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NEW INSIGHTS TO APPROXIMATE THE PARETO OPTIMAL FRONT IN EVOLUTIONARY MULTIOBJECTIVE OPTIMIZATION. AN APPLICATION TO STUDENTS' SATISFACTION

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A tí , abuela;

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Chapter One

Introduction

1.1 PhD Thesis

This PhD thesis focuses on the development of new algorithms to solve multiobjective optimization problems and an application based on real data about the economics of education, specifically a problem relate to student satisfaction in secondary school. The thesis consists of three papers. The first paper (Chapter 2) studies the improvement of an evolutionary multiobjective optimization (EMO) algorithm called Global Weighting Achievement Scalarizing Function Genetic Algorithm (GWASF-GA) (Saborido, Ruiz, and Luque, 2017). The study of improved algorithm entails an analysis of the parameters producing the most promising results regarding the quality of approximation found and the efficiency of the method. The second paper (Chapter 3) provides an exhaustive description of a new EMO algorithm to solve so-called many-objective optimization problems, based on the previous one. Finally, in the third paper (Chapter 4), the statistical-econometric analysis of real data about student satisfaction with schools is used to propose a regression model, allowing us to build a multiobjective optimization problem that is subsequently solved by means of an EMO algorithm, in order to have a better insight of the problem.

Although the GWASF-GA algorithm generates good results in a series of multiobjective optimization problems, when the problem has a complicated Pareto Optimal Front (PF), such as a nonconvex and/or discontinuous one, the approximation set obtained could be enhanced in order to adjust better to the characteristics of the PF. In this sense, in Chapter 2, we propose an improvement of GWASF-GA where the weight vectors used are dynamically adjusted depending on the nondominated solutions found during the convergence process. The general idea is that, once GWASF-GA has been run for a percentage of iterations, some of the weight vectors are recalculated in a way that weight vector directions pointing toward overcrowded areas of the PF are re-calculated to re-direct the search towards regions with a lack of solutions. We analyze several settings for the parameters of the adjustment in a large set of benchmark test problems with three, five and six objectives. These experiments enable us to conclude that GWASF-GA obtains a better approximation of the PF in the problems considered when the weight vectors are adapted than when they are not.

In Chapter 3, a new version of the GWASF-GA algorithm, called Adaptive GWASF-GA (A-GWASF-GA), is proposed to solve many-objective optimization problems (those with more than three objective functions). The new proposal follows a philosophy similar to that of the previous paper, but with a fixed parameter setting. To be more precise, a metric called scattering level is defined to identify the overcrowded areas and the areas with a lack of solutions. Depending on each solution's scattering level, and based on some theoretical results demonstrated (in this thesis) regarding the projection directions that can be defined for any solution using either the utopian or the nadir points, the weight vectors to be re-directed are selected and re-calculated. The performance of the new algorithm is tested in comparison with other state-of-the-art EMO algorithms, such as different versions of MOEA/D (Zhang and Li, 2007), RVEA (Cheng et al., 2016), and NSGA-III (Deb and Jain, 2014; Jain and Deb, 2014), showing the promising results of A-GWASF-GA in more than thirty well-known (Deb et al., 2002b; Huband et al., 2007) and novel many-objective optimization problems (Li et al., 2019).

Finally, in Chapter 4, a real application regarding the satisfaction of students with their schools is proposed, using data from Andalusian secondary school students. Specifically, a multiobjective optimization problem is built and modeled through an econometric analysis, which is subsequently solved with GWASF-GA and its variants. First, lineal regressions are researched to measure four different aspects of the academic performance as functions of the satisfaction with several aspects of schools. Next, we define the objective functions to be optimized in the problem using the regression models obtained, as well as the constraints according to certain dependencies observed between the explanatory variables. We maximize four different measures of academic performance regressed with the students' satisfaction with the schools. Once the model is built, using GWASF-GA with different metrics, a set of nondominated solutions is generated in order to show the different trade-offs existing among the objective functions. In addition, these results allow us to extract interesting conclusions with respect to policy implications to be carried out in the Spanish educational system.

The following introduction is divided in two sections: multiobjective optimization (Section 1.2) and an application to student satisfaction and school performance (Section 1.3). In Section 1.2, some concepts and notations about multiobjective optimization are given in Section 1.2.1, followed by the motivation of the new algorithm introduced in Section 1.2.2, and the drafted proposal in Section 1.2.3. Finally, Section 1.3 briefly describes the application to economics of education.

1.2 Multiobjective Optimization

1.2.1 Background

In many real life situations, we have to make decisions simultaneously involving one or more conflicting criteria. In such situations, our final choice is based on intuition and common sense. Nevertheless, many situations need a more detailed study of the problem to find a final solution. For example, many problems in economy, industry, engineering, and other sciences require the use of mathematical modeling and programming techniques in order to model the problem and solve it to obtain the best solution.

In general, the concept of optimization means, first of all, to handle a problem involving one or several criteria; and secondly, to find the solution that best fits these criteria within a set of feasible alternatives. When both the criteria and the constraints that determine the feasible set of solutions or alternatives can be mathematically expressed by functions, we talk about an optimization problem. If more than one criterion is considered, they are usually in conflict, which means that it is impossible to find a single solution that optimizes each one of them individually. In such situations, single objective optimization techniques are insufficient and methods that deal with several conflicting objectives are required.

Multiobjective optimization programming can be framed as the area of Operational Research that defines concepts, theory, and methods to solve problems with more than one objective function over a feasible region. These types of problems are called Multiobjective Optimization Problems (MOPs), and can be defined as follows[1]:

$$\begin{array}{ll} \text{minimize} & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\} \\ \text{subject to} & \mathbf{x} \in X, \end{array} \tag{1.1}$$

where $k \geq 2$ is the number of objective functions, which are denoted by $f_i : X \rightarrow \mathbb{R}$ $(i = 1, \ldots, k)$. The vector of decision variables is referred to as $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T \in X$, and $X \subset \mathbb{R}^n$ is the feasible region or feasible set. In particular, when $k > 3$, we talk about a Many-objective Optimization Problem (MaOP).

Depending on the nature of the problem, it can be solved by considering different techniques. Multiattribute Decision Analysis supplies methods for solving problems in which the feasible set is explicitly known in advance, and consists of a finite number of alternatives (for further details, see Dyer et al. (1992), Tzeng and Huang (2011), and Zionts (1992)). Nevertheless, we will deal with problems whose set of feasible solutions is infinite and un-

[1] Without loss of generality, the multiobjective optimization problem is defined using the minimization form, but if one or several objective functions must be maximized, they are transformed just by multiplying them by -1.

known, that is, with problems whose solutions are determined by the decision variable values fulfilling the mathematically modeled constraints. In addition, there are problems for which some important parameters (such as coefficients of the objective functions or the constraints) are unknown. This type of MOP is analyzed with Stochastic Multiobjective Optimization (Goicoechea, Hansen, and Duckstein, 1982; Stancu-Minasian, 1984), Fuzzy Multiobjective Optimization (Ehrgott and Gandibleux, 2002; Bellman and Zadeh, 1970; Kacprzyk and Orlovski, 1987; Slowinski and Teghem, 1990; Verdegay, 1982) or Interval Multiobjective Optimization (Oliveira and Antunes, 2007), depending on the features of the problem.

As pointed out before, in the majority of MOPs, the degree of conflict existing among the objective functions makes it difficult to find an optimal solution where all of them can reach their individual optima. Instead, the so-called Pareto optimal or efficient solutions are defined, at which no objective function can be improved without worsening, at least, one of the others. A solution $\mathbf{x} \in X$ is said to be Pareto optimal for problem (1.1) if and only if there is no other $\bar{\mathbf{x}} \in X$ such that $f_i(\bar{\mathbf{x}}) \leq f_i(\mathbf{x})$ for all $i = 1, \ldots, k$, and $f_j(\bar{\mathbf{x}}) < f_j(\mathbf{x})$ for at least one index j. The corresponding objective vector $\mathbf{f}(\mathbf{x})$ is referred to as a Pareto optimal objective vector or a nondominated solution. The set of all Pareto optimal solutions is called the Pareto optimal set (PS), denoted by E, and the set of all nondominated solutions is called the Pareto optimal front (PF), denoted by $\mathbf{f}(E)$. A solution $\mathbf{x} \in X$ is weakly Pareto optimal if there does not exist another $\bar{\mathbf{x}} \in X$ such that $f_i(\bar{\mathbf{x}}) < f_i(\mathbf{x})$ for all $i = 1, \ldots, k$. Additionally, given two vectors $\mathbf{z}, \bar{\mathbf{z}} \in \mathbb{R}^k$, $\mathbf{z}$ dominates $\bar{\mathbf{z}}$ if and only if $z_i \leq \bar{z}_i$ for all $i = 1, \ldots, k$, and $z_j < \bar{z}_j$ for, at least, one index j. A definition derived from the previous one is the ε-dominance, for a small positive real value $\varepsilon > 0$. We say that $\mathbf{z}$ ε-dominates $\bar{\mathbf{z}}$ if and only if $z_i \leq \bar{z}_i + \varepsilon$ for all $i = 1, \ldots, k$, and $z_j < \bar{z}_j + \varepsilon$ for at least one index j. This type of domination is more relaxed than the strict dominance previously defined.

Multiobjective optimization has been an active area of research since the middle of the 20^{th} century. Some recommended readings about its history can be found in Gal and Hanne (1997), Köksalan, Wallenius, and Zionts (2011), and Stadler (1979). In order to solve MOPs, many methods have been developed over the years, of which we can highlight two extensively developed methodologies: Multiple Criteria Decision Making (MCDM) (Ehrgott and Gandibleux, 2002; Hwang and Masud, 1979; Miettinen, 1999; Steuer, 1986) and Evolutionary Multiobjective Optimization (EMO) (Abraham, Jain, and Goldberg, 2005; Coello, Lamont, and Veldhuizen, 2007; Deb, 2001; Ehrgott and Gandibleux, 2002; Gandibleux et al., 2004; Tan, Khor, and Lee, 2005). Some real applications of multiobjective optimization can be found in Ferreira, Ilander, and Ferreira (2019). Even though both methodologies provide good results, they have certain disadvantages. On the one hand, some MCDM techniques are not designed to deal with integer-valued variables, or discontinuous, nondifferentiable or nonconvex objective functions. Additionally, their computational cost is high for computationally demanding problems, because such techniques have to use an exact method to solve a single objective problem to find the final solution. On the other hand, EMO algorithms do not guarantee the efficiency of the solutions that they obtain. Indeed, not all the existing EMO algorithms are able to manage problems with all kinds of objective functions (Zou et al., 2008), and currently much research is being devoted to handling MaOPs.

From a mathematical point of view, all Pareto optimal solutions can be considered equivalent and non-superior to each other, hence it is necessary to incorporate a decision maker to the solution process. A decision maker (DM) is a person concerned with solving the multiobjective problem, who can indicate his/her preferences regarding the conflicting objectives and who is able to make a decision based on his/her point of view. With an analysis of all the nondominated solutions, the DM can choose one solution according to his/her preferences. However, studying the trade-offs observed among the objectives to select one satisfactory final solution for the problem implies a high-cognitive effort for the DM that is not trivial, and this decision-making stage deserves special attention when solving a MOP. To ease this decision-making task, many methods include information about the DM's preferences in the solution process to successfully converge to the most preferred solution. In this sense, the DM's preferences can be introduced by providing a reference point, $\mathbf{q} = (q_1, \ldots, q_k)^T$, that comprises desirable objective function values q_i $(i = 1, \ldots, k)$.

In recent years, in order to converge to the PF and to reflect the DM's preferences in the best possible way, many research studies have tried to combine the best of MCDM and EMO. Several algorithms have emerged in both fields from the hybridization of techniques, of which we can distinguish two. The first one is "EMO in MCDM", in which population-based approaches like evolutionary algorithms are considered to solve the scalarized functions involved in MCDM philosophy. Disadvantages such as binary or integer variables, discontinuity and nonconvexity have been overcome under this scheme (see e.g.Imai et al. (2006) and Miettinen (2007)). Secondly, there is "MCDM in EMO", which attempts to approximate a region of interest proposed by the DM (by means of her/his preferences), rather than obtaining an approximation of the whole PF (Ben-Said, Bechikh, and Ghedira, 2010; Branke, Kaussler, and Schemeck, 2001; Deb and Kumar, 2007a; Deb and Kumar, 2007b; Deb et al., 2006; Ruiz, Saborido, and Luque, 2015; Thiele et al., 2009). In this thesis we focus on EMO in MCDM to try to approximate PF in all kinds of multiobjective optimization problems, including MaOP problems.

