehicle starts the service only at 10:15. The vehicle needs to unload all the samples that were collected at the health centers and takes fifteen minutes to do this task finishing the unloading at 10:30. The vehicle is ready to start the third trip at or after 10:30.

In Figure 5 we can look at the route of the last trip that is performed. The third trip starts at 10:30, immediately after unloading the vehicle with the samples collected on the second trip. In the third trip all health centers are visited after normal working hours to ensure that every and all extractions are collected. The visits to the health centers are done in the same order as it was done on the second trip. The vehicle travels from node 0 to node 1 in twelve minutes and arrives at node 1 at 10:42, collecting the samples in five minutes and leaving node 1 at 10:47. Moving forward from node 1 to node 2 in five minutes, thus arriving at node 2 at 10:52 and waiting (8 minutes) until 11:00 to do the pickup of the samples since this is the closing time of node 2. The service of ten minutes is performed, and the vehicle leaves node 2 at 11:10. The next visit is done to node 4 and the itinerary from node 2 to node 4 is done in five minutes arriving at node 4 at 11:15. The collection of the blood in node 4 is executed in ten minutes and the vehicle leaves this node at 11:25, performing the ride to the last health center, node 3, in two minutes and arriving at node 3 at 11:27. The service time in node 3 is six minutes and the vehicle leaves this node at 11:33 in order to go back to the central laboratory, node 5. The last path takes five minutes and the vehicle arrives at the central laboratory at 11:38. The vehicle needs to unload all the samples collected during the third trip taking fifteen minutes to do this and finishing this task at 11:53. At 11:53 we can conclude that all the samples were collected from all health centers within the two and a half hours limit of our problem together with all the constraints of our model mentioned in previous chapter.

From the analysis of the schemes of the three trips that are necessary to perform in order to solve our problem we immediately realize that there are several different solutions possible for the same objective function. Since we have waiting times at some health centers that are visited, we have a range of different arrival and departure times for the same total traveled distance that is minimized, this means that there are several possible solutions taking into account our objective of minimization of the distance. The objective function of our problem is 67. The minimum total travel distance to satisfy all our constraints is 67 km. This and other information regarding the statistic of our implementation can be found in Figure 6:

Statistic	Value
Cplex	solution (optimal) with objective 67
Constraints	783
Variables	504
Binary	420
Integer	84
Non-zero coefficients	2986
MIP	
Objective	67
Nodes	21389
Remaining nodes	0
Incumbent	67
Iterations	972241
Solution pool	
Count	12
Mean objective	129.166667

Figure 6: Image from the Statistic provided by the implementation of the model in CPLEX.

In our perspective it makes no sense that the vehicle waits at the health centers. In our opinion the vehicle should wait at the central laboratory so that the vehicle driver could support with other tasks that might be needed instead of waiting at the health centers.

A possible way so that the first trip doesn’t begin at 00:00 at node 0 is to impose a working schedule for the central laboratory so as the one that exists for the health centers. An opening and closing time could be used as an input for the model.

In our model the vehicle has several waiting times associated because he always starts the trips as soon as possible doing the waiting times at the health centers and not at the central laboratory. Since the vehicle starts the trips always in the earliest possible time, we will try to change the objective function of the model to see the impact of it on our solution.

A second version of the model presented in Chapter 3 will be present. Our new model differs from the previous one in the objective function and has a new variable and a new constraint.

Our new variable is variable u that represents the time of the last visit among all centers.

The new constraint is:

$$u \geq \sum_{t \in T} sp_i^t, \qquad \forall i \in V, \tag{28}$$

Our new model has a new objective function that is:

$$Minimize\ u \tag{29}$$

We solved the new model, after adding the new variable and constraint to the model and changing the objective function. CPLEX took more time to provide a solution for this new model (around 5 and a half hours). The solution obtained still consists of three trips. The first trip now visits nodes 3, 1 and 4 in this order. Trips 2 and 3 visit the same nodes as before and in the same order.