1.2.2 Motivation

As mentioned above, EMO algorithms are developed to approximate the whole PF. The nondominated solutions in the approximation generated must be as close as possible to the PF (convergence) and as evenly distributed as possible in the PF (diversity). Nevertheless, the definition of an EMO algorithm whose approximation accomplishes both properties, diversity and convergence, at the same time is difficult, particularly when handling many-objective problems. For this reason, this field of research is in constant evolution, with many existing and widely-tested EMO methods being improved. The evolutionary strategy of an EMO algorithm simulates the process of natural evolution: selection, reproduction and mutation. The first step in solving a MOP using evolutionary algorithms is to randomly generate an initial population. Then, the algorithm follows an iterative process adapting the current population and defining a new population through the selection, crossover, and mutation operators.

In general, EMO algorithms can be classified as follows, depending on the selection strategy used: dominance-based (whose selection strategy is based on a relation of dominance), indicator-based (the value of a performance indicator metric is used to select the solutions), and aggregation-based or decomposition-based approaches (in which the original problem is transformed into a set of single-objective optimization subproblems). Some examples of dominance-based EMO algorithms are NSGA-II (Deb et al., 2002a), NSGA-III (Deb and Jain, 2014; Jain and Deb, 2014), and their different versions (Köppen and Yoshida, 2007; Li, Yang, and Liu, 2014). Regarding indicator-based EMO algorithms, we can mention HypE (Bader and Zitzler, 2011), TwoArch2 (Wang, Jiao, and Yao, 2015), and SMS-MOEA(Wagner and Neumann, 2013). Finally, within the aggregation-based type, we can cite MOEA/D (Zhang and Li, 2007) and different improvements suggested in the literature, such as MOEA/D-AWA (Qi et al., 2014) and MOEA/DD (Li, Deb, and Kwong, 2015), among others, and GWASF-GA (Saborido, Ruiz, and Luque, 2017).

Precisely, part of this thesis is concerned with the enhancement of the aggregation-based EMO algorithm GWASF-GA, proposed to solve all kinds of MOP, particularly to handle MaOPs. The main feature of this type of algorithms is to transform the MOP into a set of the single objective optimization problems. For this purpose, GWASF-GA considers an initial predefined set of weight vectors to be used in the achievement scalarizing function (ASF) proposed by Wierzbicki (Wierzbicki, 1980). This function, widely known in the MCDM field, is formulated as:

$$s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \mu) = \max_{i=1,\ldots,k} \{ \mu_i(f_i(\mathbf{x}) - q_i) \} + \rho \sum_{i=1}^{k} \mu_i(f_i(\mathbf{x}) - q_i), \tag{1.2}$$

where $\mu = (\mu_1, \ldots, \mu_k)^T$ is a vector of strictly positive weights, $\mathbf{q} = (q_1, \ldots, q_k)^T$ is the so-called reference point and $\rho \geq 0$ is a real value called augmentation coefficient. By minimizing (1.2) over the feasible region X:

$$\begin{aligned} \text{minimize} \quad & s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \mu) \\ \text{subject to} \quad & \mathbf{x} \in X, \end{aligned} \tag{1.3}$$

it is ensured that any optimal solution of (1.3) is a Pareto optimal solution of the original MOP (1.1) (Miettinen, 1999). In addition, any Pareto optimal solution of (1.1) can be found by solving (1.3) using different reference points and/or weights (Steuer, 1986). When this ASF is minimized over the feasible set of solutions, in practice, the reference point is projected onto the Pareto optimal front in a direction defined by the inverses of the weights (for further details, see Miettinen (1999)).

Even though all objective functions are differentiable, the ASF (1.2) is usually nondifferentiable. If needed, this disadvantage can be overcome by reformulating problem (1.3) as follows:

$$\begin{aligned} \text{minimize} \quad & \alpha + \rho \sum_{i=1}^{k} (f_i(\mathbf{x}) - q_i) \\ \text{subject to} \quad & \mu_i (f_i(\mathbf{x}) - q_i) \leq \alpha, \quad \text{for all } i = 1, \ldots, k \\ & \mathbf{x} \in X, \alpha \in \mathbb{R}. \end{aligned} \tag{1.4}$$

In GWASF-GA, two reference points, the so-called utopian and nadir points, are considered simultaneously and denoted by $\mathbf{z}^{\star\star} = (z_1^{\star\star}, \ldots, z_k^{\star\star})^T$ and $\mathbf{z}^{\text{nad}} = (z_1^{\text{nad}}, \ldots, z_k^{\text{nad}})^T$ respectively. Formally, the utopian point can be calculated through the ideal point $\mathbf{z}^{\star}$, for every $i = 1, \ldots, k$. The ideal point is calculated as $z_i^{\star} = \min_{\mathbf{x} \in E} f_i(\mathbf{x})$ and, thus, a utopian point can be obtained slightly modifying each component of the ideal point by $z_i^{\star\star} = z_i^{\star} - \epsilon$, where $\epsilon > 0$ is a small real value. The nadir point is defined as $\mathbf{z}_i^{\text{nad}} = \max_{\mathbf{x} \in E} f_i(\mathbf{x})$, for every $i = 1, \ldots, k$. The ideal and nadir points provide lower and upper bounds for the objective function values in X respectively, and the utopian point can be very useful given that, by definition, it strictly dominates the ideal point. In practice, the nadir point is estimated at each generation from the first frontier generated by the algorithm, so that, if we consider this set of points as an approximation of the Pareto optimal front, the worst values of each objective function are the nadir values.

As previously mentioned, a set of weight vectors are considered in the weight space $[0,1]^k$, taking into account that they must define projection directions as evenly distributed as possible. At each generation of GWASF-GA, in the ASF given in (1.2), half of the weight vectors in the predefined set are used with the nadir point as reference point, and the other half are considered with the utopian point as reference point. The solutions obtained after applying the selection, crossover, and mutation operators (offspring), along with their parents, are divided into several subfronts depending on the values achieved in the ASF, for the weight vectors and using simultaneously the nadir and utopian points. The lower the values of (1.2) reached by a solution for one of these two reference points, the more this solution is highlighted. Afterwards, the solutions for the next generation are those in the lower level subfronts. To some extent, these solutions can be considered as the best individuals of the current generation for minimizing the ASF (1.2) with respect to the weight vectors used, using both the utopian and the nadir points.

It has been demonstrated that GWASF-GA produces promising results in well-known test problems with two, three and five objectives (for further details see Saborido, Ruiz, and Luque (2017)). However, there are several drawbacks that have been addressed in new versions proposed. The first one is related to the fact that the set of weight vectors used in GWASF-GA remains unchanged along the whole evolutionary process. Actually, as mentioned above, their projection directions are evenly distributed, but they are defined without taking into account the characteristics of the PF (convexity, discontinuities, degenerated parts, sharp tails, etc.). In this sense, some researchers have proposed and demonstrated that the adjustment of the weight vectors taking into account the PF's complexity improves the algorithm's performance (Qi et al., 2014; Gu, Liu, and Chen Tan, 2012; Miettinen, 1999).

On this basis, this thesis proposes a new version of GWASF-GA to enhance its performance for problems with a complicated PF. The idea is that some of the weight vectors used are re-calculated during the optimization process based on the sparsity of the solutions found so far. Weight vectors generating solutions in overcrowded areas of the PF are replaced by new ones pushing the search towards regions of the PF with a lower number of solutions. To be more precise, a percentage (p) of generations are carried out using the original GWASF-GA and, for the remaining number of generations, we perform n_a times an adjustment of N_a of weight vectors (where n_a and N_a are two positive integer values). Each adjustment is done using either the nadir or the utopian point, taking into account the solutions obtained so far, and using some theoretical results demonstrated regarding the ASF considered.

1.2.3 Proposal

As stated above, the new proposal for the improvement of the GWASF-GA algorithm is related to the set of weight vectors used. In Chapter 2, an exhaustive description of the suggested algorithm is provided. The adjustment of the weight vectors is based on the following philosophy: a new metric called scattering level is defined to distinguish, according to the approximation produced at any generation, overcrowded areas (well-approximated areas) of the PF from the ones which are not well-covered; then, the weight vectors projecting solutions onto the overcrowded areas are identified and replaced by new weight vectors orientating the search for new nondominated solutions towards areas with a lack of solutions. When the weight vector adjustment process is completed, GWASF-GA is run again with the new set of weight vectors. The new proposal depends on several parameters: the percentage of generations run before the first adjustment is applied (p), the number of adjustments to be performed (n_a), and the number of weight vectors to be adapted (N_a). Thus, a preliminary analysis about the setting of these parameters is also provided in Chapter 2, in order to gain knowledge about their impact on the performance of the algorithm. We compare the original GWASF-GA with our proposal using different configurations of the parameters. The parameter values studied are $p = 0.6, 0.7, 0.8$, $n_a = 2, 4, 6$, and $N_a = 5, 20, 25, 30, 50, 60, 75, 100$. Overall, 46 test problems are considered from the DTLZ (Deb et al., 2002b) and WFG (Huband et al., 2007) families, with 3, 5 and 6 objective functions.

The hypervolume metric (Zitzler and Thiele, 1999) is calculated in order to measure the algorithm's performance. The results show that the weight vector adjustment improves the performance of GWASF-GA in the cases considered.

The new version of GWASF-GA, which is called Adaptive GWASF-GA (A-GWASF-GA), is further described and analyzed in Chapter 3, where an exhaustive computational study is presented to measure its effectiveness in a wider variety of MOPs, including MaOPs. In comparison to the previous work, a more detailed description of the new algorithm is provided, including several issues that motivate and justify our proposal. Two theoretical results are also proved regarding the dominance of the solutions generated using the new weight vectors built at each adjustment. Besides, a much larger set of benchmark problems, with a higher number of objectives, have been considered in the computational experiment. Specifically, we have used a total of 36 different problems from the novel MaOP family (Li et al., 2019), and from the DTLZ (Deb et al., 2002b) and WFG (Huband et al., 2007) families, with up to 10 objectives. In addition, we have considered two performance indicators (instead of just one) to measure the quality of the approximations: the hypervolume (Zitzler and Thiele, 1999) and the Inverted Generational Distance (IGD) (Zitzler et al., 2003). Currently, A-GWASF-GA has been tested against four well-known state-of-the-art EMO algorithms: RVEA (Cheng et al., 2016), NSGA-III (Deb and Jain, 2014; Jain and Deb, 2014), MOEA/D-DE (Zhang and Li, 2007), MOEA/DD (Li, Deb, and Kwong, 2015), and MOEA/D-DE-AWA (Qi et al., 2014). This computational experiment allows us to conclude that our proposal improves the performance of the other algorithms, especially in multiobjective optimization problems with many objectives.

1.3 An application to student satisfaction and school performance

The academic performance of Spanish students is one of the main worries of the government regarding education. Particularly, Spain does not achieve good results in educational rankings such as PISA[2] (OECD, 2018; OECD, 2019). Following the results of PISA, Andalusia, the second biggest region of Spain, is below the average scores for Spain and the OECD countries[3]. Besides, Andalusia shows high repetition and dropout rates in secondary school. On the other hand, many papers in the literature show that educational outputs are related to economic growth in a country. Actually, (Mankiw, Romer, and Weil, 1992) argue that human capital increases with the quality of education and hence, economic growth also improves, and (Barro, 2001) claims that education is considered one of the main factors in economic growth. Therefore, it is evident that analyzing which factors enable the academic performance of Andalusian students to be improved is a very important issue for the Spanish authorities, and in particular for the Andalusian authorities.

Over the years, the concept of academic performance has changed. Conventionally, academic performance is measured through scores in the main subjects (Hanushek and Woessmann, 2010); but academic assessment can also be measured through socio-demographic characteristics and other educational indicators, i.e. retention rate (Calero and Escardíbul, 2013; Rumberger and Palardy, 2005). This fact has motivated an increase of the research work and studies published in recent years regarding student performance and satisfaction of pupils in general. Natvig, Albrektsen, and Qvarnstrøm (2003) report a positive correlation between good assessment and life satisfaction. More specifically, the school environment is positively correlated with academic achievement in English and Mathematics (Uline and

[2]PISA: Programme for International Student Assessment. https://www.oecd.org/pisa/

[3]The OECD consists of 36 countries that collaborate on key global issues at national, regional and local levels. The member countries are from Europe, North and South America, and Asian-Pacific.

Tschannen-Moran, 2008). Interestingly, parent satisfaction also influences the progression and academic assessment of students (Gibbons and Silva, 2011).

Taking into account this previous research work, we were interested in finding an optimum balance among different measures of academic performance depending on the student satisfaction with school. For this purpose, we combined econometric and multiobjective optimization techniques. There are several previous papers in the literature where a socio-economic modeling problem is built using econometric analysis, and is subsequently solved using a technique of multiobjective optimization. For example, in Luque, Marcenaro-Gutiérrez, and López-Agudo (2015), an optimum of student performance in terms of several subjects is found as a function of students' and families' socio-demographic characteristics. In a similar way, a balance is achieved among several aspects of educational outputs considering teacher satisfaction, using data provided by 4183 students, 200 teachers and 151 schools (Marcenaro-Gutiérrez, Luque, and López-Agudo, 2016).

Therefore, in Chapter 4 we present the analysis of the socio-economic phenomenon under study, i.e. the analysis of the academic performance of students in Andalusia using several indicators of their satisfaction with the school. For this purpose, we use data from 162 Andalusian schools and four explained variables to measure the academic performance: student scores in math and reading, and the percentage of students reaching level four[4] in both subjects. On the other hand, variables of student satisfaction with different aspects of the school are used as explanatory variables in the estimations. They include, amongst others, satisfaction with the facilities, information processes, teacher respect and attention, complementary activities, etc. Using econometric techniques, a multiobjective optimization problem is built, whose aim is to maximize the four measures of academic performance. In contrast with previous work (Luque, Marcenaro-Gutiérrez, and López-Agudo, 2015; Marcenaro-Gutiérrez, Luque, and López-Agudo, 2016), the resulting problem is solved using an EMO algorithm (in our case, GWASF-GA), which is a novel contribution of this

[4]Level four is reached by students with 559 points in a subject.

thesis. To the best of our knowledge, this type of techniques has not been previously applied under this econometric-multiobjective optimization perspective.

Chapter Two

Paper 1: An Improvement Study of the Descomposition-Based Algorithm Global WASF-GA for Evolutionary Multiobjective Optimization

González-Gallardo, S., Saborido, R., Ruiz, A. B., and Luque, M. (2018). “An Improvement Study of the Decomposition-Based Algorithm Global WASF-GA for Evolutionary Multiobjective Optimization”. In: Advances in Artificial Intelligence. Ed. by F. Herrera, S. Damas, R. Montes, S. Alonso, Ó. Cordón, A. González, and A. Troncoso. Springer International Publishing, pp. 219–229. DOI: 10.1007/978-3-030-00374-6_21

An Improvement Study of the Decomposition-Based Algorithm Global WASF-GA for Evolutionary Multiobjective Optimization

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Abstract. The convergence and the diversity of the decomposition-based evolutionary algorithm Global WASF-GA (GWASF-GA) relies on a set of weight vectors that determine the search directions for new non-dominated solutions in the objective space. Although using weight vectors whose search directions are widely distributed may lead to a well-diversified approximation of the Pareto front (PF), this may not be enough to obtain a good approximation for complicated PFs (discontinuous, non-convex, etc.). Thus, we propose to dynamically adjust the weight vectors once GWASF-GA has been run for a certain number of generations. This adjustment is aimed at re-calculating some of the weight vectors, so that search directions pointing to overcrowded regions of the PF are redirected toward parts with a lack of solutions that may be hard to be approximated. We test different parameters settings of the dynamic adjustment in optimization problems with three, five, and six objectives, concluding that GWASF-GA performs better when adjusting the weight vectors dynamically than without applying the adjustment.

Keywords: Evolutionary multiobjective optimization
Decomposition-based algorithm · GWASF-GA · Weight vector

1 Introduction

In general, *multiobjective optimization problems* (MOPs) can be defined as:

$$\begin{array}{ll} \text{minimize} & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\} \\ \text{subject to} & \mathbf{x} \in S, \end{array} \tag{1}$$

F. Herrera et al. (Eds.): CAEPIA 2018, LNAI 11160, pp. 219–229, 2018.
https://doi.org/10.1007/978-3-030-00374-6_21

Chapter Three

Paper 2: Adaptive Global WASF-GA to handle many-objective optimization problems

Luque, M., Gonzalez-Gallardo, S., Saborido, R., and Ruiz, A. B. (2020). "Adaptive Global WASF-GA to handle many-objective optimization problems". In: Swarm and Evolutionary Computation, 54, p. 100644. DOI: 10.1016/j.swevo.2020.100644

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Adaptive Global WASF-GA to handle many-objective optimization problems

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ABSTRACT

In this paper, a new version of the aggregation-based evolutionary algorithm Global WASF-GA (GWASF-GA) for many-objective optimization is proposed, called *Adaptive Global WASF-GA* (A-GWASF-GA). The fitness function of GWASF-GA is defined by an achievement scalarizing function (ASF) based on the Tchebychev distance, which considers two reference points (the nadir and utopian points) and a set of weight vectors. Despite of the benefits of using these two reference points simultaneously and a well-distributed set of weight vectors, it is necessary to go a step further to get better approximations in problems with complicated Pareto optimal fronts. For this, in A-GWASF-GA, some of the weight vectors are re-calculated during the optimization process based on the sparsity of the solutions found so far, and taking into account some theoretical results demonstrated in this paper regarding the ASF considered. Different strategies are carried out to accelerate the convergence and to maintain the diversity. The computational results, carried out in comparison with RVEA, NSGA-III, and different versions of MOEA/D, show the potential of A-GWASF-GA in well-known but also in novel many-objective optimization benchmark problems.

1. Introduction

In many real applications, a *multiobjective optimization problem* (MOPs) arises, consisting of the simultaneous optimization of several objective functions, subject to several constraints that determine the feasible set of solutions. As finding a single solution optimizing all the objectives at the same time is usually impossible because of the conflict existing among the objectives, the interest is focused on the so-called *Pareto optimal solutions*. At them, an objective function improvement can only be achieved at the expense of worsening, at least, one of the others. Problems handling more than three objectives, referred to as *many-objective optimization problems* (MaOPs), are more difficult to solve and lately they have drawn much attention of the multiobjective optimization community [1,2].

Along the years, *Evolutionary Multiobjective Optimization* (EMO) algorithms have demonstrated their ability for solving MOPs [3,4] and MaOPs [1,2,5,6]. They are aimed at finding a subset of non-dominated solutions approximating the *Pareto optimal front* (PF) (the set of all Pareto optimal solutions in the objective space). The approximation set must be formed by solutions as evenly distributed as possible in the PF (diversity), and as close as possible to the PF (convergence). However, achieving convergence and diversity simultaneously is not always easy for an EMO algorithm, specially when handling MaOPs or problems with complicated PFs (such as e.g. non-convex, degenerated, or discontinuous). Actually, several works propose to modify the dominance relationship in order to enhance the convergence, such as e.g. the fuzzy dominance suggested in Ref. [7] or the ϵ-dominance proposed in Ref. [8].

Based on the selection strategies used, EMO algorithms can be categorized into three main classes: indicator-based approaches (whose selection strategy is based on the computation of a performance indicator metric), dominance-based approaches (which use a dominance relation -the Pareto dominance or any other one- for the comparison of solutions), and aggregation-based or decomposition-based approaches (which transform the original problem into a set of single-objective optimization subproblems).

In the indicator-based class, we can mention SMS-MOEA [9], in which each solution is evaluated by its hypervolume contribution, distance-based algorithms such as e.g. IBEA [10], or algorithms defined according to the R2 indicator such as e.g. MOMBI [11]. Other indicator-

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Chapter Four

Paper 3: Evaluating the potential trade-off between students' satisfaction and school performance using evolutionary multiobjective optimization

Marcenaro-Gutierrez, O.D., González-Gallardo, S., and Luque, M. (2020) "Evaluating the potential trade-off between students' satisfaction and school performance using evolutionary multiobjective optimization" In: RAIRO - Operations Research. DOI: 10.1051/ro/2020027

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EVALUATING THE POTENTIAL TRADE-OFF BETWEEN STUDENTS' SATISFACTION AND SCHOOL PERFORMANCE USING EVOLUTIONARY MULTIOBJECTIVE OPTIMIZATION

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Abstract. In this article, we carry out a combined econometric and multiobjective analysis using data from a representative sample of Andalusian schools. In particular, four econometric models are estimated in which the students' academic performance (scores in math and reading, and percentage of students reaching a certain threshold in both subjects, respectively) are regressed against the satisfaction of students with different aspects of the teaching-learning process. From these estimates, four objective functions are defined which have been simultaneously maximized, subject to a set of constraints obtained by analyzing dependencies between explanatory variables. This multiobjective programming model is intended to optimize the students' academic performance as a function of the students' satisfaction. To solve this problem we use a decomposition-based evolutionary multiobjective algorithm called Global WASF-GA with different scalarizing functions which allows generating an approximation of the Pareto optimal front. In general, the results show the importance of promoting respect and closer interaction between students and teachers, as a way to increase the average performance of the students and the proportion of high performance students.

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1. Introduction

The relative low academic performance of secondary education students is a main concern for the educational authorities in Spain, as this country leads the ranking within the UE countries in terms of educational dropout rate and grade retention[1]. Specifically, Andalusia is among the Spanish regions that shows lower academic

Keywords. Evolutionary multiobjective optimization students' performance, students' satisfaction, achievement scalarizing function, econometric analysis.

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[1] The percentage of students (aged between 6 and 16 years) who have repeated in Spain, at least once, is around 32%. This figure is well above the actual average of OCDE, around 12% [18].

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Chapter Five

Results, conclusions and future research work

5.1 Results

After presenting all the research articles that constitute this thesis, the results obtained are analyzed and discussed in this chapter.

Let us start discussing the results of the first paper, in which a weight vector adaptation is proposed for the GWASF-GA algorithm, and where different parameter settings are studied in comparison against the original GWASF-GA. Regarding the hypervolume (HV) metric, the results achieved in 30 independent runs indicate a good performance of the new proposal in the majority of test problems considered in the computational study. Specifically, Figure 2 of Chapter 2 depicts the results of the 72 different parameter configurations analyzed for the adaptation of the weight vectors. It also indicates the number of problems in which GWASF-GA is significantly better (▲) and worse (▽) with the weight adjustment than the original GWASF-GA. These results were obtained applying the Wilcoxon rank-sum test (Wilcoxon, 1945). The Wilcoxon test checks whether the HV average values in the 30 runs obtained by the new proposal is significantly different from the remaining algorithms. We consider the difference to be significant if the p-value obtained is lower than $\alpha = 0.05$. We use the `wilcox.test` function from the R software, to compute the Wilcoxon rank-sum test.[1]

Regarding the three-objective problems, the GWASF-GA version with the adjustment of weight vectors wins, at least, in 12 of 18 problems. Indeed, with some configurations, it obtains better results than the original GWASF-GA in the highest number of cases, reaching simultaneously an equal performance for the rest (these cases are highlighted in gray color). Overall, the hypervolume achieved by GWASF-GA when adapting the weight's vectors is higher than the original GWASF-GA in 81% of the problems.

In relation to the five-objective problems, we tested a total of 14 problems. It is noteworthy that with the weight adaptation GWASF-GA is significantly better than the original algorithm in 13 problems (achieving the same performance in the remaining problem) in a total of 37 out of the 72 parameter configurations. On average, the new proposal is better in 76% of the cases.

For the six-objective problems, we also considered 14 problems. Here, the hypervolume achieved by GWASF-GA is higher when using the weight vector adjustment in most of the problems, although there is one configuration ($n_a = 4$, $p = 0.8$ and $N_a = 75$) for which the original GWASF-GA wins in 11 problems. In summary, the adjustment of the weight vectors allows GWASF-GA to obtain better results than the original algorithm in 74% of the problems.

According to these results, we can say that, when GWASF-GA is executed using the adaptation of weight vectors, the approximations found outperform those obtained by the original algorithm in most of the problems considered, regardless of the parameter setting used.

Next, the results obtained in the second paper (Chapter 3) are described. This paper consists of two different types of contributions: theoretical and computational results. On the one hand, the theoretical results comprise the following two theorems (Appendix A of Chapter 3).

[1] https://stat.ethz.ch/R-manual/R-devel/library/stats/html/wilcox.test.html

Theorem 1 *Given a feasible solution* $\mathbf{x} \in X$*, then, either* $\mathbf{x}$ *is an optimal solution to problem* (1.3) *considering* $\mathbf{q} = \mathbf{z}^{\star\star}$ *and the weight vector* μ^U *defined by*

$$\mu_i^U = \frac{1}{f_i(\mathbf{x}) - z_i^{\star\star}}, \quad \textit{for each } i = 1, \ldots, k.$$

or the objective vector of any optimal solution to problem (1.3) *with these parameters strictly dominates or* ε*-dominates* $\mathbf{f}(\mathbf{x})$*, for some* $\varepsilon > 0$.