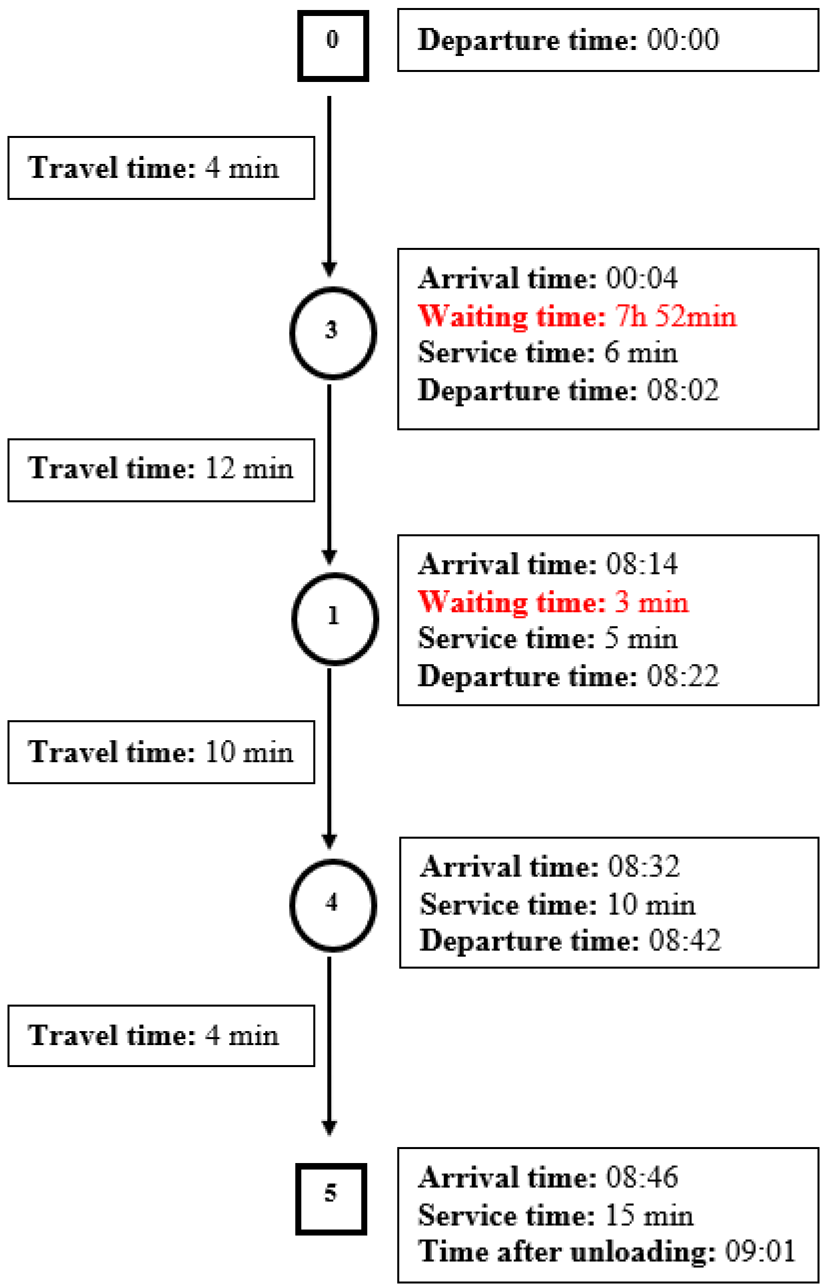


Figure 7: Scheme of the new solution for trip 1.