Theorem 2 *Given a feasible solution* $\mathbf{x} \in S$*, then, either* $\mathbf{x}$ *is an optimal solution to problem* (1.3) *considering* $\mathbf{q} = \mathbf{z}^{\mathrm{nad}}$ *and the weight vector* μ^N *defined by*

$$\mu_i^N = \frac{1}{z_i^{\mathrm{nad}} - f_i(\mathbf{x})}, \quad \textit{for each } i = 1, \ldots, k.$$

or the objective vector of any optimal solution to problem (1.3) *with these parameters strictly dominates or* ε*-dominates* $\mathbf{f}(\mathbf{x})$*, for some* $\varepsilon > 0$.

Both theorems demonstrate that the adjustment of the weight vectors carried out in A-GWASF-GA enables the generation of new nondominated solutions in less populated areas of the PF.

On the other hand, the results of the computational experiments constitute the paper's second contribution described in Chapter 3. Here, A-GWASF-GA is compared with respect to well-known EMO algorithms such as RVEA, NSGA-III, MOEA/D-DE, MOEA/DD and MOEA/D-DE-AWA, whereas in Chapter 2 the comparative study was carried out regarding the original GWASF-GA. Besides, a larger set of problems has been considered in this study, including many-objective problems with up to ten objective functions. In addition, as we are interested in the quality of the approximation of the Pareto fronts instead of the execution time of algorithms, we compared algorithms in terms of well-known performance metrics such as IGD and HV metrics. The results included in Tables 1-6 of the second paper are summarized below, and are described in Tables 5.1-5.6.

Firstly, Tables 5.1 and 5.2 depict the cases in which A-GWASF-GA is better than (▲), equal to (⊙) and worse than (▽) the other algorithms regarding the IGD metric for the MaOP problems using $n = 20$ and $n = 50$ (where n denotes de number of variables), respectively. Based on these tables, let us discuss the performance of A-GWASF-GA in comparison to each of the other algorithms included in our study.

Table 5.1 Performance of A-GWASF-GA compared to several EMO algorithms in 30 independent runs regarding the IGD metric for the MaOP problems with $n = 20$.

k	RVEA	NSGA-III	MOEA/D-DE	MOEA/DD	MOEA/D-DE-AWA
3	▲**3** ⊙**1** ▽**6**	▲**7** ⊙**0** ▽**3**	▲**1** ⊙**0** ▽**9**	▲**3** ⊙**0** ▽**7**	▲**1** ⊙**0** ▽**9**
5	▲**8** ⊙**1** ▽**1**	▲**7** ⊙**1** ▽**2**	▲**5** ⊙**0** ▽**5**	▲**8** ⊙**0** ▽**2**	▲**4** ⊙**2** ▽**4**
8	▲**8** ⊙**0** ▽**2**	▲**8** ⊙**1** ▽**1**	▲**9** ⊙**0** ▽**1**	▲**8** ⊙**0** ▽**2**	▲**7** ⊙**0** ▽**3**
10	▲**7** ⊙**0** ▽**3**	▲**9** ⊙**1** ▽**0**	▲**9** ⊙**1** ▽**0**	▲**7** ⊙**0** ▽**3**	▲**9** ⊙**0** ▽**1**

Table 5.2 Performance of A-GWASF-GA compared to several EMO algorithms in 30 independent runs regarding the IGD metric for the MaOP problems with $n = 50$.

k	RVEA	NSGA-III	MOEA/D-DE	MOEA/DD	MOEA/D-DE-AWA
3	▲**8** ⊙**1** ▽**1**	▲**7** ⊙**0** ▽**3**	▲**2** ⊙**1** ▽**7**	▲**4** ⊙**1** ▽**5**	▲**3** ⊙**0** ▽**7**
5	▲**9** ⊙**1** ▽**0**	▲**8** ⊙**0** ▽**2**	▲**6** ⊙**0** ▽**4**	▲**8** ⊙**1** ▽**1**	▲**5** ⊙**1** ▽**4**
8	▲**8** ⊙**0** ▽**2**	▲**8** ⊙**1** ▽**1**	▲**8** ⊙**0** ▽**2**	▲**9** ⊙**0** ▽**1**	▲**7** ⊙**0** ▽**3**
10	▲**9** ⊙**0** ▽**1**	▲**9** ⊙**0** ▽**1**	▲**9** ⊙**0** ▽**1**	▲**9** ⊙**0** ▽**1**	▲**8** ⊙**0** ▽**2**

In relation to RVEA, for the three-objective problems and $n = 20$, this algorithm wins in 6 out 10 of problems (indicated as 6/10 hereinafter) regarding the IGD metric. However, when $n = 50$, A-GWASF-GA wins in 8/10 cases. For the many-objective problems (with five, eight and ten objectives), the A-GWASF-GA's performance is evidently better than RVEA's, since A-GWASF-GA produces better IGD values than RVEA, at least, in 7/10 problems, both when $n = 20$ and $n = 50$.

With respect to NSGA-III, A-GWASF-GA is better in all of the dimensions concerning the IGD metric, regardless of the number of decision variables ($n = 20$ and $n = 50$). It is noted that, at least, A-GWASF-GA obtains better results than NSGA-III's in 7/10 problems, independently of the number of objective functions, particularly for the ten-objective problems, A-GWASF-GA outperforms NSGA-III in 9/10 cases for both $n = 20$ and $n = 50$.

Finally, in comparison to the MOEA/D versions considered, the IGD results produced by

A-GWASF-GA are worse than those of MOEA/D-DE, MOEA/DD and MOEA/D-DE-AWA in the three-objective problems, using both $n = 20$ and $n = 50$. However, concerning the five-objective problems, A-GWASF-GA is significantly better than MOEA/DD in 8/10 problems for both $n = 20$ and $n = 50$, while it performs similarly to MOEA/D-DE and MOEA/D-DE-AWA. Nonetheless, for the eight- and ten-objective problems, A-GWASF-GA's performance is significantly better than the three versions of MOEA/D regarding the IGD metric. In particular, for the eight-objective problems and $n = 20$, A-GWASF-GA is better than MOEA/D-DE, MOEA/DD and MOEA/D-DE-AWA in 9/10, 8/10 and 7/10 problems, respectively. When $n = 50$ (in the problems with eight objectives), A-GWASF-GA wins in 8/10 compared to MOEA/D-DE, in 9/10 with respect to MOEA/DD, and in 7/10 in comparison with MOEA/D-DE-AWA. Regarding the ten-objective problems, A-GWASF-GA performs better than MOEA/D-DE in 9/10 cases (for both $n = 20$ and $n = 50$), wins in 7/10 (when $n = 20$) and 9/10 (when $n = 50$) compared to MOEA/DD, and obtains better IGD values in 9/10 (when $n = 20$) and in 8/10 (when $n = 50$) with respect to MOEA/D-DE-AWA.

Secondly, in Tables 5.3 and 5.4, we can see the number of problems in which A-GWASF-GA is better than (▲), equal to (⊙) and worse than (▽) the other algorithms regarding the HV metric for the MaOP problems using $n = 20$ and $n = 50$, respectively. It is important to note that, owing to the computational cost that HV computation requires, this metric has not been calculated for ten-objective problems.

Table 5.3 Performance of A-GWASF-GA compared to several EMO algorithms in 30 independent runs regarding the HV metric for the MaOP problems with $n = 20$.

k	RVEA	NSGA-III	MOEA/D-DE	MOEA/DD	MOEA/D-DE-AWA
3	**▲9 ⊙1 ▽0**	**▲7 ⊙2 ▽1**	**▲9 ⊙1 ▽0**	**▲10 ⊙0▽0**	**▲8 ⊙2 ▽0**
5	**▲8 ⊙2 ▽0**	**▲8 ⊙0 ▽2**	**▲8 ⊙2 ▽0**	**▲10 ⊙0▽0**	**▲7 ⊙3 ▽0**
8	**▲9 ⊙1 ▽0**	**▲7 ⊙2 ▽1**	**▲9 ⊙1 ▽0**	**▲10 ⊙0▽0**	**▲8 ⊙2 ▽0**

Concerning RVEA, A-GWASF-GA generates significantly better HV values regardless

Table 5.4 Performance of A-GWASF-GA compared to several EMO algorithms in 30 independent runs regarding the HV metric for the MaOP problems with $n = 50$.

k	RVEA	NSGA-III	MOEA/D-DE	MOEA/DD	MOEA/D-DE-AWA
3	**▲9 ⊙1 ▽0**	**▲5 ⊙2 ▽3**	**▲4 ⊙1 ▽5**	**▲8 ⊙1 ▽1**	**▲5 ⊙2 ▽3**
5	**▲10⊙0 ▽0**	**▲9 ⊙0 ▽1**	**▲10⊙0 ▽0**	**▲10⊙0 ▽0**	**▲10⊙0 ▽0**
8	**▲9 ⊙1 ▽0**	**▲8 ⊙1 ▽1**	**▲9 ⊙1 ▽0**	**▲10 ⊙0▽0**	**▲9 ⊙1 ▽0**

the number of objectives, for both $n = 20$ and $n = 50$. In particular, A-GWASF-GA wins, at least, in 8/10 (when $n = 20$) and in 9/10 (when $n = 50$) problems for each dimension.

In regard to NSGA-III, for the three-objective problems, A-GWASF-GA performs very similarly to this algorithm. But for the five- and eight-objective problems, A-GWASF-GA obtains significantly better HV values than NSGA-III, at least, in 7/10 (when $n = 20$) and in 8/10 (when $n = 50$) problems.

Regarding MOEA/D versions, the performance of A-GWASF-GA is statistically better than the other algorithms concerning the HV metric, specifically in the five- and eight-objective problems. In these two dimensions, A-GWASF-GA wins, at least, in 7/10 (when $n = 20$) and in 9/10 (when $n = 50$) problems, respectively.

Next, we analyze the results for the DTLZ and WFG problems. Tables 5.5 and 5.6 contain the number of cases in which A-GWASF-GA is better than (▲), equal to (⊙) and worse than (▽) the other algorithms for the DTLZ and WFG problems regarding the IGD and HV metric, respectively.

Table 5.5 Performance of A-GWASF-GA compared to several EMO algorithms in 30 independent runs regarding the IGD metric for DTLZ and WFG problems.

k	RVEA	NSGA-III	MOEA/D-DE	MOEA/DD	MOEA/D-DE-AWA
3	**▲1⊙2 ▽11**	**▲2⊙0 ▽12**	**▲11⊙0 ▽3**	**▲8 ⊙1 ▽5**	**▲5 ⊙1 ▽8**
5	**▲2⊙0 ▽12**	**▲3⊙0 ▽11**	**▲11⊙0 ▽3**	**▲5 ⊙0 ▽9**	**▲5 ⊙1 ▽8**
8	**▲3⊙0 ▽11**	**▲5 ⊙1 ▽8**	**▲14⊙0 ▽0**	**▲5 ⊙2 ▽7**	**▲7 ⊙0 ▽7**
10	**▲2⊙0 ▽12**	**▲7 ⊙0 ▽7**	**▲14⊙0 ▽0**	**▲9 ⊙1 ▽4**	**▲8 ⊙1 ▽5**

Concerning the IGD metric (Table 5.5), RVEA obtains better results than A-GWASF-GA regardless of the number of objective functions. In particular, A-GWASF-GA only wins in

Table 5.6 Performance of A-GWASF-GA compared to several EMO algorithms in 30 independent runs regarding the HV metric for the DTLZ and WFG problems.

k	RVEA	NSGA-III	MOEA/D-DE	MOEA/DD	MOEA/D-DE-AWA
3	**▲12⊙0 ▽2**	**▲6 ⊙1 ▽7**	**▲12⊙1 ▽1**	**▲13⊙1 ▽0**	**▲12 ⊙1 ▽1**
5	**▲6 ⊙1 ▽7**	**▲9 ⊙0 ▽5**	**▲14⊙0 ▽0**	**▲14⊙0 ▽0**	**▲14 ⊙0 ▽0**
8	**▲8 ⊙2 ▽4**	**▲11 ⊙0 ▽3**	**▲13⊙1 ▽0**	**▲14⊙0 ▽0**	**▲12 ⊙2▽0**

3/14 and in 2/14 problems for the eight- and ten-objective problems, respectively. Besides, NSGA-III performs better than A-GWASF-GA for the three-, five- and eight-objective problems, and both algorithms perform similarly for the ten-objective problems. Nevertheless, A-GWASF-GA provides better results in comparison to MOEA/D-DE. For the three- and five-objective problems A-GWASF-GA wins in 11/14 problems and in 14/14 problems for the eight- and ten-objective problems, respectively. In respect to MOEA/DD, A-GWASF-GA obtains better results for the three- and ten-objective problems (winning in 8/14 and in 9/14 problems, respectively), although the results are not as good for the five- and eight-objective problems. Regarding MOEA/D-DE-AWA, A-GWASF-GA obtains better IGD results for the eight- and ten-objective problems, given that it wins in 7/14 and 8/14 problems, respectively. However, this does not happen for the three- and five-objective problems. These results suggest that, for the DTLZ and WFG problems and regarding the IGD metric, A-GWASF-GA does not report results that are as good as for MaOP problems.

However, in relation to the HV metric (Table 5.6), A-GWASF-GA provides statistically better results for the DTLZ and WFG problems. For the three-objective problems, A-GWASF-GA wins in 12/14 problems with respect to RVEA, in 6/14 problems as compared to NSGA-III, in 13/14 problems in comparison with MOEA/DD, and in 12/14 problems in contrast to both MOEA/D-DE and MOEA/D-DE-AWA. Regarding the five-objective problems, A-GWASF-GA is better in 6/14 problems against RVEA, in 9/14 problems with respect to NSGA-III, and in 14/14 problems for all the versions of MOEA/D. Finally, for the eight-objective problems, A-GWASF-GA achieves significantly better HV results in 8/14 problems in comparison with RVEA, in 11/14 problems compared to NSGA-III, and in 13/14, 14/14 and 12/14 problems with respect to MOEA/D-DE, MOEA/DD and MOEA/D-DE-AWA, respectively.

Overall, according to the results presented above, A-GWASF-GA has generated very promising results, especially as the number of objective functions increases, achieving better IGD and HV values in comparison to the other algorithms.

Next, the results obtained for the application problem proposed in Chapter 4 are shown. In a first stage, a multiobjective optimization problem is built by means of an econometric analysis. Four regressions have been carried out, in which the explained variables are mean scores in math, mean scores in reading, percentage of students above level four in math, and percentage of students above level four in reading; and the explanatory variables are the students' satisfaction with several aspects of the school. To make the problem more realistic, some constraints are defined considering the dependency of some explanatory variables and some bounds are defined (upper and lower) taking into account the maximum and the minimum values achieved by the explanatory variables.

To obtain different solutions and analyze the different tradeoffs among them, this problem has been solved with an improved version of GWASF-GA using different scalarizing functions (ASFs). On the one hand, we use the ASF proposed by Wierzbicki (1980) and given in (1.2) as in the original algorithm. However, we have also added the L_p metric for several values for $p = 1, 2, 5$ as ASFs. As previously mentioned, GWASF-GA is based on the use of two reference points, the utopian and the nadir points. To solve this problem, the nadir point is calculated internally by the algorithm (using the worst objective function values obtained by the solutions generated). We set the utopian point using the ideal point, which is obtained by maximizing each objective function individually.

Given that we execute GWASF-GA four times (using the four different scalarizing functions), we obtain four sets of solutions, which are then mixed to select only the nondominated solutions among all the solutions generated. As a result, 31 nondominated solutions are obtained. The normalized values [2] for the objective functions in these solutions are given in Table 7 of Chapter 4. However, in order to better understand these results, this set of solutions appears in Table A.3 of Appendix A of Chapter 4 with their original (non-normalized) values. Note that when one of the objective functions approaches its ideal value, the other objectives attain values far removed from their respective ideal values. This demonstrates the degree of conflict existing among the objectives, i.e., achieving optimal values for the students' scores in math and reading and, the percentage of students reaching level four in both subjects.

According to the solutions obtained, the values for the mean scores in maths vary from 547.246 to 553.621, and for the mean scores in reading from 553.832 to 557.097. The percentage of students above level four in math moves from 38% to 40%, and in reading from 47.6% to 48.6%. These values suggest that it is more difficult to obtain higher scores in mathematics than in reading. To summarize these results, Table 5.7 provides the descriptive statistics of the 31 solutions generated, using both their normalized and their non-normalized values. The average value of the mean scores in math is 551.434 and the mean for the mean scores in reading is 547.246 points. Regarding the proportion of students that obtain more than 559 points, the mean in math (% $\geq$ 559 scores in math) is 39.3%, and in reading (% $\geq$ 559 scores in reading) is 48.1%.

Table 5.7 Descriptive statistics of the objective values for the nondomianted solutions generated for the student satisfaction problem solved in Chapter 4.

	Mean		Min		Max	
	Norm.	Non-norm.	Norm.	Non-norm.	Norm.	Non-norm.
Mean scores in math	1.655	551.434	1.553	547.246	1.709	553.621
Mean scores in reading	1.635	555.279	1.591	553.460	1.679	557.097
% $\geq$ 559 scores in math	1.162	0.393	1.072	0.380	1.213	0.400
% $\geq$ 559 scores in reading	1.297	0.481	1.261	0.475	1.335	0.486

[2]The normalized values of variables that represent the objective functions are obtained by subtracting the mean and dividing by the standard deviation of each variable.

Indeed, Table A.4 (Appendix A of Chapter 4) shows the values achieved by the decision variables of each solution. The variable respect by teacher obtains a value close to its mean value (7.14). However, all the other decision variables obtain values close to their minimum values. In this sense, enhancing the variable respect by teacher, i.e. obtaining a better score on the satisfaction questionnaire, would improve students' performance in the output.

5.2 Conclusions

Multiobjective optimization consists of the search for a solution by optimizing multiple conflicting objectives at the same time, over a feasible set. On the one hand, this thesis proposes a new aggregation-based evolutionary algorithm for multiobjective optimization, called A-GWASF-GA. On the other hand, a real application is described, in which the solution process of a multiobjective optimization problem related to the student satisfaction in schools has been addressed by means of evolutionary multiobjective optimization.

A-GWASF-GA attempts to improve the diversity and the convergence of the original GWASF-GA algorithm. The aggregation function used in GWASF-GA consists of an ASF based on the Tchebychev distance, in which two reference points (the nadir and utopian points) are used with a predefined set of weight vectors. Despite the promising results obtained by GWASF-GA in many problems, the diversity of the solutions generated can be improved, since the use of a prefixed set of weight vectors does not guarantee the generation of a uniformly distributed set of solutions in the PF. For this purpose, in A-GWASF-GA the weight vectors used are dynamically adjusted, taking into account the distribution of the solutions in the whole PF. In practice, A-GWASF-GA detects the overcrowded areas and the areas with a lack of solutions through a distance metric defined for each solution, which is called the scattering level. According to this metric, the weight vectors projecting towards the overcrowded areas are re-directed in order to search for new nondominated solutions in the less-populated areas. While the algorithm converges, the weight vector adjustment is performed several times. Mention should be made that, in this process, the number of weight vectors internally associated with the utopian and the nadir points is automatically adjusted. This enables the convergence to adapt better according to the complexity of the PF, with a more accurate approximation of either convex or concave parts of the PF. Actually, this is supported by the theoretical results proved in the Chapter 3.

Owing to the fact that the performance of A-GWASF-GA mainly depends on three parameters (the percentage of generations run until implementing the first weight vector adjustment, the number of weight vectors adapted at each adjustment, and the number of adjustments to be performed), in Chapter 2 a total of 72 different configurations of these parameters have been tested in comparison to the original GWASF-GA. These results have demonstrated the new algorithm's outstanding performance. Furthermore, based on the insights gained with these computational tests, a higher computational analysis of A-GWASF-GA (with a fixed parameter setting) is carried out in Chapter 3. Here, A-GWASF-GA is compared to well-known EMO algorithms such as RVEA, NSGA-III and three versions of MOEA/D, using three-, five-, eight- and ten-objective benchmark problems. The computational experiments have shown the promising results of A-GWASF-GA in respect to the HV and IGD metrics, particularly when the number of objectives increases.

The application regarding economic of education presented in Chapter 4 demonstrates the benefits of solving socio-economic multiobjective optimization problems, built through the use of econometric techniques, by means of EMO algorithms. With the solutions obtained, we have been able to identify the profile of a school that achieves, as in this case, the optimum balance among the average scores in maths and reading, and the percentage of students achieving level four or more in math and reading. Based on the results obtained, some educational policies can be suggested.

Focusing on the problem proposed and solved, a set of approximately nondominated solutions have been generated using the GWASF-GA algorithm with which to analyze the trade-offs among the objective functions, i.e. among the mean scores in math, mean scores in reading, percentage of students above level four in math, and percentage of students above level four in reading. Furthermore, we have also obtained the ranges of variation of each objective. The results encourage us to consider certain measures to improve education in Andalusia. In this sense, the satisfaction with the respect shown by the teachers has proven to be very important to achieve good academic performances. Hence we suggest that measures to promote dialogue between teachers and students should be established. In addition, the authorities could control the satisfaction with mutual understanding (students-teachers) through periodical evaluations.

5.3 Future research lines

This thesis focuses on aggregation-based evolutionary multiobjective algorithms and on the application of these algorithms to an economics of education problem. Several aspects of both research lines can be improved. On the one hand, by focusing on evolutionary multiobjective optimization, we can continue the current research as follows:

- Since GWASF-GA depends on the utopian and the nadir points, as reference points in the ASF, we can study how the GWASF-GA procedure could be adapted to work with a set of well-distributed reference points. The idea would be to progressively project them onto the PF, using different weight vectors depending on the PF's complexity.

- EMO algorithms can be developed to consider preferential information in order to provide a set of nondominated solutions approximating only a region of interest (ROI) rather than the whole PF. In this sense, a possible research line would be to propose a preference-based EMO algorithm handling preferences in the form of an aspiration point (comprising desirable aspiration values for the objectives) and a reservation point (comprising values that avoid worsening the objectives). In our future proposal, we could approximate the ROI associated with both the aspiration and the reservation point. In the first generations, the reservation point and a pre-generated set of weight vectors are considered. Then, the weight vectors could be re-calculated in order to re-direct the search towards the desired ROI. Finally, the remaining number of generations would be performed using the aspiration point and the new calculated weight vectors.

On the other hand, regarding the application of economics of education, the following could be future lines of research:

- The same type of socio-economic context could be studied by defining multiobjective optimization problems considering uncertainty. In other words, the coefficients of the objective functions and constraints would be built with intervals instead of fixed values. The resulting problems would be solved using interval multiobjective programming.

- We could also propose new educational multiobjective problems using different per student data provided by the PISA database. In the last two reports (2015 and 2018), PISA contains information about students' well-being (such as e.g. anxiety, motivation, sense of belonging and bullying at school). Therefore, a multiobjective problem could be defined to study socio-demographic and well-being data of Spanish and Portuguese students.

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Chapter Six

Summary of the thesis in Spanish (resumen de la tesis en español)

La tesis presentada se basa en el desarrollo de nuevos algoritmos evolutivos para resolver problemas de optimización multiobjetivo, especialmente problemas con más de tres funciones objetivos, y en la modelización y resolución de un problema de economía de la educación. Dicha tesis está realizada en la modalidad de compendio de artículos y se compone de tres de los mismos. Los dos primeros relacionados con el desarrollo de un nuevo algoritmo evolutivo. En ellos, partiendo del algoritmo Global Weighting Achievement Scalarizing Fucntion Genetic Algorithm (GWASF-GA) (Saborido, Ruiz, and Luque, 2017), se plantea y desarrolla un nuevo algoritmo centrado en la adaptación de los vectores de pesos durante el proceso de ejecución, que ofrece muy buenos resultados en comparación con algoritmos muy conocidos y muy contrastados dentro del campo de los algoritmos evolutivos. El tercer artículo se centra en la modelización y resolución de un problema multiobjetivo obtenido a partir del análisis econométrico de datos referidos al rendimiento académico y satisfacción de los estudiantes andaluces con diferentes aspectos del proceso enseñanza-aprendizaje en los colegios de secundaria.

En general, el concepto de optimización multicriterio se basa en el manejo de varios criterios a la vez con el fin de encontrar la solución que más se ajuste a dichos criterios dentro de un conjunto de soluciones factibles. Cada vez que en la vida real tomamos una decisión en la que hay más de un criterio en conflicto, estamos resolviendo de alguna forma un problema multicriterio. A veces la intución es suficiente para elegir la solución final, pero cuando estos problemas son complejos se necesitan técnicas analíticas que permitan llegar a una buena solución final. Desde hace más de cuarenta años, se aplican métodos y técnicas multicriterio a campos muy diversos de la ciencia como son: economía, ingenieria, informática, salud, etc.