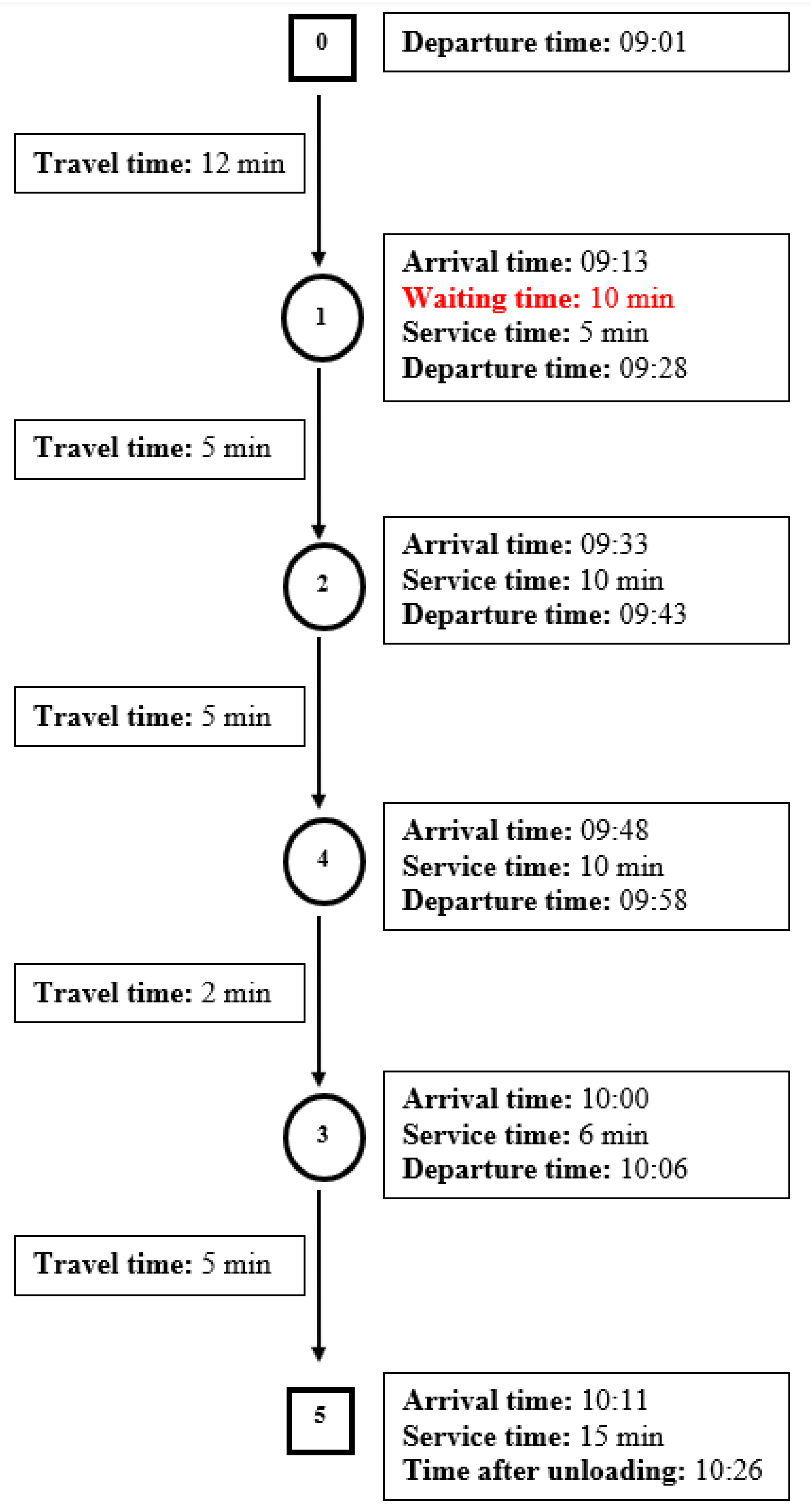


Figure 8: Scheme of the new solution for trip 2.