La Optimización Multiobjetivo, que se encuentra dentro del área de investigación llamada Investigación Operativa, se centra en la definición de conceptos, teoría y métodos para resolver problemas con más de un objetivo sobre una región factible. Dependiendo de la naturaleza del problema (región factible desconocida, finita o infinita, incertidumbre en los coeficientes, etc) existen diferentes técnicas para la resolución de dichos problemas como Análisis de Decisión Multiatributo (Dyer et al., 1992), Optimización Multiobjetivo Estocástica (Goicoechea, Hansen, and Duckstein, 1982), Optimización Multiobjetivo (Bellman and Zadeh, 1970) o Optimización Multiobjetivo Intervalar (Oliveira and Antunes, 2007).

Los problemas de optimización multiobjetivo (MOP) se definen como sigue[1]:

$$\begin{aligned} \text{minimize} \quad & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\} \\ \text{subject to} \quad & \mathbf{x} \in X, \end{aligned} \tag{6.1}$$

donde $f_i : X \to \mathbb{R}$ con $i = 1, \ldots, k$ son las funciones objetivo y $X \in \mathbb{R}^n$ es la región factible o conjunto factible. En los últimos años se les ha prestado mucha atención a los problemas con muchos objetivos ya que son bastante habituales en los modelos, sin embargo, son difíciles de resolver y el coste computacional suele ser bastante alto. A un problemas multiobjetivo con tres o más funciones objetivo se le llama Many-objective Optimization Problem (MaOP).

Una solución factible $\mathbf{x} \in X$ se dice que es un óptimo de Pareto o solución eficiente si no existe otra $\mathbf{x}' \in X$ que mejore a la anterior en al menos un objetivo. Al conjunto de

[1]Sin pérdida de generalidad, definimos el problema multiobjetivo en sentido de mínimo, pero si una o más funciones objetivos quieren maximizarse estas se multiplican por -1.

todas los óptimos de Pareto o soluciones eficientes se le llama conjunto óptimo de Pareto. Al vector objetivo correspondiente a una solución eficiente se le llama solución no dominada, y el conjunto de las soluciones no dominadas de un problema se llama frente óptimo de Pareto (PF). Además, dados dos vectores $\mathbf{z}, \bar{\mathbf{z}} \in \mathbb{R}^k$, decimos que $\mathbf{z}$ *domina a* $\bar{\mathbf{z}}$ sí y solo sí $z_i \leq \bar{z}_i$ para todo $i = 1, \ldots, k$, and $z_j < \bar{z}_j$ para, al menos, un índice j. Nos referiremos a conjunto no dominado, al conjunto de soluciones cuyos vectores objectivo no estén dominados por el resto de soluciones del cojunto. Una definición más relajada de la estricta dominancia es la ε-dominancia, para un valor real pequeño y posivito $\varepsilon > 0$. Decimos que $\mathbf{z}$ ε-domina a $\bar{\mathbf{z}}$ sí y solo sí $z_i \leq \bar{z}_i + \varepsilon$ para todo $i = 1, \ldots, k$, y $z_j < \bar{z}_j + \varepsilon$ para, al menos, un índice j. Además, una solución $\mathbf{x} \in S$ es débilmente Pareto optima si no existe otra $\bar{\mathbf{x}} \in S$ tal que $f_i(\bar{\mathbf{x}}) < f_i(\mathbf{x})$ for all $i = 1, \ldots, k$.

Existen muchos métodos para resolver problemas de optimización multiobjetivo, pero no todos son capaces de resolver cualquier tipo de problema (problemas no convexos, problemas mixtos enteros, etc.). Con el desarrollo de la computación, los algoritmos evolutivos han demostrado ser muy útiles y eficientes a la hora de resolver determinados problemas multiobjetivo que las técnicas exactas no son capaces de resolver.

Desde un punto de vista matemático, todas las soluciones óptimas de Pareto pueden considerarse equivalentes y no superiores entre sí, por lo que es necesario incorporar a un responsable de la toma de decisiones al proceso de solución. Un decisor (DM) es una persona preocupada por resolver el problema multiobjetivo, que puede indicar sus preferencias respecto a los objetivos en conflicto y que es capaz de tomar una decisión basada en su punto de vista. Con un análisis de todas las soluciones no dominadas, el DM puede elegir una solución según sus preferencias. Sin embargo, el estudio de los trade-off observados entre los objetivos para seleccionar una solución final satisfactoria para el problema implica un esfuerzo altamente cognitivo para el DM que no es trivial, y esta etapa de toma de decisiones merece especial atención cuando se resuelve un MOP. Para facilitar esta tarea de toma de decisiones, muchos métodos incluyen información sobre las preferencias del DM en el proceso de solución para converger con éxito en la solución más preferida. En este sentido, las preferencias del DM pueden introducirse proporcionando un punto de referencia, definido de la siguiente forma $\mathbf{q} = (q_1, \ldots, q_k)^T$, que comprende los valores deseables de la función objetivo q_i $(i = 1, \ldots, k)$.

Esta tesis se centra en los algoritmos evolutivos como método de resolución de problemas de optimización multiobjetivo y concretamente, en los métodos que intentan aproximar todo el frente óptimo de Pareto (PF). Se busca que la aproximación generada sea lo más cercana a la verdadera PF (convergencia) y que las soluciones estén distribuidas lo más uniforme posible dentro del PF (diversidad). Dentro del grupo de algoritmos evolutivos podemos diferenciar tres tipos. El primero de ellos son los algoritmos basados en indicadores (indicator-based algorithms), estos seleccionan las soluciones teniendo en cuenta el valor de una métrica o indicador, algunos ejemplos son HypE (Bader and Zitzler, 2011), TwoArch2 (Wang, Jiao, and Yao, 2015) y SMS-MOEA (Wagner and Neumann, 2013). En el segundo grupo, llamados algoritmos basados en dominancia,lo encuadramos aquellos en los que la selección de soluciones se basa en una relación de dominancia definida previamente (dominance-based algorithms), entre ellos destacan NSGA-II (Deb et al., 2002a), NSGA-III (Deb and Jain, 2014; Jain and Deb, 2014), y algunas de sus versiones (Köppen and Yoshida, 2007; Li, Yang, and Liu, 2014). Finalmente, podemos diferenciar los algoritmos de agregación o basados en descomposición, en estos el problema multiobjetivo original se transforma en un conjuto de subproblemas con un unico objetivo (aggregation-based o descomposition-based algorithms), de entros destacan el MOEA/D (Zhang and Li, 2007) y diferentes versiones que lo mejoran como MOEA/D-AWA (Qi et al., 2014) y MOEAD/DD (Li, Deb, and Kwong, 2015), entre otros, y GWASF-GA (Saborido, Ruiz, and Luque, 2017).

Como hemos dicho anteriormente, GWASF-GA es un algoritimo evolutivo basado en agregación o descomposición. Su principal característica es transformar el problema original en un conjunto de subproblemas monoobjetivos y para ello hace uso de la función de logro propuesta por (Wierzbicki, 1980) definida de la siguiente forma:

$$s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \mu) = \max_{i=1,\ldots,k} \{ \mu_i(f_i(\mathbf{x}) - q_i) \} + \rho \sum_{i=1}^{k} \mu_i(f_i(\mathbf{x}) - q_i), \tag{6.2}$$

donde $\mu = (\mu_1, \ldots, \mu_k)^T$ es un vector de pesos estrictamente positivos, $\mathbf{q} = (q_1, \ldots, q_k)^T$ es el llamado punto de referencia y $\rho \geq 0$ es un valor real llamado coeficiente de aumento.

De esta forma, minimizando (6.2) sobre la región factible X:

$$\begin{aligned} &\text{minimize} \quad && s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \mu) \\ &\text{sujeto a} \quad && \mathbf{x} \in X, \end{aligned} \tag{6.3}$$

aseguramos que cualquier solucion óptima de (6.3) es una solución Pareto óptima del problema multiobjectivo original (6.1) (Miettinen, 1999). Además, cualquier solución Pareto óptima de (6.1) se puede encontrar resolviendo (6.3) usando diferentes puntos de referencia y/o vectoress de peso (Steuer, 1986). En la practica, cuando minimizamos la función de logro sobre la región factible, el punto de referencia se proyecta sobre el frente de Pareto (PF) en la dirección que definen los inversos de los vectores pesos (para más detalles, ver Miettinen (1999)).

Para usar la función de logro se necesita del uso de un cojunto de vectores de pesos uniformemente distribuidos a lo largo del PF y dos puntos referencia que acotan el PF (el punto nadir y el punto utopía). Formalmente, el punto utopía se calcula haciendo uso del punto ideal. Para cada objetivo $i = 1, \ldots k$, el punto ideal se calcula como $z_i^\star = \min_{\mathbf{x} \in E} f_i(\mathbf{x})$, y por lo tanto, el punto utopía se define como $z_i^{\star\star} = z_i^\star - \epsilon$, donde $\epsilon > 0$ es un valor real pequeño. El punto nadir se define comos sigue $z_i^{\text{nad}} = \max_{\mathbf{x} \in E} f_i(\mathbf{x})$, por cada $i = 1, \ldots, k$. De esta forma, los puntos definidos proporcionan una cota superior e inferior para los objetivos.

En GWASF-GA para definir el conjunto de subproblemas de un unico objetivo, la mitad de pesos se usan con el punto nadir y la otra mitad se utilizan con el punto utopía. Por esta razon, los vectores de pesos tienen un papel esencial en este algoritmo, ya que estos definen las direcciones de búsqueda de nuevas soluciones no dominadas tanto si proyectamos desde el punto nadir como del utopía. Para seleccionar los mejores individuos (soluciones), en cada generación de GWASF-GA las soluciones obtenidas se dividen en diferentes fronteras. Esta clasificación se hace teniendo en cuenta el valor que cada solucion obtiene en la función de logro, para cada uno de los vectores de pesos.

A pesar de los buenos resultados que demuestra GWASF-GA en comparación con otros algoritmos evolutivos muy conocidos (para más detalles, ver Saborido, Ruiz, and Luque, 2017), este algoritmo presenta algunos inconvenientes en determinados problemas. El principal inconveniente es que el conjunto de vectores de pesos iniciales es fijo a lo largo de todo el proceso de resolución. Por lo tanto, las direcciones de proyección no se ajustan a la forma de la PF (no-convexa, discontinua, etc). Por este motivo y teniendo en cuenta varias investigaciones que van en esta línea (Qi et al., 2014; Gu, Liu, and Chen Tan, 2012), proponemos un nuevo algoritmo en el que se adapten los vectores de pesos de forma dinámica con el propósito de redirigir la búsqueda a la forma real del PF.

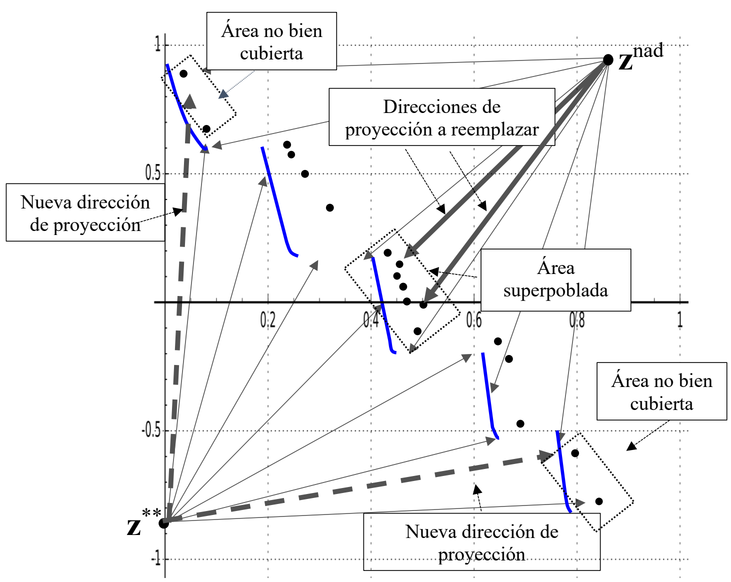

Figure 6.1 Prodecimiento de adapatación de los vectores de pesos

Así pues, proponemos un nuevo algoritmo basado en GWASF-GA en el que trás realizar un porcentaje de iteraciones haciendo uso de GWASF-GA original se procede a adaptar los vectores de pesos. De esta forma el nuevo algoritmo depende de tres parámetros: el porcentaje (p) de iteracciones realizadas con GWASF-GA original, el número de veces que se hace la adaptación de los vectores de pesos (n_a) y el número de vectores de pesos que se adaptan (N_a). En consecuencia, el nuevo algoritmo debe aplicar $p\%$ del total de iteraciones con GWASF-GA original y, tras esto, se realiza la adaptación de N_a pesos que se repite n_a veces.