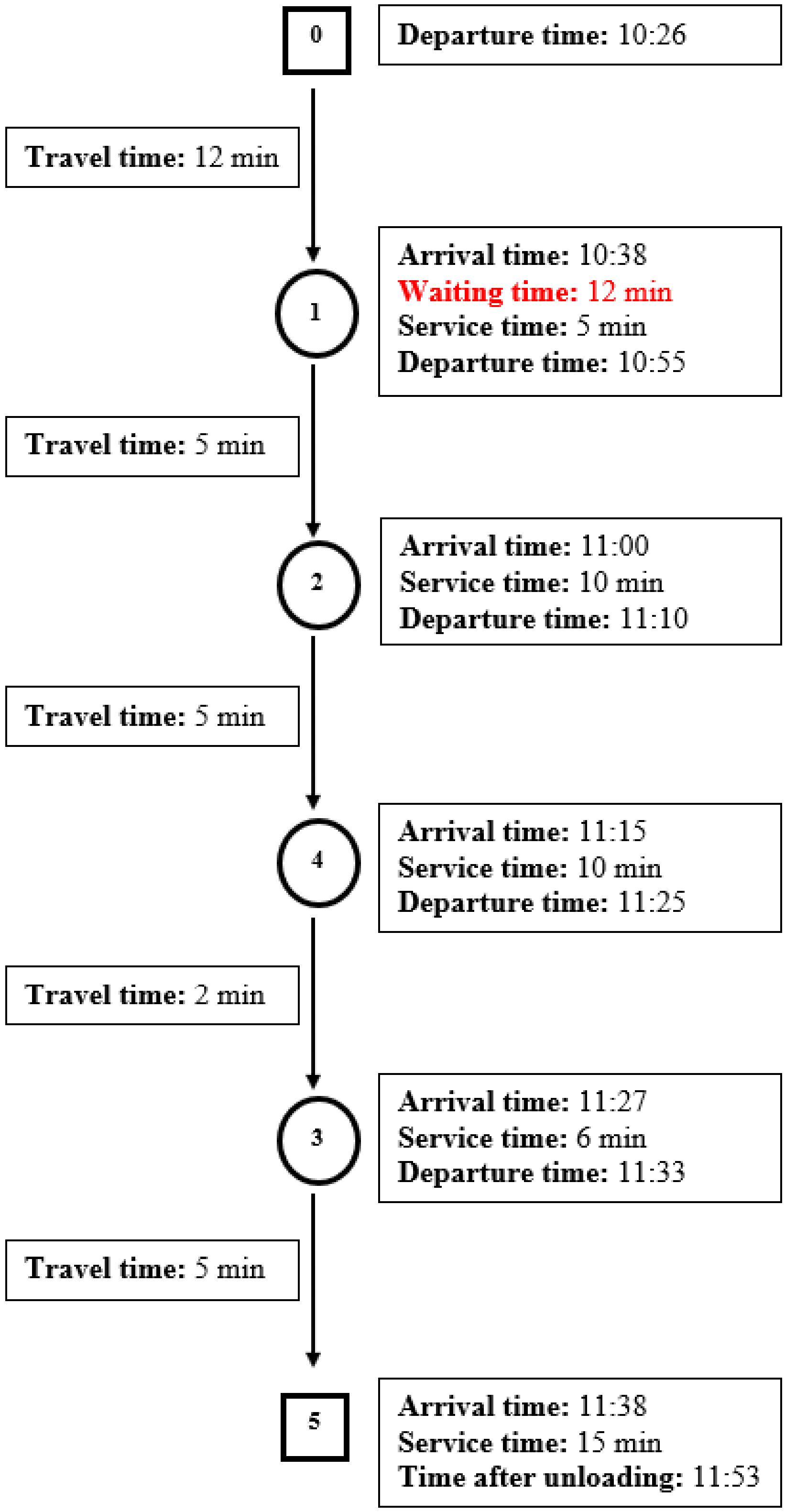


Figure 9: Scheme of the new solution for trip 3.

The objective function for this new version of the initial model is 687 minutes, that corresponds to 11:27 that represents the time of the last visit among all health centers. This time is the last arrival time at node 3 that is the last node that is visited in trip 3. Since there is no waiting time in the third trip in node 3, the last node that is visited, the pickup time is the same as the arrival time, that is 11:27.

As it can be seen, the solution obtained is different however, the time at which the complete service is finished is the same. This means the earliest possible time by which all blood samples are collected within the time imposed between production and laboratory delivery is 11:53. Regarding the total time used, the second solution requires a longer time span: from 07:56 to 11:53 instead of 08:45 to 11:53. In addition, the total travelled distance is now larger.

This shows that there is much more to explore regarding both, the model, and the problem.

Chapter 5 - Conclusions

This dissertation addresses a Vehicle Routing Problem with Multiple Trips and Time Constraints and was applied to a specific case study that fits the human health sector, more specifically the clinical analysis. The purpose is to create a model capable of improving the distribution of blood and other biological products collected from several health centers and carried to a central warehouse. A MILP model was formulated and afterwards implemented in CPLEX to solve the problem.

This work will help the clinical analysis sector not to waste blood samples and consequently reduce the negative impact on the environment because if the two and a half hours time limit to collect, transport, and deliver the blood after extraction is guaranteed, no additional extractions need to be done. This time limit imposed in our model is a big improvement for the daily work regarding companies of this sector.

From our analysis we concluded that the first big waiting time is not very relevant because this happens since there is no time schedule for the central laboratory and therefore the first trip starts as soon as possible. Starting the first trip at 00:00 makes no sense. The first trip can start at: (arrival time to the first node that is visited) – (travel time from the central laboratory to the first node that is visited). Regarding the other waiting times that happen in some stages of our solution it would be interesting that those waiting times are shift to occur always at the central laboratory so that the employees can use this time to perform other tasks and help the colleagues that are always at the central laboratory. If this shift is not possible for some other unknown reason, another possibility can be picking up other things or delivery other things in the “dead/blank times” for example medicines or correspondence (results of the blood analysis). Our model does not consider any costs. This could also be introduced into the model and analyzed in future work. As we can see there is lot of future work that can be done to improve this model and the day-to-day of the employees of the laboratories. There is always enough space to improve and learn more about something.

We decided to begin this work based on this small case study since this constitutes a real case study, this is the main reason why this case study was used. Unfortunately, there was no time to do a deeper analysis of our problem, model, and results in order to improve more our initial proposed model.

Future work may be further explorations of this specific problem and model. Multi objective functions can be considered instead of having only the objective of minimization of the total traveled distance. We have analyzed a second version of our initial model with some small changes that have changed the solution obtained in the initial model. We have changed the objective function from minimizing the total travel distance to minimizing the last visit of all health centers independently of the trip.

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