Para realizar la adaptación, primero calculamos la densidad a cada una de las soluciones obtenidas tras ejecutar el $p\%$ del total de las iteracioens, haciendo uso de una métrica llamada scattering level y denotada por $s(\mathbf{x})$, que se define como sigue:

$$s(\mathbf{x}) = \prod_{j=1}^{k} \sqrt{\sum_{i=1}^{k} (f_i(\mathbf{x}) - f_i(\mathbf{x}^j))^2}, \tag{6.4}$$

donde $\mathbf{x}^1, \ldots, \mathbf{x}^k$ son las k soluciones con los vectores objectivos más cercados a $\mathbf{f}(\mathbf{x})$ considerando la distancia L_2. El valor más alto (respectivamente, el más bajo) de $s(\mathbf{x})$, representa la región menos (respectivamente, más) poblada en lo que respecta al espacio objetivo. De esta forma, detectamos las soluciones que están en zonas superpobladas del PF, es decir, donde hay aglomeración de soluciones, y aquellas zonas que no están bien aproximadas, es decir, en las que hay falta de soluciones. Así, los vectores de pesos asociados a las N_a soluciones con menor scattering level (aquellas que están en zonas superpobladas) serán adaptados de forma que proyecten hacia las zonas con ausencia de soluciones (aquellas que tienen mayor scattering level).

Estos nuevos vectores pueden venir definidos de dos formas dependiendo de si el vector de pesos a adaptar proviene de una solución obtenida desde el nadir o desde el utopía. En la figura 6.1 se muestra la idea gráfica de este procedimiento, donde se indican las distintas zonas mencionadas y el redireccionamiento de determinados vectores de pesos. Además estos vectores de pesos serán proyectados desde el punto nadir o utopía dependiendo también de cómo se ha obtenido la solución. De esta forma, a diferencia de el algoritmo GWASF-GA original que asigna el mismo número de vectores de pesos al punto nadir que al punto ideal, después de la primera adaptación el número de vectores de pesos que se proyectan desde el nadir o desde el utopía es diferente. En este sentido, en el caso que el frente presente una forma convexa, es probable que el proceso asigne automáticamente una mayor cantidad de vectores de pesos desde el punto nadir, dando más importancia a la proyección de la misma. Pero si es cóncavo, asocie más vectores de pesos nuevos con el punto utopía, lo que significa que su proyección es más adecuada. Esta es otra de las mejoras que incorpora el nuevo algoritmo.

Tras exponer la idea de adaptación de pesos para este nuevo algoritmo, en el primer artículo de esta tesis (Capítulo 2) se hace una descripción y una primera prueba del mismo considerando 72 combinaciones diferentes del conjunto de parámetros (porcentaje de iteraciones con GWASF-GA original, número de pesos a adaptar, número de veces que se realiza la adaptación). Este artículo muestra los resultados de un experimento en el que se compara GWASF-GA original con el nuevo algoritmo utilizando las diferentes configuraciones de los parámetros. Dichas configuraciones provienen de hacer todas las combinaciones posibles de los siguientes valores $p \in \{60\%, 70\%, 80\%\}$, $n_a \in \{2, 4, 6\}$ y $N_a \in \{5, 20, 25, 30, 50, 60, 75, 100\}$. En cuanto a los problemas, se utilizan un total de 46 problemas test con tres, cinco y seis funciones objetivo pertenecientes a las familias DTLZ (Deb et al., 2002b), WFG (Huband et al., 2007) UF (Zhang et al., 2008) y LZ09 (Li and Zhang, 2009). La métrica utilizada para medir el comportamiento del nuevo algoritmo es el hipervolumen (HV) (Zitzler and Thiele, 1999). La métrica HV mide el volumen del espacio objetivo que está dominado por las soluciones en una población y está delimitado por un punto de referencia dominado por todos los vectores objetivos Pareto-óptimos.

Los resultados muestran si el algoritmo propuesto presenta estadisticamente mejores o peores resultados que GWASF-GA original considerando la media del HV obtenida en 30 ejecuciones independientes. Estos resultados se obtienen a través del test de suma de rangos de Wilcoxon (Wilcoxon, 1945). Para cada problema, la hipóteis nula es que la distribución de la media del HV en las 30 ejecuciones independientes difiere con un nivel de significación de α. Consideraremos que la diferencia es significativa si obtenemos un p-value menor que $\alpha = 0.05$.

Para calcular el test de suma de rangos de Wilcoxon utilizamos la función `wilcox.test` del software R[2].

En relación con los problemas de tres objetivos, el nuevo algoritmo gana, al menos, en 12 de los 18 problemas. De esta forma, en algunas configuraciones, obtiene mejores resultados que GWASF-GA original en el mayor número de casos, alcanzando simultáneamente un rendimiento igual para el resto, es decir, no pierde en ningún problema. En general, el hipervolumen alcanzado por GWASF-GA al adaptar los vectores de pesos es superior al de GWASF-GA original en el 81% de los problemas.

Para cinco objetivos se consideran un total de 14 problemas. Cabe destacar que con la adaptación de vectores de pesos GWASF-GA es significativamente mejor que el algoritmo original en 13 problemas (logrando el mismo rendimiento en los problemas restantes) considerando 37 de las 72 configuraciones de parámetros posibles. En promedio, la nueva propuesta para cinco objetivos es mejor en el 76% de los casos.

Para los problemas de seis objetivos, también se han considerado 14 problemas. Aquí, el hipervolumen alcanzado por GWASF-GA es mayor cuando se utiliza el ajuste de los vector de pesos en la mayoría de los problemas, aunque hay una configuración ($n_a = 4$, $p = 0,8$ y $N_a = 75$) en la que el GWASF-GA original gana en 11 problemas. En resumen, el ajuste de los vectores de peso permite a GWASF-GA obtener mejores resultados que el algoritmo original en el 74% de los problemas.

Basandonos en los resultados podemos apuntar que esta nueva propuesta ajustando los vectores de pesos a lo largo del proceso de ejecución obtiene una mejor aproximación que el algoritmo original GWASF-GA independientemente de la configuración de parámetros considerada.

Teniendo en cuenta los resultados obtenidos anteriormente, en el segundo artículo que compone esta tesis se presenta un nuevo algoritmo evolutivo denomidado Adaptive GWASF-GA (A-GWASF-GA). En este algoritmo se considera una configuración fija de los parámet-

[2]https://stat.ethz.ch/R-manual/R-devel/library/stats/html/wilcox.test.html

ros p (porcentaje de iteracciones realizadas con GWASF-GA original antes del proceso de adaptación), n_a (número de veces que se hace la adaptación) y N_a (número de vectores de pesos a adaptar) dependiendo de la dimensión del problema. Además, A-GWASF-GA está respaldado por dos teoremas propuestos y demostrados (con sus correspondientes corolarios) que aseguran la obtención de soluciones no dominadas con la redefinición de los vectores de pesos adaptados. La redefinición de los nuevos vectores de pesos permite proyectar y obtener soluciones en las zonas de la PF que no están bien cubiertas, es decir, que tienen ausencia de soluciones.

Teorema 1 *Dada una solución factible* $\mathbf{x} \in S$*, entonces, o* $\mathbf{x}$ *es una solución óptima del problema* (6.3) *considerando* $\mathbf{q} = \mathbf{z}^{\star\star}$ *y el vector de peso* μ^U *definido por*

$$\mu_i^U = \frac{1}{f_i(\mathbf{x}) - z_i^{\star\star}}, \text{ para cada } i = 1, \ldots, k. \tag{6.5}$$

o el vector objetivo de cualquier solución óptima del problema (6.3) *con esos parámetros domina estrictamente o* ε*-domina* $\mathbf{f}(\mathbf{x})$*, para algún* $\varepsilon > 0$.

Corolario 1 *Si* $\mathbf{x} \in S$ *es una solución débilmente Pareto óptima, entonces* $\mathbf{x}$ *es una solución óptima del problema* (6.3) *considerando* $\mathbf{q} = \mathbf{z}^{\star\star}$ *y usando el vector de pesos* μ^U *definido por* $\mu_i^U = \frac{1}{f_i(\mathbf{x}) - z_i^{\star\star}}$*, para cada* $i = 1, \ldots, k.$.

Teorema 2 *Dada una solución factible* $\mathbf{x} \in S$*, entonces, o* $\mathbf{x}$ *es una solución óptima del problema* (6.3) *considerando* $\mathbf{q} = \mathbf{z}^{\mathrm{nad}}$ *y el vector de pesos* μ^N *definido por*

$$\mu_i^N = \frac{1}{z_i^{\mathrm{nad}} - f_i(\mathbf{x})}, \text{ para cada } i = 1, \ldots, k. \tag{6.6}$$

o el vector objetivo de cualquier solución óptima del problema (6.3) *con esos parámetros domina estrictamente o* ε*-domina* $\mathbf{f}(\mathbf{x})$*, para algún* $\varepsilon > 0$.

Corolario 2 *Si* $\mathbf{x} \in S$ *es una solución débilmente Pareto óptima, entonces* $\mathbf{x}$ *es una solución óptima del problema* (6.3) *considerando* $\mathbf{q} = \mathbf{z}^{\mathrm{nad}}$ *y usandno el vector de peso* μ^N *definido por* $\mu_i^N = \frac{1}{z_i^{\mathrm{nad}} - f_i(\mathbf{x})}$*, para cada* $i = 1, \ldots, k.$.

Bansandonos en los resultados teóricos anteriores, las regiones del PF que no están bien aproximadas pueden cubrirse redefiniendo algunos subproblemas haciendo uso de los nuevos vectores de pesos (que reorientan la búsqueda), usando el punto nadir (6.6) o el punto utopía (1) (dependiendo de cómo se han obtenido las soluciones consideradas para calcular los nuevos vecctores de pesos, como hemos mencionado anteriormente). Además, las soluciones que aún no están suficientemente cerca del PF pueden mejorarse y acercarse a dicho frente (PF) en base a estos resultados teóricos.

Para probar el funcionamiento de A-GWASF-GA, lo comparamos con otros algortimos muy reconocidos en este campo como son RVEA (Cheng et al., 2016), NSGA-III (Deb and Jain, 2014; Jain and Deb, 2014), MOEA/D-DE (Zhang and Li, 2007), MOEA/DD (Li, Deb, and Kwong, 2015) y MOEA/D-DE-AWA (Qi et al., 2014). Además de los problemas test, ya conocidos, como son los DTLZ y WFG, se utiliza una nueva familia de problemas llamados MaOP (Li et al., 2019), considerando dos variantes dependiendo del número de variables de decisión $n = 20$ y $n = 50$. Todos estos problemas se consideran para tres, cinco, ocho y diez funciones objetivos, siendo así la mayoría de ellos problemas many-objective. Las métricas utilizadas para medir el funcionamiento de los diferentes algoritmos son el hipervolumen (HV, anteriormente descrito) y la Inverted Generational Distance (IGD) (Zitzler et al., 2003).

La métrica IGD proporciona información combinada sobre la convergencia y la diversidad de las soluciones obtenidas por cada algoritmo. Esta métrica se utiliza ampliamente para evaluar los conjuntos de soluciones aproximadas tanto para los MOP como para los MaOP y se define como:

$$IGD(S_{pf}, R) = \frac{1}{|S_{pf}|} \sum_{y \in S_{pf}} dist(y, R),$$

donde S_{pf} es la aproximación del PF encontrada por un algoritmo para un problema dado, R es un conjunto de puntos de referencia uniformemente distribuidos sobre el PF en el espacio objetivo, y $dist(y, R) = \min_{z \in R} \|y - z\|_2$. Cuanto más bajo es el valor IGD, mejor funciona el algoritmo.

Los resultados obtenidos muestran la media de dichas métricas en 30 ejecuciones independientes. También se muestran los resultados en los que A-GWASF-GA obtiene mejores, iguales o peores medias que el resto de algoritmos comparándolas a través del test de sumas de rangos de Wilcoxon (Wilcoxon, 1945).

Para los problemas con tres objetivos, en cuanto a IGD, podemos ver que generalmente A-GWASF-GA no obtiene muy buenos resultados con respecto al resto de algoritmos. En cuanto a los problemas de cinco objetivos, A-GWASF-GA obtiene mejores resultados, particularmente en los problemas MaOP con $n = 50$ ganando, por ejemplo, contra RVEA en 9/10 problemas. Sin embargo, para esta misma dimensión pero en los problemas DTLZ y WFG, A-GWASF-GA obtiene peores resultados que el resto de algortimos exceptuando al MOEA/D-DE, en el que gana en 11/14 problemas. Si nos centramos en los problemas de ocho objetivos, A-GWASF-GA gana con respecto a todos los algoritmos en los problemas MaOP para $n = 20$ y $n = 50$ variables decisión, obteniendo mejores resultados en 9/10 problemas con respecto a MOEA/DD para $n = 50$. Sin embargo, algo parecido a los problemas de cinco objetivos ocurre con los problemas DTLZ y WFG, donde A-GWASF-GA no obtiene muy buenos resultados, aunque gana en 7/14 problemas con respecto a MOEA/D-DE-AWA. Por último, para los problemas de diez objetivos, podemos decir que A-GWASF-GA obtiene mejores resultados que todos los algoritmos considerados independientemente del tipo de problema examinado, excepto contra RVEA para los problemas DTLZ y WFG.

Por otro lado, la métrica HV solo ha sido calculada para los problemas de tres, cinco y ocho objetivos (y no para los problemas de diez objetivos) debido al alto coste computacional del mismo. En general, el HV obtenido por A-GWASF-GA es mejor casi para todos los problemas independientemente del algoritmo con el que se ha comparado. Dentro de los problemas con tres funciones objetivo, centrándonos en los MaOP, A-GWASF-GA obtiene significativamente mejores valores con respecto a todos los algoritmos, excepto para $n = 50$ contra MOEA/D-DE, en los que gana en 4/10 problemas. Para los problemas DTLZ y WFG, A-GWASF-GA es significativamente mejor con respecto a todos los algoritmos menos

NSGA-III, en el que solo obtiene mejores resultados para 6/14 problemas. En cuanto a los problemas de cinco funciones objetivo, A-GWASF-GA alcanza mejores resultados para todos los problemas MaOP independientemente del número de variables de decisión (n = 20 y n = 50), ganando al menos en 7/10 problemas. Además, A-GWASF-GA también obtiene significamente mejores resultados que el resto de algoritmos para los problemas DTLZ y WFG, exceptuando el algoritmo RVEA en el que A-GWASF-GA gana en 6/14 problemas. Finalmente, A-GWASF-GA alcanza significamente mejores resultados para todos los problemas (MaOP con n = 20 y n = 50, DTLZ y WFG) contra todos los algoritmos considerados, ganando al menos en 7/10 problemas.

En resumen, con los resultados obtenidos y teniendo en cuenta los algoritmos considerdos, aunque los frentes óptimos de Pareto aproximados por A-GWASF-GA no sean los mejores en todos los casos (especialmente para los problemas con tres funciones objetivo), podemos asegurar que el nuevo algoritmo algoritmo evolutivo aquí propueto (A-GWASF-GA) muestra resultados muy prometedores en problemas con más de tres funciones objetivo. De esta forma, A-GWASF-GA se autodefine como un algoritmo para trabajar con problemas many-objective (con más de tres objetivos).

Como señalamos anteriormente, la segunda parte de esta tesis (tercer artículo, Capítulo 4) consta de la modelización y resolución de un problema de optimización multiobjetivo a partir de un análisis econométrico sobre la satisfacción de los estudiantes y su rendimiento académico.

Un aspecto que preocupa a las autoridades educativas españolas es el relativo bajo rendimiento de los estudiantes españoles en cuanto a las puntuaciones en las evaluaciones internacionales, las tasas de repetición de curso y el abandono escolar (OECD, 2018; OECD, 2019), ya que la educación es uno de los principales factores del crecimiento económico (Barro, 2001). Esta preocupación es aún mayor en el caso de la región española más poblada, Andalucía, que obtiene aún peores resultados que la media española y que constituye el tema tratado en este capítulo.

En este contexto, el principal objetivo del tercer artículo y último trabajo presentado en esta tesis ha sido encontrar qué aspectos cualitativos del proceso de enseñanza-aprendizaje, en términos de satisfacción del estudiante, permiten obtener mejores resultados para: rendimiento académico de los estudiantes medido por las puntuaciones en matemáticas y lectura, y porcentaje de estudiantes que alcanzan un determinado umbral en ambas materias (por separado). Cada vez son más los estudios educativos relacionados con la satisfacción de los estudiantes, bienestar con el colegio y global, ya que muestran significatividad en el aprendizaje y educación de los mismos. En este sentido, Natvig, Albrektsen, and Qvarnstrøm (2003) reporta la correlación positiva existente entre el rendimiento académico alto y la satisfacción con la vida. Además, en Uline and Tschannen-Moran (2008) se muestra que el ambiente escolar está positivamente correlacionado con el rendimiento académico en inglés y matemáticas. Asimismo, la satisfacción de los padres también influye en la progresión y la evaluación académica de los estudiantes (Gibbons and Silva, 2011).

En primer lugar, hemos realizado un análisis econométrico usando datos de una muestra representativa de escuelas andaluzas de secundaria y posteriormente construido un modelo multiobjetivo. Especificando, estos datos provienen de la Agencia Andaluza de Evaluación Educativa (AGAEVE) y nos dan información de la satisfacción de los estudiantes de 8º grado (aquellos entre 13 – 14 años) con 36 aspectos sobre el proceso de enseñanza-aprendizaje, por ejemplo, satisfacción con la imagen del centro, comunicación interna y externa, entre otras. Sin embargo, hemos excluido del análisis aquellas variables que tienen al menos 5% de valores perdidos (missing values) y aquellos colegios con un número de respuestas por debajo del 5% del total de los estudiantes. Por lo tanto, finalmente, tenemos 20 variables de satisfacción correspondiente a estudiantes matriculados en 162 colegios de secundaria. Además, utilizamos datos del Diagnostic Assessment Test (DAT), que nos proporcionan las puntuaciones estandarizadas de una forma similar a las desarrolladas por PISA[3].

A través de la regresión lineal (utilizando mínimos cuadrados ordinarios), hemos expre-

[3]PISA: Programme for International Student Assessment. https://www.oecd.org/pisa/

sado los cuatro objetivos (puntuaciones en matemáticas y lectura, y porcentaje de alumnos que alcanzan un determinado umbral-nivel 4 de aprendizaje establecido por PISA- en ambas asignaturas) en función de un conjunto de variables explicativas (aquellas que han presentado significatividad al menos de un 90%): respeto y atención recibidos por los profesores, evaluación global de la forma de enseñar del profesor, información recibida sobre la progresión personal y académica, actividades iniciales dirigidas al conocimiento de los compañeros y a la unión del grupo y conocimiento de los proyectos y actividades educativas de la escuela secundaria. Las variables dependientes han sido normalizadas usando la media y la desviación típica de la muestra correspondiente a las variables estudiadas. Los coeficientes obtenidos de estimación de los modelos de regresión se usan para construir las cuatro funciones objetivo que queremos maximizar simultaneamente, construyendo así un problema de programación multiobjetivo.

De esta forma, si i representa el centro educativo y las cuatro funciones objetivos se representan por j, el modelo se define como sigue:

$$P_j(i) = \widehat{\alpha}^j + \widehat{\beta}_1^j x_1(i) + \widehat{\beta}_2^j x_2(i) + \widehat{\beta}_3^j x_3(i) + \widehat{\beta}_4^j x_4(i) + \widehat{\beta}_5^j x_5(i) \tag{6.7}$$

con $j = 1, \ldots, 4$ e $i = 1, \ldots, 162$.

Para ajustar de la mejor manera posible el modelo a la realidad, hemos definido un conjunto de restricciones que delimitan los valores de las variables. Dichas restricciones se definen basándonos en la relación existente entre las variables dependientes. Tomamos de dos en dos dichas variables y determinamos los intervalos de confianza al 99% de confinaza existente entre ellas. De esta forma, obtenemos un total de 20 restricciones. Además, consideramos como cota inferior y superior, el mínimo y máximo (respectivamente) que cada variable obtiene en la muestra.

Para resolver el problema de optimización multiobjectivo con cuatro funciones objetivo, veinte restricciones y diez cotas (5 inferiores y 5 superiores) hemos utilizado GWASF-GA (Saborido, Ruiz, and Luque, 2017) con diferentes funciones logro, que nos han permitido generar una buena aproximación del frente óptimo de Pareto. Dichas funciones de logro son la propuesta por Wierzbicki (1980) definida por (6.2), y las determinadas por la distancia L_p definidas por $s(z, f(x), \mu) = \left[\sum_{j=1}^{k} \mu_j (z_j^* - f_j(x))^p\right]^{1/p}$ con $p = 1, 2, 5$. Este método hace uso de dos puntos de referencia (el nadir y el utopía) como hemos mencionado anteriormente. Particularmente, en este problema hemos utilizado como utopía el punto ideal del modelo (maximizando cada objetivo de forma indivual), y el punto nadir se calcula internamente en el proceso de resolución. Este algoritmo proporciona un conjunto de soluciones que permiten identificar las trade-offs entre las funciones objetivo y los valores alcanzados por las variables de decisión en las diferentes soluciones.

Dado que el problema se resuelve una vez para cada función de logro considerada, el conjunto de soluciones finales se constituye filtrando las soluciones no dominadas obtenidas de las cuatro resoluciones. Finalmente, el conjunto de soluciones está formado por 31 soluciones. En ellas podemos observar que cuando en una solución una función objetivo está cerca del ideal, el resto de funciones objetivo no alcanzan valores tan cercanos al ideal. La media de los resultados obtenidos en el conjunto de soluciones nodominadas para cada función objetivo son: 551.434 puntos en matemáticas, 55.279 puntos en lectura, 39.3% y 48.1% alumnos por encima del nivel 4 de aprendizaje en matemáticas y lectura, respectivamente.

Con respecto a los valores que toman las variables de decision en las soluciones, podemos decir que la variable respecto recibido por parte de los profesores alcanza un valor cercano a la media que esta tiene en la muestra (7.14 puntos), a diferencia de las demás (las cuatro restantes), que toman un valor cercano al mínimo que estas tienen en la muestra. Este hecho es lógico ya que estas variables tienen una influencia negativa en la maximización de las funciones objetivo.

Los resultados obtenidos nos llevan a pensar que hay margen para mejorar el sistema educativo andaluz en base a nuestras principales conclusiones. En concreto, afirmamos que el respeto y la atención recibida por los profesores es muy importante para promover mayores niveles de rendimiento académico en los estudiantes. Por ello, se sugiere a las autoridades establecer políticas educativas que promuevan el respeto y una interacción más estrecha entre estudiantes y profesores. Una posible forma de mejorar esto es poner a disposición de los estudiantes unas horas en las que estos puedan conversar con sus tutores y profesores, para facilitar el entendimiento mutuo estudiante-profesor y fomentar un mejor ambiente de enseñanza-aprendizaje. Además, las autoridades deberían establecer evaluaciones periódicas para comprobar la satisfacción de los profesores y los estudiantes en lo que respecta a la interacción mutua.

Estos resultados demuestran el buen comportamiento de la combinación de técnicas econométricas y multiobjetivo, especialmente cuando ultilizamos algoritmos evolutivos, para la resolución de problemas socio-económicos con la finalidad de encontrar la compensación (trade-offs) entre los objetivos estudiados y así poder sugerir mejoras, en este caso, en economía de la educación.

Finalmente, aunque se hayan hecho nuevas aportaciones en las dos líneas de investigación de esta tesis, estas pueden mejorarse en varios aspectos. Por un lado, centrándonos en los algoritmos evolutivos donde primero, proponer un algoritmo evolutivo que considere un conjunto de puntos de referencia (más de dos) desde los que proyectar y que estén más cerca del propio PF para que la proyección sea más precisa; segundo, desarrollar un algoritmo en el que solo aproximemos la región de la PF que nos interese (ROI), basandonos en valores que deseamos alcanzar (nivel de aspiración) y otros que no queremos empeorar (nivel de reserva).

Por otro lado, en cuanto a economía de la educación, se puede mejorar tanto la definicion y diseño de los problemas multiobjetivo como su ámbito de aplicación. Para ello, podemos hacer el estudio a nivel de estudiante con datos propocionados por PISA 2015 (OECD, 2018) y 2018 (OECD, 2019), en los que por primera vez se facilitan datos referidos a bienestar de los estudiantes (ansiedad, motivación, sentido de pertenencia y acoso escolar en el colegio). Además de esto, definir nuevos modelos considerando incertidumbre en los coeficientes (de las funciones objetivo y en las restricciones). De esta forma, dichos coeficientes estarán formados por intervalos en lugar de por un valor fijo. Por lo tanto, los problemas definidos serán resueltos haciendo uso de técnicas de programación multiobjetivo intervalar.

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